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المستخلص

ان الهدف الرئيسي من هذا العمل هو تقديم نوع خاص (حسب علمنا) من الدوال السديدة أسميناه الدوال السديدة بقوة- N وأعطينا بعض الخواص عن هذا المفهوم .

Strongly N-proper Functions

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Abstract

The main goal of this work is to create special type of proper functions namely , strongly
N-proper functions .Also give some properties and equivalent statements of this concept .

Introduction

One of the very important concepts in a topology is the concept of functions .There are several types of functions in this work , we study an important class of functions namely , strongly N-proper functions . Proper function was introduced by Bourbaki.

AL-Omari , A . and Noorani , M.S.M in [2] defined an N-open set .

This work consists of three sections .Section one includes the fundamental concepts in general topology and proves some results which we think that , we will be needed in the next sections .In section two we review some basic definitions ,examples and propositions about some functions which are needed in section three .Section three introduces the definition of strongly N-proper function and proves some results on this subject .

Finally , a word "space" in this work means a topological space .

1. Basic concepts

1.1 Definition[2] :

A subset A of space X is said to be an N-open if for every $x \in A$ there exists an open subset in X contains x such that U_x-A is a finite set. The complement of an N-open set is said to be an N-closed set . The family of all N-open subsets of a space (X,T) is denoted by T^N

1.2 Remark[2] :

Every open (closed) set is an N-open(N-closed) set .
The converse is not true in general as the following example shows :
Let X any set contains more than one point and T be indiscrete topology on X , let $x \in X$ .Then $X-\{x\}$ is an N-open set in X but not an open and $\{x\}$ is an N-closed set in X but not a closed set **.**

1.3 Theorem [2]:

Let (X,T) be a space . Then (X,T^N) is a topological space .

1.4 Definition[2]:

Let X be a space and $A \subseteq X$. The intersection of all N-closed sets of X containing A is called the N-closure of A and is denoted by $\overline{A}^N$.

1.5 Proposition :

Let (X,T) be a space and $A,B \subseteq X$ .Then :

i. $\overline{A}^N$ is an N-closed set .

ii. A is an N-closed set if and only if $A = \overline{A}^N$.

iii. $x \in \overline{A}^N$ if and only if for every an N-open set U containing $x, U \cap A \neq \phi$.

iv. $\overline{A}^N \subseteq \overline{A}$.

Proof :

Clear .

1.6 Remark :

The product of two non empty N-open sets is not necessary be an N-open set as the following example shows :

Let N be the set of all positive integer numbers , T be final segment topology i.e $T = \{\phi, N\} \cup \{U_n : n \in N\}$ such that $U_n = \{n, n+1, ...\}$ and let $A = \{0,2,3,4,\ldots\}$ then A and N are N-open sets . But $A \times N$ not an N-open set since $N \times N$ only open set contain $(0,0) \in A \times N$ but $N \times N - A \times N$ be infinite set .

1.7 Proposition:

Let X and Y be spaces and let A , B are non empty subsets of X and Y (respectively) such that $A \times B$ is an N-open set in $X \times Y$ .Then A and B are N-open sets in X and Y (respectively) .

Proof :

Let $x \in A$ .Then $(x, y) \in A \times B$ for some $y \in B$ and since $A \times B$ is an N-open set in $X \times Y$ then there exist basic open set $U_1 \times U_2$ containing (x, y) in $X \times Y$ such that $(U_1 \times U_2) - (A \times B)$ be a finite set , but $(U_1 \times U_2) - (A \times B) = [(U_1 - A) \times U_2] \cup [U_1 \times (U_2 - B)]$, therefore $(U_1 - A) \times U_2$ is a finite set , and $U_1 - A$ is a finite set , thus A is an N-open set in X .

In similar way we can prove that B is an N-open set in Y .

Also the product of two non empty N-closed sets is not necessary be an N-closed set as the following example shows:

1.8 Example:

Let Z be the set of integer numbers , Ze be the set of even integer numbers and $T=\{\phi,Z,Ze\}$ be a topology on Z .Then $\{1\}$ and $Z_\circ$ are N-closed sets . But $\{1\}\times Z_\circ$ is not an N-closed set .

