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Fuzzy α-Translations of KUS-algebras

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Abstract. Fuzzy α-translations, (normalized, maximal) fuzzy extensions and fuzzy β-multiplications of fuzzy KUS-subalgebras of KUS-algebras are discussed. Relations among fuzzy α-translations and fuzzy extensions of fuzzy KUS-ideals are investigated.

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1. Introduction

Several authors ([1],[2],[3]) have introduced of BCK-algebras as a generalization of the concept of set-theoretic difference and propositional calculus and studied some important properties. The concept of a fuzzy set, was introduced by L.A. Zadeh [4]. In ([5],[6],[7],[8]) , they applied the concept of fuzzy set to BCK/BCI-algebras and gave some of its properties. Areej Tawfeeq Hameed, [9] introduced KUS-ideals in KUS-algebras and introduced the notions fuzzy KUS-subalgebras, fuzzy KUS-ideals of KUS-algebras and investigated relations among them . In this paper, we discuss fuzzy α-translation, (normalized, maximal)fuzzy S-extension of fuzzy KUS-subalgebras in KUS-algebra. We discuss fuzzy α-translation and fuzzy extension of fuzzy KUS-ideals in KUS-algebra.

2. Preliminaries

Now, we introduced the concept of algebraic structure of KUS-algebra and we give some results
and theorems of it .

Definition 2.1([9]). Let $(X;*,0)$ be an algebra of type (2,0) with a single binary operation $(*)$. X is
called a KUS-algebra if it satisfies the following identities: for any x, y, z∈X ,

(kus_1) : $(z*y)*(z*x) = y*x$,
(kus_2) : $0*x = x$,
(kus_3) : $x*x = 0$,
(kus_4) : $x*(y*z) = y*(x*z)$.

In X we can define a binary relation $(\leq)$ by : $x \leq y$ if and only if $y*x = 0$.

In what follows, let $(X;*,0)$ denote a KUS-algebra unless otherwise specified. For brevity we also call X a KUS-algebra.

Lemma 2.2 ([9]). In any KUS-algebra $(X;*,0)$, the following properties hold: for all x, y, z∈X;

a) $x*y=0$ and $y*x=0$ imply $x=y$,
b) $y*[(y*z)*z] = 0$,
c) $x \leq y$ implies that $y*z \leq x*z$,
d) $x \leq y$ implies that $z*x \leq z*y$,
e) $x \leq y$ and $y \leq z$ imply $x \leq z$,
f) $x*y \leq z$ implies that $z*y \leq x$.

Definition 2.3([9]). Let X be a KUS-algebra and let S be a nonempty subset of X. S is called a KUS-subalgebra of X if $x*y \in S$ whenever x , y ∈ S.

Definition 2.4([9]). A nonempty subset I of a KUS-algebra X is called a KUS-ideal of X if it satisfies: for x , y, z ∈ X,

$(Ikus_1)$ $(0 \in I)$,
$(Ikus_2)$ $(z*y) \in I$ and $(y*x) \in I$ imply $(z*x) \in I$.

Definition 2.5([4]). Let X be a nonempty set, a fuzzy subset μ in X is a function $\mu : X \to [0,1]$.

Proposition 2.6([9]). Every KUS-ideal of KUS-algebra X is a KUS-subalgebra of X.

Definition 2.7([9]). Let X be a KUS-algebra, a fuzzy subset μ in X is called a fuzzy KUS-subalgebra of X if for all x , y ∈ X , $\mu(x * y) \geq min\{\mu(x), \mu(y)\}$.

Definition 2.8([9]). Let X be a KUS-algebra , a fuzzy subset μ in X is called a fuzzy KUS-ideal of X if it satisfies the following conditions: , for all x , y, z ∈ X ,

($Fkus_1$) $\mu(0) \geq \mu(x)$,

($Fkus_2$) $\mu(z * x) \geq min\{\mu(z * y), \mu(y * x)\}$.

