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Ideal convergence and ideal equivalence of bounded linear functions in an n-normed linear space

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ABSTRACT:

An exploratory study of advanced convergence structures in linear n-norm spaces is presented in this paper with specific emphasis on examining bounded b-linear functionals. Building upon the classical theory of convergence, we develop both l-convergence and l-Cauchy sequences in linear n-normed spaces. An important result of our research is the concept of ideal equivalent sequences for bounded b-linear functionals which provides a strong theoretical framework for analyzing the equivalence of sequences of bounded b-linear functionals using ideals as a basis of comparison. Thus, using aspects of ideals, we characterize limiting behaviors in multidimensional normed structures more accurately.

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1. Introduction

The development of convergence theory has greatly improved and added to the area of functional analysis, most notably via the introduction of Ideal Convergence (I-convergence) by Kostyrko et al. [1,2]. The new method of I-convergence builds upon the notion of statistical convergence [4,5,9] and provides a highly useful conceptual framework for studying sequences where classical limit points cannot be clearly defined. Many researchers have looked into statistical convergence and its properties in many different areas of mathematics [10-17].

In the realm of mathematics, studies have been conducted about n-normed spaces, an extension of normed spaces launched by Gähler [22] in multi-dimensional directions. The properties of these spaces have been stated in literature [23, 25], but more investigations require further investigation. Since n-norms define distance using multiple vectors, it makes determining the convergence of functionals in these spaces more complex than determining convergence in normed spaces.

This paper will look at the properties of bounded b-linear functionals that are both I-convergent and I-Cauchy. By using ideal filters, we will gain a better understanding of the nature of these functionals, including the concept of ideal equivalents for sequences

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2. Preliminaries

Definition 2.1 [7]: Allow X over the field to be linear space. The complex or real number field is K. that has $dimX \geq n$, such that n is apositive integer. on X an n-norm is defined as the areal valued function

$\|.,\ldots,.\|: X^n \to R$ if

(1) $\|x1, x2, \ldots, xn\| = 0$ if andonly if $x1, x2, \ldots, xn$ have a linear dependence

(2) $\|x1, x2, \ldots, xn\|$ Invariant under Permutations of $x_1, \ldots, x_n$

(3) $\|\alpha x1, x2, \ldots, xn\| = |\alpha| \, \|x1, x2, \ldots, xn\| \quad \forall \alpha \in K$.

(4) $\|x1 + y, x2, \ldots, xn\|, \leq \|y, x2, \ldots, xn\| + \|x1, x2, \ldots, xn\|$ hold $\forall\ x1, x2, \ldots, xn, y \in X$.

The pair $(X, \|.,\ldots,.\|)$ is said a linear n-normed space
The linear space n-normed over field K connected to the n-norm will be denoted by X throughout this paper.

Definition 2.2 [7]: let $\{a_n\}$ is asquence in X, then is called Converge to a in X, if

$\lim_{n\to\infty}\|a_n - a, h_{2,}, h_3, \ldots, h_n\| = 0$, for all $h_{2,}, h_3, \ldots, h_n \in X$, and is said that Caushy sequence if

$\lim_{n,k\to\infty}\|a_n - a_k, h_{2,}, h_3, \ldots, h_n\| = 0$,for each $h_{2,}, h_3, \ldots, h_n \in X$ If all of Cauchy sequences in X are convergent, the space is referred to as n-Banach space or complete space.

Definition 2.3 [22]: The open and closed balls following are defined in X:

$B_{\{e_2, e_3, \ldots, e_n\}}(a, \varepsilon) = \{x : \|x - a, e_2, e_3, \ldots, e_n\| < \varepsilon\}$, $x \in X$.

$B_{\{e_2, e_3, \ldots, e_n\}}[a, \varepsilon] = \{x : \|x - a, e_2, e_3, \ldots, e_n\| \leq \varepsilon\}, x \in X$.

