

On Connectedness Via Nano Generalized Open Sets

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ARTICLE INFO

Article history:

Received: 10 /06/2026

Revised form: 20 /06/2026

Accepted : 21 /06/2026

Available online: 30 /09/2026

Keywords:

Nano topology

Nano generalized open set

NG- connected space

NG-hyperconnected space

NG-ultraconnected space

Connectedness

ABSTRACT

In this paper, the concept of connectedness is explored through Nano Generalized Open Sets (NGOs) in nano topological spaces (NTS). A new class of spaces, named NG-connected spaces, is introduced and evaluated. Several characterizations of NG-connectedness are established, including formulations in terms of NGOs, NGCs and NGOs. Furthermore, two stronger forms of NG- connectedness, namely NG- hyper connectedness (NGHCD) and NG-ultraconnectedness (NGUCD), are introduced and investigated. It is observed that both NGHCD and NGUCD spaces imply NG-connectedness. Various examples and counter examples are provided to illustrate the relationships among these classes and to show the failure of several conserve implications. The obtained results prolong the study of connectedness from NOS to NGOs and present a broader framework for investigating generalized connectedness in NTS. These findings contribute to the advancement of NT and open new directions for further research including generalized topological structures.

<https://doi.org/10.29304/jqcm.2026.18.32972>

1. Introduction

Rough Set Theory, introduced by Pawlak [11], has become an important mathematical tool for handling uncertainty, vagueness, and incomplete information. The theory is based on lower and upper approximations induced by equivalence relations and has found numerous applications in knowledge discovery, information systems, machine learning, and data analysis.

Motivated by rough set approximations, Thivagar and Richard [5] introduced the concept of nano topological spaces, thereby establishing a connection between topology and rough set theory. Since then, nano topology has attracted considerable attention from researchers, leading to the development of various nano versions of classical topological concepts, including nano- open sets, nano-closed sets, continuity, compactness, and connectedness.

The notion of generalized closed sets was originally introduced by Levine [7] as an extension of ordinary closed sets in topological spaces. This concept inspires the development of several generalized nano topology. NGOs were introduced and illustrated by Bhuvaneshwari and Mythili Gnanapriya [2,3], presenting a broader framework for investigating topological properties beyond classical nano open sets.

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Communicated by 'sub etitor'

In recent years, extensive research has been conducted to the study of GO and GC sets in NTS. Rodrigo and Subithra [8] introduced a new class of NGC and evaluated their intrinsic properties. Ganesan [9] studied nano Δ -generalized closed sets and established several relationships with existing classes of generalized sets. Furthermore, Sharma et al. [10] introduced nano h-open sets and examined their topological properties. More recently, El Sayed [4] proposed the concept of nano- simply open sets and explored how they relate to other generalized open structures.

Connectedness constitutes fundamental concepts in topology. It concerns the possibility of decomposing space into disjoint components and plays a key role in understanding the overall framework of topological spaces. Various generalized forms of connectedness have been studied in both classical and nano topological settings. Ahmed and Pirbal [6] introduced nano S_B -connectedness and investigated several of its properties, thereby extending the study of connectedness in nano topology. Similar developments have appeared in soft NTS, where new notions of connectedness and compactness have emerged recently [1].

Since NGO sets form a wider class than ordinary nano-open sets, they provide a natural framework for developing stronger and more flexible notions of connectivity.

The main objective of this paper is to introduce and investigate the concept of NGO-connected spaces. Several characterizations of NG-connectedness are established in terms of NGO, NGC sets, and NG-clopen sets. Moreover, two stronger forms of connectedness, namely NGHCD and NGUCD, are introduced and studied. The relationships among these classes are examined, and examples and counterexamples are provided to illustrate the results obtained.

The rest sections can be structured as follows: section 2 shows a set of previous routing strategies in the vehicular networks, section 3 outlines the system model, section 4 illustrates the designed routing algorithm, and section 5 displays the simulation scenarios and findings, while the conclusions and future plans are presented in the last section.

2. Preliminaries

Definition 2.1[5]: Let μ, R be an approximation space and let $X \subseteq \mu$. Defined the NT as $T_R(X) = \{\mu, \emptyset, L_{R(X)}, U_{R(X)}, B_{R(X)}\}$ on μ , we express it $(\mu, T_R(X))$. Every set A in this topology is NO (nano-open) and its complement is NC (nano-closed).

