

Modeling of Position-Dependent Viscosity in Carreau Fluid Flow under the Influence of a Magnetic Field through a Curved Channel

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ABSTRACT

The peristaltic motion is studied in this work by assuming the viscosity as a distance-dependent behavior for the Carreau fluid (a non-Newtonian fluid) inside a compliant wall curved channel. An external magnetic field is also considered. A. Importance of the study in biomedical and engineering applications (mechanical behavior of physiological systems. The mathematical formulation of the problem results in a system of nonlinear partial differential equations with corresponding boundary conditions. These equations are non-dimensionalized, and simplified under the assumptions of low Reynolds number and long wavelength to produce a more analytically tractable model. The resulting problem is solved numerically using Mathematica, and the accuracy of the results is validated through comparison with previously published studies.

MSC..

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1. Introduction

The flow resulting from the contraction or expansion of the walls of a flexible channel or tube is known as peristaltic flow. Peristaltic flow is observed in many physiological systems, such as the gastrointestinal, reproductive, and circulatory systems, where it plays a crucial role in transporting fluids through vital organs. Industrial applications of peristaltic flow include diabetic pumps, heart-lung machines, peristaltic pumps with rollers and fingers, and viscous fluid transport systems in engineering processes. Researchers have studied these flows since the early investigations by Latham [15] and Mishra et al. [16]. Additionally, numerous studies have focused on peristaltic flow in curved channels [5–12].

At present, the magnetic field plays a significant role in magnetic therapy, magnetic resonance imaging, and magnetic devices. The motion of conducting fluids through a magnetic field generates electric currents, which in turn affect the moving field, and the flow is influenced by the Lorentz force [19]. Based on these applications, many researchers have investigated the effects of magnetic fields on peristaltic flow in curved channels.

Mohammed Rehan Khan et al. (2021) conducted an analytical study on the flow of Carreau-type fluids in a curved channel under the influence of a magnetic field, focusing on the effect of the magnetic field on velocity and flow characteristics[13]. Similarly, Sami and Zainab (2021) examined the flow of hybrid nanofluids in curved channels

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under a magnetic field, analyzing the impact of these factors on flow properties[3]. In addition, the effects of activation energy and thermal radiation on bio-convective peristaltic transport of a Sutterby nanofluid in a two-dimensional flexible porous channel, including heat and mass transfer, were also studied [4].

Furthermore, Raheel Ahmed et al. (2020) studied the non-isothermal peristaltic flow phenomenon of a hydromagnetic bio-fluid in a curved channel with an application to physiological system [2]. Recently, Hamad et al. [1] on January 17th, 2023 analyzed the peristaltic flow of a Powell–Eyring nanofluid through porous medium along with slip effects under the influence of magnetic field [1].

Moreover, most classical studies have assumed straight channels, even though many biological systems such as the intestines and arteries possess inherently curved geometries. Channel curvature alters the velocity distribution and generates centrifugal forces that influence vortex structure and heat transfer characteristics. Recent research has examined peristaltic flow in curved channels for non-Newtonian fluids, including the Carreau, Williamson, and Jeffrey models [17,18,14].

The Carreau fluid is an important rheological model for representing biological fluids such as blood, as it exhibits Newtonian behavior at low shear rates and power-law behavior at high shear rates. Although several studies have investigated the peristaltic flow of Carreau fluid in straight or curved channels, most of them assumed constant viscosity or did not fully incorporate the influence of a transverse magnetic field.

Therefore, the present study aims to examine the peristaltic flow of a Carreau fluid in a curved channel under the effect of a transverse magnetic field, while accounting for spatially variable viscosity. The effects of key physical parameters such as the Weissenberg number, Hartmann number, and Darcy number are analyzed on the velocity distribution, temperature field.

The significance of this study lies in integrating three fundamental aspects that have not been comprehensively addressed in the recent literature: geometric curvature, Carreau-type non-Newtonian behavior, and direct magnetic field effects, in addition to spatial viscosity variation within the channel.

2. Mathematical Formulation

In this work we define the incompressible Carreau fluid in curved channel because of a repeating waves (peristaltic waves) having wavelength and an amplitude. The curved channel is made in a circular shape that has radius R with center O and has uniform magnetic field B_0 radiating through flow (see Fig. 1). The channel walls are flexible. The velocity field $V = [V(r^*, x^*, t^*), U(r^*, x^*, t^*), 0]$ is specified by its components V in radial r and U in axial x directions respectively. While moving, it can sense the following shape of a peristaltic wave:

$$r = \pm \eta(x, t) = \pm \left[d + b \sin \frac{2\pi}{\lambda} (x - ct) \right], \quad (1)$$

where $\pm \eta$ represents the displacements of the upper and lower walls, respectively. Here x shows axial measurement r for radial measuring d to mean radius of the curved channel which does not move, and b wave amplitude, λ is absolutely the wave length, t as well time.

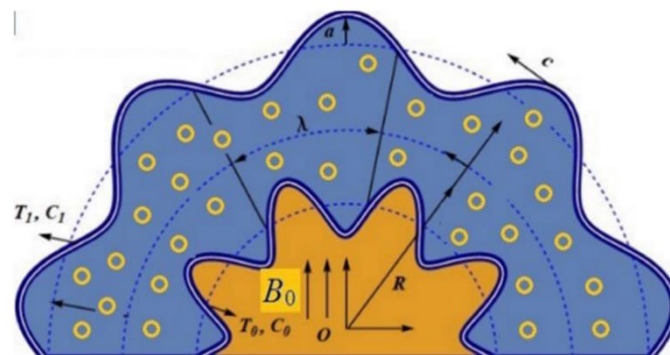


Fig. 1 The problem Geometry

Where is the distance travelled. Carreau fluid with variable viscosity the equation of incompressible carreau fluid can be expressed as follows on a steady slope close to zero:

$$\sigma = -\bar{P} \bar{I} + \bar{S}, \quad (2)$$

$$\bar{S} = \mu(\bar{r}) \left[1 + \left(\frac{n-1}{2} \right) \Gamma^2 \bar{\alpha}^2 \right] \bar{\alpha}. \quad (3)$$

Take σ is "the Cauchy stress tensor", $\bar{S}$ is "extra stress tensor", $\bar{P}$ "pressure", $\bar{I}$ "identity tensor", μ "dynamic viscosity", Γ "time constant", n "dimensionless power law index" and $\bar{\alpha}$ defined;

$$\bar{\alpha} = \sqrt{\frac{1}{2} \sum_{i=1}^3 \sum_{j=1}^3 \dot{\alpha}_{ij} \dot{\alpha}_{ji}}. \quad (4)$$

The model can exist simplified to a Newtonian model $n = 1$ or $\Gamma = 0$, and we will consider the case for $\Gamma \neq 0$.

The components of shear stress tensor S are:

$$\bar{S}_{r^*r^*} = 2\mu(\bar{r}) \left[1 + \left(\frac{n-1}{2} \right) \Gamma^2 \bar{\alpha}^2 \right] \left(\frac{\partial V}{\partial r^*} \right), \quad (5)$$

$$\bar{S}_{r^*x^*} = \bar{S}_{x^*r^*} = \mu(\bar{r}) \left[1 + \left(\frac{n-1}{2} \right) \Gamma^2 \bar{\alpha}^2 \right] \left(\frac{1}{r^*+R} \left(R \frac{\partial V}{\partial x^*} - U + (r^*+R) \frac{\partial U}{\partial r^*} \right) \right), \quad (6)$$

$$\bar{S}_{x^*x^*} = 2\mu(r) \left[1 + \left(\frac{n-1}{2} \right) \Gamma^2 \bar{\alpha}^2 \right] \left(\frac{1}{r^*+R} \left(R \frac{\partial U}{\partial x^*} + V \right) \right). \quad (7)$$

Where $\bar{S}_{ij}$ implies "a stress component on the face" whose usually is i -axis and acting in a direction of j -axis. The key governing equations of the problem system

The governing equations of the problem system

The continuity equation:

$$\frac{\partial V}{\partial r^*} + \frac{R}{r^*+R} \frac{\partial U}{\partial x^*} + \frac{V}{r^*+R} = 0. \quad (8)$$

The momentum equation:

$$\begin{aligned} \rho \left(\frac{\partial V}{\partial t^*} + V \frac{\partial V}{\partial r^*} + \frac{RU}{r^*+R} \frac{\partial V}{\partial x^*} - \frac{U^2}{r^*+R} \right) &= -\frac{\partial p^*}{\partial r^*} + \frac{1}{r^*+R} \frac{\partial}{\partial r^*} ((r^*+R) \bar{S}_{r^*r^*}) + \frac{R}{r^*+R} \frac{\partial \bar{S}_{r^*x^*}}{\partial x^*} - \frac{S_{x^*x^*}}{r^*+R}, \\ \rho \left(\frac{\partial U}{\partial t^*} + V \frac{\partial U}{\partial r^*} + \frac{RU}{r^*+R} \frac{\partial U}{\partial x^*} - \frac{UV}{r^*+R} \right) &= -\frac{R}{r^*+R} \frac{\partial p^*}{\partial x^*} + \frac{1}{(r^*+R)^2} \frac{\partial}{\partial r^*} ((r^*+R)^2 \bar{S}_{r^*x^*}) + \frac{R}{r^*+R} \frac{\partial \bar{S}_{x^*x^*}}{\partial x^*} - \sigma \left(\frac{R}{r^*+R} \right)^2 B_0^2. \end{aligned} \quad (9)$$

