

On Nano Hexa Topological Spaces

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ABSTRACT

In this paper, introduces a novel topological structure known Nano Hexa topology. The construction of this topology is based on 3 independent equivalence relations defined on the universal set and its corresponding subsets . Fundamental properties of this topology are discussed supported by theorems and examples, also styding the properties Nano Hexa extremely disconnected.

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1. Introduction and Preliminaries

Nano topology was introduced by Thivager in 2013[3] and its elements were termed Nano open sets. In the same year Nano closed set and interior (closure) Nano set were defined [4] has undergone significant development over the years, evolving into Bi, Triple, Quad spaces [1,10, 9] culminating in Nano penta topological spaces in 2021 [7] this evolution relies on "lower and upper estimations of a subset of the universe set", who paved the way for the Nano continuous function and its real-world applications[5].This work focuses on defining and exploring some of the properties of this space by expanding on the concepts introduced by Thivagar and other ,as well as introducing the concept Nano Hexa extremely disconnected and some of its characteristic. Now need "some definitions which are necessary for our study".

1.1. Definition [3]

Suppose that finite universe set $U \neq \emptyset$ and $\hat{R}$ is an equivalence relation on U , where $Y \subseteq U$. Then $\mathcal{L}_{\hat{R}}(Y) = \bigcup_{y \in U} \{[y] : \hat{R}(y) \subseteq Y\}$ is the lower appraise of Y it relates to equivalence class $\hat{R}$, $\mathcal{U}_{\hat{R}}(Y) = \bigcup_{y \in U} \{[y] : \hat{R}(y) \cap Y \neq \emptyset\}$ is the upper appraise of Y and $\mathcal{B}_{\hat{R}}(Y) = \mathcal{U}_{\hat{R}}(Y) - \mathcal{L}_{\hat{R}}(Y)$ is the boundary region of Y . We get

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$\Gamma_{\tilde{R}}(Y) = \{U, \emptyset, \mathcal{L}_{\tilde{R}}(Y), \mathcal{A}_{\tilde{R}}(Y), \mathcal{B}_{\tilde{R}}(Y)\}$; satisfies the axioms of topology [5]. So, $\Gamma_{\tilde{R}}(Y)$ is Nano topology (NI) on U it relates to Y . $(U, \Gamma_{\tilde{R}}(Y))$ is N - topology space ($NI_{\tilde{R}}$), main component $\Gamma_{\tilde{R}}(Y)$ is said Nano_ open sets (η - os).

1.2.Note [3]

The $NI_{\tilde{R}}$ on U it relates to Y , then $\mathcal{B} = \{U, \mathcal{L}_{\tilde{R}H}(Y), \mathcal{B}_{\tilde{R}H}(Y)\}$ is basis for $\Gamma_{\tilde{R}}(Y)$.

1.3. Definitions [3]: Let F be a subset of a $(U, \Gamma_{\tilde{R}}(Y))$, then:

1. The union of all η - os contained is η _interior ($\text{int}_{\eta}(\dot{F})$), thus $\text{int}_{\eta}(\dot{F})$ is the largest η - o subset $\dot{F}$.
2. "The intersection of all η _closed sets containing $\dot{F}$ is η _closure of $\dot{F}$ " ($\text{cl}_{\eta}(\dot{F})$), so, "minimal η _closed set superset $\dot{F}$ ".

1.4. Definition [8]

A space $(U, \Gamma_{\tilde{R}}(Y))$ is η _extremely disconnected (η _ED)," if η _closure for each" η - os is η - os.

1.5. Propositions [8]

A space $(U, \Gamma_{\tilde{R}}(Y))$ is η _ED , if $\mathcal{L}_{\tilde{R}}(Y) = \mathcal{A}_{\tilde{R}}(Y)$ or $\mathcal{L}_{\tilde{R}}(Y) = \emptyset$.

1.6. Theorem [8]

A space $(U, \Gamma_{\tilde{R}}(Y))$ is η _ED if and only if $\mathcal{A}_{\tilde{R}}(Y) = U$.

The aim to study Nano Hexi _ open set on Nano Hexa _Topological Space($\eta_{H-} \Gamma_S$) & to prove some of its properties.

