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An Equation Related To Jordan *-Centralizers

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معادلة ترتبط في تمركزات *-جوردان

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Abstract

Let R be a *-ring, an additive mapping T: R →R is called A left (right) Jordan *-centralizer of a *-ring R if satisfies $T(x^2)=T(x)\, x^*$ ($T(x^2)= x^*T(x)$) for all $x \in$ R. A Jordan *-centralizer of R is an additive mapping which is both left and right Jordan *-centralizer. The purpose of this paper is to prove the result concerning Jordan *-centralizer. The result which we refer state as follows: Let R be a 2-torsion free semiprime *-ring and let T: R → R be an additive mapping such that $2T(x^2) = T(x)\, x^* + x^*\, T(x)$ holds for all $x \in$ R. In this case, T is a Jordan *-centralizer

ألمســـتخلص

لتكن R حلقة-*, تدعى الدالة التجميعية T: R→R تمركزات-*جوردان اليسرى(اليمنى) إذا حققت الشرط الأتي : لكل x في R ($T(x^2) =x^*T(x)$) $T(x^2) = T(x)x^*$, وتسمى تمركزات-*جوردان اذا كانت T تمركزات-*جوردان اليسرى واليمنى, في هذا البحث سنبرهن الاتي: لتكن R حلقة-* شبه أوليه طليقة الالتواء من النمط 2 ولتكن T: R→R دالة تجميعية تحقق الشرط الاتي: $2T(x^2)$ $=T(x)x^*+x^*T(x)$ لكل x في R. فان T تكون دالة تمركزات-* جوردان .

1. Introduction

Throughout, R will represent an associative ring with center Z(R). A ring R is n-torsion free, if $nx = 0$, $x \in$ R implies $x = 0$, where n is a positive integer. Recall that R is prime if aRb = (0) implies $a = 0$ or $b = 0$, and semiprime if aRa = (0) implies a =0. An additive mapping $x \to x^*$ on a ring R is called an involution if $(xy)^* = y^* x^*$ and $(x)^{**} = x$ for all $x, y \in$ R. A ring equipped with an involution is called *-ring (see [1]). As usual the commutator $xy - yx$ will be denoted by $[x, y]$. We shall use basic commutator identities $[xy, z] = [x, z]y + x[y, z]$ and $[x, yz] = [x, y]z + y[x, z]$ for all $x,y,z \in$R, (see [1, P.2]). Also we write $x o\, y=xy+yx$ for all x, y

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$\in$R (see [1]). An additive mapping d: R$\rightarrow$R is called a derivation if d(xy) = d(x)y + xd(y) holds for all pairs x,$y$$\in$R, and is called a Jordan derivation in case d(x^2) = d(x)x + xd(x) is fulfilled for all $x \in$ R(see [2]). Every derivation is a Jordan derivation, but the converse is in general not true. A classical result of Herstein [3] asserts that every Jordan derivation on a prime ring of characteristic different from 2 is a derivation. Cusack [4] generalized Herstein's theorem to 2-torsion free semiprime ring. A left (right) centralizer of R is an additive mapping T: R$\rightarrow$R which satisfies T(xy) = T(x)y (T(xy) = xT(y)) for all x, $y \in$ R. A centralizer of R is an additive mapping which is both left and right centralizer. A left (right) Jordan centralizer of R is an additive mapping T: R$\rightarrow$R which satisfies T(x^2) = T(x)x (T(x^2)= xT(x)) for all $x \in$ R. A Jordan centralizer of R is an additive mapping which is both left and right Jordan centralizer (see [5,6,7, and 8]). Every centralizer is a Jordan centralizer. B. Zalar [8] proved the converse when R is 2-torsion free semiprime ring. Inspired by the above definition we define. A left (right) reverse *-centralizer of a *-ring R is an additive mapping T: R$\rightarrow$R which satisfies T(yx)=T(x)y* (T(yx) =x*T(y)) for all x,$y$$\in$R. A reverse *-centralizer of R is an additive mapping which is both left and right reverse *-centralizer. A left (right) Jordan *-centralizer of R is an additive mapping T: R$\rightarrow$R which satisfies T(x^2)=T(x) x* (T(x^2)= x*T(x)) for all $x \in$ R. A Jordan *-centralizer of R is an additive mapping which is both left and right Jordan *-centralizer. Every reverse *-centralizer is a Jordan *-centralizer. In this work we will study an Identity on a Jordan *-centralizers of semiprime *-rings. We will prove in case T: R$\rightarrow$R be an additive mapping, satisfies 2T(x^2) = T(x) x* + x* T(x) holds for all $x \in$ R, where R be a 2-torsion free semiprime *-ring, then T is a Jordan *-centralizers.

