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$b-ind$ and $b-Ind$

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Abstract

We study some functions by using $b-open\ \ set$ which are small and large inductive dimension (ind and Ind) and called by $b-ind$ and $b-Ind$. Also we study some relations between them .

Introduction

The concept of $b-open$ set in topological space was introduced in [2]. We recall the definition of ind and Ind in [7]. In this paper we study similar definitions using $b-open$ sets which are called $b-ind$ and $b-Ind$ and we study some relations between them .

Section one : On $b-open$ sets

1.1.Definition [2]

Let X be a topological space and $A \subseteq X$. A is called $b-open$ set in X if $A \subseteq \overline{\mathring{A}} \cup \mathring{\overline{A}}$. The complement of $b-open$ set is called $b-closed$ set ,that is , A is $b-closed$ set if $\mathring{\overline{A}} \cap \overline{\mathring{A}} \subseteq A$

It is clear that every open set is $b-open$ and every closed set is $b-closed$,but the converse may be not true in general .As the following example .

1.2.Example

Let $X=\{a,b,c\}$, $T=\{\{a\},X,\phi\}$. The $b-open$ sets are :- $\{a\},\{a,b\},\{a,c\},\phi,X$. Then $\{a,b\}$ is $b-open$ set but not open.

1.3.Proposition [4]

Let X be topological space then G is an open set in X iff $\overline{G\cap \overline{\overline{A}}}=\overline{G\cap A}$ for each $A\subset X$.

1.4.Remark [6]

The intersection of an $b-open$ set and an open set is $b-open$.

1.5.Example

The intersection of two $b-open$ sets may be not $b-open$ in general . Example let $X=\{a,b,c\}$, $T=\{\{a\},\{b\},\{a,b\},X,\phi\}$. Then each of $\{a,c\},\{b,c\}$ is an $b-open$ set , where as $\{a,c\}\cap\{b,c\}=\{c\}$ is not $b-open$.

1.6.Proposition [6]

Let $\{A_\lambda\}_{\lambda\in\wedge}$ be a collection of $b-open$ set in a topological space X , then $\bigcup_{\lambda\in\wedge} A_\lambda$ is $b-open$.

1.7.Definition [3]

Let X be a topological space and $A\subseteq X$. A is called semi-open ($s-open$) set in X if $A\subseteq \overline{\overset{\circ}{A}}$. The complement of $s-open$ set is called semi-closed ($s-closed$) that is, A is $s-closed$ set if $\overset{\circ}{\overline{A}}\subseteq A$. The intersection of all $s-closed$ subsets of X containing A is called semi-closur ($s-closur$) *of* A and the union of all $s-open$ subsets of X contained in A is called semi- interior ($s-\text{int}\,erior$) of A , and are denoted by $\overline{A}^s$, $A^{\circ s}$ respectively.

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1.8.Definition [5]

Let X be a topological space and $A \subseteq X$. A is called pre-open set in X if $A \subseteq \overline{\mathring{A}}$. The complement of $pre-open$ set is called

$pre-closed$ that is A is $pre-closed$ set if $\mathring{\overline{A}} \subseteq A$. The intersection of all $pre-closed$ subsets of X containing A is called $pre-closur\ of\ A$ and the union of all $pre-open$ subsets of X contained in A is called $pre-\text{int}erior\ of\ A$, and denoted by $\overline{A}^p$, $A^{\circ p}$ respectively.

1.9.Proposition [1]

Let X be a topological space and $A \subseteq B \subseteq X$,then :

(i) $\overline{A}^p \subseteq \overline{B}^p$

(ii) $A^{\circ p} \subseteq B^{\circ p}$

1.10.Proposition [2]

For any subset A of a space X the following statements are equivalent:-

(i) A is $b-open$ set

(ii) $A \subseteq A^{\circ p} \cup A^{\circ s}$

(iii) $A \subseteq (A^{\circ p})^{-p}$

1.11.Proposition

For any subset A of a space X , the following statements are equivalent :-

(i) A is $b-open$ set

(ii) $\overline{A}^p = \overline{A^{\circ p}}^p$

(iii) There exists an $pre-open$ set G in X such that $G \subseteq A \subseteq \overline{G}^p$.

