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Hybrid Harris Hawks Optimization with Ant Lion Optimization Algorithm for Mathematical Intelligence Systems: A Computer Vision Approach

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ABSTRACT

This research is a comparative study of Harris Hawks Optimization (HHO), Ant Lion Optimization (ALO) and Hybrid HHO-ALO algorithm to solve Computer Vision (CV) Approach problem as a complex numerical optimization problem. The proposed hybrid algorithm marries the best aspects of both algorithms, namely good exploitative performance of ALO and the good exploration performance of HHO, while nominally addressing the drawback of each algorithm: premature convergence and slow convergence near the global optimum. To assess the performance of the algorithms in terms of convergence speed, robustness and solution accuracy, some well-known benchmark functions have been used, including Sphere, Rastrigin, Ackley, Rosenbrock and Griewank. The simulation results show that the hybrid HHO-ALO algorithm can achieve better convergence speed and reduce the mean square error (MSE) than single HHO and ALO algorithm under various optimization landscapes, and the algorithm has better optimization performance than any other algorithm no matter which landscape. Furthermore, the hybrid approach proves itself better suited for dealing with multimodal, non-convex and trickily designed search spaces, as it seems to be very effective in escaping local minimum and thus ensuring more stable performance in optimization. Based on the findings of this study, HHO-ALO can be an effective and robust optimization algorithm to be used in complex engineering and computational problems, which further shows the potential of using the hybrid optimization algorithm when the adaptive optimization can be constructed and adapted to real constraints.

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1. Introduction

Optimization is an essential component of contemporary engineering systems, especially when it comes to control applications where there must be several performance targets that have to be met concurrently [1]. New technologies, such as artificial intelligence (AI) and mathematical intelligence systems, are used increasingly for designing control systems due to their increasing complexity and nonlinear characteristics and the limited knowledge of operating conditions [2-5]. The mathematical intelligence systems, which are based on the advanced mathematical modeling, adaptive learning, and intelligent optimization strategies, offer a strict approach to studying

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the dynamics of complex systems and allow exploring the high-dimensional parameter space with efficiency [6]. These systems can be used to improve the capability of improving dynamic performance measures like stability, speed of response, robustness, and precision by incorporating analytical underpinnings and clever search systems [7, 8]. Consequently, AI-based and mathematically smart optimization strategies have become the key to getting beyond the constraints of the traditional ways of tuning the control system and attaining higher control system performance in harsh and uncertain conditions [8, 9].

The nature of control systems that work in practical conditions usually demonstrates nonlinear dynamics, uncertainties in the parameters, and strict performance requirements, complicating the task of controller parameter tuning [10]. The damping ratio (ζ) is one of the parameters that influences the behavior of a system, especially when it comes to effecting transient response of dynamic systems [11]. Proper choice of this parameter is critical to guarantee positive attributes of low overshoot, short settling time, and constant system behaviour, particularly when using a closed-loop arrangement [12].

Figure 1 depicts a typical closed-loop second-order control system with the system dynamics being characterized by the natural frequency, ω and the damping ratio, ζ . The system response to external inputs is altered variably by the effects of the system on transient performance as well as steady-state performance through the influence of zeta. Poor tuning of the damping ratio can result in excessive oscillations, a long settling period, or even too slow responses, none of which is good in terms of general system performance and reliability [13, 14].

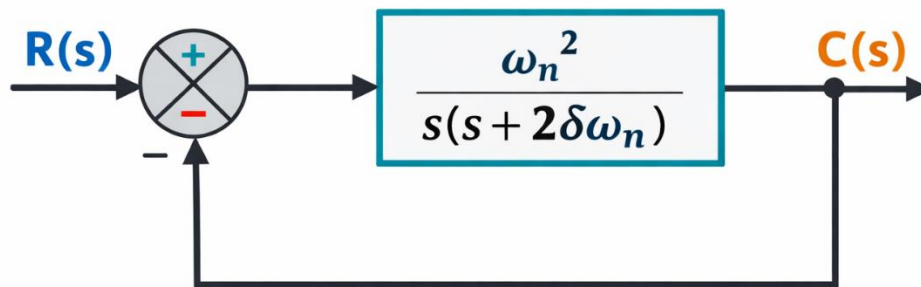


Figure 1. Closed-loop block diagram of the second-order control system.

The unit step responses in Figure 2 also show the sensitivity of the system behavior to the damping ratio of the system. As the values of zeta become smaller there is a substantial amount of overshoot and oscillatory performance, with the damping ratio larger values being employed by reducing oscillations and improving set-off properties. However, small damping ratio will give an over-damped response with less overshoot, and the response will be slower. From these observations it is seen that there is always a compromise between the response speed and stability, and the proper determination of optimal damping ratio that would meet the required performance is key [15].

Usually, the nonlinear and non-convex nature of the performance landscape of such problems poses a problem for the traditional tuning and optimization techniques (e.g. gradient ASC, and analytical) to be applied to them. Typically, these are some sort of local assumption or simplifying assumption, which make them less useful, less powerful, in the real world. More attention has thus been given to meta-heuristic and AI-inspired optimization algorithms that are more flexible, adaptive, and resilient in solving more complex optimization problems of the control system [16].

Algorithms derived from nature (e.g., Harris Hawks Optimization (HHO), Ant Lion Optimization (ALO), and other metaheuristic algorithms) have received extensive research because of their unique search mechanisms [17]. HHO is noted to have a high level of worldwide exploration due to adaptive hunting as opposed to ALO, which focuses on regional exploitation by use of antlion trapping to prey. Their differing nature can be traced to the general issue of intelligent optimization to strike a balance between exploration and exploitation, especially in the control system parameter tuning, where there are many competing dynamic performance criteria that may need to be met [18].

