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Analysis the oscillatory sutter by Fluid flow Through a flexible inclined channel

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ABSTRACT

In this research paper, we examined a type of fluid known as the Sutterby fluid, while accounting for the elasticity present in the channel walls. The governing equations were solved using techniques for solving differential equations, and the resulting plots were interpreted using the Mathematica software

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1. Introduction

The study of mechanics of fluids, gases, and plasmas, and the forces a cting on them is known a sfluid mechanics. Fluid mechanics is used in many fields, such as biology, geophysics, oceanography, meteorology, astrophysics, mechanical engineering, aviation, civil engineering, chemistry, and biomedicine [1]. Fluid dynamics, one of the oldest branches of physics, is the study of what is known as the foundation for understanding many fields of engineering and other applied sciences. Humans first thought of establishing scientific laws governing the movement of water and air nearly 200 years ago. The primary purpose of these laws was to help humans understand these elements and protect themselves from their wrath during natural disasters such as floods and hurricanes. These laws also enabled humans to harness the powers of these elements to develop fields such as marine engineering and civil engineering. [2]. There are many applications that can be listed when searching for

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fluid mechanics applications. For example, the following applications are created and improved using fluid mechanics (clean drinking water supply systems, safe aviation, hydrostatic dams, injection of plastic fluids into injection molds, power plants, wastewater transportation and treatment systems, and diesel pumping and transportation systems) [3]. TheSutterby fluid is the one that designates the highlyapplicable diluted solutions, researchers have exhibited pervasive in tent in examining Sutterby fluid indifferent physical configurations and under different constraints that designate the highly applicable diluted solutions, [4 - 9]. Our study is concerned with analyzing the oscillatory flow of a special type of fluid, "Sutter by", as quite a few have been interested in studying this type of oscillatory flow of different fluids, and we have adopted some of them in our study, [10] and [11].

NOMENCLATURE

Mathematical Formulation

Basic Equations

Fundamental Equations

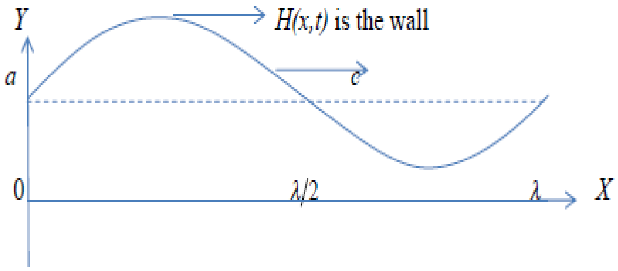
Method of Solution

Flexible Wall

Results and Discussion

2 Mathematical Formulation

Let us consider the flow of Sutter by fluid in a channel of width h. We are considering cartesian coordinate system ,such that $(u(y,t),0,0)$ is avelocity vector in which u is the x-component of velocity and y is perpendicular to the x-axis.



The wall deformation is given by:

$$H(\bar{x},\bar{t}\,) = \; a - \overline{\emptyset}\, cos^2 \frac{\pi}{\lambda}(\bar{x} - c\bar{t})$$

where $\bar{h}$, , $\bar{x}$, , $\overline{\emptyset}$,, $\bar{t}$,, λ, and c represent transferse Vibration of the wall, axial coordinate time, half width ofthe channel, a mplitude of the wave, wave length ,and wave a velocity respectively.

3. Basic Equations

Thebasic equations governing thenon-Newtonian fluid flow in three-dimension coordinate are defined as:

Continuity equation is given by

$$\frac{\partial\bar{u}}{\partial\bar{x}} + \frac{\partial\bar{v}}{\partial\bar{y}} + \frac{\partial\bar{w}}{\partial\bar{z}} = 0 \qquad (1)$$

The x -direction:

$$\rho\left(\bar{w}\frac{\partial\bar{u}}{\partial\bar{z}} + \bar{u}\frac{\partial\bar{u}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{u}}{\partial\bar{y}} + \frac{\partial\bar{u}}{\partial\bar{t}}\right) = -\frac{\partial\bar{p}}{\partial\bar{x}} + \frac{\partial\bar{\tau}_{\bar{x}\bar{x}}}{\partial\bar{x}} + \frac{\partial\bar{\tau}_{\bar{x}\bar{y}}}{\partial\bar{y}} + \frac{\partial\bar{\tau}_{\bar{x}\bar{z}}}{\partial\bar{z}} \qquad (2)$$