Proof :

$(i) \to (ii)$ since $A \subseteq \overline{A^{\circ p}}^p$ by proposition 1.10 , then $\overline{A}^p \subseteq \overline{A^{\circ p}}^p$ and since $\overline{A^{\circ p}}^p \subseteq \overline{A}^p$, then $\overline{A}^p = \overline{A^{\circ p}}^p$.

$(ii) \to (iii)$ let $G = A^{\circ p}$ .Since $\overline{A}^p = \overline{A^{\circ p}}^p$ and $A^{\circ p} \subseteq A \subseteq \overline{A^{\circ p}}^p$. Then $G \subseteq A \subseteq \overline{G}^p$.

$(iii) \rightarrow (i)$ Suppose that there exists $pre-open$ set G such that $G \subseteq A \subseteq \overline{G}^{p}$. Then $G = G^{\circ p}$ and since $A \subseteq \overline{G}^{p} = \overline{G^{\circ p}}^{p} \subseteq \overline{A^{\circ p}}^{p}$, then A is $b-open$ set by proposition 1.10 .

1.12.Proposition

For any subset A of a space X , the following statements are equivalent :-
(i) A is $b-open$ set

(ii) There exists an open set G in X such that $G \subseteq A \subseteq \overline{G} \cup \overline{\overset{\circ}{A}}$.

Proof :

(i)→(ii) suppose that A is $b-open$ set . let $G = \overset{\circ}{A}$, then $\overset{\circ}{A} = G \subseteq A \subseteq \overline{A^{\circ}} \cup \overline{\overset{\circ}{A}} = \overline{G} \cup \overline{\overset{\circ}{A}}$. Hence $G \subseteq A \subseteq \overline{G} \cup \overline{\overset{\circ}{A}}$.

(ii)→(i) Suppose that there exists an open set G in X such that $G \subseteq A \subseteq \overline{G} \cup \overline{\overset{\circ}{A}}$.Then $A \subseteq \overline{G} \cup \overline{\overset{\circ}{A}} = \overline{\overset{\circ}{G}} \cup \overline{\overset{\circ}{A}} \subseteq \overline{\overset{\circ}{A}} \cup \overset{\circ}{\overline{A}}$ by definition 1.1 **.**Hence $A \subseteq \overline{\overset{\circ}{A}} \cup \overset{\circ}{\overline{A}}$, therefore A is $b-open$ set .

1.13.Proposition

Let X be a topological space . let Y be an open subset of X and A is $b-open$ set in Y. Then there exists a $b-open$ set B in X such that $A = B \cap Y$.

Proof:

Let A is $b-open$ set in Y, then by proposition 1.12 there exists an open set U in Y such that $U \subseteq A \subseteq \overline{U}^{Y} \cup \overline{A}^{\circ Y\, Y}$,then there exists an open set W in X such that $W \cap Y = U$. Let $B = A \cup W$, then $B \cap Y = (A \cup W) \cap Y = (A \cap Y) \cup (W \cap Y) = A \cup U = A$ to prove $W \subset B \subset \overline{W} \cap \overline{\overset{\circ}{B}}$, since $W \subset A \cup W = B$, then $W \subset B$, since $B = AUW \subset \overline{W} \cup A$

$$\subset \overline{W} \cup \overline{U}^{Y} \cup \overline{A}^{\circ Y\, Y} \subseteq \overline{W} \cup (\overline{U} \cap Y) \cup \overline{A}^{\circ Y\, Y} \subset \overline{W} \cup (\overline{W} \cap Y) \cup \overline{A}^{\circ Y\, Y}$$

$$\subset \overline{W} \cup \overline{A}^{Y\, \circ Y} = \overline{W} \cup (\overline{A} \cap Y)^{\circ Y} = \overline{W} \cup (\overline{\overset{\circ Y}{A}} \cap \overset{\circ Y}{Y}) = \overline{W} \cup (\overline{\overset{\circ Y}{A}} \cap Y) = \overline{W} \cup (\overline{\overset{\circ Y}{A}} \cap \overset{\circ}{Y})$$

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(since $Y=\mathring{Y}$)$=\overline{W}\cup\overset{\circ}{\overline{A}}\subset\overline{W}\cup\overset{\circ}{\overline{B}}$ (since $A\subset A\cup W=B$).Therefore $W\subseteq B\subseteq\overline{W}\cup\overset{\circ}{\overline{B}}$.

1.14.Proposition

Let X be a topological space and $Y\subseteq X$. If G is a $b-open$ set in X and Y is an open set in X, then $G\cap Y$ is $b-open$ set in Y.