2- Characteristics of Nano Hexa open sets in $\eta_H - \Gamma_S$

2. 1. Definition

A non-empty universe finite set " U " together six "different η - topologies on U " with respect to Y . we get $(U, \sum_{H=1}^6 \Gamma_{\tilde{R}H}(Y))$ is $\eta_{H-} \Gamma_S$. So, the $(U, \sum_{H=1}^6 \Gamma_{\tilde{R}H}(Y))$ briefly $(U, \Gamma_{\tilde{R}H}(Y))$.

2.2 . Definition

A sub set G of $(U, \Gamma_{\tilde{R}H}(Y))$ is said to be Nano Hexi _ open set ($\eta_{H-} os$), if $G \in \cup \Gamma_{\tilde{R}H}$, where $H \in \{1,2,3,4,5,6\}$ and it is complement is Nano Hexa_ closed ($\eta_{H-} cs$). The collection of all ($\eta_{H-} os$) and ($\eta_{H-} cs$) sub sets of $(U, \Gamma_{\tilde{R}H}(Y))$ will be denoted by $(\eta_{HO}(U))$ ($\eta_{HC}(U)$) .

2.3. Examples

IF $U = \{1,2,3,4,5\}$ and $Y \subseteq U$, with $U / R_1 = \{\{1,2\}, \{3\}, \{4,5\}\}$

" $U / R_2 = \{\{1,2\}, \{3\}, \{4\}, \{5\}\}$ "; " $U / R_3 = \{\{1\}, \{2,3\}, \{4,5\}\}$ "

" $U / R_4 = \{\{1,3,4\}, \{2\}, \{5\}\}$; $U / R_5 = \{\{1\}, \{2\}, \{3,4\}, \{5\}$ and $U / R_6 = \{\{2,3\}, \{1,4\}, \{5\}\}$.

Where $H = 1,2,3,4,5,6$

$H = \{1,2,3,4,5,6\}$	Y_H	$\mathcal{L}_{\tilde{R}H}(Y)$	$\mathcal{A}_{\tilde{R}H}(Y)$	$\mathcal{B}_{\tilde{R}H}(Y)$	$\Gamma_{\tilde{R}H}(Y)$
U / R_1	$\{1,2,3\}$	$\{1,2,3\}$	$\{1,2,3\}$	$\emptyset$	$\{U, \emptyset, \{1,2,3\}\}$
U / R_2	$\{2,3,4\}$	$\{3,4\}$	$\{1,2,3,4\}$	$\{1,2\}$	$\{U, \emptyset, \{3,4\}, \{1,2\}, \{1,2,3,4\}\}$

U/R_3	$\{4,5\}$	$\{4,5\}$	$\{4,5\}$	$\emptyset$	$\{U, \emptyset, \{4, 5\}\}$
U/R_4	$\{2,4,5\}$	$\{2,5\}$	U	$\{1,3,4\}$	$\{U, \emptyset, \{2, 5\}, \{1, 3, 4\}\}$
U/R_5	$\{1,3,5\}$	$\{1,5\}$	$\{1,3,4,5\}$	$\{3,4\}$	$\{U, \emptyset, \{3, 4\}, \{1, 5\}, \{1, 3, 4, 5\}\}$
U/R_6	$\{5\}$	$\{5\}$	$\{5\}$	$\emptyset$	$\{U, \emptyset, \{5\}\}$

The six topologies are independently satisfying the axioms of Nano Topology. So, the

$$\eta_H O(U) = \left\{ U, \emptyset, \{1,2,3\}, \{3,4\}, \{1,2\}, \{1,2,3,4\}, \{1,3,4\}, \{5\}, \{4,5\}, \{2,5\}, \{1,5\}, \{1,3,4,5\} \right\}$$

But, not from a Nano Topology because $\{1,2,3\} \cap \{3,4\} = \{3\} \notin \eta_H - os$

2.4. Example

" $U / \hat{R} = \{\{\hat{a}, b\}, \{c\}, \{d\}, \{e\}\}$ "on $U = \{\hat{a}, b, c, d, e\}$