Definition 2.2[5]: Let $A \subseteq \mu$. The nano -closure of A ($NCl(A)$), is the intersection of all NCs containing A . The nano interior of A ($NInt(A)$) is the union of all NOs contained in A .

Definition 2.3 [7]: A subset A of NTS $(\mu, T_R(X))$ is called NGC (nano generalized closed) if $NCl(A) \subseteq U$ Whenever $A \subseteq U$ and U is NOS.

Definition 2.4 [7]: A subset $A \subseteq \mu$ is called NGOs if its complement is NGC; that is $\mu - A$. Collection NGOs of μ is $NGO(\mu, X)$.

Definition 2.5[11]: Assume $(\mu, T_R(X))$ is an NTS, it is called nano-connected if it cannot be represented as a disjoint union of two non- empty

This observation plays an important role in establishing the relationship between nano connectedness and NG-connectedness. Part 3 should contain the main original contribution of the paper for a journal article. We should begin with the definition of NG-separation, then introduce NG- connectedness followed by the first two fundamental characterizations.

3. NGO Connected Space

In this part, the concept of NG -connectedness is introduced and investigated. Several characterizations of NG-connected spaces are established in terms of NGOs and NGCs.

Definition 3.1: Let $(\mu, T_R(X))$ be an NTS. Two nonempty NGOs A and B are called NG- separation of μ if $A \cap B = \emptyset$ And $A \cup B = \mu$.

Definition 3.2: An NTS $(\mu, T_R(X))$ is called NG- connected if it admits no NG- separation. Otherwise, μ is said to be NG -disconnected.

Example 3.3: Let $\mu = \{\alpha, \beta, \gamma\}$, With equivalence relation

$\mu/R = \{\{\alpha\}, \{\beta, \gamma\}\}$ And let $X = \{\alpha\}$. Then $T_R(X) = \{\emptyset, \mu, \{\beta, \gamma\}\}$, And

$NGO(\mu, X) = \{\emptyset, \mu, \{\beta\}, \{\gamma\}, \{\beta, \gamma\}\}$. Since every two nonempty proper NGOs intersect separation of μ . Hence μ is NG- connected.

Remark 3.4: Every NOs is NGOs .

Proof : The proof follows directly from definition.

Theorem 3.5: An NTS $(\mu, T_R(X))$ is NG -connected iff it cannot be represented as the union of two nonempty disjoint NGCs.

Proof: Assume that μ is NG-connected, suppose that $\mu = F \cup G$, where F and G are nonempty NGCs satisfying $F \cap G = \emptyset$. Since the complements of NGCs are NGOs, we have $F = \mu - G$ and $G = \mu - F$. Thus, F and G are NGOs. Hence is the union of two disjoint nonempty NGOs, contradicting NG -connectedness.

Conversely, suppose μ that cannot be decomposed into the union of two disjoint nonempty NGCs. Assume that μ is NG - disconnected. Then there exist nonempty NGOs A and B such that $A \cap B = \emptyset$ and $A \cup B = \mu$. Also $A = \mu - B$ and $B = \mu - A$. Both A and B are NGCs. This contradicts the assumption. Therefore μ is NG -connected.

Theorem 3.6: An NTS $(\mu, T_R(X))$ is NG- connected iff the only subsets that are both NGOs and NGCs are $\emptyset$ and μ .

Proof: Assume that μ is NG -connected and consider a nonempty proper subset A of μ that is both NGOs and NGCs. Since A is NGCs, $\mu - A$ is NGOs. Moreover, $A \cap \mu = \emptyset$ And $A \cup (\mu - A) = \mu$. Hence μ admits an NG-separation, contradicting NG -connectedness. Therefore, the only NG-clopen set are $\emptyset$ and μ . Conversely, suppose that the only NGOs subsets are $\emptyset$ and μ . Assume that μ is NG-disconnected. Then there exist nonempty NGOs A and B such that: $A \cap B = \emptyset$, $A \cup B = \mu$. Consequently, $A = \mu - B$, $B = \mu - A$. Thus, A and B are both NGOs and NGCs, contradicting the assumption. Then μ is NG -connected.

Theorem 3.7: An NTS $(\mu, T_R(X))$ is NG -disconnected iff it contains a proper nonempty NG-clopen subset.