The y -direction:

$$\rho\left(\bar{w}\frac{\partial\bar{v}}{\partial\bar{z}} + \bar{u}\frac{\partial\bar{v}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{v}}{\partial\bar{y}} + \frac{\partial\bar{v}}{\partial\bar{t}}\right) = -\frac{\partial\bar{p}}{\partial\bar{y}} + \frac{\partial\bar{\tau}_{\bar{y}\bar{x}}}{\partial\bar{x}} + \frac{\partial\bar{\tau}_{\bar{y}\bar{y}}}{\partial\bar{y}} + \frac{\partial\bar{\tau}_{\bar{y}\bar{z}}}{\partial\bar{z}} \qquad (3)$$

The z -direction:

$$\rho\left(\bar{w}\frac{\partial\bar{w}}{\partial\bar{z}} + \bar{u}\frac{\partial\bar{w}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{w}}{\partial\bar{y}} + \frac{\partial\bar{w}}{\partial\bar{t}}\right) = -\frac{\partial\bar{p}}{\partial\bar{z}} + \frac{\partial\bar{\tau}_{\bar{z}\bar{x}}}{\partial\bar{x}} + \frac{\partial\bar{\tau}_{\bar{z}\bar{y}}}{\partial\bar{y}} + \frac{\partial\bar{\tau}_{\bar{z}\bar{z}}}{\partial\bar{z}} \qquad (4)$$

where $\bar{u}$, $\bar{v}$, and $\bar{w}$ are the components of the fluid velocity field, $\bar{\tau}$ is the shear stress of the fluid, ρ is the fluid density, $\bar{p}$ pressure.

4. Fundamental Equations

The fundamental equations for Sutterbyfluid are given [9]

$$\bar{\tau} = \frac{\mu_0}{2}\left[\frac{sinh^{-1}(b\dot{\gamma})}{b\dot{\gamma}}\right]^n \Pi \qquad (5)$$

where $\bar{\tau}$ is theextra stress tensor, μ_0 is thezero-shearrate viscosity, and $\dot{\gamma}$ is the second in variant strain tensor which is definedas:

$$\dot{\gamma} = \sqrt{\frac{1}{2}\sum_i \sum_j \dot{\gamma}_{ij}\dot{\gamma}_{ij}} = \sqrt{\frac{1}{2}\, trac(\Pi^2)}$$

here $\Pi = \nabla\bar{U} + (\nabla\bar{U})^T$ is the first RivlinErickson tensor where $(\nabla\bar{U})$ is a gradient of thefluid velocity, and $(\nabla\bar{U})^T$ is thetranspose of the gradientvelocity in the Cartesian coordinates system (x, y, z).

5. Method of Solution

Considering that the unsteady velocity field is represented by the triple $(u(y,t), 0,0)$ and then substituting with the main equations governing the fluid flow equations, we obtain the following equations

$$\frac{\partial\bar{u}}{\partial\bar{x}} + \frac{\partial\bar{v}}{\partial\bar{y}} = 0 \qquad (6)$$

$$\rho\frac{\partial\bar{u}}{\partial\bar{t}} = -\frac{\partial\bar{p}}{\partial\bar{x}} + \frac{\partial\bar{\tau}_{\bar{x}\bar{x}}}{\partial\bar{x}} + \frac{\partial\bar{\tau}_{\bar{x}\bar{y}}}{\partial\bar{y}} + \frac{\partial\bar{\tau}_{\bar{x}\bar{z}}}{\partial\bar{z}} \qquad (7)$$

$$0 = \frac{\partial\bar{p}}{\partial\bar{y}} + \frac{\partial\bar{\tau}_{\bar{y}\bar{x}}}{\partial\bar{x}} + \frac{\partial\bar{\tau}_{\bar{y}\bar{y}}}{\partial\bar{y}} + \frac{\partial\bar{\tau}_{\bar{y}\bar{z}}}{\partial\bar{z}} \qquad (8)$$

With the stress component are given by:

$$\bar{\tau}_{\bar{x}\bar{x}} = \bar{\tau}_{\bar{y}\bar{y}} = \bar{\tau}_{\bar{x}\bar{z}} = \bar{\tau}_{\bar{y}\bar{z}} = 0$$

$$\bar{\tau}_{\bar{x}\bar{y}} = \bar{\tau}_{\bar{y}\bar{x}} = \frac{\mu_0}{2}\left[1 - \frac{nb^2}{6}\left(\frac{\partial\bar{u}}{\partial\bar{y}}\right)^2\right]\left(\frac{\partial\bar{u}}{\partial\bar{y}}\right) \qquad (9)$$

For the purpose of solving the system of equations (6) – (8)

6. Flexible Wall

The governing equation ofmotion of the flexible wall maybe expressed as:

$$L^* = \bar{P} - \overline{P_0}$$

where L^* is aoperator, which is usedto represent the motionof stretched membrane with viscosity damping forces such that

$$L^{*} = -K\frac{\partial^2}{\partial x^2} + m\frac{\partial^2}{\partial t^2} + A\frac{\partial}{\partial t}$$

where K is the elastic tension in the membrane, m is the mass per unit area, A is the coefficient of viscous damping forces.