The temperature equation:

$$\rho C_p \left(\frac{\partial T}{\partial t^*} + V \frac{\partial T}{\partial r^*} + \frac{RU}{r^*+R} \frac{\partial T}{\partial x^*} \right) = k_1 \left(\frac{1}{r^*+R} \frac{\partial T}{\partial r^*} + \frac{\partial^2 T}{\partial r^{*2}} + \left(\frac{R}{r^*+R} \right)^2 \frac{\partial^2 T}{\partial x^{*2}} \right) + (\bar{S}_{r^*r^*} - \bar{S}_{x^*x^*}) \frac{\partial V}{\partial r^*} + \bar{S}_{r^*x^*} \left(\frac{\partial U}{\partial r^*} + \frac{R}{r^*+R} \frac{\partial V}{\partial x^*} - \frac{U}{r^*+R} \right). \quad (10)$$

The concentration equation:

$$\begin{aligned} \frac{\partial C}{\partial t^*} + V \frac{\partial C}{\partial r^*} + \frac{RU}{r^*+R} \frac{\partial C}{\partial x^*} &= D_n \left(\frac{1}{r^*+R} \frac{\partial C}{\partial r^*} + \frac{\partial^2 C}{\partial r^{*2}} + \left(\frac{R}{r^*+R} \right)^2 \frac{\partial^2 C}{\partial x^{*2}} \right) + \frac{D_n K_T}{T_n} \left(\frac{1}{r^*+R} \frac{\partial T}{\partial r^*} + \frac{\partial^2 T}{\partial r^{*2}} + \right. \\ &\quad \left. \left(\frac{R}{r^*+R} \right)^2 \frac{\partial^2 T}{\partial x^{*2}} \right) \end{aligned} \quad (11)$$

Where ρ is a density, D_n the diffusion coefficient in diffusing elements, T_n being an average temperature of the fluid, K_T stands for thermal diffusion ratio and T and C show respectively denote thermal diffusion ratio. g is Gravitational acceleration, β_1 is the thermal expansion coefficient, β_2 is the Solute expansion coefficient.

The boundary conditions for the present problem are

$$u = 0, \quad T = \begin{Bmatrix} T_1 \\ T_0 \end{Bmatrix}, \quad C = \begin{Bmatrix} C_1 \\ C_0 \end{Bmatrix} \quad \text{at } r = \pm \eta, \quad (13)$$

The equation which we use to analyze the behavior of an elastic wall

$$L = \bar{P} - \bar{p}_0$$

Where $\bar{p}_0$ the pressure that can be found at outside channel while L is used to explain an elastic effect in sense of viscous damping actions. It is expressed as:

$$L = B \frac{\partial^4}{\partial x^{*4}} - \tau \frac{\partial^2}{\partial x^{*2}} + m \frac{\partial^2}{\partial t^{*2}} + d \frac{\partial}{\partial t^{*}} + A_L$$

Where B is the flexural rigidity of the wall, where τ represents how tense stretched membrane is, m is called a mass per area unit and d stands for the viscous damping constant also A_L is spring stiffness. We put $p_0 = 0$ for simplicity.

It is known as condition of continuity of stress at $r = \pm \eta(x, t)$, the walls exert pressure on the fluid and this pressure is equal in magnitude to that exerted by the fluid. Mathematically,

$$\begin{aligned} \frac{R}{r^{*+R}} \left[B \frac{\partial^5}{\partial x^{*5}} - \tau \frac{\partial^3}{\partial x^{*3}} + m_1 \frac{\partial^3}{\partial t^{*2} \partial x^{*}} + d_1 \frac{\partial^2}{\partial t^{*} \partial x^{*}} + \frac{\partial}{\partial x^{*}} A_L \right] \eta^{*} = -\rho \left(\frac{\partial U}{\partial t^{*}} + V \frac{\partial U}{\partial r^{*}} + \frac{RU}{r^{*+R}} \frac{\partial U}{\partial x^{*}} - \frac{UV}{r^{*+R}} \right) + \frac{1}{(r^{*+R})^2} \frac{\partial}{\partial r^{*}} ((r^{*} + R)^2 \bar{S}_{r^{*}x^{*}}) \\ + \frac{R}{r^{*+R}} \frac{\partial \bar{S}_{x^{*}x^{*}}}{\partial x^{*}} - \sigma \left(\frac{R}{r^{*+R}} \right)^2 B_0^2 U, \end{aligned}$$

$$\text{at } r^{*} = \pm \eta^{*}. \quad (14)$$

3. Solution Method

To make it easier to simplify the control equations for movement, heat and how much of an something is present, we use these dimensionless variables:

$$\begin{aligned} x = \frac{x^{*}}{\lambda}, u = \frac{U}{c}, v = \frac{V}{\delta c}, r = \frac{r^{*}}{d}, t = \frac{ct^{*}}{\lambda}, \delta = \frac{d}{\lambda}, \eta = \frac{\eta^{*}}{d}, k = \frac{R}{d}, S_{ij} = \frac{d}{c\mu_v} \bar{S}_{i^{*}j^{*}}, Re = \frac{\rho cd}{\mu_v}, p = \frac{d^2 p^{*}}{\lambda \mu_v c}, M^2 = \frac{\sigma d^2 B_0^2}{\mu_v}, Q_1 = \frac{B d^3}{\lambda^5 \mu_v c}, \\ Q_2 = -\frac{\tau d^3}{\lambda^3 \mu_v c}, Q_3 = \frac{m_1 c d^3}{\lambda^3 \mu_v}, Q_4 = \frac{d_1 d^3}{\lambda^2 \mu_v}, Q_5 = \frac{A_L d^3}{\lambda \mu_v c}, W_e = \frac{\Gamma c}{d}, \alpha = \frac{d \bar{\alpha}}{c}, H = \frac{T - T_0}{T_1 - T_0}, \xi = \frac{C - C_0}{C_1 - C_0}, \phi = ad, \mathbb{M}(r) = \frac{\mu(\bar{r})}{\mu_v}, p_r, E_c = B_r, S_1 = \frac{\rho D_n K_T(T_0)}{\mu_v T_n(C_0)}, S_2 = \frac{\mu_v}{\rho D_n}, p_r = \frac{\mu_v C_p}{k_1}, E_c = \frac{C^2}{C_p(T_1 - T_0)}, u = -\frac{\partial \psi}{\partial r}. \end{aligned} \quad (15)$$

Where Re is the Reynolds number, k curvature parameter, δ the wave number, W_e is the Weissenberg number, Q_1 will flexural stiffness, Q_2 the tension characterizing parameter, Q_3 the mass characterizing parameter, Q_4 the damping parameter, Q_5 is spring stiffness, S_1 is the Schmidt number, S_2 is the sorot number, k_1 the thermal conductivity of fluid.

Now substituting (3.15) into equations (3.8) - (3.14) and into equations (3.5) - (3.7) we have:

Using the variables (13), Eqs. (6)–(12) can be expressed as below.

$$\left(\frac{c}{\lambda} \right) \left(\frac{\partial v}{\partial r} + \frac{k}{r+k} \frac{\partial u}{\partial x^{*}} + \frac{v}{r+k} \right) = 0, \quad (16)$$

$$Re \delta \left(\delta^2 \frac{\partial v}{\partial t} + \delta^2 v \frac{\partial v}{\partial r} + \delta^2 \frac{ku}{r+k} \frac{\partial v}{\partial x} - \frac{u^2}{r+k} \right) = -\frac{\partial p}{\partial r} + \delta \frac{1}{r+k} \frac{\partial}{\partial r} ((r+k) S_{rr}) + \delta^2 \frac{k}{r+k} \frac{\partial S_{xr}}{\partial x} - \delta \frac{S_{xx}}{r+k}, \quad (17)$$

$$Re \left(\delta \frac{\partial u}{\partial t} + \delta v \frac{\partial u}{\partial r} + \delta \frac{ku}{r+k} \frac{\partial u}{\partial x} - \delta \frac{uv}{r+k} \right) = -\frac{k}{r+k} \frac{\partial p}{\partial x} + \frac{1}{(r+k)^2} \frac{\partial}{\partial r} ((r+k)^2 S_{rx}) + \frac{k}{r+k} \delta \frac{\partial S_{xx}}{\partial x} - M^2 \left(\frac{k}{r+k} \right)^2 (u), \quad (18)$$

$$Re P_r \left(\delta \frac{\partial H}{\partial t} + \delta v \frac{\partial H}{\partial r} + \frac{\delta k u}{r+k} \frac{\partial H}{\partial x} \right) = \left(\frac{1}{r+k} \frac{\partial H}{\partial r} + \frac{\partial^2 H}{\partial r^2} + \delta^2 \left(\frac{k}{r+k} \right)^2 \frac{\partial^2 H}{\partial x^2} \right) + E_c p_r \delta (S_{rr} - S_{xx}) \frac{\partial v}{\partial r} + E_c p_r S_{xr} \left(\frac{\partial u}{\partial r} + \delta^2 \frac{k}{r+k} \frac{\partial v}{\partial x} - \frac{u}{r+k} \right), \quad (19)$$