$H = \{1, 2, 3, 4, 5, 6\}$	$\mathcal{L}_{RH}(Y)$	$\mathfrak{A}_{RH}(Y)$	$\mathcal{B}_{RH}(Y)$	$\Gamma_{RH}(Y)$
$Y_1 = \{\hat{a}, d', e\}$	$\{d', e\}$	$\{\hat{a}, b, d', e\}$	$\{\hat{a}, b\}$	$\{U, \phi, \{\hat{a}, b\}, \{d', e\}, \{\hat{a}, b, d', e\}\}$
$Y_2 = \{d'\}$	$\{d'\}$	$\{d'\}$	$\emptyset$	$\{U, \phi, \{d'\}\}$
$Y_3 = \{e\}$	$\{e\}$	$\{e\}$	$\emptyset$	$\{U, \phi, \{e\}\}$
$Y_4 = \{\hat{a}, c, d'\}$	$\{c, d'\}$	$\{\hat{a}, b, c, d'\}$	$\{\hat{a}, b\}$	$\{U, \phi, \{\hat{a}, b\}, \{c, d'\}, \{\hat{a}, b, c, d'\}\}$
$Y_5 = \{\hat{a}, b, e\}$	$\{\hat{a}, b, e\}$	$\{\hat{a}, b, e\}$	$\emptyset$	$\{U, \phi, \{\hat{a}, b, e\}\}$
$Y_6 = \{b, c\}$	$\{c\}$	$\{\hat{a}, b, c\}$	$\{\hat{a}, b\}$	$\{U, \phi, \{\hat{a}, b\}, \{c\}, \{\hat{a}, b, c\}\}$

Then

$$\eta_H O(U) = \left\{ U, \phi, \{d', e\}, \{d'\}, \{e\}, \{\hat{a}, b\}, \{c, d'\}, \{\hat{a}, b, e\}, \{c\}, \{\hat{a}, b, d', e\}, \{\hat{a}, b, c\}, \{\hat{a}, b, c, d'\} \right\}, \text{ but not form } \eta_H - \Gamma \text{ because } \{c\} \cup \{e\} = \{c, e\} \notin \eta_H O(U)$$

2.5. Remark

The union of the six topologies on a set is not necessarily a topology on same set. So, $\Gamma_1 \vee \Gamma_2 \vee \Gamma_3 \vee \Gamma_4 \vee \Gamma_5 \vee \Gamma_6$ is the supremum topology on U contains Γ_H , where $(H = 1, 2, 3, 4, 5, 6)$ and $(U, \sum_H^6 \Gamma_H)$ topological spaces. Thus $\eta_H - os$ fulfilling the conditions of a supremum space in a Nano Hexa topology $(\eta_H - \Gamma)$ on U it relates to Y , as $(U, \Gamma_{RH}(Y))$ is $\eta_H - \Gamma_S$.

As, from examples (2.3), (2.4) such that

$$\Gamma_{RH} = \left\{ \begin{array}{l} U, \emptyset, \{1,2,3\}, \{3,4\}, \{1,2\}, \{1,2,3,4\}, \\ \{4,5\}, \{2,5\}, \{1,3,4\}, \{1,5\}, \{1,3,4,5\}, \{5\}, \{1,2,3,5\}, \{3,4,5\}, \\ \{2,3,4,5\}, \{1,4,5\}, \{1,2,4,5\}, \{2,4,5\}, \{1,2,5\}, \{3\}, \{1\}, \{2\}, \{1,3\}, \{4\} \end{array} \right\} \text{ and}$$

$$" \Gamma_{RH} = \left\{ U, \phi, \{\hat{a}, b\}, \{c, d'\}, \{e\}, \{c, d', e\}, \{\hat{a}, b, c, e\}, \{c, e\}, \{\hat{a}, b, c, d'\}, \{\hat{a}, b, e\}, \{c\}, \{\hat{a}, b, c\}, \{d'\}, \{\hat{a}, b, d', e\}, \{\hat{a}, b, d'\} \right\} "$$

2.6. Theorem: Every $\eta - os$ is $\eta_H - os$.

Proof: Suppose B is $\eta - os$ in $(U, \Gamma_{RH}(Y))$. It is an $\eta - os$ in one of the space $(U, \Gamma_{RH}(Y))$, we get $B \in \Gamma_{RH}(Y)$ and $\eta_H - os$.

2.7. Remarks

- 1- U, ϕ are η_H -clopen sets.
- 2- $\Gamma_{RH}(Y) = \{U, \phi, \mathcal{L}_{RH}(Y), \mathcal{A}_{RH}(Y), \mathcal{B}_{RH}(Y)\}$ is η_H - Γ_S
- 3- The set $\mathcal{B} = \{U, \cup \mathcal{L}_{RH}(Y), \mathcal{B}_{RH}(Y)\}$ is the basis for $\Gamma_{RH}(Y)$.