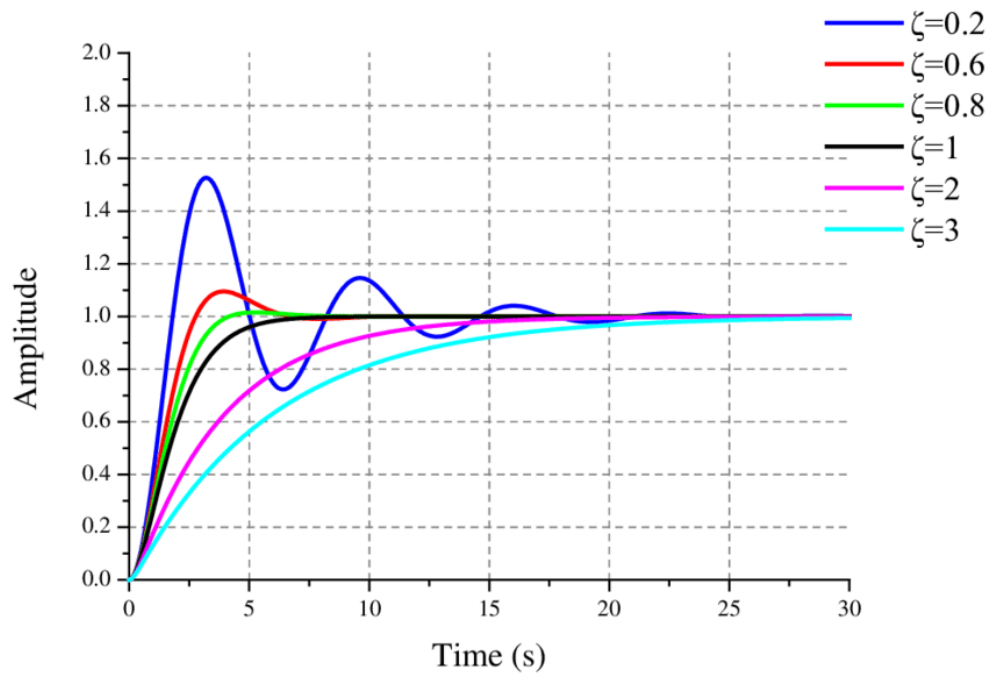


Figure 2. Effect of damping ratio on the unit step response of a second-order system.

Recent studies of nature-inspired and mathematical intelligence optimization techniques have also shown a longstanding focus on developing, improving, and combining HHO and ALO algorithms to solve nonlinear, multimodal, complex optimization problems in engineering and control-related problems. The hybrid Harris Hawks Optimizer-Arithmetic Optimization Algorithm is presented by the authors (CetinbaS, Tamyurek, and Demirtas, 2022) to optimize the choice of microgrid system sizes and design to optimize its quality through a hybridization of the algorithms, and in this case the hybridization significantly enhanced the convergence and quality of the solutions in microgrid systems [19]. Based on Harris Hawks optimization, Zeng et al. (2023) introduced a mixed pinhole imaging strategy with Harris Hawks Optimization (MHHO) algorithm that uses a pinhole imaging strategy and other search algorithms to improve global exploration and convergence and yield better results on benchmark test functions and real constrained optimization [20]. Gogula and Vakula (2024) used a multi-objective Harris Hawks Optimization algorithm to optimize the location and rating of distributed generation units in power systems to create better voltage profiles and lower active power losses than traditional optimization mechanisms in multi-objective environment [21]. Kumar et al. (2024) proposed a hybrid HGO-Sine Cosine algorithm to solve the problem of congestion control in power transmission networks, achieving a better balance between exploration and exploitation and better optimization results than traditional metaheuristics in the field of hybrid variants [22]. Similar work has also been common in ALO and variants; Son (2023) has suggested a hybrid algorithm Ant Lion Optimizer, which adds more search methods to overcome traps in local optima and deliver more accurate solutions to benchmark functions [23], and Kalita et al. (2024) have proposed a Many-Objective Ant Lion Optimizer (MaOALO) to many-objective engineering problems, which exhibits stronger convergence and offers diversity in complex test suites [24]. To complement these advancements, Sathyamurthy et al. (2025) introduced a spatial hybrid Ant Lion optimization strategy, which admits to a successful adaptable equilibrium between ant and antlion search dynamics in order to attain strong performance on challenging objective landscapes [25]. All these publications represent an obvious research direction of hybridization, the improvement of algorithms, and multi-objective adaptation of HHO and ALO-based frameworks, which is increasingly relevant in solving high-dimensional and real-world optimization problems in which classical methods are often insufficient.

2. Proposed Methodology

The research paper will be based on a detailed comparative study of the HHO, ALO, and a hybrid HHO-ALO solution to the optimization of the damping ratio (zeta) of a second-order control system, where the damping ratio in the control system should be tuned with high accuracy to reach desired dynamic performance in terms of low

overshoot and reduced settling time. The optimal damping ratio is calculated using each algorithm, minimizing the deviation of predefined performance levels of the system and the hybrid HHO-ALO algorithm is developed by combining the robust global exploration power of HHO and the efficient local exploitation principle of ALO to address the flaws of the individual algorithms. Simulations using the MATLAB language are carried out to assess and compare the algorithms on convergence behavior, objective functions minimization, robustness and consistency of the algorithms in repeated independent runs. The reliability and stability of each algorithm in the confined optimization settings are measured by using statistical performance indicators, such as mean objective value, standard deviation, and the mean square error (MSE). The simulation findings have shown that the hybrid HHO-ALO algorithm is always better when compared to other HHO and ALO algorithms in that it gets to the convergence faster, better, and more robust to eliminate overshoot and settling time. The results demonstrate the possibilities of hybrid metaheuristic optimization methods as efficient instruments of advanced control system design and the possibility of application of these methods to real-world engineering issues with specific parameter optimization needs. The principal contributions of this work can be presented as follows:

- The comparative study of HHO, ALO and hybrid HHO-ALO algorithms in optimization of the damping ratio of the second-order control system.
- Creation and execution of a hybrid HGO-ALO optimization framework which can effectively balance global exploration and local exploitation.
- Complete evaluation of the performance in terms of statistical measures like mean objective value, standard deviation and MSE and repeated simulation runs.
- Evidence of the increased accuracy of convergence, robustness, and stability of the hybrid HHO-ALO algorithm over the independent standalone optimization tools.
- Confirmation of the suggested hybrid strategy to improve the work of the control system by reducing overshoot and minimization of the settling time.
- Determination of the possible relevance of hybrid metaheuristic optimization methods in such engineering areas as industrial automation, robotics, and power systems.

The rest of this paper will be structured in the following way. Section 3 represents the Model Setup and Methodology, with Section the simulation and performance metrics setup. Section 4 describes the Hybrid Optimization Process, including the major mechanisms of the proposed method. Section 5 is the Results and Discussion on which the performance of the optimization algorithms is discussed and compared. Section 6 includes Recommended Developments, which show the possible future research and improvements. Finally, Section 7 presents the paper to the conclusion through the closing remarks with the key findings and contributions.

3. Model Setup and Methodology

In this section, we will give a detailed description of the model configuration in which the HHO, ALO, and a hybrid of the two, the HHO-ALO algorithm, is used in optimizing a control system. The mathematical expressions, algorithmic processes and pseudo-code of each optimization method are described, the key phases of the optimization program being the initializations, system modeling, and parameters optimization, and performance measures. The section will attempt to show clearly the implementation of each algorithm and how the hybrid HHO-ALO can combine the merits of both HHO and ALO to optimize the accuracy of optimization, convergence behavior and robustness in the control system parameter tuning.

3.1. Initialization and System Design

The optimization process will start with the determination of critical parameters to the control system such as desired ζ , overshoot and settling time. These parameters are used to ensure the optimization process is done so that it can be stable and an efficient control system can be realized.

3.1.1. Control System Definition

A standard second-order mathematical model is used to model the dynamic behavior of the control system in question and is commonly utilized to examine transient performance properties such as overshoot, settling time, and stability. The representation describes the main dynamics of the system and makes it possible to evaluate

directly the effect of the damping ratio on the total response. The second-order transfer function is used to model the system to be optimized [26]:

$$H(s) = \frac{1}{s^2 + 2\zeta s + 1} \quad (1)$$

Zeta is known as the damping ratio (optimization variable), Laplace transform variable is s .

