Journal of Al-Qadisiyah for Computer Science and Mathematics Vol.18.(1) 2026,pp.Math 174–180

Available online at www.qu.edu.iq/journalcm
JOURNAL OF AL-QADISIYAH FOR COMPUTER SCIENCE AND MATHEMATICS
ISSN:2521-3504(online) ISSN:2074-0204(print)

Comparative Review of Ranking Functions in Fuzzy Number

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ARTICLE INFO

Article history:
Received: 11 /03/2026
Rrevised form: 25 /03/2026
Accepted : 29 /03/2026
Available online: 30 /03/2026

Keywords: Ranking Function (RF), Fuzzy Number (FN), Triangular Fuzzy Numbers (TriFN), Trapezoidal FuzzyNumbers(TraFN).

ABSTRACT

Fuzzy numbers represent vague numerical quantities, making them difficult to organize according to their magnitude. For making critical decisions, scheduling these numbers is essential. Because fuzzy numbers are often in a state of uncertainty, alternatives are calculated, and comparing them essentially involves choosing between alternatives. Previous studies abound with methods and methodologies addressing fuzzy number (FN) comparison. Many different, simple, and consistent methods are used in research and studies for FN comparison. The purpose of this procedure review is to understand the concepts of different ordering functions for triangular and trapezoidal fuzzy numbers, as presented in previous studies and research. This research will present the results applied to triangular and quaternary fuzzy numbers and their various ordering functions, and compare them.

MSC.

https://doi.org/ 10.29304/jqcsm.2026.18.12727

1. Introduction

Decision-making (DM) is considered the core and most important (MI) aspect of mathematical modeling in various scientific fields. However, the practical application of these decisions is sometimes fraught with ambiguity [1]. Developing a method is described as a category of decision-making, where decisions can be made to develop one or more objectives based on a specific set of conditions. Fuzzy development is a technique that deals with ambiguity in imprecise criteria, represented by fuzzy elements belonging to an uncertain set. The most important advantage of fuzzy development is its ability to handle a wide range of uncertainties in the issue environment, such as the varying ambitions of DM regarding objectives [2].

Fuzzy numbers play a pivotal task in DM processes, and there is a wealth of research on this topic [3–8]. Problems constantly arise in diverse fields, such as fuzzy control (FC), optimization, and business and economic systems, highlighting the critical need for various comparisons in fuzzy environments. In these environments, humans encounter situations of fuzzy randomness expressed through linguistic variables. In these models, the DM represents these situations using FN, and after scheduling them, determines make a choice. The comparison of FN serves to rank the corresponding situations. Jain [9] first proposed fuzzy number ranking functions, and over time, various methods have emerged with varying limitations. These methods are only applicable in specific contexts, and some can only be implemented when the membership functions (MFs) have particular properties, such as regular, trigonometric, and other types of functions. Bortolon and Degani [10] provided a review and proactive study of a range of these methods.

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A group of researchers studied the identification of FN based on their center coordinates, and this classification method has been used in various ways.

Recently, a study was conducted comparing the center of fuzzy numbers with their order, as shown in the source [11]. The first to explore the concept of fuzzy number center and order was Yager [12]. Yager suggested the horizontal coordinate with the fuzzy number center $\tilde{x}$ as an indicator for classification. The study showed that a fuzzy number is larger when its corresponding $\tilde{x}$ value is bigger. This methodology does not accurately identify FN if the $\tilde{x}$ value is the identical but the $\tilde{y}$ value, which represents the vertical coordinate of the FN center. Nejad and Mashinchi investigated identifying FN based on the distances between the left & right sides. This method is characterized via its simple mathematical operations and ease of use compared to other methods [13]. Identifying fuzzy numbers is crucial for DM. Because most of the proposed strategies cannot distinguish between sets of FN, and ultimately, we cannot classify them because they are only partially ordered, and therefore they cannot be compared like actual value [14].