Proof :

Since G is a $b-open$ set in X, then $G\subseteq\overset{\circ}{\overline{G}}\cup\overline{\overset{\circ}{G}}$. But $\overline{(G\cap Y)}^{Y\circ Y}\cup\overline{(G\cap Y)^{\circ Y}}^{Y}\supseteq\overline{(G\cap Y)}^{Y\circ}\cup\overline{(G\cap Y)^{\circ}}^{Y}=(\overline{G\cap Y}^{\circ}\cap Y^{\circ})\cup(\overline{G^{\circ}\cap Y^{\circ}}\cap Y)=$ $(\overline{(\overline{G}\cap Y)}^{\circ}\cap Y^{\circ})\cup(\overline{(\overline{G^{\circ}}\cap Y^{\circ})}\cap Y)$ by proposition 1.3 $\supseteq((\overline{G}\cap Y)^{\circ}\cap Y^{\circ})\cup(\overline{G^{\circ}}\cap Y^{\circ}\cap Y)=(\overline{G}^{\circ}\cap Y^{\circ})\cup(\overline{G^{\circ}}\cap Y^{\circ})$ $=(\overset{\circ}{\overline{G}}\cup\overline{G^{\circ}})\cap Y^{\circ}=(\overset{\circ}{\overline{G}}\cup\overline{G^{\circ}})\cap Y\supseteq G\cap Y$ .Then $G\cap Y$ is a $b-open$ in Y

1.15.Proposition

Let X be a topological space .Let Y be an open subset of X and A is $b-open$ set in Y .Then A is $b-open$ in X .

Proof :
Let A is $b-open$ set in Y, then by proposition 1.12 there exist an open set G in Y such that $G\subseteq A\subseteq\overline{G}^{Y}\cup\overline{A}^{Y\circ Y}$,then there exist an open set W in X such that $W\cap Y=G$, $A\subset\overline{A}^{Y\circ Y}\cup\overline{A^{\circ Y}}^{Y}=(\overline{A}^{\circ Y}\cap Y^{\circ Y})\cup(\overline{A^{\circ Y}}\cap Y)\subseteq$ $\overline{A}^{\circ}\cup(\overline{\overline{A^{\circ Y}}\cap Y})=\overline{A}^{\circ}\cup(\overline{A^{\circ Y}\cap Y^{\circ}})=\overline{A}^{\circ}\cup\overline{A^{\circ}}$. Then $A\subseteq\overline{A}^{\circ}\cup\overline{A^{\circ}}$. Hence A is $b-open$ in X .

1.16.Proposition

For any subset A of a space X, the following statements are equivalent :
(i) A is $b-closed$ set in X .
(ii)There exist a closed set C in X such that $C^{\circ}\cap\overline{A}^{\circ}\subseteq A\subseteq C$.
Proof :

(i)→(ii) Suppose that A is $b-closed$ set , then A^c is $b-open$ set . By proposition 1.12 there exists an open set G such that $G \subseteq A^c \subseteq \overline{G} \cup \overline{\overset{\circ}{A}}^{\,c}$,

$(\overline{G} \cup \overline{\overset{\circ}{A^c}})^c \subseteq A \subseteq G^c$

$(\overline{G} \cup \overline{\overset{\circ}{A^c}})^c = \overline{G}^{\,c} \cap \overline{A^c}^{\circ c}$

$= G^{c^\circ} \cap \overline{A^{c^{c^{\circ c^c}}}}^{\,c^c} = G^{c^\circ} \cap \overline{A^\circ}$

Then $\overline{G^c} \cup \overline{A^\circ} \subseteq A \subseteq G^c$,let $C = G^c$. Then $C^\circ \cap \overline{A^\circ} \subseteq A \subseteq C$.

(ii)→(i) Suppose that there exists a closed set C such that $C^\circ \cap \overline{A^\circ} \subseteq A \subseteq C$,
then $C^c \subseteq A^c \subseteq (C^\circ \cup \overline{A^\circ})^c$,

$(C^\circ \cup \overline{A^\circ})^c = C^{\circ^c} \cup \overline{A^\circ}^{\,c} = \overline{C^c} \cup \overline{A^c}^{\,c^{c^{\circ c^c}}}$, let $C^c = G$, $G \subseteq A^c \subseteq \overline{G} \cup \overline{\overset{\circ}{A^c}}$,
then by proposition 1.12 A^c is $b-open$ set . Therefore A is $b-closed$ set .