2. The Main Results

If T: R $\rightarrow$ R is a Jordan *-centralizer, where R is an arbitrary *-ring, then T satisfies the relation 2T(x^2) = T(x) x* + x* T(x) for all $x \in$ R. It seems natural to ask whether the converse is true. More precisely, we are asking whether an additive mapping T on a *-ring R satisfying 2T(x^2) = T(x) x* + x* T(x)for all $x \in$ R, is a Jordan *-centralizer. It is our aim in this paper to prove that the answer is affirmative in case R is a 2-torsion free semiprime *-ring.

Theorem 2.1.

Let R be a 2-torsion free semiprime *-ring and let T: R$\rightarrow$R be an additive mapping such that 2T(x^2) = T(x) x* + x* T(x) holds for all $x \in$ R. In this case T is a Jordan *-centralizer.

For the proof of the above theorem we shall need the following.

Lemma 2.2. [6].

Let R be a semiprime ring. Suppose that the relation $axb + bxc = 0$ holds for all $x \in$ R and some a, b, c $\in$ R. In this case $(a + c)xb = 0$ is satisfied for all $x \in$ R.

Proof of Theorem 2.1:

We have

$$2T(x^2) = T(x)\, x^* + x^*T(x) \quad \text{for all } x \in R, \tag{1}$$

We intend to prove the relation

$$[T(x), x^*] = 0 \quad \text{for all } x \in R \tag{2}$$

In order to achieve this goal we shall first prove a weaker result that T satisfies the relation

$$[T(x), x^{*2}] = 0 \qquad \text{for all } x \in R \tag{3}$$

Since the above relation can be written in the form $[T(x), x^*]x^*+x^*[T(x),x^*]= 0$, it is obvious that T satisfies the relation (3) if T is satisfies (2) .

Putting in the relation (1) $x^* + y^*$ for x one obtains

$$2T((xy+yx)^*)=T(x^*)y+ xT(y^*) + T(y^*)x + yT(x^*) \quad \text{for all } x,y\in R. \tag{4}$$

Our next step is to prove the relation

$$8T(xyx) = T(x)((yx)^* + 3(xy)^*) + ((xy)^* + 3(yx)^*)T(x) + 2x^*T(y)\, x^*$$
$$-x^{*2}\, T(y) - T(y)\, x^{*2} \quad \text{for all } x, y \in R \tag{5}$$

For this purpose, we put in the relation (4) $2(xy + yx)$ for y, then using (4) we obtain

$$4T((x(xy+yx)+(xy+yx)x)^*) = 2T(x^*)(xy+yx) + 2xT((xy+yx)^*) + 2T((xy+yx)^*)x +$$
$$2(xy + yx)T(x^*) = 2T(x^*)\,(xy + yx)+ xT(x^*)y + x^2\, T(y^*) + xT(y^*)x + (xy)T(x^*) +$$
$$T(x^*)(yx)+ xT(y^*)x+ T(y^*)\, x^2 + yT(x^*)\, x + 2(xy + yx)\, T(x^*)$$

Thus, we have

$$4T((x(xy+yx) + (xy+yx)x)^*) = T(x^*)(2xy+3yx) +(3xy+2yx)\, T(x^*) +$$
$$xT(x^*)y + yT(x^*)x + 2xT(y^*)x + x^2T(y^*)$$
$$+ T(y^*)\, x^2 \qquad \text{for all } x, y \in R, \tag{6}$$