Proof: The proof follows directly from Theorem 3.6 .

Theorem 3.8: An NTS $(\mu, T_R(X))$ if μ is NG -connected, then it is N-connected.

Proof: Assume that μ is NG -connected, suppose that μ is not nano -connected. Since there exist a non empty proper nano -clopen subset ω of μ . Since every NOs is NGOs Thus ω is NG-clopen set which is contradiction. Therefore ω is nano- connected.

Remark 3.9: The converse is not true in general.

Example 3.10: Let $\mu = \{\alpha, \beta, \gamma, \delta\}$ and let the equivalence relation R on μ be given by $\mu/R = \{\{\alpha, \beta\}, \{\gamma, \delta\}\}$ consider the subset $X = \{\alpha, \beta\}$. Then $L_R(X) = \{\alpha, \beta\}$ and $U_R(X) = \{\alpha, \beta\}$. Hence $B_R(X) = \emptyset$. Therefore, the NT generated by X is $T_R(X) = \{\mu, \emptyset, \{\alpha, \beta\}\}$. It follows the family of NGOs is

$NG(\mu, X) = P(\mu)$ (Set of all subsets of μ) we have any two nonempty NGOs have a nonempty intersection. Therefore μ is N-connected but not NG-connected.

Hyperconnectedness represents one of the strongest forms of connectedness. In this part, we introduce NGHCD spaces and investigate their basic properties and relationships with NG-connected spaces.

4. NANO Generalized Hyperconnected Spaces

Hyperconnectedness represents one of the strongest forms of connectedness. In this section, we introduce NGHCD spaces and investigate their basic properties and relationships with NG-connected spaces.

Definition 4.1: An NTS $(\mu, T_R(X))$ is called NGHCD if the intersection of any two nonempty NGOs is nonempty.

Example 4.2: Consider the nano-topological space $\mu = \{\alpha, \beta, \gamma\}$, with equivalence relation $\mu/R = \{\{\alpha\}, \{\beta, \gamma\}\}$ And $X = \{\alpha\}$, thus $NGO(\mu, X) = \{\emptyset, \mu, \{\alpha\}, \{\alpha, \beta\}, \{\alpha, \gamma\}\}$. is NGHCD

Theorem 4.3: In $(\mu, T_R(X))$ if $T_R(X) = \{\mu, \emptyset\}$ then μ is NGHCD.

Proof: Since $\emptyset, \mu$ are only NGO, the only nonempty NGOs is μ , therefore, it is impossible to find two distinct nonempty intersection. Hence, the intersection of any two nonempty NGOs is nonempty. Therefore, μ is NGHCD.

Theorem 4.4: In $(\mu, T_R(X))$ if $U_R(X) = L_R(X) = \{x\}$ for sum $x \in \mu$ and μ contains at most two elements, then μ is NGHCD.

Proof: Since $U_R(X) = L_R(X) = \{x\}$ then $B_R(X) = \emptyset$. Hence, $T_R(X) = \{\mu, \emptyset, \{x\}\}$ therefore, the intersection of any two nonempty NGOs contains x . Hence, μ is NGHCD.

Theorem 4.5: Every NGHCD space is NG-connected.

Proof: Suppose μ is NGHCD. Assume that μ is NG-disconnected. Then, there exist two disjoint nonempty NGOs A, B s.t. $\mu = A \cup B$. Thus, $A \cap B = \emptyset$, which contradicts NGHCD. Therefore, μ is NG-connected.

Example 4.6: Let $\mu = \{\alpha, \beta, \gamma, \delta\}$ and $\mu/R = \{\{\alpha\}, \{\beta, \gamma, \delta\}\}$ consider the subset $X = \{\alpha\}$. Then $L_R(X) = \{\alpha\}$ and $U_R(X) = \{\alpha\}$. Hence $B_R(X) = \emptyset$. Therefore, $T_R(X) = \{\mu, \emptyset, \{\alpha\}\}$. The family of NGCs is $NGCs = \{\emptyset, \mu, \{\beta\},$

$\{\gamma\}, \{\delta\}, \{\alpha, \beta\}, \{\alpha, \gamma\}, \{\alpha, \delta\}, \{\beta, \gamma\}, \{\beta, \delta\}, \{\gamma, \delta\}, \{\alpha, \beta, \gamma\}, \{\alpha, \beta, \delta\}, \{\alpha, \gamma, \delta\}, \{\beta, \gamma, \delta\}\}$, NGOs = $P(\mu)/\{\beta, \gamma, \delta\}$. There for μ is NG-connected but is not NGUCD.