Proposition 2.9([9]). Every fuzzy KUS-ideal of KUS-algebra X is a fuzzy KUS-subalgebra of X .

3. Fuzzy α-translations of fuzzy KUS-subalgebras .

We study the relations among fuzzy α-translation,(normalized, maximal) fuzzy S-extension of KUS-subalgebras of KUS-algebra X.

In what follows let (X;∗,0) denote a KUS-algebra, and for any fuzzy set μ of X, we denote $T = 1 - \sup\{\mu(x) \mid x \in X\}$ unless otherwise specified.

Definition 3.1([1]). Let X be a nonempty set and μ be a fuzzy subset of X and let $\alpha \in [0,T]$. A mapping $\mu_\alpha^T : X \to [0,1]$ is called a **fuzzy subset α-translation** of μ if it satisfies: $\mu_\alpha^T(x) = \mu(x) + \alpha$, for all x ∈ X.

Theorem 3.2. Let X be a KUS-algebra and μ be a fuzzy KUS-subalgebra of X and $\alpha \in [0,T]$. Then the fuzzy subset α-translation μ_α^T of μ is a fuzzy KUS-subalgebra of X .

Proof: Assume μ be a fuzzy KUS-subalgebra of X and $\alpha \in [0,T]$, let x, y ∈ X. Then $\mu_\alpha^T(x * y) = \mu(x * y) + \alpha \geq min\{\mu(x), \mu(y)\} + \alpha = min\{\mu(x) + \alpha, \mu(y) + \alpha\} = min\{\mu_\alpha^T(x), \mu_\alpha^T(y)\}$.

Hence μ_α^T is a fuzzy KUS-subalgebra α-translation of X. ⌂

Theorem 3.3. Let X be a KUS-algebra and μ be a fuzzy KUS-subalgebra of X such that the fuzzy subset α-translation μ_α^T of μ is a fuzzy KUS-subalgebra of X for some $\alpha \in [0,T]$. Then μ is a fuzzy KUS-subalgebra of X .

Proof: Assume μ_α^T be a fuzzy KUS-subalgebra α-translation of X for some $\alpha \in [0,T]$. Let x, y ∈ X, then $\mu(x * y) + \alpha = \mu_\alpha^T(x * y) \geq min\{\mu_\alpha^T(x), \mu_\alpha^T(y)\} = min\{\mu(x) + \alpha, \mu(y) + \alpha\} = min\{\mu(x), \mu(y)\} + \alpha$ and so $\mu(x * y) \geq min\{\mu(x), \mu(y)\}$. Hence μ is fuzzy KUS-subalgebra of X .⌂

Definition 3.4([8]). Let μ_1 and μ_2 be fuzzy subsets of a set X. If $\mu_1(x) \leq \mu_2(x)$ for all x ∈ X, then we say that μ_2 is **a fuzzy extension of** μ_1.

Definition 3.5. Let X be a KUS-algebra , μ_1 and μ_2 be fuzzy subsets of X. Then μ_2 is called **a fuzzy S-extension** of μ_1 if the following assertions are valid:

(S_i) μ_2 is a fuzzy extension of μ_1.

(S_{ii}) If μ_1 is a fuzzy KUS-subalgebra of X, then μ_2 is a fuzzy KUS-subalgebra of X.

By means of the definition of fuzzy α-translation, we know that $\mu_\alpha^T(x) \geq \mu(x)$ for all x ∈X.

Hence we have the following proposition.

Proposition 3.6. Let μ be a fuzzy KUS-subalgebra of a KUS-algebra X and $\alpha \in [0,T]$. Then the fuzzy subset α-translation μ_α^T of μ is a fuzzy S-extension of μ.

Proof: Straightforward. ⌂

In general, the converse of proposition (3.6) is not true as seen in the following example.