On the other hand, using (4) and (1), we obtain

$$4T((x(xy+yx)+(xy+yx)x)^*)=4T((x^2y+yx^2)^*)+8T((xyx)^*)=2T(x^{*2})y+2x^2 T(y^*)+2T(y^*)x^2+2y\,T(x^{*2})+8T((xyx)^*)=T(x^*)(xy)+xT(x^*)y+2x^2\,T(y^*)+2T(y^*)x^2+yT(x^*)x+(yx)T(x^*)+8T((xyx)^*)$$

for all $x,y\in R$ (7)

By comparing (6) with (7) we arrive at (5). Let us prove the relation

$$T(x)(xyx-2yx^2-2x^2y)^*+(xyx-2x^2y-2yx^2)^*T(x)+x^*T(x)(xy+yx)^*+(xy+yx)^*T(x)x^* + x^{*2}\,T(x)y^* + y^*T(x)x^{*2} = 0 \quad \text{for all } x,y\in R. \qquad (8)$$

Putting in (4) $8(xyx)^*$ for y^* and x^* for x using (5), we obtain

$$16T(x^2yx+xyx^2)=8T(x)(xyx)^*+8x^*T(xyx)+8T(xyx)x^*+8(xyx)^*T(x)= 8T(x)(xyx)^*+x^*T(x)(yx+3xy)^*+(xyx+3yx^2)^*T(x)+2x^{*2}T(y)x^*-x^{*3}T(y)-x^*T(y)x^{*2}+ T(x)(xyx+3x^2y)^*+(xy+3yx)^*T(x)x^*+2x^*T(y)x^{*2} - x^{*2}\,T(y)x^* - T(y)\,x^{*3}+ 8(xyx)^*T(x)$$

We have, therefore

$$16T(x^2yx+xyx^2)=T(x)(9xyx+3x^2y)^*+(9xyx+3yx^2)^*T(x)+x^*T(x)(yx+3xy)^* +(xy + 3yx)^*T(x)x^* + x^{*2}\,T(y)x^* + x^*T(y)x^{*2} - T(y)x^{*3} - x^{*3}T(y) \quad \text{for all } x, y \in R. \qquad (9)$$

On the other hand, we obtain first using (5) and then after collecting some terms using (4)

$$16T(x^2yx+xyx^2)=16T(x(xy)x)+16T(x(yx)x)=2T(x)(3x^2y+xyx)^*+2(3xyx+x^2y)^* T(x)+4x^*T(xy)x^*-2x^{*2}\,T(xy)-2T(xy)x^{*2}+2T(x)(3xyx+yx^2)^*+ 2(3yx^2+xyx)^*T(x)+4x^*T(yx)x^*-2x^{*2}\,T(yx)-2T(yx)x^{*2}= T(x)(6x^2y+2yx^2+8xyx)^*+(8xyx+6yx^2+2x^2y)^*T(x)+4x^*T(xy+yx)x^*-2x^{*2}\,T(xy+yx)- 2T(xy+yx)x^{*2}=T(x)(6x^2y+2yx^2+8xyx)^*+(8xyx+6yx^2+ 2x^2y)^*T(x) + 2x^*T(x)(xy)^* + 2x^{*2}\,T(y)x^* + 2x^*T(y)x^{*2} + 2(yx)^*T(x)x^* - x^{*2}\,T(x)y^* -x^{*3}T(y) - x^{*2}\,T(y)x^* - (y x^2)^*T(x) - T(x)(x^2y)^* - x^*T(y)x^{*2} - T(y)x^{*3}-y^*T(x)x^{*2} \quad \text{for all } x,y\in R,$$

We have, therefore

$$16T(x^2yx + xyx^2) = T(x)\,(5x^2y + 2yx^2 + 8xyx)^* + (5yx^2+2x^2y+8xyx)^* T(x)+2x^*T(x)(xy)^*+2(yx)^*T(x)x^*+x^{*2}\, T(y)x^*+x^*T(y)x^{*2}-x^{*2}\, T(x)y^*- y^*T(x)x^{*2}- x^{*3}T(y)- T(y)x^{*3} \quad \text{for all } x,y \in R. \tag{10}$$