$$\frac{\partial}{\partial \bar{x}} L^{*}(\bar{h}) = \frac{\partial \bar{p}}{\partial \bar{x}} = -\rho\frac{\partial \bar{u}}{\partial \bar{t}} + \frac{\partial \bar{\tau}_{\bar{x}\bar{x}}}{\partial \bar{x}} + \frac{\partial \bar{\tau}_{\bar{x}\bar{y}}}{\partial \bar{y}} + \frac{\partial \bar{\tau}_{\bar{x}\bar{z}}}{\partial \bar{z}}, \tag{10}$$

we use the following non-dimensional equations that help us simplify the equations

$$\left.\begin{array}{c} x = \frac{\bar{x}}{h},\ y = \frac{\bar{y}}{h},\ u = \frac{\bar{u}}{u_s},\ t = \frac{\bar{t}u_s}{h},\ p = \frac{\bar{p}\,h}{\mu_0 u_s}, \\ U_0 = \frac{U_h}{u_s}, \tau_{xy} = \frac{h}{\mu_0 u_s}\bar{\tau}_{\bar{x}\bar{y}}, Re = \frac{phu_s}{\mu_0}, \epsilon = \frac{{u_s}^2 n\, b^2}{2h^2} \end{array}\right\} \tag{11}$$

where u_s is the meanflow velocity, Re is Reynolds Number, ϵ Sutterby parameter. Substituting the non-dimensional equations (11) into the equations (6) - (9), we have thefollowing of non dimensional equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{12}$$

$$\frac{\partial p}{\partial x} = \frac{1}{2}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y} - \frac{\epsilon}{3}\left(\frac{\partial u}{\partial y}\right)^3\right) \tag{13}$$

$$0 = -\frac{\partial p}{\partial y} + \frac{1}{2}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y} - \frac{\epsilon}{3}\left(\frac{\partial u}{\partial y}\right)^3\right) \tag{14}$$

From the (14) equation we get $\frac{\partial p}{\partial y} = 0$, this means that the pressure p is a constant function with respect to y. Hence we have the following equation

$$E1\frac{\partial^3 h}{\partial x^3} + E2\frac{\partial^3 h}{\partial x \partial t^2} + E3\frac{\partial^2 h}{\partial x \partial t} = \frac{1}{2}\left(\frac{\partial^2\ u}{\partial y^2} - \epsilon\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2\ u}{\partial y^2}\right) \tag{15}$$

where

$$E_1 = -\frac{\kappa a^2}{\mu c \lambda^3},\ E_2 = \frac{m_1 c a^2}{\mu \lambda^3},\ E_3 = \frac{C a^2}{\mu \lambda^2},$$

and

$$h(x,t) = 1 - \phi\cos^2\pi(x - t)$$

7. Results and Discussion

This section discusses graphically oscillatory flow and its effect on Sutterby fluid through a flexible channl. Through the MATHEMATICA-13 program to find the solution to equation (15) , we discuss all the solutions we obtain graphically. Under the influence of different parameters related to the fluid velocity, important results are displayed in figures (2,5,7) **increases.** Figures (3,4,6) **decreases.**

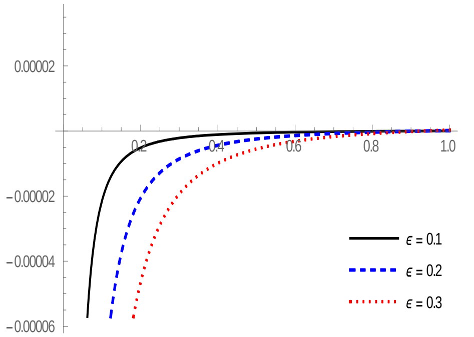


Figure 2

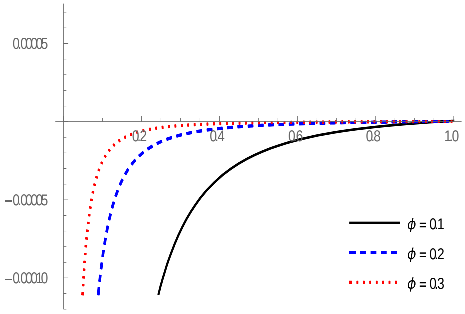


Figure 3

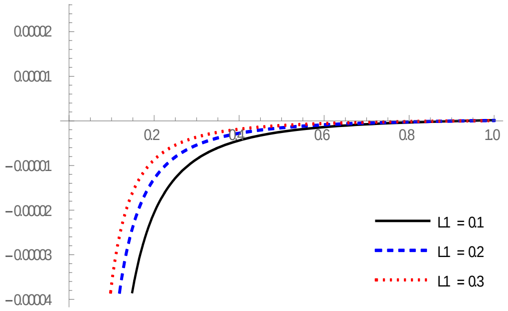


Figure 4.