Example 3.7. Consider a KUS-algebra X = {0, a, b, c} with the following Cayley table:

∗	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

By [9]. Define a fuzzy subset μ of X by:

X	0	a	b	c
μ	0.8	0.5	0.6	0.5

Then μ is a fuzzy KUS-subalgebra of X. Let ν be a fuzzy subset of X given by

X	0	a	b	c
ν	0.94	0.66	0.78	0.66

Then ν is a fuzzy S-extension of μ. But ν is not the fuzzy subset α-translation μ_α^T of μ for all α ∈ [0,T].

Proposition 3.8. the intersection of fuzzy S- extensions of a fuzzy subset μ of X is a fuzzy S-extension of μ.

Proof: Let { μ_i | i ∈ Λ.} be a family of fuzzy KUS-subalgebras of KUS- algebra X , then for any x, y ∈ X , i ∈Λ, $\left(\bigcap_{i\in\wedge}\mu_i\right)(x*y) =$

$$\inf\left(\mu_i\,(x*y)\right)$$

$$\geq \inf\left(\min\left\{\mu_i\,(x),\ \mu_i\,(y)\right\}\right)$$

$$= \min\{\inf\left(\mu_i\,(x)\right), \inf\left(\mu_i\,(y)\right)\}$$

$$= min\,\{(\bigcap_{i\in\wedge}\mu_i\,)(x)), (\bigcap_{i\in\wedge}\mu_i\ (y)\}\ .\ ⌂$$

Clearly, the union of fuzzy S-extensions of a fuzzy subset μ of X. μ is not a fuzzy S-extension of μ as seen in the following example.

Example 3.9. Let X = {0, a, b, c} be a KUS-algebra which is given in Example 3.7 , and consider a fuzzy subalgebra μ of X that is defined in Example 3.7 .Let ν and δ be fuzzy subsets of X given by

X	0	a	b	c
ν	0.9	0.6	0.6	0.8
δ	0.9	0.6	0.7	0.6

Then ν and δ are fuzzy S-extensions of μ. But the union ν ∪ δ is not a fuzzy S-extension of μ since (ν ∪ δ)(c ∗ b) = 0.6 < 0.7 = min{(ν ∪ δ)(c), (ν ∪ δ)(b)}.

Definition 3.10. For a fuzzy subset μ of a KUS-algebra X, α ∈ [0,T] and t ∈ [0, 1] with t ≥ α, let U_α (μ; t) := {x ∈ X | μ(x) ≥ t − α}.

If μ is a fuzzy KUS-subalgebra of X, then it is clear that U_α (μ; t) is a KUS-subalgebra of X, for all t∈Im(μ) with t ≥ α. But if we do not give a condition that μ is a fuzzy KUS-subalgebra of X, then U_α (μ; t) is not a KUS-subalgebra of X as seen in the following example.

Example 3.11. Let X = {0, a, b, c} be a KUS-algebra which is given in Example (3.9). Define a fuzzy subset λ of X by

X	0	a	b	c
λ	0.7	0.6	0.4	0.3

Then λ is not a fuzzy KUS-subalgebra of X since λ(a∗b) = λ(c) = 0.3 < 0.4 = min{λ(a), λ(b)}. For α = 0.1 and t = 0.5, we obtain U_α (λ; t) = {0, a, b} which is not a KUS-subalgebra of X since a ∗ b = c ∉ U_α (λ; t).

Proposition 3.12. Let μ be a fuzzy subset of a KUS-algebra X and α ∈ [0,T]. Then the fuzzy subset α-translation μ_α^T of μ is a fuzzy KUS-subalgebra of X if and only if U_α (μ; t) is a KUS-subalgebra of X for all t ∈ Im(μ) with t ≥α.