By comparing (9) with (10), we obtain (8). Replacing in (8) y by xy, we obtain

$$T(x)(x^2yx-2xyx^2-2x^3y)^*+(x^2yx-2x^3y-2xyx^2)^*T(x)+x^*T(x)(xyx+x^2y)^*+ (xyx+x^2y)^* T(x)\, x^*+ x^{*2}\, T(x)\,(xy)^* +(xy)^*\, T(x)\, x^{*2} = 0 \quad \text{for all } x,y\in R. \tag{11}$$

Right multiplication of (8) by x^*gives

$$T(x)\,(x^2yx - 2xyx^2 - 2x^3y)^* +(xyx - 2x^2y -2yx^2)^*\, T(x)\, x^*+ x^*T(x) (xyx + x^2y)^* + (xy + yx)^*\, T(x)\, x^{*2} + x^{*2}\, T(x)\,(xy)^* + y^*\, T(x)x^{*3} = 0 \quad \text{for all } x, y \in R \tag{12}$$

Subtracting (12) from (11), we obtain

$$(xyx)^*[x^*,T(x)]+2(x^2y)^*[T(x),x^*]+2(yx^2)^*[T(x),x^*]+(xy)^*[x^*,T(x)]x^*+ (yx)^*[x^*,T(x)]x^*+y^*[x^*,T(x)]x^{*2} = 0, \quad \text{for all } x, y \in R.$$

This reduces after collecting the first and the five terms together to

$$(yx)^*[x^{*2},T(x)]+2(x^2y)^*[T(x),x^*]+2(yx^2)^*[T(x),x^*]+(xy)^*[x^*,T(x)]x^*+ y^*[x^*,T(x)]x^{*2} = 0 \quad \text{for all } x, y \in R. \tag{13}$$

Substituting $y\,(T(x))^*$ for y in the above relation gives

$$x^*T(x)y^*[x^{*2},T(x)] + 2T(x)(x^2y)^*[T(x),x^*] +2x^{*2}\, T(x)y^*[T(x),x^*] + T(x) (xy)^*[x^*,T(x)]x^*+ T(x)y^*[x^*,T(x)]\, x^{*2} = 0 \quad \text{for all } x, y \in R. \tag{14}$$

Left multiplication of (13) by $T(x)$ leads to

$$T(x)(yx)^*[x^{*2},T(x)]+2T(x)(x^2y)^*[T(x),x^*]+2T(x)(yx^2)^*[T(x),x^*]+T(x) (xy)^*[x^*,T(x)]x^*+ T(x)y^*[x^*,T(x)]x^{*2} = 0, \text{ for all } x, y \in R \tag{15}$$

Subtracting (15) from (14), we arrive at

$$[T(x),x^*]y^*[T(x),x^{*2}]-2[T(x),x^{*2}]y^*[T(x), x^*] =0 \quad \text{for all } x, y \in R.$$

From the above relation and Lemma 1.2.3 it follows that

$$[T(x),x^*]y^*[T(x),x^{*2}] = 0 \quad \text{for all } x, y \in R. \tag{16}$$

From the above relation one obtains easily

$$([T(x), x^*]\, x^*+ x^*[T(x),x^*])\, y^*[T(x), x^{*2}] = 0 \quad \text{for all } x, y \in R.$$

Replace y by y^*, we get

$$[T(x),x^{*2}]\, y\ \ [T(x),x^{*2}] = 0 , \quad \text{for all } x, y \in R.$$

This implies (3). Substitution $x + y$ for x in (3) gives

$$[T(x), y^{*2})]+[T(y),x^{*2}]+[T(x),(xy+yx)^*]+[T(y),(xy+yx)^*] = 0 \qquad (17)$$

Putting in the above relation $-x$ for x and comparing the relation so obtained with the above relation, we obtain

$$[T(x),(xy + yx)^*] + [T(y),x^{*2}] = 0 \quad \text{for all } x, y \in R. \qquad (18)$$

Putting in the above relation $2(xy+yx)$ for y we obtain according to (4) and (3)

$$0 =2[T(x) ,(x^2y + yx^2 + 2xyx)^*] + [T(x)y^* + x^*T(y) + T(y)x^* + y^*T(x), x^{*2}] =2 x^{*2}$$
$$[T(x) , y^*] + 2 [T(x) ,y^*] x^{*2} + 4 [T(x) ,(xyx)^*] + T(x) [y^*,x^{*2}] + x^*[T(y),x^{*2}] +$$
$$[T(y),x^{*2}]x^*+[y^*,x^{*2}]T(x) \qquad \text{for all } x, y \in R,$$