1.9 proposition:

Let X and Y be spaces and let A, B are non empty subsets of X and Y (respectively) such that $A\times B$ be an N-closed set in $X\times Y$ then A and B are N-closed sets in X and Y (respectively) .

Proof :

To prove A is an N-closed set in X , must prove A^c is an N-open set , let $x\in A^c$, then $(x,y)\in A^c\times Y$ for some $y\in Y$ hence $(x,y)\in(A^c\times Y)\bigcup(X\times B^c)=(A\times B\ )^c$, since $(A\times B)^c$ is an N-open set , hence there exist basic open set $U_1\times U_2$ containing (x,y) in $X\times Y$ such that $(U_1\times U_2)-(A\times B)^c$ is a finite set . But $(U_1\times U_2)-(A\times B)^c=(U_1\times U_2)\bigcap(A\ \times B)\ =(U_1\bigcap A)\ \times\ (U_2\bigcap B)$ $=(U_1-A^c)\times(U_2-B^c)$, hence (U_1-A^c) is a finite set . Thus A is an N-closed set .

In similar way we can prove that B is an N-closed set in Y .

1.10 Definition:

Let Y be a subspace of a space X . A subset B of Y is said to be an N-open set in Y if for each $y\in B$ there exist an open set U in Y containing y such that $U-B$ is a finite set .

1.11 Proposition :

Let X be a space ,$Y\subseteq X$ and if B is an N-open set in X then $B\bigcap Y$ is an N-open set in Y .

Proof :

Let $x\in B\bigcap Y$ then $x\in B$, since B is an N-open set in X , then there exist an open set U contain x such that U-B is a finite set . Hence $U\bigcap Y-B\bigcap Y$ is a finite set .Thus $B\bigcap Y$ is an N-open in Y .

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1.12 Proposition:

Let X be a space and Y be an N-open set of X , if A is an N-open set in Y then A is an N-open in X .

Proof:

Let $x \in A$,since A is N-open in Y then there exist open set W in Y contain x such that W-A is a finite set , since W an open set in Y then $W = U \bigcap Y$ (where U an open set in X) by theorem (1.3) then $U \bigcap Y$ is an N-open in X, hence for each $x \in U \bigcap Y$ there exist an open set V_x in X such that $V_x - (U \bigcap Y)$ is a finite set , thus $V_x - A$ is a finite. Therefore A is an N-open in X .

1.13 Remark:

If Y is not an N-open in X ,then proposition (1.12) is not true in general as the following example shows :

Let R be the set of real numbers , U be usual topology on R and let $Y = \{1,2\}$ then $\{1\}$ is an N-open in a subspace Y . But $\{1\}$ not an N-open in R .

1.14 proposition:

Let X be a space and Y be an N-closed set of X . If A is an N-closed set in Y , then A is an N-closed set in X .

Proof :

To show that $X - A$ is an N-open set in X , let $x \in X - A$ then either $x \in X - Y$ or $x \in Y - A$, if $x \in X - Y$, since Y is an N-closed set in X then $X - Y$ is an N-open set in X , hence there exist an open set U in X contain x such that $U - (X - Y)$ be a finite set and since $A \subseteq Y$ then $X - Y \subseteq X - A$,hence $U - (X - A)$ is finite set .Thus $X - A$ is an N-open set in X .

Now if $x \in Y - A$, since $(Y - A)$ is an N-open set in Y then there exist an open set in Y V such that $V - (Y - A)$ be a finite set. Hence $V = W \bigcap Y$ (W is an open set in X) and $W \bigcap Y - (Y - A) = W \bigcap Y \bigcap A$ be a finite set ,since $A \subseteq Y$ thus $W \bigcap A$ be a finite set .Therefore $W - (X - A)$ be a finite set .

1.15 Definition [9]*:*

A net in a set X is a function $\chi : D \to X$ where D is directed set .The point $\chi(d)$ is usually denoted by χ_d .

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1.16 Theorem [9]:

Let X be a space and A be a subset of X , $x \in X$. Then $x \in \overline{A}$ if and only if there is a net in A which converges to x.