Such that ε be a positive number and $e_2, e_3, \ldots, e_n, a \in X$

Definition 2.4 : **[22]** if E is a subset in X, then $\bar{E} = \{x \in X$ such that $\{x_k\} \in E$ with $\lim_{k\to\infty} x_k = x\}$ if $E = \bar{E}$ then the set E is considered closed.

Definition 2.5 [20]: suppose that W is a subspace in X and a fixed element is $b_{2,}, b_3, \ldots, b_n$ in X, $\langle b_i \rangle$ represent subspace of X produced b_i, forthe $i = 2, 3, \ldots, n,$ the function $\mathcal{F}$ is said a b-linear function such that $\mathcal{F}: W \times \langle b_2 \rangle \times \langle b_3 \rangle \times \ldots \times \langle b_n \rangle \to K$ if the conditions is satisfied for each $x, b \in W$ and $\alpha \in K$, :

$1 - \mathcal{F}(x + a, b_{2,}, b_3, \ldots, b_n) = \mathcal{F}(x, b_{2,}, b_3, \ldots, b_n) + \mathcal{F}(a, b_{2,}, b_3, \ldots, b_n)$

$2 - \mathcal{F}(\alpha x, b_{2,}, b_3, \ldots, b_n) = \alpha \mathcal{F}(x, b_{2,}, b_3, \ldots, b_n)$.

IF $m > 0$ is a real number, there is a bounded b-linear functional.,then

$$|\mathcal{F}(x, b_{2,}, b_3, \ldots, b_n| \leq m \, \|x, b_{2,}, b_3, \ldots, b_n\| \qquad \text{For each } x \in W.$$

The norm $\mathcal{F}$ of the bounded b-linear functional is defined by
$\|\mathcal{F}\| = \inf. \{m : m > 0;\ |\mathcal{F}(x, b_{2,}, b_3, \ldots, b_n| \leq m. \|x, b_{2,}, b_3, \ldots, b_n\| \ , \ for\ each\ x \in W\}$

Any one of the following equivalent formulas able to used to express the norm of T.

$1 - \|\mathcal{F}\| = \sup\{|\mathcal{F}(x, b_{2,}, b_3, \ldots, b_n)| : \|x, b_{2,}, b_3, \ldots, b_n\| \leq 1\}$

$2 - \|\mathcal{F}\| = \sup\{|\mathcal{F}(x, b_{2,}, b_3, \ldots, b_n)| : \|x, b_{2,}, b_3, \ldots, b_n\| = 1\}$

$3 - \|\mathcal{F}\| = \sup\left\{\frac{|\mathcal{F}(x, b_{2,}, b_3, \ldots, b_n)|}{\|x, b_{2,}, b_3, \ldots, b_n\|} : \|x, b_{2,}, b_3, \ldots, b_n\| \neq 0\right\}$

Moreover,it has

$$\left|\mathcal{F}(x, b_{2,}, b_3, \dots, b_n)\right| \leq \|\mathcal{F}\| \left\|x, b_{2,}, b_3, \dots, b_n\right\| \quad ,\forall\ x \in W$$

Regarding the aforementioned norm, assume that X_F^* defined on all Bounded b-linear functions in the Banach space, $X \times \langle b_2\rangle \times \langle b_3\rangle \times \dots \times \langle b_n\rangle$ [20,21] has been discussed in A few characteristics of the defined Bounded b-linear functional on $X \times \langle b_2\rangle \times \langle b_3\rangle \times \dots \times \langle b_n\rangle$

Definition 2.6 [25] Consider the non-trivial ideal $L \subset 2^N$ in N when each $\varepsilon > 0$ and $z \in X$ a sequence (x_n) in 2-normed Space X is said to L −converge x if for every set, $A(\varepsilon) = \{n_0 \in N;\ \| x_{n_0} - x, z\| \geq \varepsilon\}$ belongs to L. Also, we will provide a few instances of ideal and corresponding L −convergence.