Thus, we have

$$2x^{*2} [T(x),y^*] + 2[T(x),y^*]x^{*2} + 4[T(x),(xyx)^*] + T(x)[y^*,x^{*2}]$$
$$+[y^*,x^{*2}]T(x)+x^*[T(y),x^{*2}]+[T(y),x^{*2}]x^*=0 \qquad \text{for all } x,y \in R. \qquad (19)$$

For $y = x$ the above relation reduces to

$$x^{*2} [T(x),x^*] + [T(x),x^*]x^{*2} + 2[T(x),(x^2x)^*] = 0$$

This gives

$$x^{*2} [T(x),x^*] + 3[T(x),x^*]x^{*2} = 0, \ \text{for all } x \in R.$$

According to the relation $[T(x) , x^*] x^*+ x^*[T(x) , x^*] = 0$ (see (3)) one can replace in the above relation $x^{*2} [T(x) , x^*]$ by $[T(x) , x^*] x^{*2}$, which gives

$$[T(x) , x^*] x^{*2} = 0, \text{ for all } x \in R \qquad . \qquad (20)$$

And

$$x^{*2} [T(x) , x^*] = 0, \ \text{for all } x \in R \qquad (21)$$

We have also,

$$x^*[T(x) , x^*] x^*= 0 \quad \text{for all } x \in R \qquad (22)$$

Because of (18) one can replace in (19) $[T(y),x^{*2}]$ by $-[T(x),(xy+yx)^*]$, which gives

$$0 = 2x^{*2}[T(x),y^*] + 2[T(x),y^*]x^{*2} + 4[T(x),(xyx)^*] + T(x)[y^*,x^{*2}] + [y^*,x^{*2}]\, T(x) - x^*[T(x),(xy+yx)^*] - [T(x),(xy+yx)^*]x^* = 2x^{*2}\,[T(x),y^*]$$
$$2[T(x),y^*]x^{*2} + 4[T(x),x^*](xy)^* + 4x^*[T(x),y^*]x^* + 4(yx)^*[T(x),x^*] + T(x)[y^*,x^{*2}] + [y^*,x^{*2}]T(x) - x^*[T(x),x^*]y^* - x^{*2}\,[T(x),y^*] - x^*[T(x),y^*]x^* - (yx)^*[T(x),x^*] - [T(x),x^*](xy)^* - x^*[T(x),y^*]x^* - [T(x),y^*]x^{*2} - y^*[T(x),x^*]x^* = 0 \quad \text{for all } x,y\ x \in R.$$

We have, therefore

$$x^{*2}\,[T(x),y^*]+[T(x),y^*]x^{*2}+3[T(x),x^*](xy)^*+3(yx)^*[T(x),x^*] +2x^*[T(x),y^*]x^*+T(x)[y^*,x^{*2}]+[y^*,x^{*2}]T(x)-x^*[T(x),x^*]y^*- y^*[T(x),x^*]x^* = 0, \quad \text{for all } x, y \in R, \tag{23}$$

The substitution xy for y in (23) gives

$$0=x^{*2}[T(x),(xy)^*] + [T(x),(xy)^*]x^{*2}+3[T(x),x^*](x^2y)^*+3(xyx)^*\,[T(x),x^*] + 2x^*[T(x),(xy)^*]x^* + T(x)[(xy)^*,x^{*2}] + [(xy)^*,x^{*2}]\, T(x) - x^*[T(x),x^*]\,(xy)^* - (xy)^*[T(x),x^*]x^* = x^{*2}[T(x),y^*]x^*+(yx^2)^*[T(x),x^*]+y^*[T(x),x^*]x^{*2} +[T(x),y^*]x^{*3}+3(xyx)^*[T(x),x^*]+3[T(x),x^*](x^2y)^*+ 2x^*[T(x),y^*]x^{*2} + 2(yx)^*[T(x)\,,\, x^*]x^*+ T(x)[y^*\,,\,x^{*2}]x^* + [y^*\,,\,x^{*2}]x^*T(x) - x^*[T(x)\,,\,x^*]\,(xy)^* - (xy)^*\,[T(x)\,,\, x^*]\,x^*$$
for all $x, y \in R$,