The performance indicators such as percent overshoot (OS) and settling time (ST) are analyzed in terms of step response properties of the system because the two parameters represent a direct performance measure and stability of the transient performance. In order to measure the variation between the actual system response and the desired performance specifications, a scalar objective function is developed by adding the absolute error in settling time and overshoot. Such formulation allows the use of several performance criteria to be considered at the same time as the optimization process. It has the following objective structure [26]:

$$f(\zeta) = |OS_{\text{desired}} - OS_{\text{actual}}| + |ST_{\text{desired}} - ST_{\text{actual}}| \quad (2)$$

The desired overshoot is OS_{desired} and ST_{desired} is settling time, OS_{actual} and ST_{actual} are referring to the system's actual overshoot and settling time for a given zeta.

3.2. SIMULATION AND PERFORMANCE METRICS SETUP

The simulation environment that has been used in this research is in a manner that promotes fair and consistent comparison of the optimization algorithms to be considered. The key elements of the simulation framework are as follows. The process of every optimization algorithm is 50 iterations with a population size of 20 search agents (particles or wolves, depending on the algorithm) as it offers an optimal trade-off between computation speed and search performance. In order to increase the statistical validity of the received results and eliminate the effects of random fluctuations, all simulations are done three times independently, and the results are reported in the form of the average values of the obtained results.

The algorithms are tested on a number of metrics generally accepted to measure both convergence behavior and quality of the solution. These measures are the mean objective value and it is an indicator of the median fitness obtained during the optimization process; rate of convergence which is a measure of how fast the algorithm is converging towards the optimal solution and MSE which is a measure of how close the final optimized solution will be to the target values. Combined, these measures can offer a total evaluation of accuracy in optimization, efficiency and strength.

A qualitative scoring system is used to enable a comparison of the performance evaluation and it is summarized in Table 1. The following table will show relative performance of HHO, ALO and the hybrid HHO-ALO algorithm in the three criteria of convergence rate, accuracy and stability with marks on a scale of 1 (poor) to 5 (excellent). The hybrid HGO-ALO algorithm, as exhibited in Table 1, exhibits the best total score which implies that it is better than the HGO and the ALO algorithms in terms of convergence speed, accuracy, and stability.

Table 1. Presents the scoring system used to evaluate algorithm performance across key metrics:

Algorithm	Convergence Rate	Accuracy	Stability	Score (1-5)
HHO	Moderate	Good	Moderate	3
ALO	Good	Good	Good	4
Hybrid HHO-ALO	Excellent	Excellent	Excellent	5

4. HYBRID OPTIMIZATION PROCESS

The Hybrid HHO-ALO algorithm switches the exploration-based mechanisms of HHO and exploitation-based mechanisms of ALO between the optimization process. Such adaptive switching allows a balance between the search behavior, in which the global exploration is stressed during the initial stages in order to determine well-informed areas of the solution space, and the local exploitation over the developed candidate solutions and to refine the convergence precision.

4.1. HARRIS HAWKS OPTIMIZATION (HHO) PHASE

At the HHO stage, the locomotion of every search agent is controlled by an adaptive position update algorithm that takes into consideration individual experience and shared information of the population. Each agent is updated on its velocity basing on its past velocity, personal best position and the globally best position found to date. Such an update plan allows agents to share information well and even explore promising areas in the search space. The equations of updating the velocity and position are defined as follows [27]:

$$v_i^{k+1} = w \cdot v_i^k + c_1 \cdot r_1 \cdot (p_{best} - x_i^k) + c_2 \cdot r_2 \cdot (g_{best} - x_i^k) \quad (3)$$

$$x_i^{k+1} = x_i^k + v_i^{k+1} \quad (4)$$

In this formula, (w) represents the inertia weight used to balance exploration and exploitation. The acceleration coefficients are denoted by c_1 and c_2 , while r_1 and r_2 signify random numbers uniformly distributed between 0 and 1. Finally, p_{best} indicates the personal best position of an individual particle, and g_{best} represents the best global position achieved across the entire swarm.

4.2. ANT LION OPTIMIZATION (ALO) PHASE

In the ALO phase, the update of the position of every search agent is directed by the best solutions in the population, often known as alpha (α), beta (β) and delta (δ) which are the three solution candidates that have been found to be the best to date. These leaders give directional guidance that allows the good exploitation of the promising regions in the search space. The distances of the current agent position to that of each of the leading solutions are calculated to control the effect the leader's impact on the movement of the agent. The post update system can be expressed as follows [28]:

$$D_\alpha = |C_1 \cdot \alpha - x_i| \quad (5)$$

$$D_\beta = |C_2 \cdot \beta - x_i| \quad (6)$$

$$D_\delta = |C_3 \cdot \delta - x_i| \quad (7)$$

$$x_i = \frac{x_\alpha + x_\beta + x_\delta}{3} \quad (8)$$

Random coefficients guiding exploration are c_1 to c_3 .

The hybrid optimization model is based on the alternating exploitation and exploration abilities of the ALO and HHO algorithm in a systematic manner. This switching plan aims at realizing effective balance between global search and local refinement during the optimization process. Precisely, the hybrid algorithm has two primary directions of work:

- Exploration (HHO Phase): Focuses on searching for a broad solution space during early iterations.
- Exploitation (ALO Phase): Refines solutions by focusing on the best candidates during later iterations. Additionally, the hybrid approach incorporates a combination rule to adaptively switch between HHO and ALO phases based on convergence behavior.

In order to further avoid premature convergence and ensure that there is adequate diversity in the population, the strategies contributing to elitism and mutation are incorporated in the hybrid structure. Elitism is used to maintain the high-quality solutions that have the highest performance in future iterations and thus secure quality and stability of the solutions. At the same time, a small number of candidate solutions are subject to random mutations in order to add some controlled perturbation, increase diversity, and allow escape out of possible local optima. The mathematical formulation of the mutation process is as follows [29]:

$$x_{mutated} = x + mutation_factor \cdot r \cdot (\zeta_{max} - \zeta_{min}) \quad (9)$$

The pseudo-code of the suggested Hybrid HHO-ALO algorithm is structured to effectively demonstrate the sequential measurements in the optimization process, population initiation, fitness estimation, adaptive transition between exploration and exploitation sub-processes, and solution updating processes. The pseudo-code representation is organized in a structured form which makes reproducibility easier and also gives the algorithmic workflow a concise view:

Algorithm 1: proposed Pseudo – Code for Hybrid HHO – ALO

1. Initialize positions of hawks and antlions randomly within the search space
2. Evaluate the initial fitness of each solution
3. **While** stopping criteria are not met **do**
4. **If** iteration <= threshold
5. % HHO Phase
6. **Update** positions using Harris Hawks Optimization exploration equations
7. **else**
8. % ALO Phase
9. **Update** positions using ALO equations
10. **end**
11. Evaluate fitness of updated solutions
12. Apply elitism and mutation
13. **Update** best solutions (**best hawk, elite antlion**)
14. **End while**
15. **Return** optimal damping ratio (zeta_opt)

4.2.1.1. STANDARD TEST FUNCTIONS

The benchmark functions are widely used for analyzing and comparing the performance of Algorithmic Optimizers. They provide much more controlled testing of algorithms, relevant to capacity for exploring complicated search space and exploring local minima or global optima. A few of the benchmark functions that are applicable in this work are explained with mathematical representation, property and issues in detail as described below.