FN can be ordered, converted to actual value & differentiated using a ranking function. Following Zadeh's proposal (1965) [15], many ranking methods were introduced. Jain was the first to develop this strategy [16], and by that time, several strategies for determining the objective had been studied. A number of ranking methods were proposed in [17]. Chen studied the fundamentals of maximizing, minimizing, and ordering fuzzy numbers [18]. Dubois and Prade explained the mean principle for ordering fuzzy numbers [19]. Lee and Li considered a probability scale for differentiating fuzzy numbers [20].

Many researchers have studied the conversion of parameters such as transportation cost, game theory, and network analysis to fuzzy numbers. To obtain an accurate value using a ranking function, procedures were proposed [21-32]. Allahviranloo et al. developed a novel scale for studying the generalized Hausdorff area and the correlation of fuzzy numbers [33]. PhaniBushan and RaoPeddia identified two categories for maximizing and minimizing triangular fuzzy numbers and the center point [34]. Ali et al. proposed spacing for ordering fuzzy numbers with interval values [35]. Kareem and Ramadan first proposed a multidimensional strategy for ordering triangular fuzzy (TriF) numbers. They represented each FN as a row in the matrix & then found the configuration points that represented the FN in R^2 space. Since the set of points is not precisely defined, various techniques for forming and differentiating the numbers have been proposed [36].

2. Basic Definitions

This section explains the essential basic information and definitions that pertain to fuzzy sets.

Definition 2.1 [37]

Let χ be a group of elements, usually denoted by the symbol ẋ. Therefore, the fuzzy set (FS) $\tilde{A}$ in χ is described by a number of ordered pairs: $\tilde{A} = \{ (x, M_{\tilde{A}}(\chi)) : x \in \chi \}$, where $M_{\tilde{A}}(\chi)$ is called the membership function (MF) of x to $\tilde{A}$. The MF assigns each element of χ a membership between 0 & 1.

Definition 2.2 [38]

Let the FN $\tilde{\mathcal{A}}$ be a set of convex and standard fuzzy numbers on the real number line Ř such that:

(i) it exists at least one x_0 in Ř with $M_{\tilde{\mathcal{A}}}(x_0)=1$, and

(ii) $M_{\tilde{\mathcal{A}}}(X)$ is piecewise continuous.

Definition 2.3 [39]

Let the FN $\tilde{a}$ represent a triangular fuzzy (TriF) number denoted by (a,b,c) where a, b, and c are actual values & its MF $\mu_{\tilde{A}}$ is given below:

$$\mu_{\tilde{A}} = \begin{cases} \frac{x-p}{q-p} & p \le x \le q \\ 1 & x = q \\ \frac{r-x}{r-q} & q \le x \le r \end{cases}$$

Definition 2.4 [40]

A FN g̃ is a trapezoidal fuzzy (TraF) number denoted by $\tilde{g} = (t^l, t^U, v_1, v_2)$ where $t^l, t^U, \alpha_1, \alpha_2$ are real numbers and its g(x) membership function is given below,

$$g(x) = \begin{cases} \dfrac{x-(t^l - v_1)}{v_1}, & t^l - v_1 \le x \le t^l \\ 1, & t^l \le x \le t^U \\ \dfrac{(t^U + v_2) - x}{v_2}, & t^U \le x \le t^{U+v_2} \\ 0, & others \end{cases}$$

3. Ranking Functions [41]

The concept of the ordering function F(Ḳ)describes a good way to order the elements Ḳ (A˜_P) : F(Ḳ)→Ḳ is a function that transforms every natural number into a real number (RN) line of natural order.