$$Re \left(\delta \frac{\partial \xi}{\partial t} + v \frac{\partial \xi}{\partial r} + \frac{\delta k u}{r+k} \frac{\partial \xi}{\partial x} \right) = \frac{1}{S_2} \left(\frac{1}{r+k} \frac{\partial \xi}{\partial r} + \frac{\partial^2 \xi}{\partial r^2} + \delta^2 \frac{\partial^2 \xi}{\partial x^2} \right) + S_1 \left(\frac{1}{r+k} \frac{\partial H}{\partial r} + \frac{\partial^2 H}{\partial r^2} + \delta^2 \frac{\partial^2 H}{\partial x^2} \right), \quad (20)$$

$$\frac{k}{r+k} \left[Q_1 \frac{\partial^5}{\partial x^5} + Q_2 \frac{\partial^3}{\partial x^3} + Q_3 \frac{\partial^3}{\partial x \partial t^2} + Q_4 \frac{\partial^2}{\partial t \partial x} + Q_5 \frac{\partial}{\partial x} \right] \eta = -Re \left(\delta \frac{\partial u}{\partial t} + \delta v \frac{\partial u}{\partial r} + \delta \frac{k u}{r+k} \frac{\partial u}{\partial x} - \delta \frac{u v}{r+k} \right) + \frac{1}{(r+k)^2} \frac{\partial}{\partial r} ((r+k)^2 S_{rx}) + \frac{k}{r+k} \delta \frac{\partial S_{xx}}{\partial x} - M^2 \left(\frac{k}{r+k} \right)^2 (u), \quad (21)$$

$$S_{rx} = \mathbb{M}(r) \left[1 + \left(\frac{n-1}{2} \right) W_e^2 \alpha^2 \right] \left(\frac{\partial u}{\partial r} - \frac{u}{r+k} \right). \quad (22)$$

$$\text{Since } \alpha^2 = \left(\frac{\partial u}{\partial r} - \frac{u}{r+k} \right)^2,$$

$$u = 0, H = \left\{ \frac{1}{0} \right\}, \xi = \left\{ \frac{1}{0} \right\} \text{ at } r = \pm \eta = \pm(1 + \epsilon \sin 2\pi(x-t)), \quad (23)$$

The general solution of governing equations (16) - (23) in most cases can not be easily solved and it will be very hard thus we shall do the analysis under an approximation of small wavelength $\delta \ll$ high Reynolds number, then we present above equations as stream function Form:

$$- \frac{\partial p}{\partial r} = 0 \quad (24)$$

$$- \frac{k}{r+k} \frac{\partial p}{\partial x} + \frac{1}{(r+k)^2} \frac{\partial}{\partial r} ((r+k)^2 S_{rx}) + \frac{k}{r+k} \delta \frac{\partial S_{xx}}{\partial x} - M^2 \left(\frac{k}{r+k} \right)^2 (u) = 0 \quad (25)$$

$$\left(\frac{\partial^2 H}{\partial r^2} + \frac{1}{r+k} \frac{\partial H}{\partial r} \right) + B_r S_{rx} \left(\frac{\partial u}{\partial r} - \frac{u}{r+k} \right) = 0 \quad (26)$$

$$\frac{1}{S_2} \left(\frac{\partial^2 \xi}{\partial r^2} + \frac{1}{r+k} \frac{\partial \xi}{\partial r} \right) + S_1 \left(\frac{\partial^2 H}{\partial r^2} + \frac{1}{r+k} \frac{\partial H}{\partial r} \right) = 0 \quad (27)$$

$$\frac{k}{r+k} \left[Q_1 \frac{\partial^5}{\partial x^5} + Q_2 \frac{\partial^3}{\partial x^3} + Q_3 \frac{\partial^3}{\partial x \partial t^2} + Q_4 \frac{\partial^2}{\partial t \partial x} + Q_5 \frac{\partial}{\partial x} \right] \eta = \frac{1}{(r+k)^2} \frac{\partial}{\partial r} ((r+k)^2 S_{rx}) - M^2 \left(\frac{k}{r+k} \right)^2 (u) \quad (28)$$

Since

$$\frac{\partial u}{\partial r} = 0, H = \left\{ \frac{1}{0} \right\}, \xi = \left\{ \frac{1}{0} \right\} \text{ at } r = \pm \eta \quad (29)$$

4. Velocity Function

The viscosity variable with temperature is described by the Reynolds model of viscosity .The Reynolds viscosity definition in the model is:

$$\mathbb{M}(r) = e^{-\alpha r}. \quad (30)$$

By using the Maclaurin series, we obtain:

$$\mathbb{M}(r) = 1 - \alpha r \text{ when } \alpha \ll 1, \quad (31)$$

In (28), is the fixed viscosity at $\alpha = 0$, and from equation (31) where represents variable viscosity. So, the velocity equation ends up in following shape.

$$\frac{k}{r+k} \left[Q_1 \frac{\partial^5}{\partial x^5} + Q_2 \frac{\partial^3}{\partial x^3} + Q_3 \frac{\partial^3}{\partial x \partial t^2} + Q_4 \frac{\partial^2}{\partial t \partial x} + Q_5 \frac{\partial}{\partial x} \right] \eta = \frac{1}{(r+k)^2} \frac{\partial}{\partial r} \left((r+k)^2 (1-\alpha r) \left[\left(\frac{\partial u}{\partial r} - \frac{u}{r+k} \right) + \left(\frac{n-1}{2} \right) W_e^2 \left(\frac{\partial u}{\partial r} - \frac{u}{r+k} \right)^3 \right] - M^2 \left(\frac{k}{r+k} \right)^2 (u) \right) \quad (32)$$

Rewrite We the equation (32) as follows:

$$\begin{aligned} \frac{k}{r+k} \left[Q_1 \frac{\partial^5}{\partial x^5} + Q_2 \frac{\partial^3}{\partial x^3} + Q_3 \frac{\partial^3}{\partial x \partial t^2} + Q_4 \frac{\partial^2}{\partial t \partial x} + Q_5 \frac{\partial}{\partial x} \right] \eta = \frac{1}{(r+k)^2} \left\{ 2(r+k) \left((1-\alpha r) \left[\left(\frac{\partial u}{\partial r} - \frac{u}{r+k} \right) + \left(\frac{n-1}{2} \right) W_e^2 \left(\frac{\partial u}{\partial r} - \frac{u}{r+k} \right)^3 \right] \right) \right. \\ \left. + (r+k)^2 (1-\alpha r) \left[\left(\frac{\partial^2 u}{\partial r^2} - \frac{1}{r+k} \frac{\partial u}{\partial r} + \frac{1}{(r+k)^2} u \right) + 3 \left(\frac{n-1}{2} \right) W_e^2 \left(\frac{\partial u}{\partial r} - \frac{u}{r+k} \right)^2 \left(\frac{\partial^2 u}{\partial r^2} - \frac{1}{r+k} \frac{\partial u}{\partial r} + \frac{1}{(r+k)^2} u \right) \right] - \right. \\ \left. \alpha(r+k)^2 \left[\left(\frac{\partial u}{\partial r} - \frac{u}{r+k} \right) + \left(\frac{n-1}{2} \right) W_e^2 \left(\frac{\partial u}{\partial r} - \frac{u}{r+k} \right)^3 \right] \right\} - M^2 \left(\frac{k}{r+k} \right)^2 u. \end{aligned} \quad (33)$$

Equation (33) is not a linear and it is non-homogeneous partial differential equation also, it becomes so complicated that an exact solution is not able to be found and thus will apply towards, perturbations method (two times first in terms of parameter W_e also α then afterwards in terms of the parameter) to find it as follows: Initially let

$$u = u_0 + W_e^2 u_1 + O(W_e^4) \quad (34)$$

also second $u_i = u_{i0} + \alpha u_{i1} + O(\alpha^2)$, for $i = 0, 2$. Therefore, the final form of the velocity function will exist $u = (u_{00} + \alpha u_{01}) + W_e^2 (u_{10} + \alpha u_{11})$.

For similar equally forces equations configuration we will simplify W_e also α , respectively.

Zero order system (W_e^0)

$$\frac{k}{r+k} \left[Q_1 \frac{\partial^5}{\partial x^5} + Q_2 \frac{\partial^3}{\partial x^3} + Q_3 \frac{\partial^3}{\partial x \partial t^2} + Q_4 \frac{\partial^2}{\partial t \partial x} + Q_5 \frac{\partial}{\partial x} \right] \eta = \frac{1}{(r+k)^2} \left[(r+k) \left(\frac{\partial u_0}{\partial r} \right) - u_0 - \alpha r(r+k) \frac{\partial u_0}{\partial r} + \alpha r u_0 - \right. \\ \left. (r+k)^2 \frac{\partial^2 u_0}{\partial r^2} - (r+k) \frac{\partial u_0}{\partial r} - \alpha r(r+k)^2 \left(\frac{\partial^2 u_0}{\partial r^2} \right) - \alpha(r+k)^2 \left(\frac{\partial u_0}{\partial r} \right) + \alpha(r+k) u_0 - M^2 k^2 u_0 \right] \quad (34)$$

(i) Zero order system (α^0)

$$\frac{k}{r+k} \left[Q_1 \frac{\partial^5}{\partial x^5} + Q_2 \frac{\partial^3}{\partial x^3} + Q_3 \frac{\partial^3}{\partial x \partial t^2} + Q_4 \frac{\partial^2}{\partial t \partial x} + Q_5 \frac{\partial}{\partial x} \right] \eta = \frac{1}{(r+k)^2} \left[-(r+k)^2 \frac{\partial^2 u_{00}}{\partial r^2} - u_{00} - M^2 k^2 u_{00} \right]$$