4.2.1.2. SPHERE FUNCTION

The most common benchmark very frequently used and simplest functions of optimization studies is the Sphere function. It is a unimodal and convex function and has a global minimum at the origin (0, 0). The Sphere object is based on a continuous parabolic terrain, which makes it a perfectly smooth surface on which to assess the basic convergence behavior and quality of optimization algorithms. The fact that it is a simple algorithm enables researchers to measure the potential of an algorithm to find the global optimum effectively without being affected by local minima as indicated in the equation below [30]:

$$f(x, y) = x^2 + y^2 \quad (10)$$

4.2.1.3. RASTRIGIN FUNCTION

Rastrigin function is very multimodal, with local minima many due to the cosine terms that appear in it. Although it is more complex the global minimum is at the origin (0,0). The periodic variations caused by the cosine terms make it a difficult test case of optimization algorithm [30].

$$f(x, y) = 10 \cdot n + (x^2 - 10 \cdot \cos(2\pi x)) + (y^2 - 10 \cdot \cos(2\pi y)) \quad (11)$$

4.2.1.4. ACKLEY FUNCTION

Ackley function A famous benchmark function with a flattened outer shape and a basin in the middle with a high count of local minima. Despite the smooth gradient structure of the given function, its multifaceted topography with sharp peaks and deep saddles poses a serious challenge to the optimization algorithms, especially regarding the prevention of the early settlement at the local optima. The optimum of Ackley function in the whole world is at the origin (0, 0) and therefore, it is a useful test case to test the exploration and exploitation of the optimization algorithm [30].

$$f(x, y) = -20 \cdot \exp\left(-0.2 \cdot \sqrt{0.5 \cdot (x^2 + y^2)}\right) - \exp(0.5 \cdot (\cos(2\pi x) + \cos(2\pi y))) + e + 20 \quad (12)$$

4.2.1.5. ROSENBROCK FUNCTION (BANANA FUNCTION)

Rosenbrock function also known as the Banana function is a classical benchmark problem with a long and curved valley passing through to its global minimum at 11. On the one hand, the global minimum is analytically clear, on the other hand, the steepness of the walls of the function and its flat and long valley floor makes the optimization procedure difficult. These properties complicate the act of optimization algorithms sustaining a suitable direction in search and convergence rate and hence give a stringent experiment with measuring their capacity to cause a balance between exploration and exploitation. Rosenbrock function can be mathematically expressed as follows [30]:

$$f(x, y) = (1 - x)^2 + 100 \cdot (y - x^2)^2 \quad (13)$$

4.2.1.6. THE MULTI-DIMENSIONAL GREWINGK PROBLEM

Griewank function is a famous multimodal benchmark function that is represented by a high density of local minima in its search space. The closer the decision variables are to the origin, the higher the coefficient of the polymeric term of the function and the more the cosine terms give the oscillatory effect that introduces many local optimas. This complicated terrain renders the Griewank challenging especially to the optimization algorithms, which is vulnerable to early convergence. The minimum of the function is reached at the origin (0, 0) where the minimum of the function is reached. The mathematical representation of the Griewank function is the following [30]:

$$f(x, y) = 1 + \frac{4000}{x^2} + \frac{4000}{y^2} - \cos(x) \cdot \cos\left(\frac{2}{y}\right) \quad (14)$$

In a bid to offer a detailed structure of evaluation of the optimization algorithms, a number of benchmark functions that are well established are used. Such benchmark problems are currently popular in the optimization literature because they exhibit different mathematical characteristics and different degrees of challenge. They allow the systematic evaluation of the capability of an algorithm to tackle convex and non-convex landscapes, unimodal and multimodal structures, and are also associated with issues of local optima entrapment. The benchmark functions chosen are summarized in table 2 presenting the global minima, search domains, the nature of the functions, as well as the relative level of optimization difficulty of the functions.

Table 2. Standard Test Functions

Function	Global Minimum	Domain	Nature	Challenge Level
Sphere	$f(0,0) = 0$	$[-5.12, 5.12]$	Convex, Unimodal	Low
Rastrigin	$f(0,0) = 0$	$[-5.12, 5.12]$	Multimodal	High
Ackley	$f(0,0) = 0$	$[-5, 5]$	Multimodal	Moderate
Rosenbrock	$f(1,1) = 0$	$[-5, 5]$	Non-convex, Unimodal	High
Griewank	$f(0,0) = 0$	$[-5, 5]$	Multimodal	High

4.2.1.7. Applications for Optimization

The chosen benchmark functions are set to measure different and complimentary differences of optimization algorithm performances by a range of problematic search conditions. All the functions focus on certain features that are vital to evaluate the strength and efficiency of algorithms. Specifically, multimodal landscapes, exemplified by functions like Rastrigin and Griewank, place optimization algorithms in a difficult situation, having to efficiently range the large number of local minima but converge to the global minimum. The functions are particularly helpful when analyzing the global search and resistance to premature convergence of an algorithm.

Functions whose valleys are narrow and curved like the Rosenbrock function are used to test the capacity of an algorithm to follow a steady search direction, as it approaches constrained and misleading portions of the search space. It is important to have a fine control of the step size and directional accuracy in such landscapes to achieve optimum results. The Sphere problem, however, provides a simple problem that is convex and unimodal, allowing for a straightforward assessment of the basic convergence properties, and the computational efficiency of an algorithm to be easily evaluated assuming no local optima.

Ackley function: A significant one to check the balance of exploration and exploitation, since it has a relatively flat outer region, and a complex central basin containing several local minima. The ability to search for a solution in different directions and simultaneously to find a solution in interesting directions can be called functional activity of the algorithm. These benchmark functions can be used together to create a variety of optimization problems on which to evaluate the robustness, efficiency, convergence speed and adaptability of an algorithm to different problem landscapes. For such diversity to be achieved is when one needs to show the effectiveness of the more evolved optimization techniques in practice in more real-life problems.

5. Results and Discussion

This section will summarize the results and give a short analysis of the performance of the three optimization strategies (HHO, ALO, and Hybrid HHO-ALO) depending on the simulations and benchmark assessments that were performed in our MATLAB code. The results show the relative effectiveness of both algorithms in the optimisation objective, the convergence speed, the accuracy and robustness for the benchmark functions. The discussion brings

out the merits and drawbacks of both approaches and it can be said that the Hybrid HHO-ALO algorithm would be more suitable for the environment of challenging optimisation and would be capable of delivering reliable results.