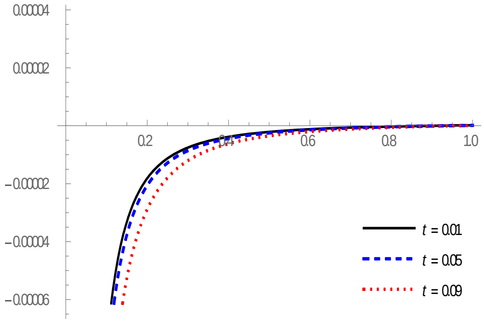


Figure 5

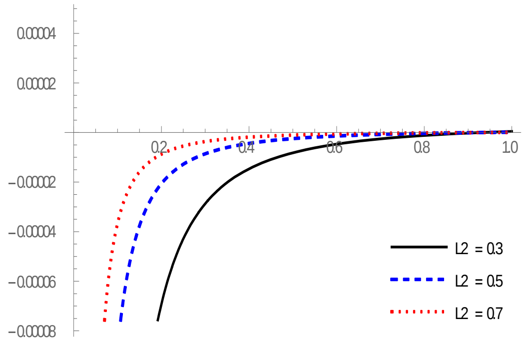


Figure 6

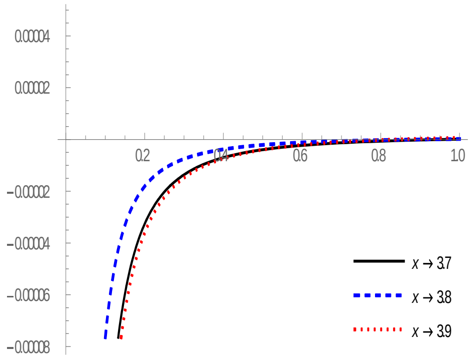


Figure 7

References

[1] JANNA, W. S. 2020. Introduction to fluid mechanics, CRC press.

[2] RATHAKRISHNAN, E. 2022. Fluid mechanics: An introduction, PHI Learning Pvt. Ltd.

[3] ELGER, D. F., LEBRET, B. A., CROWE, C. T. & ROBERSON, J. A. 2020. Engineering fluid mechanics, John Wiley & Sons.

[4] Sutterby, J.L. 1966. Laminar converging flow of dilute polymer solutions in conical sections: Part I. Viscosity data, new viscosity model, tube flow solution. AIChE J., 12, 63–68.

[5] BILAL, S., SHAH, I. A., AKGÜL, A., TEKIN, M. T., BOTMART, T. & YAHIA, I. 2022. A comprehensive mathematical structuring of magnetically effected Sutterby fluid flow immersed in dually stratified medium under boundary layer approximations over a linearly stretched surface. *Alexandria Engineering Journal,* 61, 11889-11898.

[6] ABBAS, N., SHATANAWI, W., ABODAYEH, K. & SHATNAWI, T. A. M. 2023. Comparative analysis of unsteady flow of induced MHD radiative Sutterby fluid flow at nonlinear stretching cylinder/sheet: Variable thermal conductivity. *Alexandria Engineering Journal*, 72, 451-461.

[7] Neeran Ammar, Hayat A. Ali. 2023. Mathematical Modelling for Peristaltic Flow of Sutterby Fluid Through Tube under the Effect of Endoscope. Iraqi Journal of Science, 64(5), 2368-2381.

[8] Asmaa A. Mohammed, Liqaa Zeki Hummady. 2023. Influence of Heat Transform and Rotation of Sutterby Fluid in an Asymmetric Channel. *Iraqi Journal of Science*, *64*(11), 5766-5777.

[9] Fatima Khalid Moeana, Dheia G. Salih Al-Khafajy. 2024. Analyze a temperature and MHD peristaltic flow of sutterby fluid through a porous wavechannel in a rotating frame, Journal of Al-Qadisiyah for Computer Science and Mathematics VOL.16(1), PP Math. 34–46.

[10] Wedyan A .Muhi Al-Kaabi and Dheia G. Salih Al-Khafajy. 2022. Influence of electrical conductivity on the oscillatory flow for Prandtl-Eyring fluid through porous channel, AIP Conference Proceedings 2398, 060054.

[11] Mkhatshwa, M. Triple-Diffusive Bioconvection Flow of Sutterby Nanofluid Over an Oscillatory Stretchable Surface Immersed in a Darcy-Forchheimer Porous Medium. BioNanoSci. 14, 495–516 (2024).