2.8. Proposition: Every η -closed sets in which of $(U, \Gamma_{RH}(Y))$ is η_H -c sets.

Proof: By using theorem (2.6), let $\mathcal{B}$ be η_H -os in one of the $(U, \Gamma_{RH}(Y))$, we get $\mathcal{B}^c \in \cup (\Gamma_{RH}(Y))^c$ and $\mathcal{B}^c$ is η_H -c set.

2.9. Definitions : A subset $\dot{F}$ of $(U, \Gamma_{RH}(Y))$ it relates to Y and if $\dot{F} \subseteq U$, then

- 1- η_H -interior of $\dot{F}$ is the union of all η_H -o subset contained in $\dot{F}$ ($Int_{\eta_H}(\dot{F})$) is largest η_H -o subset of $\dot{F}$.
- 2- η_H -closure of $\dot{F}$ is intersection of all η_H -cs containing $\dot{F}$ ($CL_{\eta_H}(\dot{F})$) is smallest η_H -cs contains $\dot{F}$.

2.10. Proposition

If $\mathcal{A}$ is η_H -c set and $\mathcal{A} \subseteq \mathcal{B}$, which $\mathcal{A} \subseteq CL_{\eta_H}(\mathcal{A})$, we get $\mathcal{B}$ is also η_H -c set.

Since $\mathcal{A}$ in η_H -c set if and only if $CL_{\eta_H}(\mathcal{A}) = \mathcal{A}$, then $\mathcal{A} \subseteq \mathcal{B} \subseteq \mathcal{A}$. So, $\mathcal{B} = \mathcal{A}$ therefore $\mathcal{B}$ is η_H -c set

2.11. Properties : The space $(U, \Gamma_{RH}(Y))$ then:

- 1- $CL_{\eta_H}(\mathcal{B}_{RH}(Y)) = \mathcal{A}_{RH}(Y^c)$
- 2- $CL_{\eta_H}(\mathcal{L}_{RH}(Y)) = \mathcal{L}_{RH}(Y^c)$
- 3- $CL_{\eta_H}(\mathcal{A}_{RH}(Y)) = U$.

We will now consider to discuss three situation

First case: Suppose that U/R be equivalence class defined on U . We will look at some chosen subset of U . as the example (2.4)

We get

$$\eta_H C(U) = \left\{ \phi, U, \{\hat{a}, \hat{b}, \hat{c}\}, \{\hat{c}\}, \{\hat{a}, \hat{b}, \hat{c}, \hat{e}\}, \{\hat{a}, \hat{b}, \hat{c}, \hat{d}'\}, \{\hat{c}, \hat{d}', \hat{e}\}, \{\hat{a}, \hat{b}, \hat{e}\}, \{\hat{c}, \hat{d}'\}, \{\hat{a}, \hat{b}, \hat{d}', \hat{e}\}, \{\hat{e}\}, \{\hat{d}', \hat{e}\} \right\}.$$

Second case: Using six equivalence classes on U and its six subsets. As from the example (2.3)

$$\text{We get } \eta_H O(U) = \left\{ U, \phi, \{1,2,3\}, \{3,4\}, \{1,2\}, \{1,2,3,4\}, \{4,5\}, \{2,5\}, \{1,3,4\}, \{1,5\}, \{1,3,4,5\}, \{5\} \right\}$$

$$\eta_H C(U) = \left\{ \phi, U, \{4,5\}, \{1,2,5\}, \{3,4,5\}, \{5\}, \{1,2,3\}, \{1,3,4\}, \{2,5\}, \{2,3,4\}, \{2\}, \{1,2,3,4\} \right\}$$

Third case: Using six equivalence classes on U and subsets of U . For the

2.12. Example

Let $Y = \{2, 3, 4\} \subseteq \{1, 2, 3, 4, 5\}$, so

$$U / R_1 = \{\{1\}, \{2,3\}, \{4,5\}\},$$

$$U / R_2 = \{\{1\}, \{5\}, \{2,3,4\}\},$$

$$U / R_3 = \{\{1, 2, 5\}, \{3, 4\}\},$$

$$U / R_4 = \{\{2\}, \{4, 5\}, \{1, 3\}\}, U / R_5 = \{\{4\}, \{1, 3\}, \{2, 5\}\} \text{ and } U / R_6 = \{\{3, 5\}, \{1\}, \{2, 4\}\}, \text{ we get}$$