The orders on F(Ḳ) are shown as follows:

$\tilde{\mathcal{G}}_\delta \ge \tilde{\phi}_\delta \leftrightarrow$ Ḳ $(\tilde{\mathcal{G}}_\delta) \ge$ Ḳ $(\tilde{\phi}_\delta)$

$\tilde{\mathcal{G}}_\delta \le \tilde{\phi}_\delta \leftrightarrow$ Ḳ $(\tilde{\mathcal{G}}_\delta) \le$ Ḳ $(\tilde{\phi}_\delta)$

$\tilde{\mathcal{G}}_\delta = \tilde{\phi}_\delta \leftrightarrow$ Ḳ $(\tilde{\mathcal{G}}_\delta) =$ Ḳ $(\tilde{\phi}_\delta)$

4. Ranking Methods (RM) for Fuzzy Numbers (FN)

4.1. Ranking Methods (RM) for Triangular Fuzzy Numbers (TriFN)

There are many strategies for ordering TriFN. All the ordering methods identified here have been studied previously for triangular fuzzy numbers, and a number of studies were compiled from the existing literature. The most important of these were identified:

1. The Ranking function (RF) of TriFN [42]

$$\mathcal{R}(\tilde{\mathcal{A}}) = \frac{a+6*b+c}{8}$$

2. The Ranking function (RF) of TriFN [43]

$$\mathcal{R}(\tilde{\mathcal{A}}) = \frac{a+4*b+c}{6}$$

3. Subinterval Mean Method for TriFN [44]

$$\mathcal{R}(\tilde{\mathcal{A}}) = \frac{4(a+b+c)}{12}$$

4. Method of summing subintervals of TriFN [45]

$$\mathcal{R}(\tilde{\mathcal{A}}) = \frac{4(a + b + c)}{6}$$

5. Pascal's triangular graded mean of TriFN [46]

$$\mathcal{R}(\tilde{\mathcal{A}}) = \frac{(a + 2b + c)}{4}$$

4.2. Ranking Methods (RM) for Trapezoidal Fuzzy Numbers (TraFN)

There are many methods for arranging functions, including trapezoidal fuzzy numbers. In the past, a number of arrangements for trapezoidal fuzzy numbers, as described here, were discovered through a review of various research papers.

1. Subinterval Mean Method for TraFN [44]

$$R(\tilde{A}) = \frac{5(a + b + c + d)}{20}$$

2. Method of summing subintervals of TraFN [45]

$$R(\tilde{A}) = \frac{5(a + b + c + d)}{10}$$

3. Pascal's triangular graded mean of TraFN [47]

$$R(\tilde{A}) = \frac{(a + 3b + 3c + d)}{8}$$

4. The Ranking function (RF) of TraFN [48]

$$R(\tilde{A}) = \frac{(2a + 7b + 7c + 2d)}{18}$$

5. The Ranking function (RF) of TraFN [49]

$$R(\tilde{A}) = \frac{(2a + 2b - c + d)}{4}$$

5. Comparative Reviews

This section will present several different sorting functions on fuzzy numbers, most commonly used in reviews and previous studies comparing fuzzy numbers, and triangular and trapezoidal sorting functions.

Example 5.1 it briefly describes the types that were evaluated of ranking functions and TriFN & reference sources.

Example 5.1. Considering a pair of two TriFN

A_1 = (1,4,5) and A_2= (2,3,6),,taken from Nguyen [50].

Table 1. Comparative ranking results (RRe) of the TriFN of Ex. 5.1

No	Researcher	ranking value of the TriFN (objective function)
1	Hasan I and Hasan A [42]	7
2	Kumar and Subramanian [43]	7
3	Dinagar et al. [44]	7
4	Dinagar and Kamalanathan [45]	14
5	Srinivasan et al. [46]	7

The objective function (OF) values were calculated for several possible solutions, yielding the following results: 7, 7, 7, 14, 7. we observe that most values are equal to 7, while one value, 14, is higher. If the optimization problem is of the maximization type, the best value for the OF is 14, and therefore the solution associated with this value is the optimal solution (OS). However, if the issue is of the minimization type, the smallest value for the OF is 7. Furthermore, the similarity of the values indicates the stability of the results and the comparable performance of the suggested solutions.

Example 5.2 briefly shows the evaluated types of ranking functions and TraFN & reference sources.

Example 5.2. Consider the following TraFN, given in Nasseri et al. [51].