(ii) First order system (α)

$$\frac{\partial^2 u_{01}}{\partial r^2} + \frac{(1+M^2 k^2)}{(r+k)^2} u_{01} = \frac{1}{(r+k)^2} \left[-r(r+k)^2 \left(\frac{\partial^2 u_{00}}{\partial r^2} \right) - (r+k)(2r+k) \left(\frac{\partial u_{00}}{\partial r} \right) + (2r+k) u_{00} \right]$$

Second order system (W_e^2)

$$\begin{aligned} 0 = \frac{1}{(r+k)^2} \left[(r+k) \left(\frac{\partial(u_1)}{\partial r} \right) - (u_1) - \alpha r(r+k) \frac{\partial(u_1)}{\partial r} + \alpha r u_1 - (r+k)^2 \frac{\partial^2(u_1)}{\partial r^2} - (r+k) \frac{\partial(u_1)}{\partial r} - \alpha r(r+k)^2 \frac{\partial^2(u_1)}{\partial r^2} - \right. \\ \alpha(r+k)^2 \left(\frac{\partial(u_1)}{\partial r} \right) + \alpha(r+k)(u_1) + \left(\frac{n-1}{2} \right) W_e^2 \left[-(r+k) \left(\frac{\partial(u_0)}{\partial r} \right)^3 + \left(\frac{\partial(u_0)}{\partial r} \right)^2 (u_0) + 3 \frac{1}{r+k} \left(\frac{\partial(u_0)}{\partial r} \right) (u_0)^2 + \right. \\ \left. \frac{1}{(r+k)^2} (u_0)^3 + \alpha r(r+k) \left(\frac{\partial(u_0)}{\partial r} \right)^3 - 3\alpha r \left(\frac{\partial(u_0)}{\partial r} \right)^2 (u_0) + 3\alpha r \frac{1}{r+k} (u_0)^2 \left(\frac{\partial(u_0)}{\partial r} \right) - \alpha r \frac{1}{(r+k)^2} (u_0)^3 + 3(r+k)^2 \left(\frac{\partial(u_0)}{\partial r} \right)^2 \left(\frac{\partial^2(u_0)}{\partial r^2} \right) - \right. \\ \left. 6(r+k)(u_0) \left(\frac{\partial(u_0)}{\partial r} \right) \left(\frac{\partial^2(u_0)}{\partial r^2} \right) - 3(u_0)^2 \left(\frac{\partial^2(u_0)}{\partial r^2} \right) - 3\alpha r(r+k)^2 \left(\frac{\partial(u_0)}{\partial r} \right)^2 \left(\frac{\partial^2(u_0)}{\partial r^2} \right) + \right. \end{aligned}$$

$$6\alpha r(r+k)(u_0) \left(\frac{\partial(u_0)}{\partial r} \right) \left(\frac{\partial^2(u_0)}{\partial r^2} \right) - 3\alpha r(u_0)^2 \left(\frac{\partial^2(u_0)}{\partial r^2} \right) - \alpha(r+k)^2 \left(\frac{\partial(u_0)}{\partial r} \right)^3 + 3\alpha(r+k) \left(\frac{\partial(u_0)}{\partial r} \right)^2 (u_0) - 3\alpha \left(\frac{\partial(u_0)}{\partial r} \right) (u_0)^2 + \frac{1}{r+k} (u_0)^3 \Big] - M^2 k^2 (u_1) \Big] \quad (35)$$

(i) Zero order system (α^0)

$$\frac{\partial^2(u_{10})}{\partial r^2} + \frac{(1+M^2 k^2)}{(r+k)^2} u_{10} = \left(\frac{n-1}{2} \right) \frac{1}{(r+k)^2} \left[-(r+k) \left(\frac{\partial u_{00}}{\partial r} \right)^3 + \left(\frac{\partial u_{00}}{\partial r} \right)^2 (u_{00}) + 3 \frac{1}{r+k} \left(\frac{\partial u_{00}}{\partial r} \right) (u_{00})^2 + \left(1 + \frac{1}{r+k} \right) \frac{1}{r+k} (u_{00})^3 + 3 \left((r+k)^2 \left(\frac{\partial u_{00}}{\partial r} \right)^2 - 2(r+k)(u_{00}) \left(\frac{\partial u_{00}}{\partial r} \right) - (u_{00})^2 \right) \left(\frac{\partial^2 u_{00}}{\partial r^2} \right) \right]$$

(ii) First order system (α)

$$\frac{\partial^2 u_{11}}{\partial r^2} + \frac{(1+M^2 k^2)}{(r+k)^2} (u_{11}) = \frac{1}{(r+k)^2} \left[-r(r+k)^2 \frac{\partial^2 u_{10}}{\partial r^2} - (2r+k)(r+k) \frac{\partial u_{10}}{\partial r} + (2r+k)(u_{10}) + \left(\frac{n-1}{2} \right) \left[3 \left((r+k)^2 \left(\frac{\partial u_{00}}{\partial r} \right)^2 - (u_{00})^2 \right) \frac{\partial^2 u_{01}}{\partial r^2} + \left(-3(r+k) \left(\frac{\partial u_{00}}{\partial r} \right)^2 + 2 \frac{\partial u_{00}}{\partial r} u_{00} + 3 \frac{1}{r+k} (u_{00})^2 \right) \frac{\partial u_{01}}{\partial r} + \left(\left(\frac{\partial u_{00}}{\partial r} \right)^2 + 6 \frac{1}{r+k} \frac{\partial u_{00}}{\partial r} u_{00} + 3 \frac{1}{(r+k)^2} (u_{00})^2 + 3 \frac{1}{r+k} (u_{00})^2 \right) u_{01} + \left(6(r+k)^2 \frac{\partial u_{00}}{\partial r} \frac{\partial u_{01}}{\partial r} - 6(r+k) u_{00} \frac{\partial u_{01}}{\partial r} - 6(r+k) u_{01} \frac{\partial u_{00}}{\partial r} - 6 u_{00} u_{01} - 3r(r+k)^2 \left(\frac{\partial u_{00}}{\partial r} \right)^2 + 6r(r+k)(u_{00}) \left(\frac{\partial u_{00}}{\partial r} \right) - 3r(u_{00})^2 \right) \frac{\partial^2 u_{00}}{\partial r^2} - (r+k)^2 \left(\frac{\partial u_{00}}{\partial r} \right)^3 + r(r+k) \left(\frac{\partial u_{00}}{\partial r} \right)^3 + 3(r+k) \left(\frac{\partial u_{00}}{\partial r} \right)^2 (u_{00}) - 3r \left(\frac{\partial u_{00}}{\partial r} \right)^2 (u_{00}) + 3r \frac{1}{r+k} (u_{00})^2 \left(\frac{\partial u_{00}}{\partial r} \right) - 3 \left(\frac{\partial u_{00}}{\partial r} \right) (u_{00})^2 - r \frac{1}{(r+k)^2} (u_{00})^3 \right] \right]$$

And we get the equations of u_0 and u_1 because its solution is too long so it was not mentioned. We take the equations for u_0 , and u_1 and then substitute them into equation (34), to get u . Because the solution to the equation is too long, it is not mentioned. So we only discuss the solution by drawing.

5. Temperature and Concentration Function

The answers for heat fluid equation (26) and the concentration fluid equation (27) are given with used boundary condition equations (29) and velocity equation (34), to obtain The solution to the equation is quite long so it will not be mentioned. So we simply speak of the solution through drawing.

6. Results and Discussion

This section provides a graphical explanation of various important numerical and computational results obtained for the Carreau fluid non-Newtonian incompressible viscous fluid problem involve distance-dependent viscosity in a curved channel, by considering the impact of a radial magnetic field. We use the program (MATHEMATICA) to get numeric results and plots.

6.1. Velocity distribution

Effect of many parameters on the velocity distribution, are shown inside figures (3-2) - (3-11). These figures are scratched at the fixed value of ($x = 0.1, t = 0.2$).

It is well known in magnetohydrodynamics (MHD), the study of electrically conducting fluids under the influence of a magnetic field, that the velocity increases with an increase in the magnetic parameter. This effect is clearly illustrated in Fig. (3-2). The velocity decreases with an increase in the curvature parameter K . Physically, as the curvature increases, the channel becomes more curved, which complicates the flow path and enhances the effect of centrifugal forces on the fluid particles. These forces push the fluid toward the channel walls, increasing the frictional resistance between the fluid and the surface, and consequently reducing the overall velocity, as shown in

Fig. (3-3). Number 10 reference Flow pores in blood capillary walls permit the passage of water, oxygen and other vital substances between blood and tissues. It is observed that the axial velocity is a increasing function of velocity slip parameter see Figure(3-4). The axial velocity is seen to increase with a higher value of the parameter Q_1 , representing the flexural stiffness of the wall. In contrast, the parameters Q_2 , Q_3 , Q_4 , and Q_5 which correspond to the tension, mass characteristic, damping effect, and spring stiffness, respectively—oppose the motion, and thus a decreasing behavior in velocity is observed with their increase. The results in Figure (3.10), indicate the effect of the Weber number on velocity, showing that the velocity decreases with an increase in the Weber number. Figure (3-11), illustrates the effect of the parameter ϕ on velocity, we see that velocity u increase with an increase of ϕ .