I- in N the admissible ideal is l_f, its Converge is conceded with regular Converge if l_f is the family of every Finite subset of N[19].

II- A formula is $l_s = \{A \subset N: \delta(A) = 0\}$. Then l_s convergence yields statistical convergence, and l_s in [29] is the admissible ideal in N

3. Main results

In this section ,we study the statistical convergence, ideal convergence, ideal Cauchy, and statistical Cauchy of bounded b-linear functionals on $X \times \langle b_2\rangle \times \langle b_3\rangle \times \dots \dots \langle b_n\rangle$, Throughout the paper, I is taken to be an admissible ideal in N

Definition3.1: Let X_F^* define on $X \times \langle b_2\rangle \times \langle b_3\rangle \times \dots \dots \langle b_n\rangle$ is the space of bounded b-linear functionals, such that X be an n-normed space, and o a sequence $\{T_i\} \subset X_F^*$ It is said that is statistically Convergent to $T \in X_F^*$ if for each $\in > 0$ and for all $b_2, \dots, b_n \in X$ the set $\{i \in N: \|T_i - T\| \geq \varepsilon\}$ has a zero asymptotic density.

Here, we write $st - \lim_{i\to\infty} \|T_i - T\| = 0$, and then T is said to be the st_limit of $\{T_i\}$ in X_F^* .

Definition3.2: let X be an n-normed space and X_F^* be the space of bounded b-linear functionals on $X \times \langle b_2\rangle \times \langle b_3\rangle \times \dots \dots \langle b_n\rangle$, a sequence $\{T_i\}$ is said Statistically Cauchy if each $\in > 0$ and for all $b_2, \dots, b_n \in X$ which are linearly independent there exist $N = N(\varepsilon, b_2, \dots, b_n)$such that the set $\{i \in N: \|T_i - T_s\| \geq \varepsilon\}$ has asymptotic density zero.

Definition 3.3: A sequence $\{T_i\}$ in X_F^*, is said to be ideal convergent (I −convergence) to $T \in X_F^*$ when for all $\varepsilon >$ 0and for any set of linearly independent elements $b_2, \dots, b_n \in X$, the set $\{i \in N: \|T_i - T\| \geq \varepsilon\}$ belongs to I. Here, we write $I - \lim_{i\to\infty} T_i = T$

Definition 3.4: A sequence $\{T_i\}$ in X_F^* is said to be an ideal Cauchy sequence(I –cauchy) to if for all $\varepsilon > 0$ and linearly independent elements $b_2, \dots, b_n \in X$, there exists a number k $\in N$such that

$\{i \in N: \|T_i - T_k\| \geq \varepsilon\}$ belong to I .

Theorem 3.5 : in n-Banach space$X$ if $\{T_i\} \subseteq X_F^*$ then the statements that follow are equivalent :

- i- $I - \lim_{i\to\infty} \|T_i - T\| = 0$,for all $b_2, \dots, b_n \in X$.
- ii- $\{i \in N: \|T_i - T\| \geq \varepsilon\} \in I$ for all $b_2, \dots, b_n \in X$ and each $\varepsilon > 0$.
- iii- $\{i \in N: \|T_i - T\| < \varepsilon\} \in \mathcal{F}(I)$ for all $b_2, \dots, b_n \in X$ and each $\varepsilon > 0$.

Proof: (i)⟺(ii)

By the definition of ideal convergence in the context of normed structures, a sequence $\{k_i\}$ in R is said to be I-convergent to k if for every $\varepsilon > 0$,the set of indices $\{i \in N: |k_i - k| \geq \varepsilon\}$ belong to the ideal I.

In our setting , let $k_i = \|T_i - T\|$ and $k = 0$. Since the norm $\|.\|$ of the bounded b-linear functional is defined as the supremum over the n-normed space , we have :

$I - \lim_{i\to\infty} \|T_i - T\| = 0 \iff \{i \in N: \|T_i - T\| - 0 \geq \varepsilon\} \in I$,which is precisely :

$\{i \in N: \|T_i - T\| \geq \varepsilon\} \in I, \quad \forall \varepsilon > 0, \forall\ b_2, \dots, b_n \in X.$

This proves the equivalence between $(i)$and(ii).