1.17 Definition:

Let X be a space and B is any subset of X. N- neighborhood of B is any subset of X contains an N- open set containing B . N- neighborhoods of a subset {x} consisting of single point are also called N- neighborhoods of the point x. The collection of all N-neighborhoods of the subset B of X is denoted by $N_N(B)$ .In particular, the collection of all N-neighborhoods of x is denoted by $N_N(x)$.

1.18 Definition:

let $(\chi_d)_{d\in D}$ be a net in a space X, $x \in X$ .Then $(\chi_d)_{d\in D}$ N-converges to x. if $(\chi_d)_{d\in D}$ is eventually in every N-neighborhood of x (written $\chi_d \xrightarrow{N} x$). The point x is called N-limit point of $(\chi_d)_{d\in D}$

1.19 Definition :

Let $(\chi_d)_{d\in D}$ be a net in a space X , $x \in X$ .Then $(\chi_d)_{d\in D}$ is said to have x as an N-cluster point if $(\chi_d)_{d\in D}$ is frequently in every an N-neighborhood of x . (written $\chi_d \overset{N}{\propto} x$).

1.20 Proposition:

Let (X,T) be a space , $x \in X$ and $A \subseteq X$,Then $x \in \overline{A}^N$ if and only if there is a net $(\chi_d)_{d\in D}$ in A such that $\chi_d \overset{N}{\propto} x$.

Proof :Let $x \in \overline{A}^N$ then $U \cap A \neq \phi \ \ \forall U \in T^N$, $x \in U$ .Notice that $(N_N(x), \subseteq)$ is a directed set .
such that for all $U_1, U_2 \in N_N(x)$, $U_1 \geq U_2$ if and only if $U_1 \subseteq U_2$. Since for all $U \in N_N(x)$, $U \cap A \neq \phi$. Then we can define a net $\chi : N_N(x) \to X$ as follows $\chi(U) = \chi_u \in U \cap A$, $U \in N_N(x)$. Now to prove that $\chi_u \overset{N}{\propto} x$, let $B \in N_N(x)$ and

let $U \in N_N(x)$ then $B \bigcap U \in N_N(x)$. Since $B \bigcap U \subseteq U$, then $B \bigcap U \geq U$, $\chi(B \bigcap U) = \chi_{B \cap U} \in B \bigcap U \subseteq B$. Hence $\chi_d \overset{N}{\propto} x$.

Conversely:

suppose that there is a net $(\chi_d)_{d \in D}$ in A such that $\chi_d \overset{N}{\propto} x$, and let $U \in T^N$ $x \in U$, since $\chi_d \overset{N}{\propto} x$ then $(\chi_d)_{d \in D}$ is frequently in U. Thus $U \bigcap A \neq \phi \quad \forall U \in T^N$ $x \in U$. Hence $x \in \overline{A}^N$.

1.21 Definition :

A space X is called N-compact if every N-open cover of X has a finite sub cover .

1.22 Proposition[2]:

A space X is a compact if and only if every N-open cover of X has a finite sub cover .

1.23 Theorem [2]:

Let X any space then X is a compact if and only if every proper N-closed set is a compact with respect to X .

2- Certain Types Of N- functions

2.1 Definition:

Let $f : X \to Y$ be a function from a space X into a space Y .Then :

i. f is called an N-continuous function if $f^{-1}(A)$ is an N-open set in X for every open set A in Y .[2]

ii. f is called an N-irresolute function if $f^{-1}(A)$ is an N-open set in X for every N- open set A in Y.

2.2 Example:

Let f be a function of a space (X,T) into a space (Y,T') then :

i. The constant function is an N-continuous (N-irresolute) function .
ii. If X is discrete space then f is an N-continuous (N-irresolute) function .
iii. If X is finite set and T any topology on X then f is an N-continuous **.**

2.3 Example:

Let U be usual topology on R (set of real numbers) and T be discrete topology on Y={1,2} then the function $f : R \to Y$ defined by $f(x) = \begin{cases} 1 & if x \in Q \\ 2 & if x \in Q^c \end{cases}$ is not an N-continuous. Since $f^{-1}(\{1\})$ is not an N-open set in R .

2.4 Remark:

i. Every continuous function is an N-continuous function .
ii. Every N-irresolute function is an N-continuous function .