1.17.Proposition [8]

A space X is *regular* space iff for every $x \in X$ and each open set U in X such that $x \in U$ there exists an *open* set W such that $x \in W \subseteq \overline{W} \subseteq U$.

1.18.Proposition[8]

A space X is normal space iff for every closed set $C \subseteq X$ and each open set U in X such that $C \subseteq U$ there exists an open set W such that $C \subseteq W \subseteq \overline{W} \subseteq U$.

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Section TWO

On ind and $b-ind$

2.1.Definition [7]

Let X be a topological space then $ind\ X=-1$ iff $X=\phi$, and if n is a positive integer or 0 then it is said that $ind\ X \le n$ iff the following satisfied :

For each $x \in X$ and for each open set G containing x , there exists an open set U such that $x \in U \subseteq G$ and $ind\ b(\mathrm{U}) \le \mathrm{n}-1$. If there exists no integer n for which $ind\ X \le \mathrm{n}$ then we put $ind\ X=\infty$

This suggests the following :-

2.2.Definition

Let X be a topological space. we say that $b-ind\ X=-1$ iff $X=\phi$, and if n is a positive integer or 0 then we say that $b-ind\ X \le \mathrm{n}$ iff the following satisfied:

For each $x \in X$ and for each open set G containing x, there exists an $b-open$ set U such that $x \in U \subseteq G$ and $b-ind\ b(\mathrm{U}) \le \mathrm{n}-1$.If there exists no integer n for which $b-ind\ X \le \mathrm{n}$ then we put $b-ind\ X=\infty$.

To find $b-ind\ b(\mathrm{U}) \le \mathrm{n}-1$ we have to know the topology on $b(\mathrm{U})$, then we have to get $b-ind$ of the boundary of open set in the topology of $b(\mathrm{U})$.

2.3.Example

The following example of a space X with $ind\ \mathrm{X}=\mathrm{b}\text{-}ind\ X=0$. Let $X=\{a,b,c\}$, $T=\{\{c\},\{a,b\},\phi,X\}$ the $b-open$ sets are $\{c\},\{a,b\},\phi,X,\{a\}$, $\{b\},\{b,c\},\{a,c\}$. Since $\{a,b\}$ is an open and closed then $b\{a,b\}=\phi$, $indb\{a,b\}=-1$ hence $indX \le 0$ and since $X \ne \phi$ then $indX=0$. And by theorem 2.6 then $b-indX=0$.

2.4.Example

Example 1.5 gives $ind\ \mathrm{X}=\mathrm{b}\text{-}ind\ X=1$.Since $a \in \{a\} \subseteq X$ such that $b\{a\}=\overline{\{a\}}-\{a\}^{\circ}=\{a,c\}-\{a\}=\{c\}$ .The topology on $\{c\}$ is indiscrete then $ind\ b\{a\}=-1$ if $b\{a\}=\phi$ or $indb\{a\}=0$ if $b\{a\} \ne \phi$ since X is not regular then $ind\ \mathrm{X} \ne 0$ and since $ind\ b\{a\}=0$ then $ind\ \mathrm{X}=1$.Since e $\{a\}$ is $b-open$ then $\mathrm{b}\text{-}ind\ X=1$

2.5.Proposition

Let X be a topological space .If $indX$ is exist then $b-ind\ X \le indX$

Proof:

By induction on n. It is clear $n=-1$. Suppose that it is true for $n-1$. Now suppose that $ind\,X \le n$, to prove $b-ind\,X \le n$, let $x \in X$ and G is an open set in X such that $x \in G$ since $ind\,X \le n$, then there exists an open set V in X such that $x \in U \subseteq G$ and $ind\,b(U) \le n-1$ and since each open set is $b-open$ then U is an $b-open$ set such that $x \in U \subseteq G$ and $b-ind\,b(U) \le n-1$. Hence $b-ind\,X \le n$.

2.6.Theorem

Let X be a topological space , then $ind\ X = 0$ iff $b-ind\ \mathrm{X} = 0$.