5.1. HYBRID HHO-ALO ALGORITHM FOR INTELLIGENT MATHEMATICAL SYSTEM

The optimal dynamic performance was used as the optimization objective for the control system parameters in this paper, the Hybrid HHO-ALO algorithm was used to optimize the parameters. The algorithm is proposed based on combining the excellent exploitation capability of Ant Lion Optimization (ALO) with the excellent exploration capability of Harris Hawks Optimization (HHO), and can improve Harris Hawks Optimization (HHO) and Ant Lion Optimization (ALO), respectively, and can overcome some of their shortcomings such as premature convergence and the slow convergence towards the global optimum. Careful consideration of key algorithmic parameters ensured that the computational efficiency and accuracy during optimization was balanced. Considering the need for convergence, the number of iterations was selected as 50, but not enough to cause excessive computation. Another parameter, the population size of 20 search agents, was employed because of the need to maintain diversity in the search space, as well as a low level of complexity that guarantees efficient search. The fitness function was defined to reflect how far the system response is from the desired performance specifications, particularly looking at overshoot and settling time, and it was used to draw a conclusion on how well the system is performing, thus steering the optimization towards the optimal solution versus worse one; this is due to the lower the fitness value is, the better the system response and thus, the better the system will perform. The solution position is updated by a dual algorithm that involves hybrid HHO-exploration and ALO-inspired exploitation and elite guidance, which is to ensure that the process of finding the solution positions is convergent and stable during later iterations to achieve the final global optimum.

From Figure 3, it can be seen that the Hybrid HHO-ALO algorithm showed better performance compared to the two individual algorithms: HHO and ALO. Compared to the fully sequential and parallel methods, the hybrid method showed improved convergence with increased stability about descent; as well, the hybrid method demonstrated lower objective values with less fluctuations over iterations. The objective values of ALO and HHO are higher, and their convergence is slower, but the objective reduction of the hybrid algorithm remains smooth and steady, showing its robustness and reliability in various runs. As can be seen in Figure 4, the mean final best score when using ALO is the same and worse than that of HHO, while the Hybrid HHO-ALO algorithm is setting itself to a better performance by being able to balance the exploration/exploitation aspects of the algorithm, improving the performance of HHO. The stability of the algorithm system is tested using the standard deviation of the final score in Figure 5, showing that the standard deviation for hybrid algorithm is smaller than the standard deviation for ALO, which means the algorithm system is more stable and less sensitive to initial conditions. Finally, as presented in Figure 6, the MSE comparison highlights the effectiveness of the Hybrid HHO-ALO algorithm, since it showcases the lowest MSE among all the methods, revealing that the algorithm is accurate, consistent, and has good performance in the optimization landscape for a complex problem. The obtained results validate the practicality of the proposed hybrid approach on real engineering problems of optimal control of any practical control system.

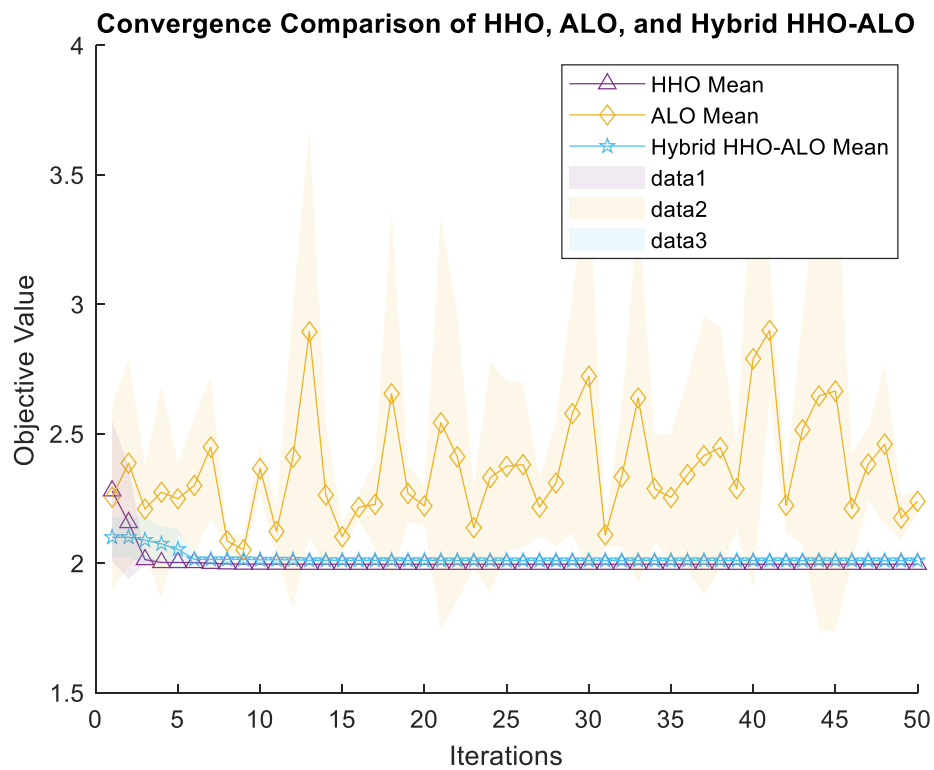


Figure 3. Convergence behavior comparison of HHO, ALO, and hybrid HHO-ALO algorithms.

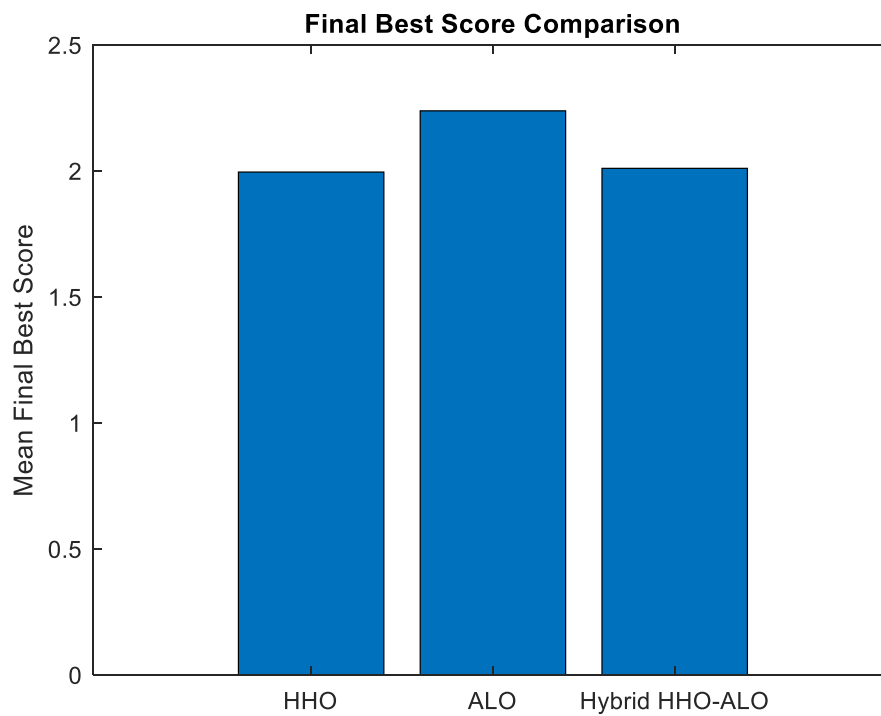


Figure 4. Comparison of mean final best scores for HHO, ALO, and Hybrid HHO-ALO.