The converse of (i,ii) is not true in general as the following examples show :

Let X={a ,b} ,Y ={c ,d}, T be indiscrete topology on X and $T' = \{\phi, Y, \{c\}\}$ be a topology on Y . Then the function $f : X \to Y$ defined by f(a)=c , f(b)=d is an N-continuous , but not a continuous.

Notice that in example (2.3) if T indiscrete topology on Y then f is an N-continuous but f is not an N-irresolute .

2.5 proposition[2]:

Let $f : X \to Y$ be one to one , continuous function from a space X into a space Y then $f^{-1}(A)$ is an N-open set in X for every N- open set A in Y **.**

2.6 Proposition :

Let $f : X \to Y$ be an N-continuous function and $A \subseteq X$ then the restriction function $f|_A : A \to Y$ is an N-continuous .

Proof: Let B is an open set in Y , since f is an N –continuous then $f^{-1}(B)$ is an N-open set in X , by proposition (1.11) $f^{-1}(B) \cap A$ is an N-open set in A , but $(f|_A(B))^{-1} = f^{-1}(B) \cap A$ hence $(f|_A(B))^{-1}$ is an N-open in A . Thus $f|_A : A \to Y$ is an N-continuous.

A composition of two N-continuous functions is not necessary be an N-continuous function as the following example shows :

2.7 Example :

Let Z be set of integer numbers , Y={1,2} ,W={3,6} .Let T and T' are indiscrete topologies on Z and Y (respectively) .If $T''=\{\phi, W, \{3\}\}$ is a topology on W then the functions

$f : Z \to Y$ defined by $f(z)=\begin{cases} 1 & if z \in Z_{\circ} \\ 2 & if z \in Z_e \end{cases}$ and $g : Y \to W$ defined by g(1)=3 , g(2)=6 are N-continuous functions . But $g \circ f$ is not an N-continuous function .Since $(g \circ f)^{-1}\{3\}$ not an N- open set in X .

2.8 Proposition :

Let X , Y and Z be spaces , Let $f : X \to Y$ and $g : Y \to Z$ be functions . Then :

i. If f and g are N-irresolute functions then $g \circ f$ is an N-irresolute function .

ii. If f is an N-irresolute and g is an N-continuous then $g \circ f$ is an N-continuous .

Proof:

Obvious .

2.9 Remark :

The product of two N-continuous functions is not necessary be an N-continuous function as the following example shows**:**

Let N be the set of positive integer numbers and T be final segment topology on N , let τ be the co-finite topology on N .Then identity functions $f_i : (N,T) \to (N,\tau)$ $i=1,2$ are N-continuous functions , but $f_1 \times f_2$ is not an N-continuous function ,since $(N-\{1,2\}) \times N$ be an open set in $N \times N$. But $(f_1 \times f_2)^{-1}(N-\{1,2\} \times N)$ is not an N-open set .

2.10 Proposition:

Let $f_i : X_i \to Y_i$, i=1,2 be functions such that $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ be an N-continuous function then f_i are N-continuous .

Proof :

Let $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ be an N-continuous .To prove $f_1 : X_1 \to Y_1$ is an N-continuous . Let V is an open set in Y_1 , then $V \times Y_2$ is an open set in $Y_1 \times Y_2$, since $f_1 \times f_2$ is an N-continuous then $(f_1 \times f_2)^{-1}(V_1 \times Y_2) = f^{-1}(V) \times f_2^{-1}(Y_2)$ is an N-open set in $X_1 \times X_2$. Hence $f_1^{-1}(V)$ is an N-open subset in X_1 therefore $f_1 : X_1 \to Y_1$ is an N-continuous **.**

In similar way we can prove that $f_2 : X_2 \to Y_2$ is an N-continuous .

2.11 Proposition :

Let $f : X \to Y$ be an N-irresolute function . The image of any compact set A in X is a compact subset of Y .