$(ii) \Leftrightarrow (iii)$:

Let $A(\epsilon) = \{i \in N: \|T_i - T\| \geq \varepsilon\}$ be the set defined in (ii).

Recall the definition of the dual filter $\mathcal{F}(I)$ associated with the ideal I, which is given by :

$$\mathcal{F}(I) = \{M \subseteq N: N\backslash M \in I\}$$

Consider the set in (iii), let call it $B(\epsilon) = \{i \in N: \|T_i - T\| < \varepsilon\}$. Observe that $B(\epsilon)$ is the absolute complement of $A(\epsilon)$ in the set of natural numbers N:$B(\epsilon) = \mathrm{N}\backslash \mathrm{A}(\epsilon)$.by the fundamental property of filters:$A(\epsilon) \in \mathrm{I} \Leftrightarrow (\mathrm{N}\backslash \mathrm{A}(\epsilon) \in \mathcal{F}(I) \Leftrightarrow B(\epsilon) \in \mathcal{F}(I)$. Thus,$\{i \in N: \|T_i - T\| \geq \varepsilon\} \in I$ if and only if $\{i \in N: \|T_i - T\| < \varepsilon\} \in \mathcal{F}(I)$. This completes the proof of the theorem.

Example 3.6: Let $X = R^n$ be a linear n −normed space. We define the n −norm on X using the absolute value of the determinant as follows:

$$\|x_1, x_2, \dots, x_n\| = abs\left(\begin{vmatrix} x_{11} & x_{12} \dots. & x_{1n} \\ x_{21} & x_{22} \dots & x_{2n} \\ \vdots & \vdots & \\ x_{n1} & x_{n2} \dots & x_{nn} \end{vmatrix}\right)$$

Let $b2, \dots, b_n$ be fixed linearly independent elements in X. We define a sequence of b-linear functionals $\{T_k\}$ where $T_k: X \times \langle b_2 \rangle \times \langle b_3 \rangle \times \dots \dots \langle b_n \rangle \to R$ is given by:

$$T_k(x_1, b_2, \dots, b_n) = \begin{Bmatrix} k.\|x_1, b_2, \dots, b_n\|, if\ k = m^2, m \in N \\ \frac{1}{K}.\|x_1, b_2, \dots, b_n\|, \quad if \quad k \neq m^2 \end{Bmatrix}$$

Consider the limit of the norm $\|T_k\|$ as $k \to \infty$. For the subsequence where k is a perfect square $(k = 1{,}4.9.16, \dots)$

We have $\|T_k\| \to \infty$. Therefore, the sequence $\{T_k\}$ dose not converge to the zero functional T_0 in the ordenary n-normed topology.

Now let $I = I_\delta$ be the admissible ideal of subsets of N with natural density zero.for any $\epsilon > 0$, let us examine the set :

$A(\epsilon) = \{k \in N: \|T_k - T_0\| \geq \varepsilon\}$,this set $A(\varepsilon)$ is containedin the union of a finite set (where $\frac{1}{k} \geq \epsilon$) and the set of perfect squares $\{m^2: m \in N\}$. Since the natural density of perfect squares is :$\delta\{m^2: m \in N\} = \lim_{n\to\infty} \frac{\sqrt{n}}{n} = 0$. It follows that $A(\epsilon) \in I$.

According to defintion 3.3 the sequence $\{T_k\}$ is I-convergent to zero function T_0.

Theorem 3.7: In X_F^* let T_i , Y_i be two Ideal Cauchy sequences; consequently, so are $\{T_i + Y_i\}$, $\{\lambda T_i\}$, when λ is a constant.