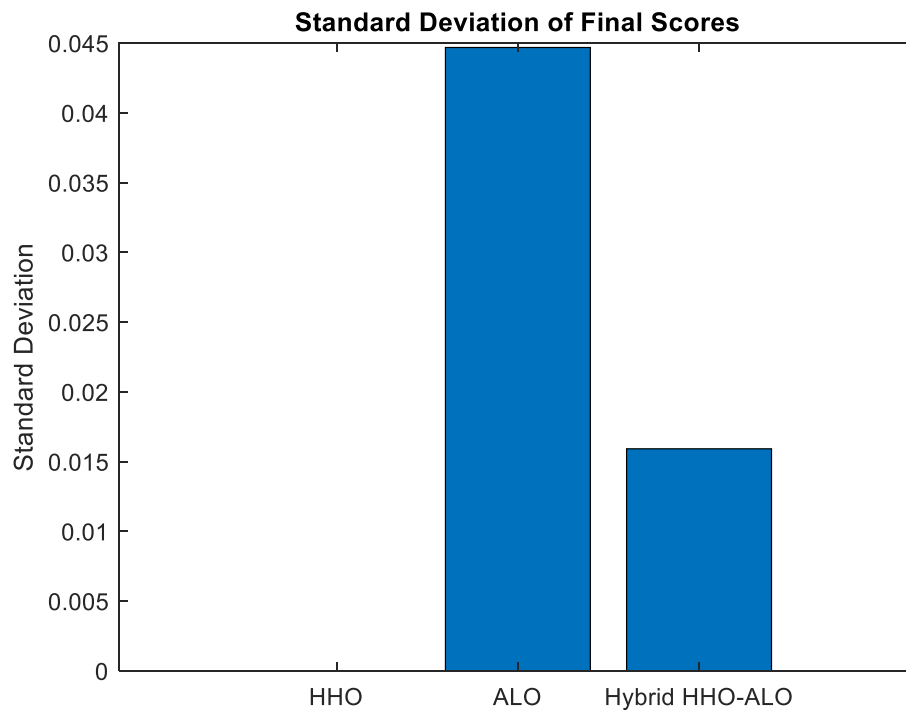


Figure 5. Standard deviation of final best scores for HHO, ALO, and hybrid HHO-ALO.

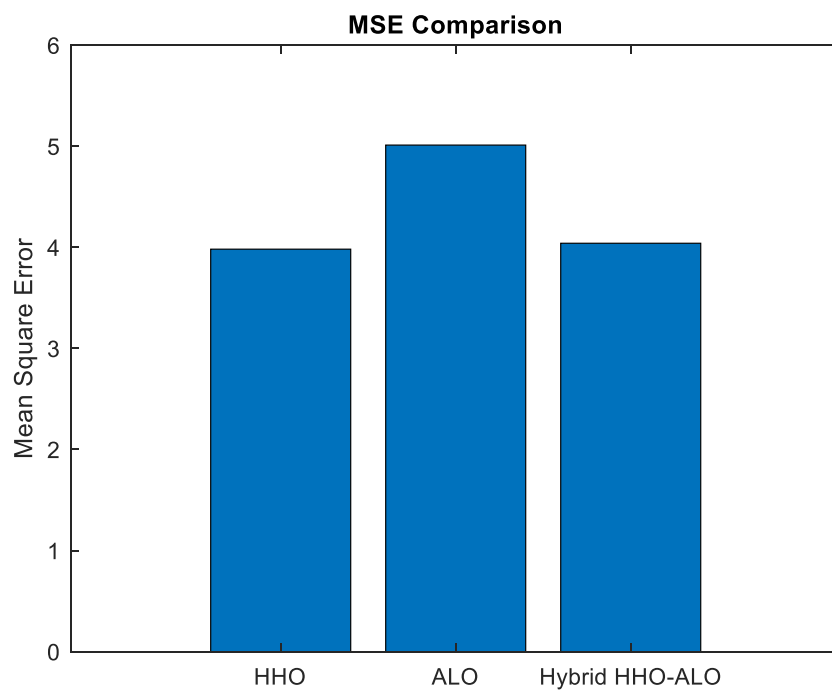


Figure 6. MSE comparison of HHO, ALO, and Hybrid HHO-ALO.

5.2. RESULTS OF BENCHMARK FUNCTIONS

In this work, a set of popular benchmark functions were employed to evaluate the performance of the proposed Hybrid HHO-ALO algorithm. The choice of these benchmark functions is to cover a large range of optimization problems, some of which are straightforward convex landscapes, and others of which are very multimodal and misleading search spaces. This diversity provides a range of evaluation of the algorithm robustness, convergence rate and the ability to seek global optima and evade immigration in local minima that is critical in the solution of complex optimization problems. Figure 7 shows the Sphere function, which is smooth and convex and this is defined by the single global minimum at the origin (0,0) and the lack of local minimum. Due to the ease of modification in shape (parabolic shape), this function is a test base of optimization algorithms. This is shown in the way the Hybrid HHO-ALO algorithm quickly and steadily approaches the global minimum with the smooth descent curve to the bottom of the figure. This behavior indicates that the algorithm has a good exploitation ability and proves to be reasonable in addressing simple convex optimization problems with high precision and low computational costs.

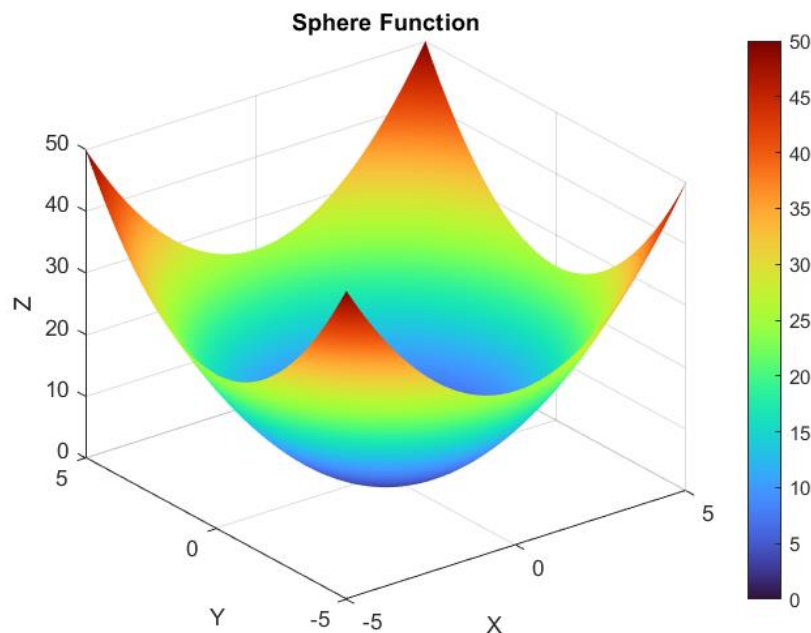


Figure 7. the 3D surface plot of the Sphere benchmark function.