Proof: Let $\{V_\propto : \propto \in \Lambda\}$ be open cover of $f(A)$ then $\{V_\propto : \propto \in \Lambda\}$ be N-open cover of $f(A)$. Since f is an N-irresolute then $\{f^{-1}(V_\propto) : \propto \in \Lambda\}$ is N-open cover of A , but A is a compact set in X then by proposition (1.22) A has finite sub cover ,i.e $A \subseteq \bigcup_{i=1}^{n} \{f^{-1}(V_{\alpha i})\}$ hence $f(A) \subseteq \bigcup_{i=1}^{n} (V_{\alpha i})$. Therefore $f(A)$ is a compact set in Y **.**

2.12 Definition :

Let f be a function from a space X into a space Y .Then :

i. f is called an N-closed(an N-open) function if $f(A)$ is an N-closed(an N-open) set in Y for every closed(open) set A in X . [2]

ii. f is called strongly N-closed(strongly N-open) function if $f(A)$ is an N-closed(N-open) set in Y for every N-closed(N-open) set A in X.

We denoted for brief strongly N-closed(strongly N-open) function by (St-N-closed) (St-N-open) respectively .

2.13 Example:

i. A constant function is N-closed (St-N-closed) function .

ii. Let $f: X \to Y$ is a function from a space X into a space Y such that Y be a finite set then f is an N-closed (St-N-closed) function .
iii. Let $f:(Z,T) \to (Z,T')$ be function defined by $f(z)=2z$ for all $z \in Z$, $T=\{\phi, Z, Ze\}$ be a topology on Z and T' be indiscrete topology on Z then f is not a St- N-closed function .

2.14 Remark :

Every closed(open) function is an N-closed (N-open) function .
The converse is not true in general as the following example shows :

2.15 Example:

Let X={1,2,3},Y={4,5} , $T=\{\phi, X, \{3\}\}$ be a topology on X and τ be indiscrete topology on Y. Then a function $f: X \to Y$ defined by f(1)=f(2)=4 , f(3)=5 is an N-closed (N-open) function . But f is not closed (open) function **.**

2.16 Proposition :

Let X , Y and Z be spaces .If $f: X \to Y$ and $g: Y \to Z$ are a St-N-closed function then $g \circ f: X \to Z$ is a St-N-closed function .

Proof:

Clear.

2.17 proposition[2] :

L et $f: X \to Y$ be an open function then f is a St-N-open function **.**

2.18 Proposition

Let $f: X \to Y$ be open , bijective function then f is a St-N-closed function .

Proof :

Let $f: X \to Y$ be open , bijective function and let A be an N-closed set in X then A^c is an N-open in X and by proposition(2.17) $f(A^c)$ is an N-open set in Y, since f is bijective then $f(A^c)=(f(A))^c$. Thus $(f(A))^c$ is an N-open set , therefore f(A) is an N-closed set in Y .

3-Strongly N-proper functions

3.1 Definition:

Let f be a function of a space X into a space Y , f is said to be Strongly N-proper function (St-N- proper) if :

i f is an N- continuous function .
ii The function $f \times I_Z : X \times Z \to Y \times Z$ be a St-N-closed for every space Z .

3.2 Example:

Let X={1,2,3}, Y={2,4,6} , let T be indiscrete and T' ={ ϕ ,Y,{4}} are topologies on X , Y (respectively) then the function $f : X \to Y$ defined as $f(x) = 2x \quad \forall x \in X$ is a St-N-proper function since

i. $f : X \to Y$ is an N- continuous function .

ii. A function $f \times I_Z : X \times Z \to Y \times Z$ is a St-N-closed for every space Z , since $f \times I_Z$ is open and bijective function then by proposition (2.18) $f \times I_Z$ is a St-N-closed function .

The following example shows not every function is a St- N-proper.

3.3 Example:

Let Z be the set of integer numbers and Z_e be set of even integer numbers , let T={ ϕ ,Z, Z_e } be a topology on Z and T' be indiscrete topology on Z then identity function $f : (Z,T) \to (Z,T')$ is not a St- N-proper function . Since if (Z, T'') be indiscrete space then $f \times I_Z : Z \times Z \to Z \times Z$ is not a St- N-closed function , since $Z_\circ \times Z$ is an N- closed set in $Z \times Z$. But $f \times I_Z\,(Z_\circ \times Z)$ is not a St- N-closed set in $Z \times Z$.