$U / R_i =$ $1, 2, 3, 4, 5, 6$						
$L_{RH}(Y)$	$\{2, 3\}$	$\{2, 3, 4\}$	$\{3, 4\}$	$\{2\}$	$\{4\}$	$\{2, 4\}$
$U_{RH}(Y)$	$\{2, 3, 4, 5\}$	$\{2, 3, 4\}$	U	U	U	$\{2, 3, 4, 5\}$
$R_{RH}(Y)$	$\{4, 5\}$	ϕ	$\{1, 2, 5\}$	$\{1, 3, 4, 5\}$	$\{1, 2, 3, 5\}$	$\{3, 5\}$

$$\text{So, } \eta_H O(U) = \left\{ U, \phi, \{2, 3\}, \{2, 3, 4, 5\}, \{4, 5\}, \{2, 3, 4\}, \{3, 4\}, \{1, 2, 5\}, \{2\}, \{1, 3, 4, 5\}, \{4\}, \{1, 2, 3, 5\}, \{2, 4\}, \{3, 5\} \right\}$$

$$\eta_H C(U) = \left\{ \phi, U, \{1, 4, 5\}, \{1\}, \{1, 2, 3\}, \{1, 5\}, \{1, 2, 5\}, \{3, 4\}, \{1, 3, 4, 5\}, \{2\}, \{1, 2, 3, 5\}, \{4\}, \{1, 3, 5\}, \{1, 2, 4\} \right\}.$$

Thus, η_H –clopen sets $\{U, \phi, \{2\}, \{3, 4\}, \{1, 2, 5\}, \{1, 3, 4, 5\}, \{4\}, \{1, 2, 3, 5\}, \{1, 2, 3, 5\}\}.$

Note: cases are independent in the structure formation in $\eta \Gamma_{RH}$.

2.13. Example

Since $\forall \eta - o$ set is $\eta_H - os$, then every relative $\eta \Gamma_R [1]$ is relative $\eta \Gamma_{RH}$.

2.14. Proposition

For every A subset of the $(\tilde{U}, \Gamma_{RH}(Y))$ then $**CL_{\eta_H \tilde{U}}(A) = **CL_{\eta_H}(A) \cap \tilde{U}$.

Proof.

A & $\tilde{D}$ are $\eta_H - c$ set. By using definition (2.9), then $**CL_{\eta_H}(A) = \cap \{\tilde{D} \cap \tilde{U} : A \subseteq \tilde{D} \cap \tilde{U}\}$, thus $\tilde{D}$ is $\eta_H - cs$ in $(U, \Gamma_{RH}(Y)) \ni A = \tilde{D} \cap \tilde{U}$. We get $A \subseteq \tilde{D} = \cap \{\tilde{D} : \tilde{D} \text{ is } \eta_H - c \text{ set in } U \text{ and } A \subseteq \tilde{D}\} \cap \tilde{U} = **CL_{\eta_H}(A) \cap \tilde{U}$.

2.15. Definition

A space $(U, \Gamma_{RH}(Y))$ is η_H –extremely disconnected ($\eta_H ED$) if the η_H –closure of every $\eta_H - os$ is $\eta_H - \eta_H - os$ in U .

As, the example (2.4) it is noticed that $\eta_H ED$.

2.16. Remark : Every N_{ED} is $\eta_H ED$.

2.17 Proposition : A space $(U, \Gamma_{RH}(Y))$ is $\eta_H ED$ if and only if $U_{RH}(Y) = U$.

Proof. By using theorems (1.7) and (2.6) are proved.

2.18 Propositions : A space $(U, \Gamma_{RH}(Y))$ is $\eta_H ED$, if

1. $L_{RH}(Y) = \phi$
2. $L_{RH}(Y) = U_{RH}(Y)$.

Proof. We use proposition (1.6) and theorem (2.6) are proved.

Conclusion:

In this work, we have provided new type topology is Nano Hexa Topological Spaces based on six different Nano Topologies. We "found that 3 cases are independently of each other". Finally, we hope that this paper will be the beginning of new -life applications in Nano Hexa Topology.

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