Proof: Necessity is clear. To prove the sufficiency, assume that there exist $x, y \in X$, $\gamma \in [0, 1]$ with $\gamma \geq \alpha$ such that $\mu_\alpha^T(x * y) < \gamma \leq min\{\mu_\alpha^T(x), \mu_\alpha^T(y)\}$. Then $\mu(x) \geq \gamma - \alpha$ and $\mu(y) \geq \gamma - \alpha$, but $\mu(x * y) < \gamma - \alpha$. This shows that $x, y \in U_\alpha(\mu; \gamma)$ and $x * y \notin U_\alpha(\mu; \gamma)$. This is a contradiction, and so $\mu_\alpha^T(x * y) \geq min\{\mu_\alpha^T(x), \mu_\alpha^T(y)\}$, for all $x, y \in X$. Hence μ_α^T is fuzzy KUS-subalgebra α-translation of X. ⌂

Proposition 3.13. Let μ be a fuzzy KUS-subalgebra of KUS- algebra X and α, $\lambda \in [0,T]$. If $\alpha \geq \lambda$, then the fuzzy KUS-subalgebra α-translation μ_α^T of μ is a fuzzy S-extension of the fuzzy KUS-subalgebra λ-translation μ_λ^T of μ.

Proof: Straightforward. ⌂

For every fuzzy KUS-subalgebra μ of a KUS- algebra X and $\lambda \in [0,T]$, the fuzzy subset λ-translation μ_λ^T of μ is a fuzzy KUS-subalgebra of X. If ν is a fuzzy S-extension of μ_λ^T, then there exists $\alpha \in [0,T]$ such that $\alpha \geq \lambda$ and $\nu(x) \geq \mu_\alpha^T(x)$ for all $x \in X$.

Proposition 3.14. Let μ be a fuzzy KUS-subalgebra of a KUS-algebra X and $\lambda \in [0,T]$. For every fuzzy S-extension ν of the fuzzy KUS-subalgebra λ-translation μ_λ^T of μ, there exists $\alpha \in [0,T]$ such that $\alpha \geq \lambda$ and ν is a fuzzy S-extension of the fuzzy KUS-subalgebra α-translation μ_α^T of μ.

Proof: Straightforward. ⌂

The following example illustrates proposition (3.14).

Example 3.15. Consider a KUS-algebra X = {0, 1, 2, 3} with the following Cayley table:

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

By [9].Define a fuzzy subset μ of X by:

X	0	1	2	3
μ	0.6	0.6	0.2	0.2

Then μ is a fuzzy KUS-subalgebra of X and T=0.3 . If we take λ = 0.2, then the fuzzy KUS-subalgebra λ-translation μ_λ^T of μ is given by :

X	0	1	2	3
μ_λ^T	0.8	0.8	0.4	0.4

Let ν be a fuzzy subset of X defined by:

X	0	1	2	3
ν	0.94	0.84	0.84	0.86

Then ν is clearly a fuzzy KUS-subalgebra of X which is fuzzy extension of μ_λ^T and hence ν is a fuzzy S-extension of fuzzy subset λ-translation μ_λ^T of μ. But ν is not a fuzzy KUS-subalgebra α-translation μ_α^T of μ for all $\alpha \in [0,T]$. Take α = 0.23, then α = 0.23 > 0.2 = λ , and the fuzzy KUS-subalgebra α-translation μ_α^T of μ is given as follows:

X	0	1	2	3
μ_α^T	0.8	0.83	0.43	0.43

Note that $\nu(x) \geq \mu_\alpha^T(x)$ for all $x \in X$, and hence ν is a fuzzy S-extension of the fuzzy KUS-subalgebra α-translation μ_α^T of μ.

Definition 3.16. A fuzzy S-extension ν of a fuzzy KUS-subalgebra μ in a KUS-algebra X is said to be **normalized** if there exists $x_0 \in X$ such that $\nu(x_0) = 1$. Let μ be a fuzzy KUS-subalgebra of X. A fuzzy subset ν of X is called a maximal fuzzy S-extension of μ if it satisfies:

(M_i) ν is a fuzzy S-extension of μ,
(M_{ii}) there does not exist another fuzzy KUS-subalgebras of a KUS-algebra X which is a fuzzy extension of ν.