6.2. Trapping phenomenon

Through the figures (3-12) – (3-20), we discuss the trapped boluses that arise as a result of the movement of the fluid through the flow channel and thus take the form of bracelets that move in the direction of the fluid movement. We discussed the influence of some important parameters on the boluses. We observed An increase in the bolus size by increasing the value of ϕ and W_e , in figures (3.12), and (3.13) respectively. Similarly, increase in the bolus size by increasing the value of Q_1 in figure (3.14). The figure (3.15) illustrates the effects of Q_2 , the bolus size decrease by increasing the value of Q_2 . When these parameters Q_3 , Q_4 , and Q_5 are increased in the figures (3.16) – (3.18), the bolus size decreases while its number remains constant with increasing values, and this effect is very weak in the figure. The bolus size decreases by increasing the value of k in figure (3.19). The effect of M in the figure (3.20), where it is noticed that the bolus size increase by increasing the value of M .

6.3 Temperature Distribution

This section discusses the variation in temperature distribution (H) through a set of physical parameters, as illustrated in Figures (3.21) - (3.29):

An increase in temperature H is observed as the magnetic field strength increases (see Figure (3.21)). It is also evident from Figure (3.22) that the effect of the Brinkman number (B_r) leads to an increase in H , as heat generation is associated with specific characteristics of viscous dissipation. It is observed that the flexural stiffness of the wall (Q_1) leads to an increase in temperature (H) in figure (3.23), whereas the parameters Q_2 , Q_3 , Q_4 , and Q_5 representing tension, mass characteristic, damping effect, and spring stiffness, respectively—act oppositely, resulting in a decrease in temperature (see Figures (3.24) – (3.27)). It illustrates the effect of the parameter (ϕ) on temperature, where it is observed that the temperature increases with an increase in (ϕ) in figure (3.28). In Figure (3.29), the effects of the curvature parameter (k) are discussed. It is observed that (H) decreases with an increase in the curvature parameter (k), indicating that the curvature parameter has a significant influence on the temperature distribution. For very large values of (k), the variation in (H) becomes negligible.

6.4. Concentration Distribution

Figures (3.30) - (3.37) reveal the effects of the relevant flow parameters Q_1 , Q_2 , Q_3 , Q_4 , Q_5 , k , M , S_1 , and S_2 on the concentration profile ξ . Figure (3.30) shows the effects of the parameter Q_1 on the concentration. This graph shows that increasing the wall curvature stiffness Q_1 causes a decrease in the concentration profile. Meanwhile, the parameters Q_2 , Q_3 , Q_4 , and Q_5 work in the opposite way as shown in the figure (3.31) – (3.34). Figure (3.35) displays how changing curvature parameter k affects the concentration levels. In the figure more curvature gives a possibility for nanoparticle concentration to get higher. Figure (3.36) is presenting what happens with Hartmann parameter on concentration, as this parameter goes up there is a reduction in concentration that happens. The effect of the parameters S_2 , and S_2 on the concentration is plotted in the figure (3.37) – (3.38), and a decrease in concentration is observed when these parameters increase.