Figure 6 shows the Rastrigin function, which is defined by a very multimodal topography produced by periodic cosine terms. Often these oscillatory words generate a high count of local minima that are evenly spaced throughout the search space making the function especially difficult to optimize with an inadequate exploration power. The Hybrid HHO-ALO algorithm can successfully move through the complex and oscillatory surface and reach the global minimum at (0, 0) as illustrated in the figure. This outcome shows that the algorithm can detect a strong balance between exploration and exploitation that helps it to overcome the problem of entrapment in local minima and converge reliably in the presence of a large number of local minima during optimization processes.

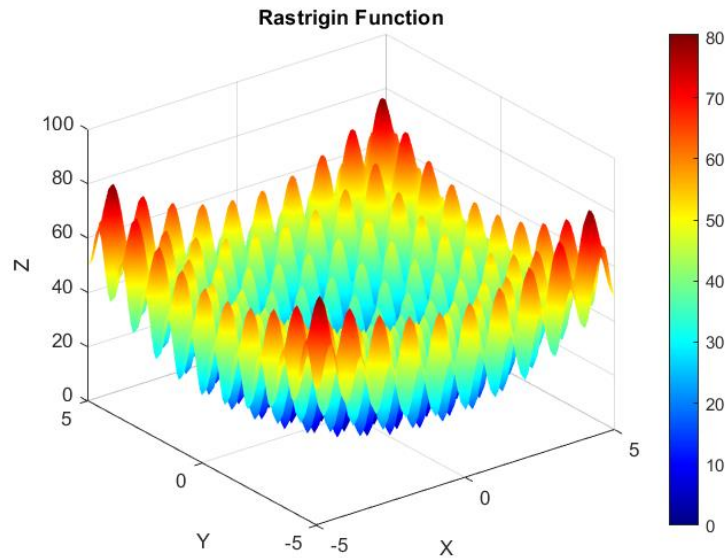


Figure 8. The 3D surface plot of the Rastrigin benchmark function.

Figure 9 illustrates the Ackley function, which has a very flat outer area and sharp depths towards the center of the globe. This kind of misleading topography may impede the convergence, especially to those algorithms which use local search mechanisms. The Hybrid HHO-ALO algorithm is efficient at addressing this difficulty as demonstrated by the fact that it traverses the flat areas and approaches the central minimum. This performance indicates the flexibility and capability of the algorithm to sustain the search speed and use potentially good areas when high gradients arise.

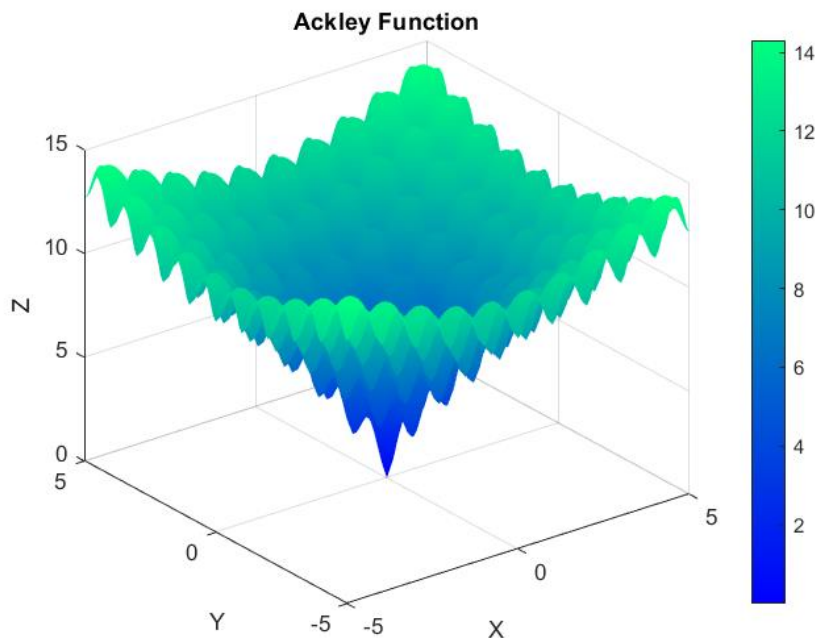


Figure 9. The 3D surface plot of the Ackley benchmark function.

Figure 8 shows the Rosenbrock function or the banana function as it is often referred to, it is a narrow and curved valley that reaches a global minimum at 11. The long, deeply curved geometry of this

valley is a major challenge to optimization algorithms in which successful convergence is achieved by control of direction and step-size to avoid going off the optimal path. As shown in the figure, Hybrid HHO-ALO algorithm can successfully follow the curvilinear valley and reach the global minimum which can prove its ability to cope with complicated and ill-posed optimization landscapes. Despite the fact that convergence process takes more iterations than simpler benchmark functions, the algorithm exhibits stable search behavior and is able to avoid premature convergence.

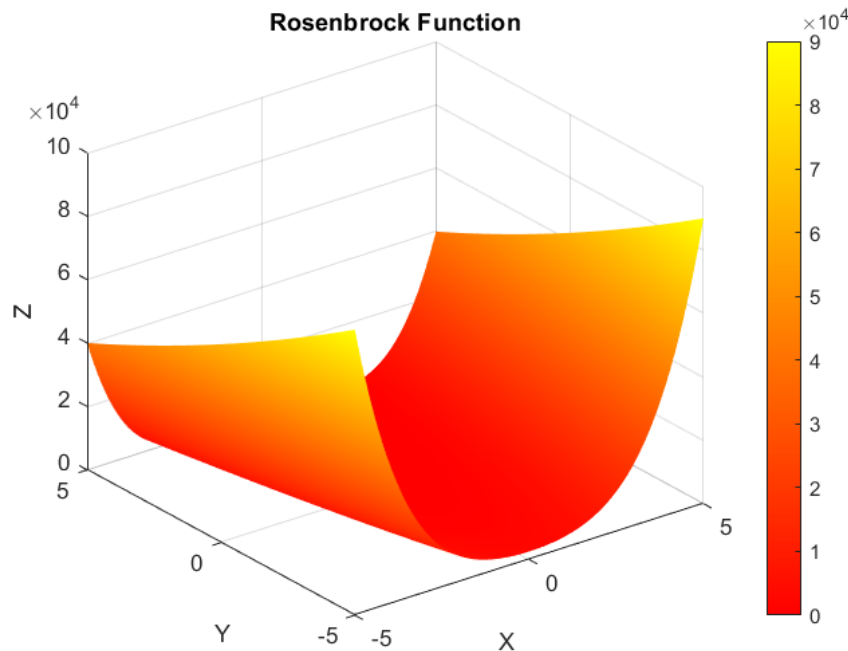


Figure 10. The 3D surface plot of the Rosenbrock benchmark function.