Example 3.17. Let X = {0, a, b, c} be a KUS-algebra which is given in Example 3.7.Let μ and ν be fuzzy subsets of X which are defined by $\mu(x) = \frac{1}{5}$ and $\nu(x) = 1$ for all $x \in X$. Clearly μ and ν are fuzzy KUS-subalgebras of X. It is easy to verify that ν is a maximal fuzzy S-extension of μ.

Proposition 3.18. If a fuzzy subset ν of a KUS-algebra X is a normalized fuzzy S-extension of a fuzzy KUS-subalgebra μ of X, then $\nu(0) = 1$.
Proof: It is clear because $\nu(0) \geq \nu(x)$ for all $x \in X$. ⌂

Proposition 3.19. Let μ be a fuzzy KUS-subalgebra of a KUS-algebra X. Then every maximal fuzzy S-extension of μ is normalized.
Proof: This follows from the definitions of the maximal and normalized fuzzy S-extensions.⌂

4. Fuzzy α-translations of fuzzy KUS-ideals .

We study the relations among fuzzy α-translation and fuzzy extension of KUS-ideals of KUS-algebra X.

Theorem 4.1. Let μ is a fuzzy KUS-ideal of a KUS-algebra X , then the fuzzy subset α-translation μ_α^T of μ is a fuzzy KUS-ideal of X, for all $\alpha \in [0,T]$.
Proof: Assume μ be a fuzzy KUS-ideal of X and let $\alpha \in [0,T]$. For all $x, y, z \in X$ and $\mu(0) \geq \mu(x)$. Then $\mu_\alpha^T(0) = \mu(0) + \alpha$

$$\geq \mu(x) + \alpha = \mu_\alpha^T(x). \text{ and } \mu_\alpha^T(z * x)$$
$$= \mu(z * x) + \alpha \geq min\{\mu(z * y), \mu(y * x)\} + \alpha$$
$$= min\{\mu(z * y) + \alpha, \mu(y * x) + \alpha\}$$
$$= min\{\mu_\alpha^T(z * y), \mu_\alpha^T(y * x)\}.$$ Hence μ_α^T is a fuzzy KUS-ideal α-translation of X .⌂

Theorem 4.2. Let μ be a fuzzy subset of KUS-algebra X such that the fuzzy subset α-translation μ_α^T of μ is a fuzzy KUS-ideal of X for some $\alpha \in [0,T]$. Then μ is a fuzzy KUS-ideal of X.
Proof: Assume μ_α^T is a fuzzy KUS-ideal α-translation of X for some $\alpha \in [0,T]$. Let $x, y, z \in X$, we have $\mu(0) + \alpha = \mu_\alpha^T(0) \geq \mu_\alpha^T(x)$
$= \mu(x) + \alpha$. So $\mu(0) \geq \mu(x)$ and $\mu(z * x) + \alpha$
$= \mu_\alpha^T(z * x) \geq min\{\mu_\alpha^T(z * y), \mu_\alpha^T(y * x)\} = min\{\mu(z * y) + \alpha, \mu(y * x) + \alpha\} = min\{\mu(z * y), \mu(y * x)\} + \alpha$ and so $\mu(z * x) \geq min\{\mu(z * y), \mu(y * x)\}$. Hence μ is a fuzzy KUS-ideal of X .⌂

Definition 4.3. Let μ_1 and μ_2 be fuzzy subsets of a KUS-algebra X. Then μ_2 is called a fuzzy extension KUS-ideal of μ_1 if the following assertions are valid:

(I_i) μ_2 is a fuzzy extension of μ_1.
(I_{ii}) If μ_1 is a fuzzy KUS-ideal of X, then μ_2 is a fuzzy KUS-ideal of X.