Proof: if the ideal Cauchy sequences in X_F^* are $\{T_i\}$ and $\{Y_i\}$ are then for each $\varepsilon > 0$, $b_2, \dots, b_n \in X$ there exist $N = N(\varepsilon, b_2, \dots, b_n)$ Suchthat($A1 = \left\{i \in N: \|T_i - T_m\| \geq \frac{\varepsilon}{2}\right\} \in I$) and $A2 = \left\{i \in N: \|Y_i - Y_m\| \geq \frac{\varepsilon}{2}\right\} \in I$)

Let's $A = \{i : \|(T_i + Y_i) - (T_m + Y_m)\| \geq \varepsilon,\ i \in N\}$.
then, the inclusion $A \subset A1 \cup A2$ holds, and the following assertion is made:

If $\lambda = 0$, it is trivial. let $(\lambda \neq 0) \in R$, since $I - \lim_{i\to\infty}\|T_i - T\| = 0$,then ,for every $\varepsilon > 0$, and $b_2, \dots, b_n \in X$ we have

$\left\{i \in N: \|T_i - T\| \geq \frac{\varepsilon}{|\lambda|}\right\} \in I$, given that X_F^* is a Banach space, then

$$\{i \in N: \|\lambda T_i - \lambda T\| \geq \varepsilon\} \subseteq \left\{i \in N: \|T_i - T\| \geq \frac{\varepsilon}{|\lambda|}\right\} \qquad (2)$$

In a set, (2) The right side belongs to I, as can be readily confirmed. By definition, the collection on (2)the left side is perfect and belongs to I, too. This suggests that. $I-\lim_{i\to\infty}\| \lambda T_i - \lambda T\| = 0$,for all $\lambda \in R$, and every $b_2,\dots,b_n \in X$.

Theorem3.8: Allow X to an n-Banach space. $\{T_n\} \in X_F^*$ the admissible ideal of N is I , then $I-limt$ of $\{T_n\}$ is unique if the sequence $\{T_n\}$ is I-convergent.

Proof: Assume that I-Lim $_{n\to\infty}\|T_n - T_1\| = 0$, and I-Lim $_{n\to\infty}\|T_n - T_2\| = 0$, such that $T_2 \neq T_1$.

Choose $\varepsilon \in \left(0, \frac{\|T_1-T_2\|}{2}\right)$ (1)

We have $A(\varepsilon) = \{ n \in N : \|T_n - T_1\| \geq \varepsilon\} \in I$,

$B(\varepsilon) = \{ n \in N : \|T_n - T_2\| \geq \varepsilon\} \in I.$

Then, based on the definition and assumption of the filter connected to the ideal I, we have $A^c(\varepsilon), B^c(\varepsilon) \in \mathcal{F}(I)$. However, the set $A^c(\varepsilon) \cap B^c(\varepsilon) \in \mathcal{F}(I)$, too. thus, $j \in N$ such that $\|T_j - T_1\| < \varepsilon$ and $\|T_j - T_2\| < \varepsilon$.

From this for all $b_2,\dots,b_n \in X$ We've, $\|T_1 - T_2\| < \|T_j - T_1\| + \|T_j - T_2\| < 2\varepsilon$.

which contradicts (1) .

Theorem 3.9: A squence $T_{i,}$ in n-banach space X_F^* is ideal convergent if it convergent .

Proof: let T_i be a sequence in n-banach space X_F^* that converges to $T \in X_F^*$. A ccording to the definition of convergence , for all $\varepsilon > 0$, and all $\{ b_2,\dots,b_n\} \in \{X\}$, There exist a positive number $k = k(\varepsilon)$ such that

$$\|T_i - T, b_2,\dots,b_n\| < \varepsilon \qquad \forall i \geq k$$

Now, let us define the set $A(\epsilon)$ as the collection of indices that do not satigfy this inequality Since

$A(\epsilon) = \{(i \in N: \|T_i - T, b_2,\dots,b_n\| \geq \epsilon\},$

Based on the condition of classical convergence, the set $A(\epsilon)$ is necessarily a subset of the finite set $\{1,2,\dots N\}$. In any n-Banach space, and since the ideal I under consideration is an admissible ideal, it must contain all finite subsets of N by its structural properties.