$A_1 = (5,7,9,10)$, $A_2 = (6,7,9,10)$

Table 2. Comparative ranking results of the TraFN of Ex. 5.2.

No	Researcher	ranking value of the TraFN (objective function)
1	Dinagar et al. [44]	15.7500
2	Dinagar and Kamalanathan [45]	31.500
3	Ponnialagan et al. [47]	15.9167
4	Rahmani [48]	15.8333
5	Hussein and, Abood [49]	13

Most of the values fall between 15.7500, 15.9167, and 15.8333, indicating a clear convergence among the majority of solutions. However, the values 31.500 and 13 are considered high and low, respectively, compared to the other values, and may indicate a difference in the solutions used in the model. Fig. 1 briefly shows the evaluated types of comparative ranking results.

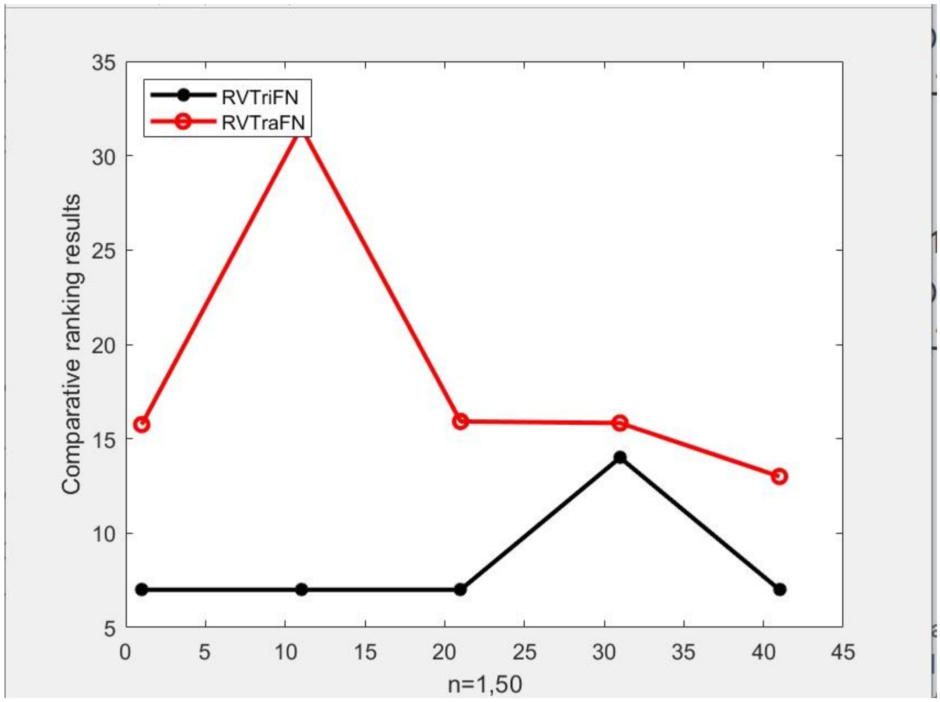

Fig.1. Explain comparative ranking results (RV) of the triangular and trapezoidal FN.

(RVTriFN) represent the ranking value of the TriFN

(RVTraFN) represent the ranking value of the TraFN

6. Conclusion

The inconsistency and complexity of calculations hinder the classification of FN. This research review highlights the importance of FN as a classification tool to reduce this complexity. Based on previous studies and research, the importance and benefits of classification functions are evident in terms of consistency and ease of calculation. Classification functions have several advantages, including: first, supporting classification results with human perception; and second, ensuring the ease of calculations regardless of the type of fuzzy number. These characteristics are crucial in various fields, particularly in multi-criteria and multi-objective decision-making, statistical data analysis, and the development of different technologies. This review will examine triangular and trapezoidal FN by studying their concepts and the types of classification functions that have developed in recent times. It will compare a range of triangular and trapezoidal classification functions and assess their importance based on market needs and decision-makers' requirements for functions aimed at increasing production and reducing costs in various real and future areas.

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