Proposition 4.4. Let μ be a fuzzy KUS-ideal of X and let $\alpha, \gamma \in [0,T]$. If $\alpha \geq \gamma$, then the fuzzy subset α-translation μ_α^T of μ is a fuzzy extension KUS-ideal of the fuzzy KUS-ideal γ-translation μ_γ^T of μ.
Proof: Straightforward. ⌂

For every fuzzy KUS-ideal μ of X and $\gamma \in [0,T]$, the fuzzy subset γ-translation μ_γ^T of μ is a fuzzy KUS-ideal γ-translation of X. If ν is a fuzzy extension KUS-ideal of μ_γ^T, then there exists $\alpha \in [0,T]$ such that $\alpha \geq \gamma$ and $\nu(x) \geq \mu_\alpha^T(x)$ for all $x \in X$.

Proposition 4.5. Let μ be a fuzzy KUS-ideal of a KUS-algebra X and γ ∈ [0,T]. For every fuzzy extension KUS-ideal ν of the fuzzy KUS-ideal γ-translation μ_γ^T of μ, there exists α ∈ [0,T] such that α ≥ γ and ν is a fuzzy extension KUS-ideal of the fuzzy KUS-ideal α-translation μ_α^T of μ.

Proof: Straightforward. ⌂

The following example illustrates proposition (4.5).

Example 4.6. Let X = {0, 1,2} in which (∗) be give by:

∗	0	1	2
0	0	1	2
1	1	0	1
2	2	2	0

Then (X;∗,0) is a KUS-algebra by [9].
Define a fuzzy subset μ of X by:

X	0	1	2
μ	0.8	0.7	0.6

Then μ is a fuzzy KUS-ideal of X and T = 0.2. If we take γ = 0.12, then the fuzzy KUS-ideal γ-translation μ_γ^T of μ is given by :

X	0	1	2
μ_γ^T	0.92	0.82	0.72

Let ν be a fuzzy subset of X defined by:

X	0	1	2
ν	0.98	0.89	0.81

Then ν is clearly a fuzzy extension KUS-ideal of the fuzzy KUS-ideal γ-translation μ_γ^T of μ .

But ν is not a fuzzy KUS-ideal α-translation μ_α^T of μ for all α ∈[0,T]. Take α = 0.17, then α = 0.17 > 0.12 = γ , and the fuzzy KUS-ideal α-translation μ_α^T of μ is given as follows:

X	0	1	2
μ_α^T	0.97	0.87	0.77

Note that ν(x) ≥ μ_α^T (x) for all x ∈ X, and hence ν is a fuzzy extension KUS-ideal of the fuzzy KUS-ideal α-translation μ_α^T of μ.

Proposition 4.7. Let μ be a fuzzy KUS-ideal of a KUS-algebra X and α∈ [0,T]. Then the fuzzy subset α-translation μ_α^T of μ is a fuzzy extension KUS-ideal of μ.

Proof: Straightforward. ⌂

A fuzzy extension KUS-ideal of a fuzzy KUS-ideal μ may not be represented as a fuzzy KUS-ideal α-translation μ_α^T of μ , that is , the converse of proposition (4.7) is not true in general , as shown by the following example.

Example 4.8. Let X = {0, 1, 2, 3} be a KUS-algebra with the following Cayley table:

∗	0	1	2	3
0	0	1	2	3
1	3	0	1	2
2	2	3	0	1
3	1	2	3	0

By [9].Define a fuzzy subset μ of X by:

X	0	1	2	3
μ	0.9	0.6	0.8	0.6

Then μ is a fuzzy KUS-ideal of X . Let ν be a fuzzy subset of X defined by:

X	0	a	b	c
ν	0.82	0.46	0.59	0.46

Then ν is a fuzzy extension KUS-ideal of X . But ν is not the fuzzy KUS-ideal which is fuzzy KUS-ideal α-translation μ_α^T of μ for all α ∈ [0,T].

Proposition 4.9. The intersection of any set of fuzzy KUS-ideals α-translation of KUS-algebra X is also fuzzy KUS-ideal α-translation of X.