3.4 proposition :

Every St - N-proper function is a St- N-closed **.**

Proof :

Let $f : X \to Y$ be a St-N-proper .Then The function $f \times I_Z : X \times Z \to Y \times Z$ be a St- N-closed for every space Z . Let Z={p} then

$f : X \overset{M}{\simeq} X \times \{p\} \xrightarrow{f \times I_p} Y \times \{p\} \overset{h}{\simeq} Y$ is a St- N-closed function .Where $M : X \to X \times \{p\}$ is homeomorphism from X into $X \times \{p\}$ and $h : Y \times \{p\} \to Y$ is homeomorphism from $Y \times \{p\}$ into Y .Since $f = h \circ (f \times I_p) \circ M$ and the function $h \circ (f \times I_p) \circ M$ is an N-closed by proposition (2.18) and proposition (2. 16) . Hence f is a St-N-closed .

The converse of last proposition is not true in general as the following example shows :

3.5 Example

Let $f : (R,U) \to (R,U)$ be function defined by $f(x) = 0$ for all $x \in R$ then f is a St- N-closed . But $f \times I_R : R \times R \to \{0\} \times R$ is not a St- N-closed function where U be usual topology on R , since if A=$\{(x, y) : xy = 1\}$ then A is an N-closed set in $R \times R$. Let V be an N-open set contain (0,0) . Then $V \cap (\{0\} \times (R\text{-}\{0\})) \neq \phi$ and by proposition (1.5 iii) $(0,0) \in \overline{(\{0\} \times (R - \{0\}))}^N$. But $(f \times I_R)(A) = \{0\} \times (R - \{0\})$. Thus $(0,0) \in \overline{(f \times I_R)(A)}^N$ and since $(0,0) \notin (\{0\} \times (R - \{0\}))$ then $(0,0) \notin (f \times I_R)(A)$. Hence $(f \times I_R)(A)$ is not an N-closed set in $R \times R$. Thus $f \times I_R : R \times R \to \{0\} \times R$ is not a St- N-closed function , and hence f is not a St- N-proper function .

The product of two St- N-proper functions is not necessary be a St- N-proper as the following example shows :

3.6 Example:

Let $f_i : (N,T) \to (N,T')$ be identity functions where i=1,2 , T be final segment topology on N and T' be co-finite topology on N then f_i are N-continuous functions and $f_i \times I_Z$ for every space Z are St- N-closed functions by using proposition (2.18) . Hence f_i are St-N-proper functions .But product of f_i is not a St- N-proper function since the product of f_i is not an N-continuous .

3.7 Proposition [4]:

Let $f : X \to p = \{w\}$ be a function on a space X . If f is a proper then X is a compact space . where w is any point which does not belong to X .

Simple verification shows that this result remain valid when $f : X \to p = \{w\}$ is a St-N-proper .

3.8 Proposition :

Let X and Y be spaces and $f : X \to Y$ be an N-irresolute function and X a compact space .Then the following statement are equivalent :

i. f is a St- N-proper function .
ii. f is a St- N-closed function and $f^{-1}\{y\}$ is a compact for each $y \in Y$.
iii. If $(\chi_d)_{d\in D}$ is a net in X and $y \in Y$ is an N-cluster point of $f(\chi_d)$ then there is an N-cluster point $x \in X$ of $(\chi_d)_{d\in D}$ such that f(x)=y .

Proof *:*

$(i) \to (ii)$ Let $f : X \to Y$ be a St- N-proper function . Then by proposition (3.4) f is a St-N-closed function . Now , let $y \in Y$ then $\{y\}$ is an N-closed set in Y , since $f : X \to Y$ be an N-irresolute then $f^{-1}\{y\}$ is an N-closed set in X and since X is a compact then by proposition (1.23) $f^{-1}\{y\}$ is a compact in X .

$(ii) \to (iii)$ Let $(\chi_d)_{d\in D}$ be a net in X and $y \in Y$ be an N-cluster point of a net $f(\chi_d)_{d\in D}$ in Y . Claim $f^{-1}(y) \neq \phi$, if $f^{-1}(y) = \phi$ then $y \notin f(X) \to y \in ((f(X))^c$ since X is an N-closed set in X . Then f(X) is an N-closed set in Y .Thus $((f(X))^c$ is an N-open set in Y . Therefore $f(\chi_d)$ is frequently in $((f(X))^c$. But $f(\chi_d) \in f(X), \forall d \in D$ then $f(X) \bigcap ((f(X))^c \neq \phi$, and this is a contradiction . Thus $f^{-1}(y) \neq \phi$. Now , suppose that the statement (iii) is not true .That means , for all $x \in f^{-1}(y)$ there exist an N-open U_x in X contains x such that (χ_d) is not frequently in U_x . Notice that $f^{-1}(y) = \bigcup_{x\in f^{-1}(y)} \{x\}$.