Proof :

By **2.5** if $ind\ X = 0$ then $b-ind\ \mathrm{X} \le 0$ and since $X \ne \phi$, then $b-ind\ \mathrm{X} = 0$. Now let $b-ind\ \mathrm{X} = 0$, let $x \in X$ and G an open set in X such that $x \in G$. Since $b-ind\ \mathrm{X} = 0$ then there exists an $b-open$ set U such that $x \in U \subseteq G$ and $b-ind\ b(\mathrm{U}) \le -1$. Then $b(\mathrm{U}) = \phi$, and thus $ind\ b(\mathrm{U}) = -1$, so that $indX \le 0$, and since $X \ne \phi$,then $indX = 0$.

It is known that if X is a topological space with $ind\ \mathrm{X} = 0$ then X is a regular space (see [7])

Then we have the following :

2.7.Corollary

Let X be a topological space , if $b-ind\ X = 0$ then X is a regular space .

Proof:

Let $x \in X$ and G an $open$ set such that $x \in G$. Since $b-ind\ X = 0$ then there exists an $b-open$ set V such that $x \in V \subseteq G$ and $b-ind\ b(V) = -1$ then $b(V) = \phi$ hence V is an open and closed set .Therefore $x \in V \subseteq \overline{V} \subseteq G$ by proposition 1.17 Then X is a regular space .

It is known that a space X satisfies $ind\ \mathrm{X} = 0$ iff it is not empty and has a base for its topology which consists of open and closed sets. (see[7])

Similarity, we have :

2.8.Corollary

A space X satisfies $b-ind\ \mathrm{X}=0$ iff it is not empty and has a base for its topology which consists of open and closed sets.

2.9.Theorem

If A is an open subspace of a space X, then $\mathrm{b}\text{-}ind\, A \le b-ind\ X$.

Proof:

By induction on n. It is clear $n=-1$. Suppose that it is true for $n-1$. Now suppose that $\mathrm{b}\text{-}ind\, X \le n$, to prove $b-ind\, A \le n$, let $x \in A$ and G is an open subset in A such that $x \in G$, since G is an open in A, then there exists U open set in X such that $U \cap A = G$. Since $x \in U$ and $b-ind\, X \le n$, then there exist an $b-open$ set W in X such that $x \in W \subseteq U$ and $b-ind\, b(W) \le n-1$, then $V = W \cap A$ is $b-open$ in A by proposition 1.14.

$$x \in V = W \cap A \subset U \bigcap A = G$$

$$b_A(V) \subseteq b(V) \cap A = (V - \overset{\circ}{V}) \cap A \subset (\overline{W} - \overset{\circ}{V}) \cap A)$$
$$= (W \cap \overset{\circ c}{V}) \cap A$$
$$= \left[\overline{W} \cap (\overset{\circ c}{W} \cup \overset{\circ c}{A}) \right] \cap A$$
$$= \left[(\overline{W} \cap \overset{\circ c}{W}) \cup (W \cap A^c) \right] \cap A \subseteq \left[b(W) \cup A^c \right] \bigcap A$$
$$= \left(b(W) \cap A \right) \bigcup (A^c \cap A)$$
$$= b(W) \cap A \subseteq b(W)$$

Since $b-ind\ b(w) \le n-1$, then $b-ind\ \mathrm{b_A}(v) \le n-1$, therefore $b-ind\ A \le n$.

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Section Three : On Ind and $b-Ind$

3.1.Definition [7]

Let X be a topological space . It is said that $Ind\ X=-1$ iff $X=\phi$ and if n is a positive integer or 0 then we say that $Ind\ X\leq n$ iff the following is satisfied :

For each closed set C in X and each open set G , $C\subseteq G$, there exists an open set U such that $C\subseteq U\subseteq G$ and $Ind\ b(\mathrm{U})\leq n-1$. If there exists no integer n for which $Ind\ X\leq \mathrm{n}$ then we put $Ind\ X=\infty$

This suggest the following :

3.2.Definition

Let X be a topological space . we say that $b-Ind\ X=-1$ iff $X=\phi$, and if n is a positive integer or 0 then we say that $b-Ind\ X\leq n$ iff the following is satisfied :

For each closed set C in X and each open set G , such that $C\subseteq G$, there exists an $b-open$ set U such that $C\subseteq U\subseteq G$ and $b-Ind\ b(\mathrm{U})\leq n-1$. If there exists no integer n for which $b-Ind\ X\leq \mathrm{n}$ then we put $b-Ind\ X=\infty$ .To find $b-Ind\ b(\mathrm{U})\leq \mathrm{n}-1$ we have to know the topology on $b(\mathrm{U})$, then we have to get $b-Ind$ of the boundary of open set in the topology of $b(\mathrm{U})$.