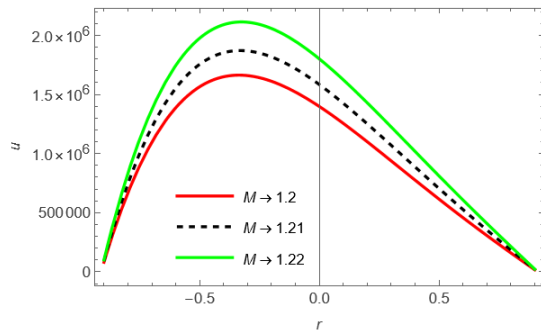


Figure (3-2): Velocity profile for various values of M

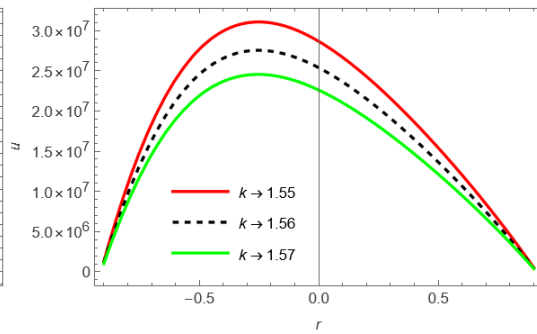


Figure (3-3): Velocity profile for various values of K

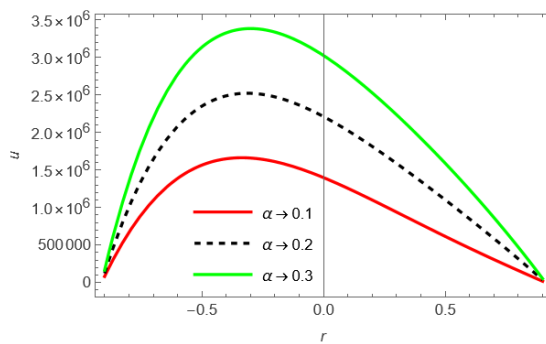


Figure (3-4): Velocity profile for various values of α

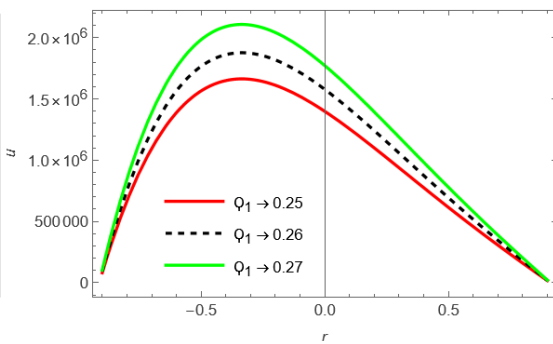


Figure (3-5): Velocity profile for various values of Q_1

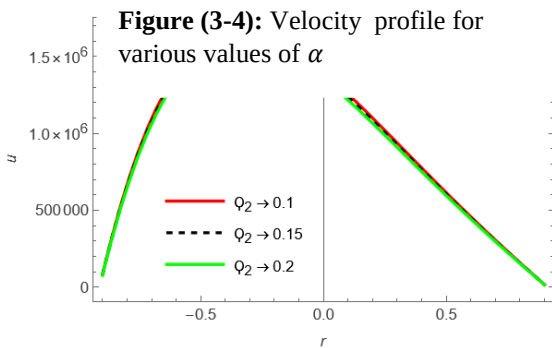


Figure (3-6): Velocity profile for various values of Q_2

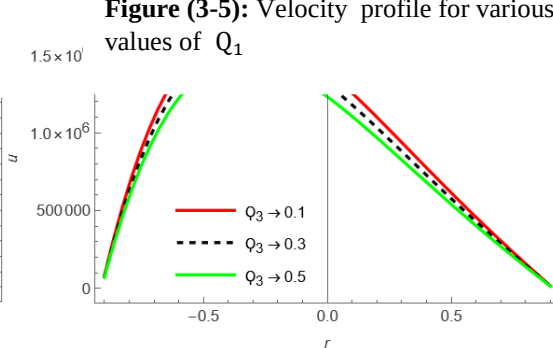


Figure (3-7): Velocity profile for various values of Q_3

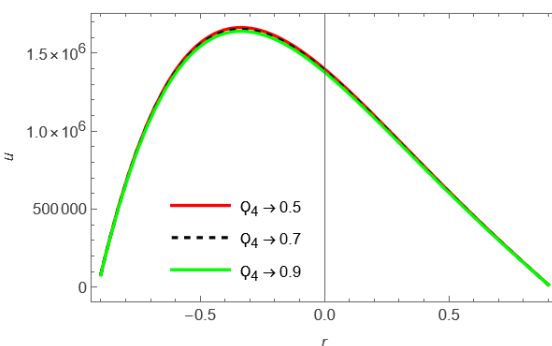


Figure (3-8): Velocity profile for various values of Q_4

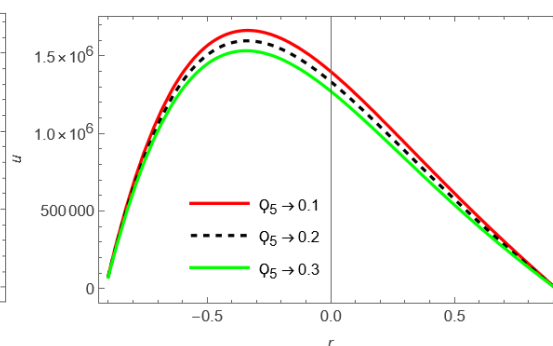


Figure (3-9): Velocity profile for various values of Q_5

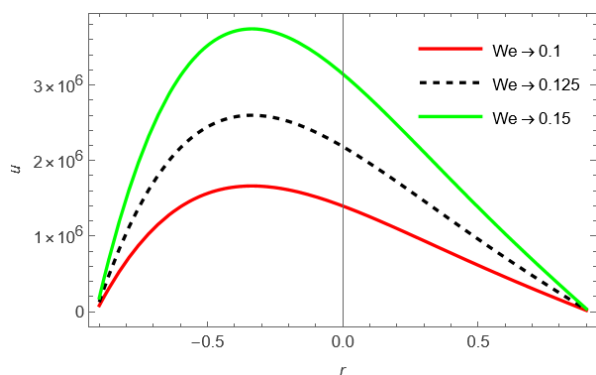


Figure (3-10): Velocity profile for various values of We

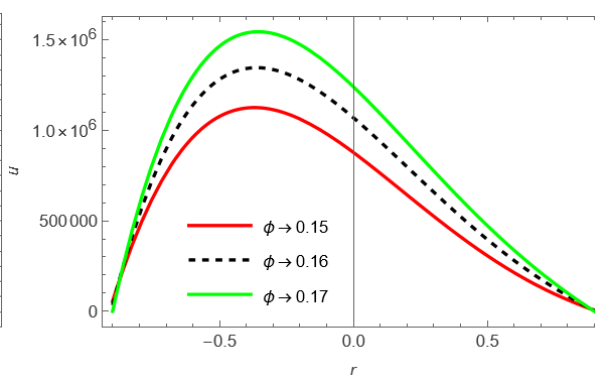


Figure (3-11): Velocity profile for various values ϕ

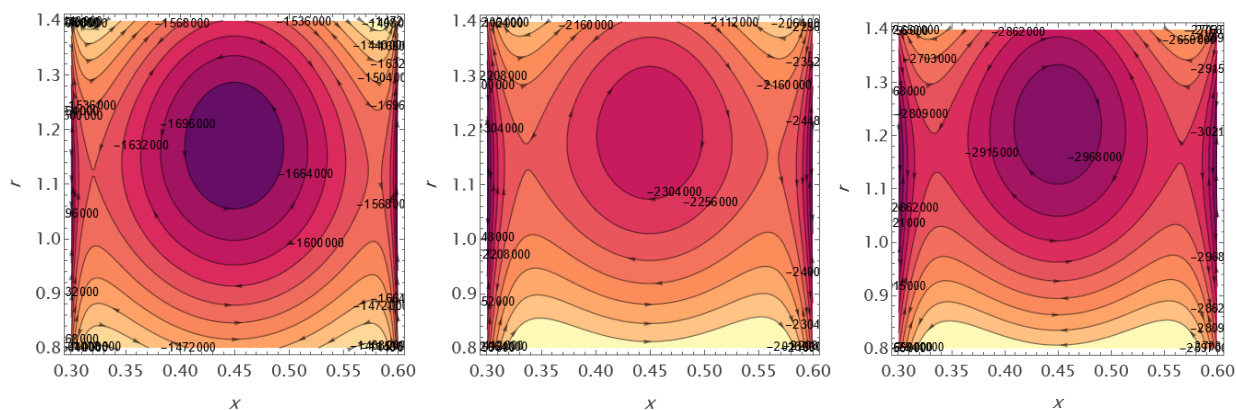


Figure (3-12): Streamlines for various values of $\phi = 0.175, 0.2, 0.225$

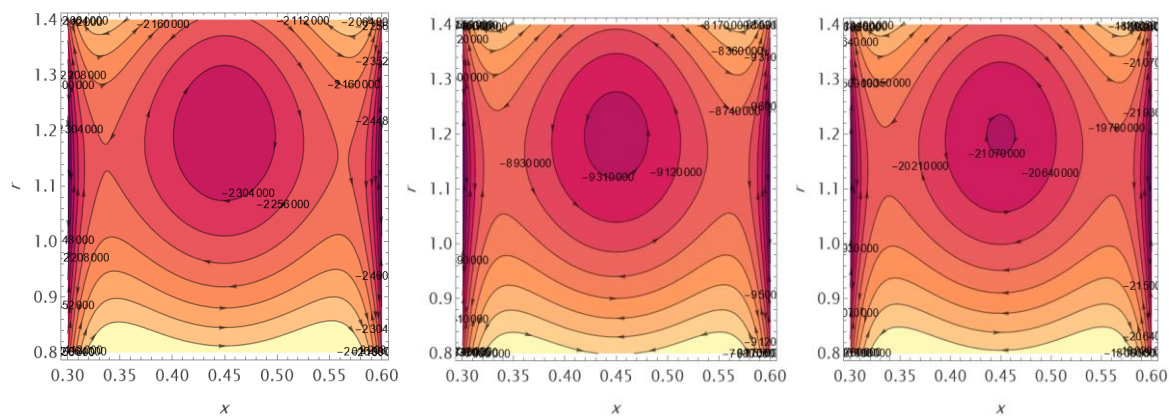
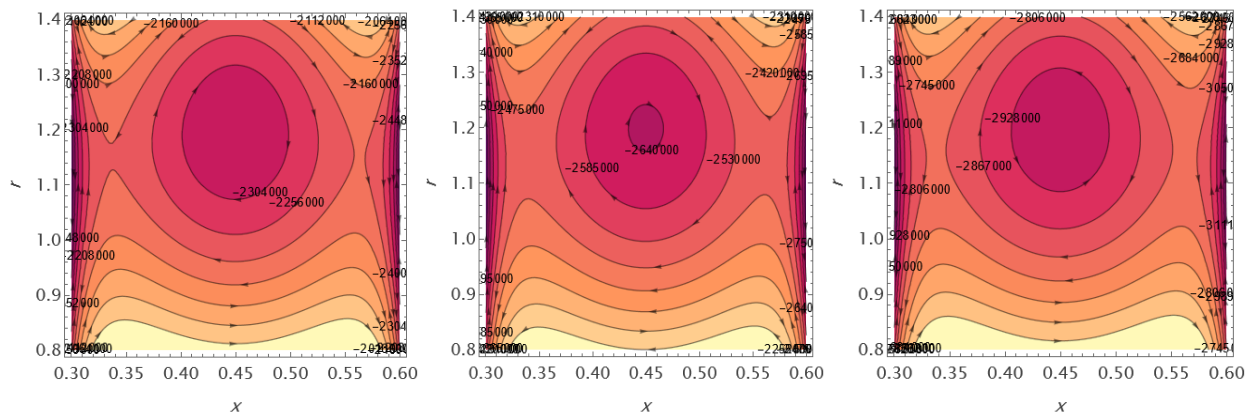
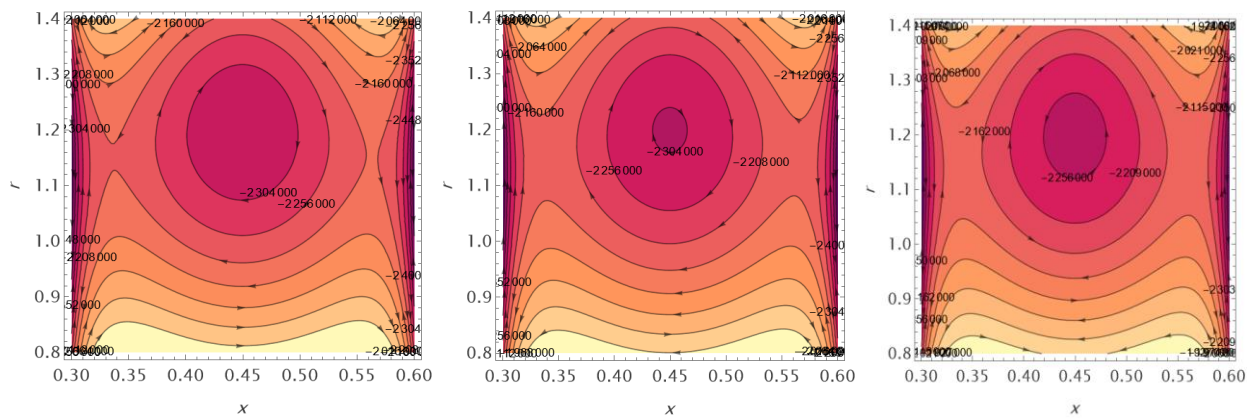
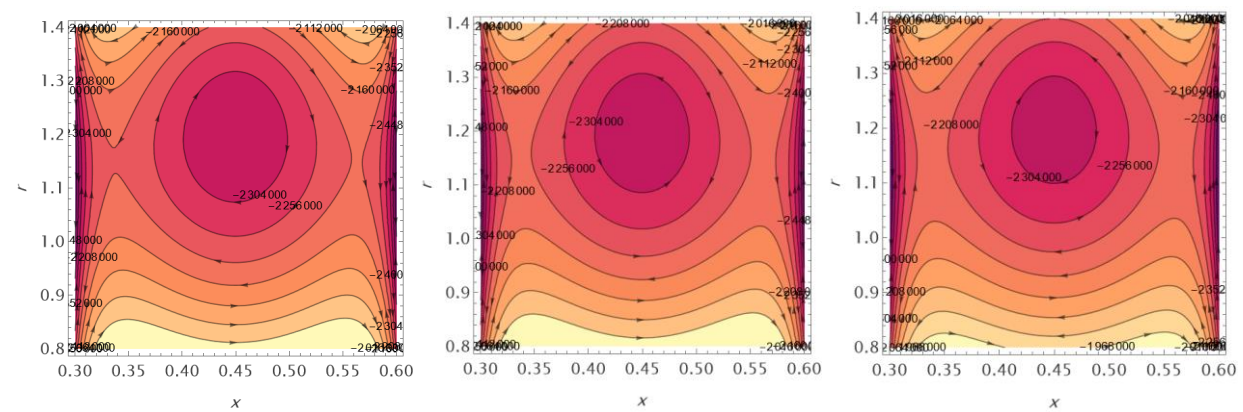
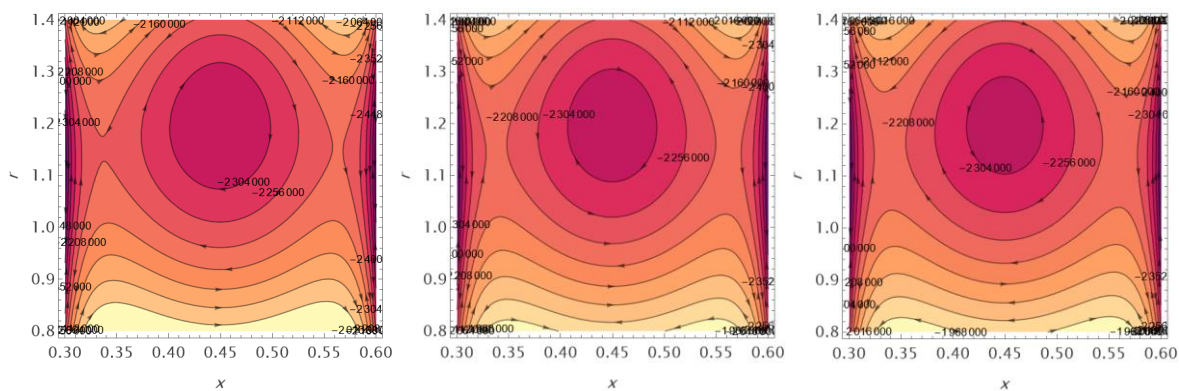