Therefore the family $\{U_x | x \in f^{-1}(y)\}$ is N- open cover of $f^{-1}(y)$. But $f^{-1}(y)$ is a compact set , thus by proposition (1.22) there exist $x_1, \ldots, x_n$ such that

$f^{-1}(y) \subseteq \bigcup_{i=1}^{n} U_{xi}$, $f^{-1}(y) \bigcap \left[\bigcup_{i=1}^{n} U_{xi}\right]^c = \phi \to f^{-1}(y) \bigcap \left[\bigcap_{i=1}^{n} U_{xi}^c\right] = \phi$. But $(\chi_i)_{i\in\Lambda}$ is not frequently in U_{xi} $\forall i = 1, \ldots, n$. Thus (χ_d) is not frequently in $\bigcup_{i=1}^{n} U_{xi}$. But $\bigcup_{i=1}^{n} U_{xi}$ is an N-open set in X , then $\bigcap_{i=1}^{n} U_{xi}^c$ is an N- closed set in X .

Thus $f(\bigcap_{i=1}^{n} U_{xi}^c)$ is an N-closed set in Y .

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Claim $y \notin f(\bigcap_{i=1}^{n} U_{xi}^{c})$, if $y \in f(\bigcap_{i=1}^{n} U_{xi}^{c})$ then there exist $x \in \bigcap_{i=1}^{n} U_{xi}^{c}$ such that f(x)=y , thus $x \notin \bigcup_{i=1}^{n} U_{xi}$. But $x \in f^{-1}(y)$ therefore $f^{-1}(y)$ is not a subset of $\bigcup_{i=1}^{n} U_{xi}$ and this a contradiction . Hence there is an N –open set A in Y such that $y \in A$ and $A \cap f(\bigcap_{i=1}^{n} U_{xi}^{c}) = \phi \rightarrow \quad f^{-1}(A) \cap f^{-1}(f(\bigcap_{i=1}^{n} U_{xi}^{c}) = \phi$ then $f^{-1}(A) \cap (\bigcap_{i=1}^{n} U_{xi}^{c}) = \phi \rightarrow f^{-1}(A) \subseteq \bigcup_{i=1}^{n} U_{xi}$. But $f(\chi_d)$ is frequently in A , then (χ_d) is frequently in $f^{-1}(A)$ and then (χ_d) is frequently in $\bigcup_{i=1}^{n} U_{xi}$. This is contradiction .Thus $x \in f^{-1}(y)$ and $\chi_d \overset{N}{\propto} x$ and f(x)=y .

$(iii) \rightarrow (i)$ Let Z be any space . To prove that $f \times I_Z : X \times Z \rightarrow Y \times Z$ is a St- N-closed function . Let F be an N- closed set in $X \times Z$. To prove that $f \times I_Z(F)$ is an N-closed set in $Y \times Z$. Let $(y,z) \in \overline{f \times I_Z(F)}^{N}$. Then by proposition (1.20) there exist a net $\{(Y_d, Z_d)\}_{d \in D}$ in $(f \times I_Z)(F)$ such that $(Y_d, Z_d) \overset{N}{\propto} (y,z)$. Then $(Y_d, Z_d) = (f \times I_z)(\chi_d, Z_d) where \ \{(\chi_d, Z_d)\}_{d \in D}$ is a net in F . Thus $(f(\chi_d), I_Z(Z_d)) \overset{N}{\propto} (y,z)$. Hence $f(\chi_d) \overset{N}{\propto} y$ and $Z_d \overset{N}{\propto} z$. Then there is $x \in X$ such that $\chi_d \overset{N}{\propto} x$ and f(x)=y . Hence $(\chi_d, Z_d) \overset{N}{\propto} (x,z)$ and $\{(\chi_d, Z_d)\}_{d \in D}$ is a net in F . Thus by proposition (1.20) $(x,z) \in \overline{F}^{N}$ since $F = \overline{F}^{N}$ then $(x,z) \in F \rightarrow (y,z) = (f \times I_Z)(x,z)$, hence $(y,z) \in (f \times I_Z)(F)$ thus $\overline{(f \times I_Z)}^{N}(F) = (f \times I_Z)(F)$. Hence $(f \times I_Z)(F)$ is an N-closed set in $Y \times Z$, hence $f \times I_Z : X \times Z \rightarrow Y \times Z$ is a St- N-closed function .Thus $f : X \rightarrow Y$ is a St- N-proper .