3.3.Example

The following example of a space X with $b-Ind\ X=Ind\ X=0$. Let $X=\{a,b,c,d\}$, $T=\{\{d\},\{b,c\},\{b,c,d\},\phi,X\}$. The $b-open$ sets are $\{b\},\{c\},\{b,d\},\{c,d\},\{a,b,d\},\{a,c,d\},\{d\},\{b,c\},\{b,c,d\},\{a,d\},\{a,b,c\},\phi,X$ .Since $\{a,b.c\}\subseteq X\subseteq X$ $,b(X)=\phi$ *then* $Indb(X)=-1$ *hence* $IndX=0$ and since X is an $b-open$ then $b-Ind\ X=0$.

3.4.Example

The following example of a space X with $Ind\ X=b-Ind\ X=1$ .Let $X=\{a,b,c,d,e\}$, $T=\{\{a\},\{c,d\},\{a,c,d\},\{a,b,d,e\},\{d\},\{a,d\},X,\phi\}$.

The $b-open$ sets are:- $\{a\},\{c,d\},\{a,c,d\},\{a,b,d,e\},\{d\},\{a,d\},\{a,b,d\},\{a,d,e\},$ $\{a,b,c,d\},\{a,c,d,e\}\{b,d,e\},\{b,c,d,e\},\{a,e\},\{b,d\},\{d,e\},\{c,d,e\},\{b,c,d\},\{a,b,e\},\{a,b\},\phi,X$ ..Since $\{c\}\subset\{c,d\}\subset X$ such that $b\{c,d\}=\overline{\{c,d\}}-\{c,d\}^{\circ}=\{b,c,d,e\}-\{a,c,d\}=\{b,e\}$ .The topology on $\{b,e\}$ is an indiscrete then $Ind\, b\{c,d\}=-1$ *if* $b\{c,d\}=\phi$ *or* $Indb\{c,d\}=0$ *if* $b\{c,d\}\neq\phi$ since X is not normal then $Ind\,\mathrm{X}\neq 0$ and since $Ind\, b\{c,d\}=0$ then $Ind\,\mathrm{X}=1$.Since $\{c,d\}$ is $b-open$ then $\mathrm{b}\text{-}ind\, X=1$

3.5.Proposition[7]

Let X be a topological space , if $Ind\ X=0$ then X is normal space.

3.6.Corollary

Let X be a topological space , if $b-Ind\ X=0$, then X is normal space .

Proof:

Let C be a closed set in X and U an open set such that $C\subseteq U$ .Since $b-Ind\ X=0$,then there exist an $b-open$ set W such that $b-Ind\ b(W)=-1$,hence W is an open and closed set .Therefore $C\subseteq W\subseteq\overline{W}\subseteq U$ by proposition 1.18 .Then X is a normal.

3.7.Proposition:

Let X be a topological space .If $IndX$ is exist then $b-Ind\ X\leq Ind\ X$.

Proof:

By induction on n. It is clear $n=-1$. Suppose that it is true for $n-1$. Now suppose that $Ind\, X\leq n$, to prove $b-Ind\, X\leq n$, let C be a closed set in X and G is an open set in X such that $C\in G$ since $Ind\, X\leq n$, then there exists an open set V in X such that $C\in U\subseteq G$ and $Ind\, b(U)\leq n-1$ and since each open set is $b-open$ then U is an $b-open$ set such that $C\in U\subseteq G$ and $b-Ind\, b(U)\leq n-1$. Hence $b-Ind\, X\leq n$.

The following analogous to theorem 2.6 and its proof is similar and hence is omitted .

3.8.Theorem

Let X be a topological space . Then $Ind\ X=0$ iff $b-Ind\ X=0$.

It is known and easy to see that if X is T_1-space, then $ind\ X\leq Ind\ X$ (see[7]).
Similarity we have :

3.9.Proposition

Let X be a topological space . If X is $T_1 - space$ then $b-ind\ X \leq b-Ind\ X$.