Consequently, $A(\epsilon) \in I$ in I for every $\epsilon > 0$. By referring to the definition of I -convergence (Definition 3.3), this condition precisely fulfills the requirement for the sequence $\{T_k\}$ to be I -convergent to T. This concludes the proof.

4. Ideal equivalent sequence in a bounded b-linear functional

Definition 4.1: In X_F^* b − linear n-normed space, two ideal Cauchy sequences $\{T_k\},\{Y_k\}$ are called ideal equivalents if in each $\mathbb{U}$ the Neighborhood of 0, the integer $N(\mathbb{U})$ exists so $\{k \in N: k \geq N(\mathbb{U})\ and\ \{T_k - Y_k \notin \mathbb{U}\} \in I$. It is equivalent $I-\lim_{k\to\infty}\|T_k - Y_k, b_2,\dots,b_n\|$ for each ($b_2,\dots,b_n \in X$). If $\{T_k\} and \{Y_k\}$ ideal. equivalent then write $\{T_k\} \approx^I \{Y_k\}$.

Theorem 4.2 : In X_F^* when $\{a_k\} \approx^I \{T_k\}$,$\{ b_k\} \approx^I \{h_k\}$then $\{a_k + b_k\} \approx^I \{ T_k + h_k\}$ and $\{\propto a_k\} \approx^I \{\propto T_k\}$ with $\propto \in R$.

Proof: since $\{a_k\} \approx^I \{T_k\}$ and$\{ b_k\} \approx^I \{h_k\}$ for all($\varepsilon > 0$) and ($b_2,\dots,b_n \in X$) ,$A1, A2 \in I$ suchthat

$$A1 = \{ a \in N: \|a_k - T_k, b_2,\dots,b_n\| \geq \frac{\varepsilon}{2} \}$$

$$A2 = \{a \in N: \|b_k - h_k, b_2,\dots,b_n\| \geq \frac{\varepsilon}{2}\}$$

Let $A = A(\varepsilon) = \{a \in N: \|(a_k + b_k) - (T_k + h_k), b_2,\dots,b_n\| \geq \varepsilon \}$

Since the proof of $A \subset A1 \cup A2$ and $\{\propto a_k\} \approx^I \{\propto T_k\}$ are routine, we leave out the specifics.

$\hat{X}$ is a representation of the collection of every ideal equivalence class for a Cauchy sequence in X. let $\hat{T}, \hat{h}$, etc, represent the components of $\hat{X}$. Define a scalar multiplication and addition on $\hat{X}$ in the following way:

(i) $\hat{T} + \hat{h}$ = set of every ideal sequences that are equivalents to $\{\hat{T}_k + \hat{h}_k\}$,Where $\{T_k\} \in \hat{T}$ and $\{h_k\} \in \hat{h}$.

(ii) $\alpha\hat{T}$ = all sequences that are ideal equivalent to $\{\alpha T_k\}$, where $T_k \in \hat{T}$.

Since these two operations are independent of the selection of components from $\hat{T}$ *and* $\hat{h}$ they are well-defined by the above theorem. The two operations are present in X, which is a linear space.

Remark 4.3: The X_F^* is a b-linear n-normed space. If it has an Ideal Cauchy Sequence $\{T_k\}$, and $(b_2, \dots, b_n \in X)$ then $I - \lim_{k\to\infty}\|T_k, b_2, \dots, b_n\|$ exist.