Which reduces because of (20) and (21) to

$$x^{*2}\,[T(x),y^*]x^*+(yx^2)^*[T(x),x^*]+[T(x),y^*]x^{*3}+3(xyx)^*[T(x),x^*]+ 3[T(x),x^*](x^2y)^*+2x^*[T(x),y^*]x^{*2} + 2(yx)^*[T(x)\,,\,x^*]x^*+ T(x)[y^*\,,\,x^{*2}]x^* + [y^*\,,\,x^{*2}]x^*T(x) - x^*[T(x)\,,\,x^*]\,(xy)^* = 0 \quad \text{for all } x,y \in R. \tag{24}$$

Right multiplication of (23) by x^*gives

$$x^{*2}[T(x),y^*]x^*+[T(x),y^*]x^{*3}+3[T(x),x^*](x^2y)^*+3(yx)^*[T(x),x^*]x^*+2x^* [T(x),y^*]x^{*2}+T(x)[y^*,x^{*2}]x^*+[y^*,x^{*2}]\,T(x)x^*-x^*[T(x),x^*](xy)^*- y^*[T(x),x^*]x^{*2} = 0 \quad \text{for all } x, y \in R, \tag{25}$$

Subtracting (25) from (24), we obtain

$$[y^*,x^{*2}][x^*,T(x)]+ 3(yx)[x^*,[T(x),x^*] + 2(yx)^*\,[T(x)\,,\,x^*]\,x^*+y^*[T(x),x^*] x^{*2}+(yx^2)^*[T(x),x^*] = 0 \quad \text{for all } x, y \in R,$$

Which reduces because of (21), (20) to

$$2(yx^2)^*[T(x),x^*]+3(yx)^*[x^*,[T(x),x^*]+2(yx)^*[T(x),x^*]x^*=0 \quad \text{for all } x,y\in R.$$

Replacing in the above relation $-[T(x),x^*]x^*$ by $x^*[T(x),x^*]$, we obtain

$$(yx^2)^*[T(x),x^*]+2(yx)^*[T(x),x^*]x^*=0 \quad \text{for all } x,y\in R.$$

Because of (3), (20), (21) and (22) the relation (13) reduces to $(yx^2)^*[T(x),x^*]=0$ for all $x, y \in R$, which gives together with the relation above $(xyx)^*[T(x),x^*]=0$ for all $x,y \in R$, whence it follows

$$x^*[T(x),x^*]y^*x^*[T(x),x^*]=0 \quad \text{for all } x,y\in R.$$

Thus, we have

$$x^*[T(x),x^*]=0, \text{ for all } x\in R. \tag{26}$$

Of course, we have also

$$[T(x),x^*]x^*=0 \quad \text{for all } x\in R. \tag{27}$$

From (26) one obtains (see the proof of (18))

$$y^*[T(x),x^*]+x^*[T(x),y^*]+x^*[T(y),x^*]=0 \quad \text{for all } x,y\in R.$$

Left multiplication of the above relation by $[T(x),x^*]$ gives because of (27)

$$[T(x),x^*]y^*[T(x),x^*]=0 \quad \text{for all } x,y\in R,$$

Whence it follows

$$[T(x),x^*]=0 \quad \text{for all } x\in R. \tag{28}$$

Combining (28) with (1), we obtain

$$T(x^2)=T(x)x^* \quad \text{for all } x\in R.$$

And also

$$T(x^2)=x^*T(x) \quad \text{for all } x\in R.$$

Which means that T is a Jordan *-centralizer. The proof of the Theorem is complete.

If R is prime ring, we get the following corollary

Corollary 2.3.

Let R be a 2-torsion free prime *-ring and let T: R $\rightarrow$ R be an additive mapping such that $2T(x^2) = T(x)x^* + x^*T(x)$ holds for all $x \in R$. In this case, T is a Jordan *-centralizer.

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