Proof: Let {μ_i| i ∈ Λ.} be a family of fuzzy KUS-ideals α-translation of KUS- algebra X , then for any x, y, z ∈ X , i ∈Λ,

$$\begin{aligned}(\bigcap_{i\in\wedge}(\mu_\alpha^T)_i)(0) &= \inf((\mu_\alpha^T)_i(0)) \\ &= \inf(\mu_i(0)+\alpha) \\ &\geq \inf(\mu_i(x)+\alpha) \\ &= \inf((\mu_\alpha^T)_i(x)) \\ &= (\bigcap_{i\in\wedge}(\mu_\alpha^T)_i)(x)\end{aligned}$$

and $(\bigcap_{i\in\wedge}(\mu_\alpha^T)_i)(z*x) = \inf((\mu_\alpha^T)i)(z*x))$
$= \inf(\mu_i(z*x)+\alpha)$
$\geq \inf(\min\{\mu_i(z*y), \mu_i(y*x)\})+\alpha$
$= \inf(\min\{\mu_i(z*y)+\alpha, \mu_i(y*x)+\alpha\})$
$= \min\{\inf(\mu_i(z*y)+\alpha), \inf(\mu_i(y*x)+\alpha\}\}$
$= \min\left\{\left(\bigcap_{i\in\Lambda}\mu_i\right)(z*y)+\alpha\right), \left(\bigcap_{i\in\Lambda}\mu_i\ (y*x)+\alpha\right\}$
$= \min\{(\bigcap_{i\in\wedge}(\mu_\alpha^T)_i)(z*y), (\bigcap_{i\in\wedge}(\mu_\alpha^T)_i)(y*x)\}$. ⌂

Clearly, the union of fuzzy extensions of a fuzzy subset α-translation of KUS-algebra X is not a fuzzy extension of μ as seen in the following example.

Example 4.10. Let X = {0, 1, 2, 3} be a KUS-algebra which is given in Example (3.15). Define a fuzzy subset μ of X by:

X	0	1	2	3
μ	0.8	0.5	0.6	0.5

Let α = 0, then μ is a fuzzy KUS-ideal α-translation of X. Let ν and δ be fuzzy subsets α-translation of X given by:

X	0	1	2	3
ν	0.9	0.6	0.7	0.6
δ	0.9	0.6	0.6	0.7

Then ν and δ are fuzzy extensions of μ. But the union ν ∪ δ is not a fuzzy extension of μ since (ν ∪ δ)(3∗2) = 0.6 < 0.7 = min{(ν ∪ δ)(3), (ν ∪ δ)(2)}.

Theorem 4.11. Let α ∈ [0,T], μ_α^T be the fuzzy subset α-translation of μ. Then the following are equivalent:

(1) μ_α^T is a fuzzy KUS-ideal α-translation of X.

(2) ∀t ∈ Im(μ) , t > α ⇒ U_α (μ; t) is KUS-ideal of X.

Proof: Assume that μ_α^T is a fuzzy KUS-ideal α-translation of X and let t ∈ Im(μ) be such that t > α. Since $\mu_\alpha^T(0) \geq \mu_\alpha^T(x)$ for all x ∈ X, we have $\mu(0)+\alpha = \mu_\alpha^T(0) \geq \mu_\alpha^T(x) = \mu(x)+\alpha$ that mean μ(0) ≥ μ(x), for all x∈ X. Let x ∈ U_α (μ; t), then μ(x) > t-α and $\mu(0) \geq \mu(x)$ imply $\mu(0) \geq \mu(x) \geq t-\alpha$. Hence 0 ∈ U_α (μ; t).