Figure 11 depicts the Griewank function which is a difficult multimodal test case with many local minima which are broadly distributed and are caused by interaction between quadratic and cosine functions. Although this landscape of search is a complicated one, the Hybrid HHO-ALO algorithm has high optimization performance and is able to search the solution space and converge to a global minimum at (0, 0). The algorithm also escapes the local minima as depicted in the figure and will undergo a steady convergence curve to the optimum solution. These findings also prove the practicality of the hybrid approach in balancing between global exploration and local exploitation in tackling issues of highly multimodal optimization.

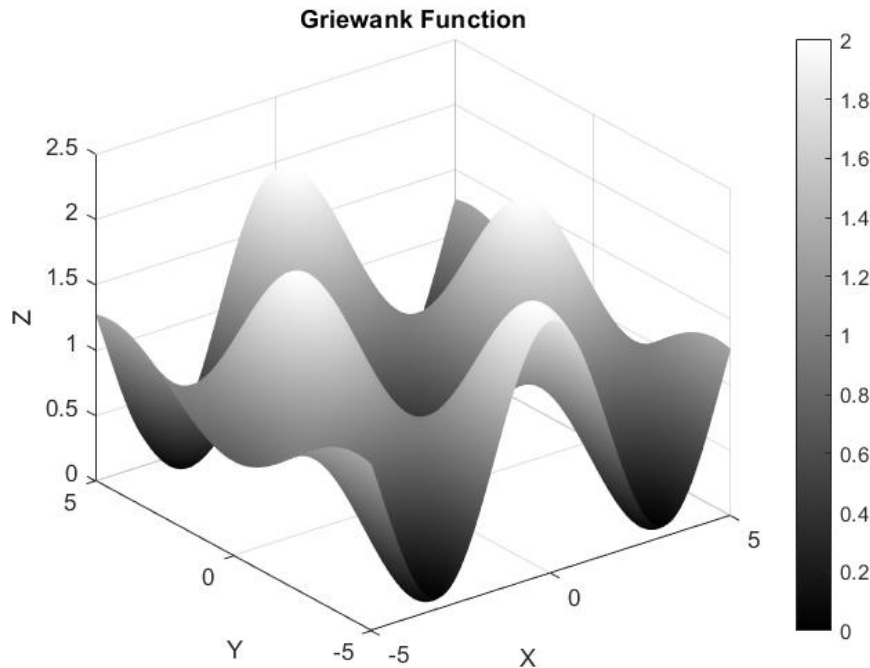


Figure 11. The 3D surface plot of the Griewank benchmark function.

6. Recommended Developments

Though the Hybrid HHO-ALO algorithm has shown encouraging performance in optimization problems on the benchmark and in such control applications, a number of its areas can be explored to improve its performance. Subsequent research can aim at enhancing the range of conditions to which the developed hybrid framework can be applicable, its strength, and flexibility to more complex and realistic conditions. First, the performance evaluation of Hybrid HHO-ALO algorithm was carried out under the realistic control systems under external disturbance, nonlinearity, modelling errors and parameters that change over time will also be extended in future research. Those studies would provide a better sense of the resilience and flexibility of the algorithm in the real world, where assumptions are often missing. Second, it is feasible to include the adaptive learning processes into the Hybrid HHO-ALO, to increase the improvement. The algorithm could be also more flexible, by, for example, varying important algorithmic parameters – exploration and exploitation rates, mutation probability or switching thresholds – during the optimization process in response to system feedback and convergence behavior to gain improved flexibility, convergence speed and solution stability between optimizations. Thirdly, a further performance boost can be gained by investigating multi-hybrid optimization technique, a hybrid combination of Hybrid HHO-ALO technique with other more advanced/expansive metaheuristic approaches such as the Genetic Algorithms (GA) or Differential Evolution (DE). This hybridization can take advantage of complementarity of features derived from the diversity of the population to implement global search along with local refinement at the fine-grained level, by boosting faster and stronger the pace in a complex optimization space. Further, the Hybrid HHO-ALO algorithm is applicable to any further advanced and intelligent controller design such as: Fractional Order Controller, Adaptive Controller, and Robust Controller. In case of these kinds of controllers, the application of the algorithm is expected to yield greater precision in the control, greater stability margins and improved disturbance rejection properties. Additionally, further studies can focus on the implementation of the Hybrid HHO-ALO algorithm in a wider context of engineering, including optimization of power systems, management of renewable energy, autonomous and robotic systems, smart grids and industrial automation. The proposed hybrid approach would also be generally applicable and scalable due to these cross-domain applications. The designing of problem specific hybrid variants utilizing domain knowledge, context-specific heuristic rules and/or constraint-handling mechanisms are also potential research fields. The Hybrid HHO-ALO algorithm could be applied to specific applications and large-scale optimization problems, where computing efficiency performance and optimization performance improvements are significant benefits, particularly in the high-dimensional and real-time scenarios.

7. CONCLUSION

Three optimization strategies, namely HHO, ALO and Hybrid HHO-ALO algorithm, were implemented and compared in optimizing the system parameters for dynamic control applications. The proposed Hybrid algorithm (HHO-ALO) aimed to combine both the strong global exploration capability of HHO and the efficient exploitation process of ALO to overcome common drawbacks of using a standalone metaheuristic algorithm, such as premature convergence and slow convergence to the global optimum. These optimization schemes were tested with benchmark test cases as well as convergence analysis with special focus on minimizing the objective value, stability and robustness. The results that were obtained prove that Hybrid HHO-ALO algorithm always performs better than HHO and ALO in terms of convergence rate, accuracy and robustness. The hybrid algorithm showed notable results in dealing with benchmark functions for which spatial location of the optimum was non-trivial; especially when a large number of benchmark functions were in complex landscapes like Rastrigin, Ackley, Rosenbrock, and Griewank, where the algorithm was able to avoid the local minimum and reach the global optimum reliably. Additionally, in terms of the MSE values and the final best score, the hybrid HHO-ALO algorithm had less error value and higher stability among the results of the several independent runs performed compared to the individual HHO and ALO algorithms. The hybrid system presented a remarkable performance in the exploration phase, excellent capability to climb up the difficult optimization landscape, and high exploitation efficiency as well. The benchmark evaluations also verify the stability and flexibility of the Hybrid HHO-ALO algorithm by successfully applying it to a variety of problems: simple convexity versus multimodal scenarios. The results confirm the efficiency of the proposed hybrid method as a reliable optimization solution in real life engineering applications where dynamic, accurate, and adaptive solutions are needed is confirmed.

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