Figure (3-13): Streamlines for various values of $We = 0.125, 0.15, 0.175$

Figure (3-14): Streamlines for various values of $Q_1 = 0.25, 0.26, 0.27$ Figure (3-15): Streamlines for various values of $Q_2 = 0.1, 0.15, 0.2$ Figure (3-16): Streamlines for various values of $Q_3 = 0.1, 0.11, 0.12$



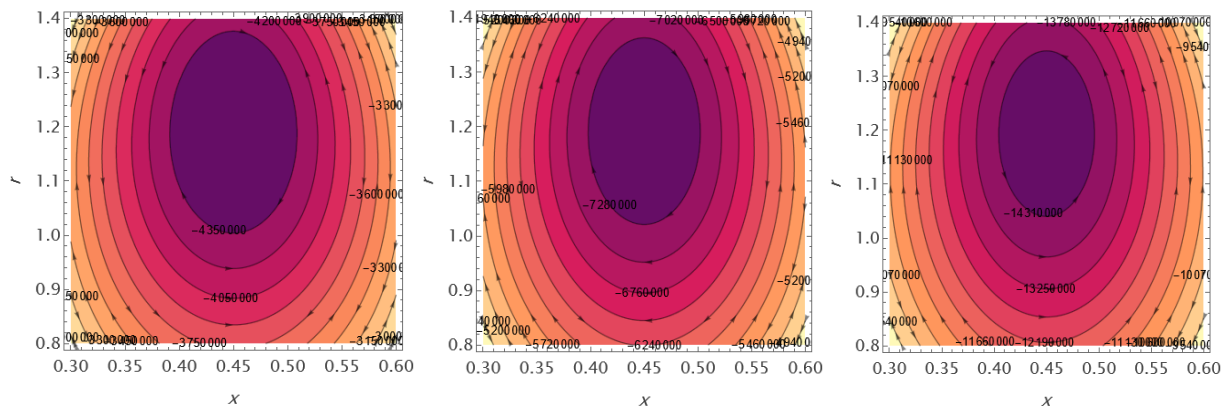


Figure (3-20): Streamlines for various values of $M = 1.3, 1.35, 1.4$

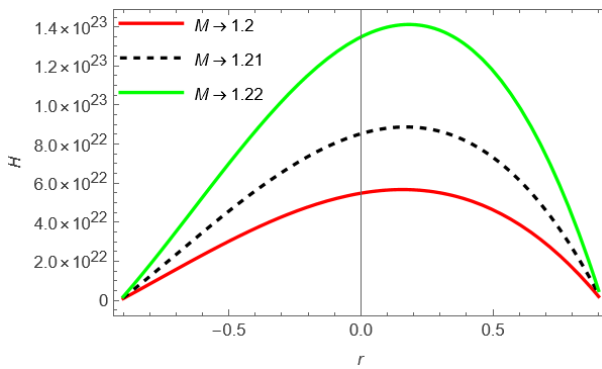


Figure (3-21): Temperature profile for different values of M

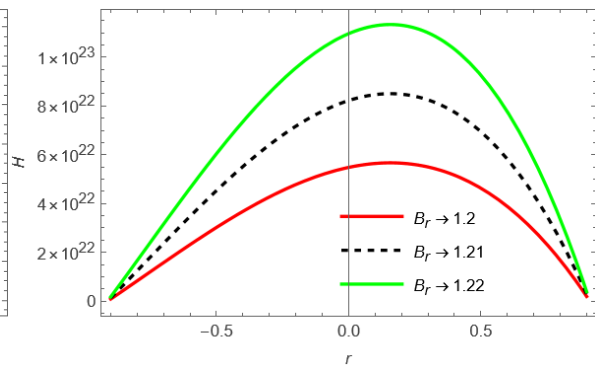


Figure (3-22): Temperature profile for different values of B_r

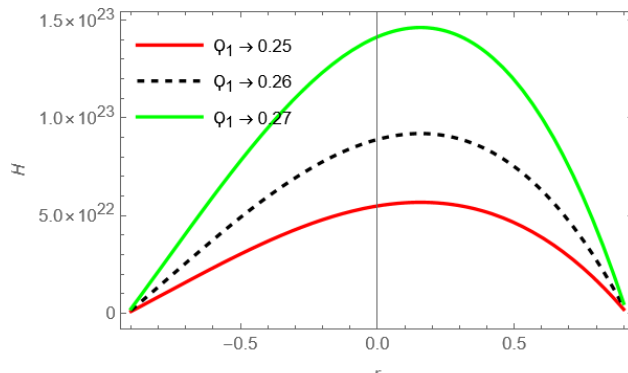


Figure (3-23): Temperature profile for different values of Q_1

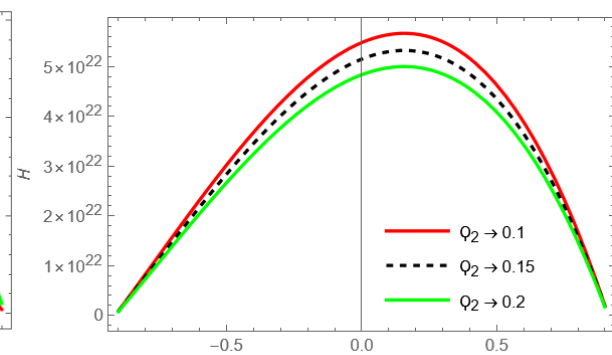


Figure (3-24): Temperature profile for different values of Q_2

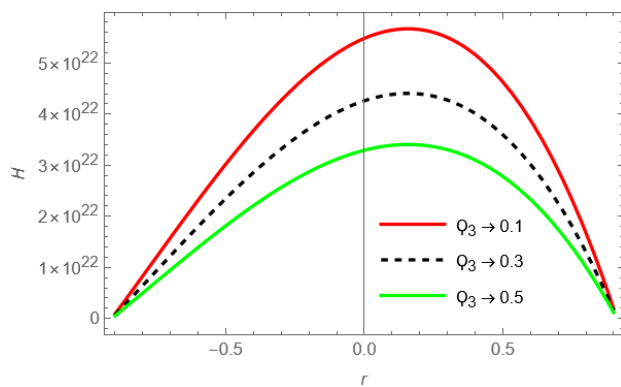


Figure (2-25): Temperature profile for different values of Q_3

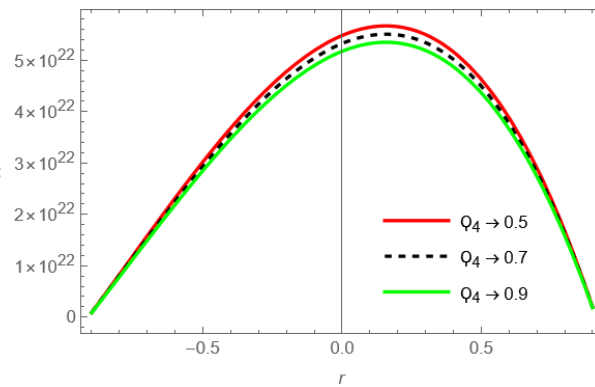


Figure (3-26): Temperature profile for different values of Q_4

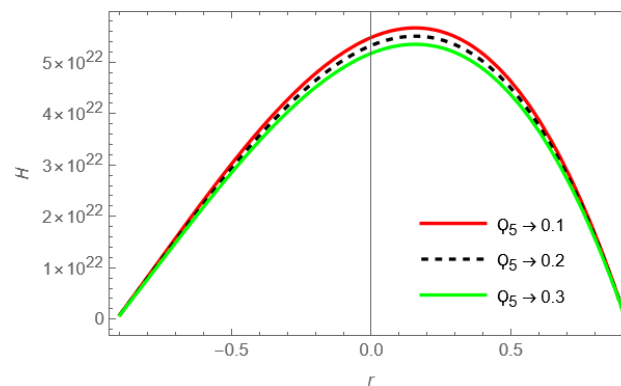


Figure (3-27): Temperature profile for different values of Q_5

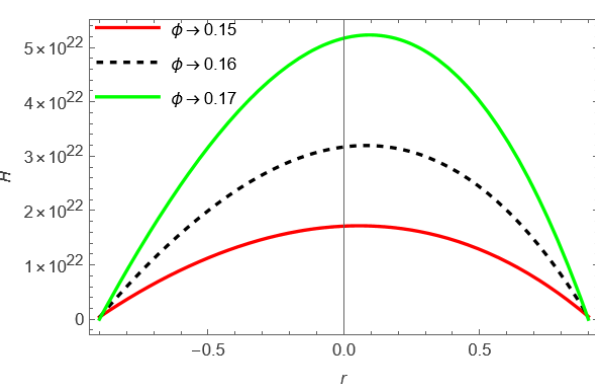


Figure (3-28): Temperature profile for different of ϕ

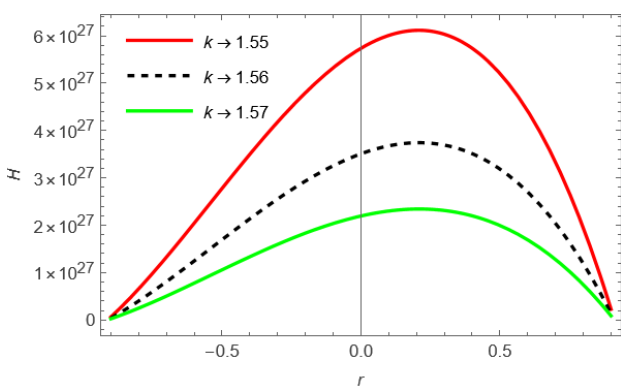


Figure (3-29): Temperature profile for different values of k

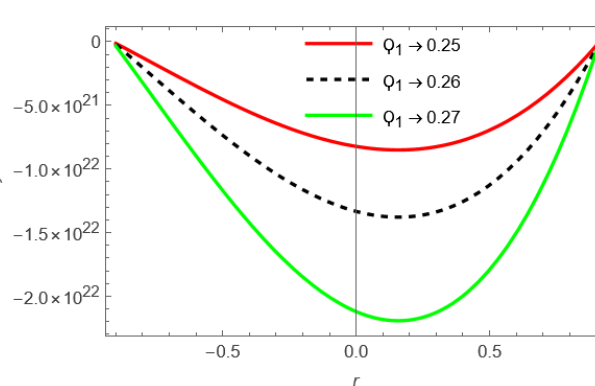


Figure (3-30): Concentration profile for different values of Q_1

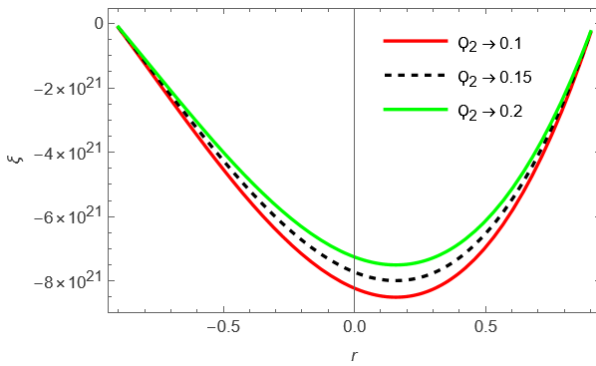


Figure (3-31): Concentration profile for different values of Q_2

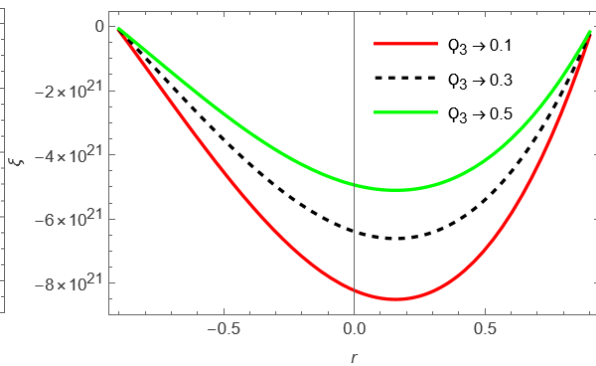


Figure (3-32): Concentration profile for different values of Q_3