Let x, y, z ∈ X be such that $(z*y) \in U_\alpha(\mu;t)$ and $(y*x) \in U_\alpha(\mu;t)$. Then $\mu(z*y) \geq t-\alpha$ and $\mu(y*x) \geq t-\alpha$, i.e., $\mu_\alpha^T(z*y) = \mu(z*y)+\alpha \geq t$ and $\mu_\alpha^T(y*x) = \mu(y*x)+\alpha \geq t$. Since μ_α^T is a fuzzy KUS-ideal α-translation of X, it follows that $\mu(z*x)+\alpha = \mu_\alpha^T(z*x) \geq \min\{\mu_\alpha^T(z*y), \mu_\alpha^T(y*x)\} \geq t$, that is, $\mu(z*x) \geq t-\alpha$ so that $(z*x) \in U_\alpha(\mu;t)$. Therefore U_α (μ; t) is KUS-ideal of X.

Conversely, suppose that U_α (μ; t) is KUS-ideal of X for every t ∈ Im(μ) with t > α. If there exists x ∈ X such that $\mu_\alpha^T(0) < \lambda \leq \mu_\alpha^T(x)$,then $\mu(x) \geq \lambda-\alpha$ but $\mu(0) < \lambda-\alpha$. This shows that $x \in U_\alpha(\mu;t)$ and $0 \notin U_\alpha$ (μ; t). This is a contradiction, and so $\mu_\alpha^T(0) \geq \mu_\alpha^T(x)$ for all x ∈ X.

Now assume that there exist x, y, z ∈ X such that $\mu_\alpha^T(z*x) < \gamma \leq \min\{\mu_\alpha^T(z*y), \mu_\alpha^T(y*x)\}$. Then $\mu(z*y) \geq \gamma-\alpha$ and $\mu(y*x) \geq \gamma-\alpha$, but $\mu(z*x) < \gamma-\alpha$. Hence $(z*y) \in U_\alpha(\mu;\gamma)$ and $(y*x) \in U_\alpha(\mu;\gamma)$, but $(z*x) \notin U_\alpha(\mu;\gamma)$. This is a contradiction, and therefore $\mu_\alpha^T(z*x) \geq \min\{\mu_\alpha^T(z*y), \mu_\alpha^T(y*x)\}$,for all x, y, z ∈ X. Hence μ_α^T is a fuzzy KUS-ideal α-translation of X. ⌂

In Theorem(4.11(2)), if t ≤ α, then U_α (μ; t) =X.

Proposition 4.12. Let μ be a fuzzy KUS-ideal of a KUS-algebra X and let $\alpha \in [0,T]$, then the fuzzy subset α-translation μ_{α}^{T} of μ is a fuzzy KUS-subalgebra of X.

Proof: Since μ be a fuzzy KUS-ideal of a KUS-algebra X , then by proposition (2.9) μ be a fuzzy KUS-subalgebra of a KUS-algebra X and let $\alpha \in [0,T]$, then by proposition (3.13), the fuzzy subset α-translation μ_{α}^{T} of μ is a fuzzy KUS-subalgebra α-translation of X. ⌂

In general, the converse of the proposition (4.12) is not true .

Example 4.13. Consider a KUS-algebra X = {0, 1, 2} with the example (4.6). Define a fuzzy subset μ of X by:

X	0	1	2
μ	0.7	0.5	0.6

Then μ is not fuzzy KUS-ideal of X. since $\mu(0*1) = \mu(1)=0.5<0.6=\min\{\mu(0*2), \mu(2*1)\}=\min\{\mu(2), \mu(2)\}$, and T=0.3.But if we take α=0.2 the fuzzy translation μ_{α}^{T} of μ is given as follows:

X	0	1	3
μ_{α}^{T}	0.9	0.7	0.8

Then μ_{α}^{T} is a fuzzy KUS-subagebra of X.

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الترجمات الضبابية من النوع α إل جبر KUS

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،

المستخلص :
الترجمات الضبابية من النوع α (سويا ،العظمى) التوسيعات الضبابية والضرب الضبابي من النوع β إلى الجبر الجزئي الضبابي من النوع KUS ونناقش العلاقات بين الترجمات الضبابية من النوع α و التوسيعات الضبابية إلى المثالية الضبابية من النوع KUS إلى الجبر من النوع KUS ويتم التحقيق فيها.