3.9 Remark:

A composition of two St- N-proper functions is not necessary be a St - N-proper function . Since the composition of two N-continuous functions is not necessary be an N-continuous function .

3.10 Proposition :

Let X , Y and Z be spaces , let $f : X \to Y$ and $g : Y \to Z$ be St-N-proper functions such that $f : X \to Y$ is an N-irresolute . Then $g \circ f : X \to Z$ is a St-N-proper function .

Proof :

i. Since f is an N-irresolute and g is an N-continuous then by proposition (2.8 ii) $g \circ f$ is an N-continuous.

ii. Let Z_1 any space .Since f is a St-N-proper then $f \times I_{Z_1} : X \times Z_1 \to Y \times Z_1$ is a St-N-closed . Similarly , since g is a St-N-proper then $g \times I_{Z_1} : Y \times Z_1 \to Z \times Z_1$ is a St-N-closed . Thus by proposition (2.16) the function $(g \times I_{Z_1}) \circ (f \times I_{Z_1}) : X \times Z_1 \to Z \times Z_1$ is a St-N-closed . But $(g \circ f) \times I_{Z_1} \equiv (g \times I_{Z_1}) \circ (f \times I_{Z_1})$, hence $(g \circ f) \times I_{Z_1} : X \times Z_1 \to Z \times Z_1$ is a St-N-closed .Thus $g \circ f : X \to Z$ is a St-N-proper function .

3.11 Proposition:

Let X , Y and Z be spaces , Let $f : X \to Y$, $g : Y \to Z$ be N-irresolute functions , such that X is a compact space and $g \circ f : X \to Z$ is a St-N-proper function .Then :

i. If f is onto then g is a St- N-proper function .

ii. If g is one to one then f is a St- N-proper function **.**

Proof:

i. Let F be an N-closed subset of Y, since f is an N- irresolute then $f^{-1}(F)$ is an N-closed in X. Since $g \circ f$ is a St-N-proper function then $g \circ f$ is an N-closed function , hence $g \circ f(f^{-1}(F))$ is an N-closed in Z . But f is onto then $g \circ f(f^{-1}(F)) = g(F)$. Hence $g(F)$ is an N-closed in Z . Thus g is a St- N-closed function . Now , let $z \in Z$, since $g \circ f$ is a St- N-proper function and X is compact , then by proposition (3.8) the set $(g \circ f)^{-1}(\{z\}) = f^{-1}(g^{-1}(\{z\}))$ is a compact set in X. Since f is an N-irresolute and onto then by proposition (2.11) $f(f^{-1}(g^{-1}(\{z\})) = g^{-1}(\{z\})$ is a compact in Y. Clear that $f(X) = Y$ is a compact . So by proposition (3.8) the function g is a St-N-proper function .

ii. To prove f is a St-N-closed function . Let F be an N- closed set of X .Then $(g \circ f)(F)$ is an N-closed set in Z . Since $g: Y \to Z$ is an N- irresolute and one to one function then $g^{-1}(g(f(F)) = f(F)$ is an N-closed in Y. Hence the function $f: X \to Y$ is a St-N-closed .Now , Let $y \in Y$ then $g(y) \in Z$, Let $z_1 = g(y)$. Since $g \circ f: X \to Z$ is a St-N-proper , X is a compact space and g one to one function then the set $(g \circ f)^{-1}(z_1) = f^{-1}(g^{-1}(z_1)) = f^{-1}\{y\}$ is a compact in X .Therefore by proposition (3.8) the function $f: X \to Y$ is a St-N-proper .

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