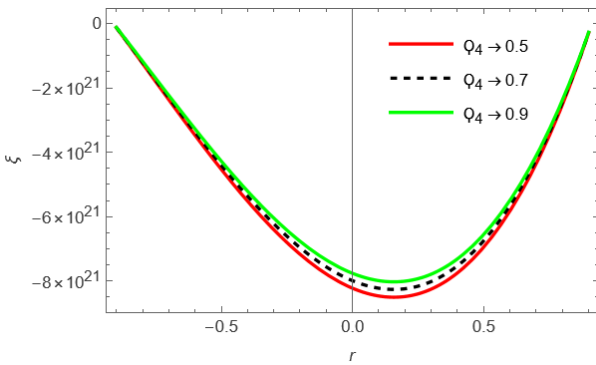


Figure (3-33): Concentration profile for different values of Q_4

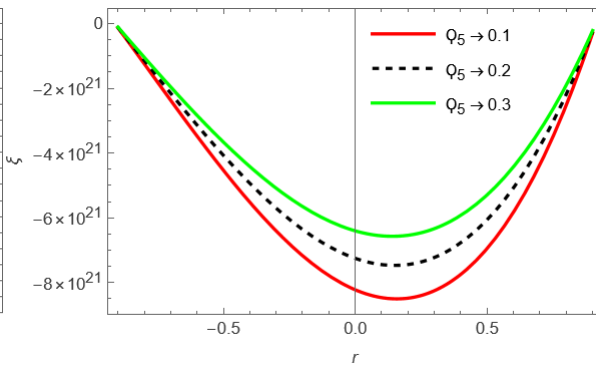


Figure (3-34): Concentration profile for different values of Q_5

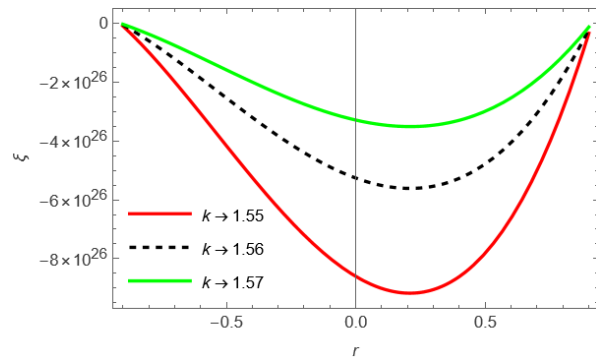


Figure (3-35): Velocity profile for various values of k

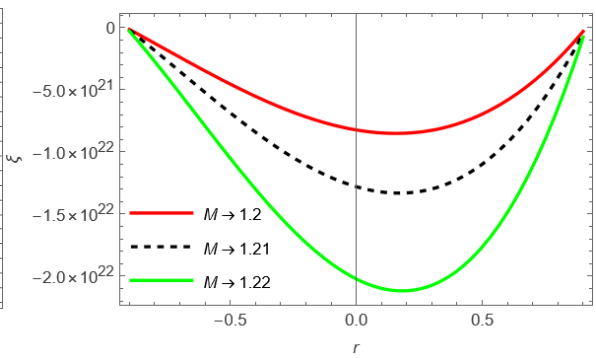


Figure (3-36): Velocity profile for various values of M

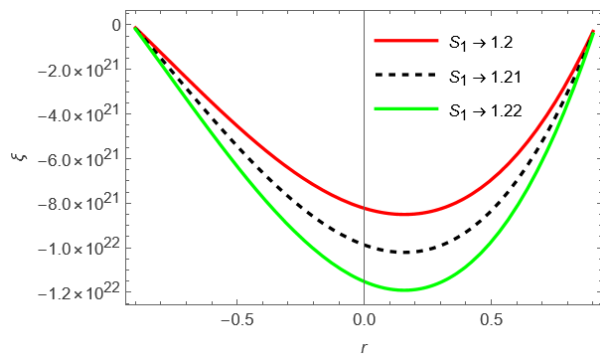


Figure (3-37): Concentration profile for different values of S_1

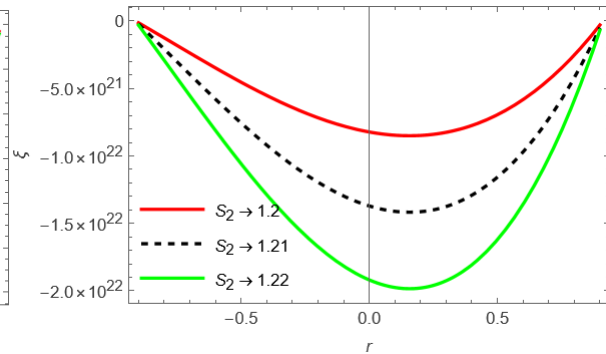


Figure (3-38): Concentration profile for different values of S_2

7. Conclusions

The above effort was made to explore the flow analysis and heat transfer of a Carreau fluid influenced by magnetism (MHD) in a curved flow configuration with distance-dependent viscosity:

- The velocity flow rises with the increase of Q_1, M, W_e, ϕ , and α , while decreases the velocity is due to the increase in parameter Q_2, Q_3, Q_4 , and Q_5 , we notice an increase in velocity under the effect of temperature on viscosity is less than the effect of distance on viscosity
- The temperature of fluid rises with increasing M, B_r, Q_1, ϕ and k while it decreases with increasing Q_2, Q_3, Q_4 , and Q_5 .
- The concentration of fluid rises with increasing Q_2, Q_3, Q_4, k , and Q_5 while it decreases with S_1, S_2, M , and Q_1 .
- We have noticed an increase in the load size by increasing the values of W_e , and ϕ , and that the number and size of vortices decrease with the increase of these parameters Q_2, Q_3, Q_4 .

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