Proof:

Let $x \in X$ and G be an open set such that $x \in G$ **,**since X is $T_1 - space$ then $\{x\} \subseteq G$ and since $b-Ind\ X \leq n$ then there exist an $b-open$ set V such that $\{x\} \subseteq V \subseteq G$ and $Ind\, b(V) \leq n-1$ **.**Hence $ind\, b(V) \leq n-1$,then $ind\ X \leq n$ **.**

It is known and easy to see that if X is a regular topological space ,then $ind\ X \leq Ind\ X$. (see[7])

Similarity , we have

3.10.Theorem

Let X be a topological space . If X is regular space then $b-ind\ X \leq b-Ind\ X$.

Proof :

By induction on n. If $n=-1$ then $b-Ind\ X=-1$ and $X=\phi$, so that $b-Ind\ X=-1$. Suppose that the statement is true for $n-1$. Now , let $b-Ind\ X \leq n$. Let $x \in X$ and G be an open set such that $x \in G$. Since X is regular space then there exists an open set V such that $x \in V \subseteq \overline{V} \subseteq G$ by proposition 1.17. Also since $b-Ind\ X \leq n$ and $\overline{V}$ is closed , $\overline{V} \subseteq G$, then there exists an $b-open$ set U such that $\overline{V} \subseteq U \subseteq G$ and $b-Ind\, b(\mathrm{U}) \leq n-1$, then $b-ind\, b(\mathrm{U}) \leq n-1$[by indication] and $b-ind\ X \leq n$.

3.11.Proposition[7]

If A is a closed subset of a space X , then $Ind\ A \leq Ind\ X$

We have the following :

3.12.Theorem

If A is an open and closed subspace of a space X , then $b-Ind\ A \leq b-Ind\ X$.

Proof :

By induction on n. It is clear if $n=-1$, suppose that it is true for $n-1$. Now suppose that $b-Ind\ X\leq n$, to prove $b-Ind\ A\leq n$, let C is a closed subset of A and G is an open subset in A such that $C\subseteq G$,since C is closed in A and A is closed in X , then C is closed in X . Since G is an open in A , then there exists U open set in X such that $U\bigcap A=G$. Since $C\subset U$ and $b-Ind\ X\leq n$, then there exists an $b-open$ set W in X such that $C\subseteq W\subseteq U$ and $b-Ind\ b(\mathrm{W})\leq n-1$, since A is an open set then $V=W\bigcap A$ is $b-open$ set in A by proposition 1.14 ,

$C\subset V=W\cap A\subset U\cap A=G$

$$b_A(V)\subseteq b(V)\cap A=(\overline{V}-V^{\circ})\cap A\subset(\overline{W}-V^{\circ})\cap A=(\overline{W}\cap V^{\circ c})\bigcap A$$

$$=\left[\overline{W}\cap(W^{\circ c}\cup A^{\circ c})\right]\cap A$$

$$=\left[(\overline{W}\cap W^{\circ c})\cup(\overline{W}\cap A^{c})\right]\cap A$$

$$\subseteq\left[b(W)\cup A^{c}\right]\cap A$$

$$=(b(W)\cap A)\cup(A^{c}\cap A)=b(W)\cap A\subseteq b(W)$$

Since $b-Ind\ b(\mathrm{W})\leq n-1$, then $b-Ind\ \mathrm{b}_{\mathrm{A}}(\mathrm{V})\leq n-1$, therefore $b-Ind\ A\leq n$.

References

[1] Najla Jabbar kaisam Al-Malaki, "Some Kinds of Weakly Connected and Pair wise Connected Space", M.Sc. Thesis University of Baghdad, College of Education \ Abn AL-Haitham ,2005.

[2] D. Andrijevic, "On $b-open$ sets", Mat. Vesnik, 48(1996), 59 – 64.

[3] P. Das, "Note on Some Applications on Semi Open sets", Progress of Mathematics , 7(1973).

[4] J. Dugundji, Topology , Allyn and Bacon Boston , 1966.

[5] A.S. Mashhour, M.E. Abd-El-Mansef and S.N. El-Deeb, "On Pre-continuous and Weak Pre-continuous Mappings" ,Proc.Math. and Phys. Egypt 53(1982),47-53.

[6] A.AL-Omari and M.S. Noorani, " Decomposition of Continuity Via $b-open$ set", Bol. Soc. Paran. Math. V. 26(1-2)(2008):53-64.

[7] A.P. Pears, "On Dimension Theory of General Spaces", Cambridge University press ,1975.

[8] N.J. Pervin, "Foundation of General Topology" A cadmic press, New York, 1964.