Theorem 4.7: An NTS μ is NGHCD iff every nonempty NGO subset of μ is NG-connected.

Proof: Assume that μ is NGHCD space and let A be a nonempty NGO subset of μ . Suppose that A is NG-disconnected. Then there exist nonempty NGO subsets G and H of A s.t. $G \cap H = \emptyset$ and $G \cup H = A$. Since G and H are NGO in μ , this contradicts the NGHCD of μ . Therefore A is NG-connected. Conversely, assume that every nonempty NGO subset of μ is NG-connected. Suppose that μ is not NGHCD. Then there exist non-empty NGOS A and B s.t. $A \cap B = \emptyset$. The set $A \cup B$ is NG and admits an NG-separation by A and B , contradicting the assumption. Therefore μ is NGHCD.

5. NANO Generalized Ultraconnected

In this part, we defined NGUCD space in NTS. Later, we study the relation between NG-connected and NGUCD.

Definition 5.1: An NTS $(\mu, T_R(X))$ is called NGUCD if the intersection of any two nonempty NGCs is nonempty.

Proposition 5.2: Every NGUCD space is NG-connected.

Proof: Suppose μ is NGUCD. Assume that μ is NG-disconnected. Then there exist two disjoint nonempty NGOs A, B of μ since that $A \cap B = \emptyset$, there complements $\mu - A$ and $\mu - B$ are nonempty NGCs. Moreover $(\mu - A) \cap (\mu - B) = \emptyset$ which contradicts NGUCD. There for μ is NG-connected.

Remark 5.3: The convers in Theorem 5.2 is not true in general.

Example 5.4: Let $\mu = \{\alpha, \beta, \gamma\}$ and let the equivalence relation R on μ be given by $\mu/R = \{\{\alpha\}, \{\beta, \gamma\}\}$ consider the subset $X = \{\alpha\}$. Then $L_R(X) = \{\alpha\}$ and $U_R(X) = \{\alpha\}$. Hence $B_R(X) = \emptyset$. Therefore, the NT generated by X is $T_R(X) = \{\mu, \emptyset, \{\alpha\}\}$. The family of NGCs is $NGCs = \{\emptyset, \mu, \{\beta\}, \{\gamma\}, \{\beta, \gamma\}\}$, NGOs = $\{\emptyset, \mu, \{\alpha\}, \{\alpha, \gamma\}, \{\alpha, \beta\}\}$ then we have any two NGOs have a nonempty intersection. There for μ is NG-connected but is not-ultraconnected.

Theorem 5.5: In $(\mu, T_R(X))$ if $T_R(X) = \{\mu, \emptyset\}$ then μ is NGUCD.

Proof: Since only NGC are $\emptyset, \mu$. There are not two disjoint nonempty NGCs. Hence, the intersection of any two nonempty NGCs is nonempty.

Therefore, μ is NGUCD.

Theorem 5.6: In $(\mu, T_R(X))$ if $U_R(X) = L_R(X) = \{x\}$ for sum $x \in \mu$, and μ contains at most two elements, then μ is NGUCD.

Proof: Since $U_R(X) = L_R(X) = \{x\}$ then $B_R(X) = \emptyset$. Hence, $T_R(X) = \{\mu, \emptyset, \{x\}\}$ therefore, all nonempty NGCs contains $\{x\}$. Hence the intersection of any two nonempty NGCs contains x . We have μ is NGUCD.

Conclusion

Connectedness is one of the most important concepts in topology due to its numerous applications in science and engineering. It plays a essential role in network analysis, communication systems, electrical circuits, image processing, robotics, geographic information systems, and data clustering. In these applications, connectedness is used to determine whether a system forms a unified structure or can be decomposed into separate components. Consequently, generalized forms of connectedness provide broader mathematical frameworks for modeling connectivity in complex and uncertain environments. Since NG-connectedness is a generalization of connectedness in nano topological spaces, it may provide a theoretical foundation for future extensions of these applications in rough-set-based information systems and uncertain data environments.

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