Proof:To establish the existence of the ideal limit for the sequence of norms, we begin by considering the sequence $\{T_k\}$ as an ideal Cauchy sequence in the b-linear n-normed space X_F^* .By applying the reverse triangle inequality for n-norms, we observe that for any $k, p \in N$, the scalar difference satisfies

$\big|\|T_k, b_2, \dots, b_n\| - \|T_p, b_2, \dots, b_n\|\big| \le \|T_k - T_p, b_2, \dots, b_n\|$. Given that$\{T_k\}$ is I-Cauchy, for every $\epsilon > 0$, the set $A(\epsilon) = \{(k,p) \in N \times N: \|T_k - T_p, b_2, \dots, b_n\| \ge \epsilon\}$,belong to the ideal I. define the set of indices for the scalar sequence as $B(\epsilon) = \{(k,p) \in N \times N: \big|\|T_k, b_2, \dots, b_n\| - \|T_p, b_2, \dots, b_n\|\big| \ge \epsilon\}$, it follow inequality that $B(\epsilon) \subseteq A(\epsilon)$. Due to the hereditary property of the admissible ideal I, the containment $B(\epsilon) \subseteq A(\epsilon)$ implies that $B(\epsilon) \in I$, thereby confirming that the sequence of norms $\|T_k, b_2, \dots, b_n\|$ is itself an ideal Cauchy sequence of real scalars. Finally, leveraging the fact that every ideal Cauchy sequence in a complete scalar field is ideally convergent, we conclude that $I - \lim_{k\to\infty}\|T_k, b_2, \dots, b_n\|$ exists and is a well-defined value in R.

Theorem 4.4: Let $\{T_{1(k)}\}$ and $\{T_{2(k)}\}$ be Ideal Cauchy sequences in X_F^* n-normed Linear space and let them be ideal equivalents, then

$I - \lim_{k\to\infty}\|T_{1(k)}, b_2, \dots, b_n\| = I - \lim_{k\to\infty}\|T_{2(k)}, b_2, \dots, b_n\|$ For each $b_2, \dots, b_n \in X$.

Proof: since

$$\lim_{k\to\infty}\|T_{1(k)}, b_2, \dots, b_n, \| = \lim_{k\to\infty}\|T_{1(k)} - T_{2(k.)} + T_{2(k.)}, b_2, \dots, b_n\|$$
$$\le \|T_{1(k)} - T_{2(k)}, b_2, \dots, b_n\| + \|T_{2(k)}, b_2, \dots, b_n\|$$

We have

$$\|T_{1(k)}, b_2, \dots, b_n\| - \|T_{2(k)}, b_2, \dots, b_n\| \le \|T_{1(k)} - T_{2(k)}, b_2, \dots, b_n\|$$

and

$$\|T_{2(k)}, b_2, \dots, b_n\| - \|T_{1(k)}, b_2, \dots, b_n\| \le \|T_{2(k)} - T_{1(k)}, b_2, \dots, b_n\|$$

As a result, we get the following

$$\{k \in N: \big|\|T_{2(k)}, b_2, \dots, b_n\| - \|T_{1(k)}, b_2, \dots, b_n\|\big| \ge \varepsilon\} \subseteq \{k \in N: \|T_{2(k)} - T_{1(k)}, b_2, \dots, b_n\| \ge \varepsilon\}.$$

Since $\{T_{1(k)}\}$ and $\{T_{2(k)}\}$ are ideal equivalent

$$\{k \in N: \|T_{2(k)} - T_{1(k)}, b_2, \dots, b_n\| \ge \varepsilon\} \in I$$

And therefore, the proof is finished with the following.

$$\{k \in N, : \big|\|T_{2(k)}, b_2, \dots, b_n\| - \|T_{1(k)}, b_2, \dots, b_n\|\big| \ge \varepsilon\} \in I .$$

5. Conclusions

This paper explores and extend the notion of convergence with in the space of bounded linear operators acting on n-normed linear spaces. We primarily investigate the properties of ideal convergence(I-convergence)and Cauchy sequences ,Additionally, the study introduces and characterizes I-equivalent sequences in this specific functional setting .Our results delineate the structural relationships between these convergence types and provide necessary and sufficient conditions for their equivalence.

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