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# Coefficient Estimates for Certain New Classes of Bi-Univalent Functions in Hilbert Space Framework

**Salah Mahdi Ali<sup>1</sup>, Sanaa Noor Al-Sharifi<sup>2</sup>, Rafid Habib Buti<sup>3</sup>**

<sup>1</sup>College of Nursing, University of Al-Qadisiyah, Iraq. [salah.mahdi@qu.edu.iq](mailto:salah.mahdi@qu.edu.iq)

<sup>2</sup>General Directorate of Babylon Education, Iraq. [sanaa.mohammed1103@sc.uobaghdad.edu.iq](mailto:sanaa.mohammed1103@sc.uobaghdad.edu.iq)

<sup>3</sup>Department of Mathematics and Computer Application, College of Sciences, Al-Muthanna University, Iraq. [sci.rafid@mu.edu.iq](mailto:sci.rafid@mu.edu.iq)

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## ABSTRACT

This paper introduces two new subclasses of the bi-univalent function class  $\Sigma$  in the open unit disk  $\mathbb{D}$ , along with estimates for the coefficients  $|a_2|$  and  $|a_3|$  for functions in these subclasses. Moreover, the study is framed within the Hilbert space setting, particularly the Hardy space  $H^{2D}$ , providing an operator-theoretic approach for analyzing coefficient bounds.

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## 1. Introduction

Let  $\mathfrak{S}$  be a collection of all analytic functions  $f$  stipulated in the open unit disk, normalized by the requirements  $f(0) = 0$  with  $f'(0) - 1 = 0$ . where  $\mathfrak{X} = \{z \in \mathcal{C}: |z| < 1\}$ . Therefore, each  $f \in \mathfrak{S}$  possesses a Taylor series expansion of the corresponding structure:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad z \in \mathfrak{X}. \quad (1.1)$$

All function  $f \in \mathfrak{S}$  possesses an inverse map  $f^{-1}$  that fulfills the aforementioned requisites:

$$f^{-1}(f(z)) = z \quad (z \in \mathfrak{X}),$$

\*Corresponding author: Salah Mahdi Ali

Email addresses: [salah.mahdi@qu.edu.iq](mailto:salah.mahdi@qu.edu.iq)

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and

$$f(f^{-1}(w)) = w \quad (|w| < r_0(f); r_0(f) \geq \frac{1}{4}).$$

The inverse function is, in reality, represented by

$$f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3) w^3 - (5a_2^3 - 5a_2 a_3 + a_4) w^4 + \dots \quad (1.2)$$

[1] identified some mapping properties for the following subclass of  $\mathcal{A}$ :  $\mathcal{R}(\alpha, \beta) = \{f \in \mathcal{A} : \operatorname{Re}((f'(z) + \beta z f''(z))) > \alpha, \text{ with the conditions } \beta > 0 \text{ and } \{0 \leq \alpha < 1; z \in \mathbb{D}\}.$  [2][Theorem 3.1, p.9], showed that  $\mathcal{R}(\alpha, \beta)$  is a subset of  $\mathcal{S}$  if and only if  $2(1 - \alpha) \sum_{m=1}^{\infty} \frac{(-1)^{m-1}}{\beta m+1} \leq 1$ .

**Definition 1.1** A single-valued complex product  $f$  is defined as univalent in a simply linked domain.  $D$  is a function that does not assign the same value to different inputs; specifically, if  $f(z_1) \neq f(z_2)$  whenever  $z_1 \neq z_2, \forall z_1, z_2 \in \mathbb{D}$ .

**Definition 1.2** A function  $f \in \mathfrak{S}$  is considered bi-univalent in  $\mathbb{D}$  if each  $f(z)$  as well as  $f^{-1}(z)$  are univalent within  $\mathbb{D}$ .

Let  $\Sigma$  represent the class of bi-univalent functions in  $\mathbb{D}$  defined by (1.1). Instances of functions within the class  $\Sigma$  are

$$\frac{z}{1-z}, -\log(1-z), \frac{1}{2} \log\left(\frac{1+z}{1-z}\right), \dots$$

It is important to observe that the well-known Koebe function is not an element of  $\Sigma$ , as it maps the unit disk  $\mathbb{D}$  univalently onto the entire complex plane, excluding the segment of the negative real axis from  $-1/4$  to  $-\infty$ . Therefore, it is evident that the image of the domain does not encompass the unit disk  $\mathbb{D}$ .

The class  $\mathcal{S}^*(\alpha)$  of starlike functions of order  $\alpha$  within the open unit disk has been thoroughly examined as well as is a subset of the function class  $\mathcal{S}$ .  $\mathcal{S}^*(\alpha)$  is defined by the equation (1.11).

Ezrohi [3] introduced the class  $\mathfrak{u}(\alpha)$ , defined as:

$$\mathfrak{u}(\alpha) = \{f : f \in \mathcal{S} \text{ and } \operatorname{Re}\{f'(z)\} > \alpha, (z \in \mathbb{D}; 0 \leq \alpha < 1)\} \quad (1.3)$$

Additionally,[4] introduced the class  $\mathcal{K}(\alpha)$ , given by:

$$\mathcal{K}(\alpha) = \left\{f : f \in \mathcal{S} \text{ and } \operatorname{Re}\left\{1 + \frac{z f''(z)}{f'(z)}\right\} > \alpha, (z \in \mathbb{D}; 0 \leq \alpha < 1)\right\}. \quad (1.4)$$

A function  $f \in \mathfrak{S}$  is classified as belonging to the class  $\mathcal{S}_{\Sigma}^*(\alpha)$  of substantially bi-starlike functions of order  $\alpha$  ( $0 < \alpha \leq 1$ ) if it fulfills the subsequent conditions, as referenced in [5] and [6]:

$$f \in \Sigma \text{ and } \left| \arg\left(\frac{z f'(z)}{f(z)}\right) \right| < \frac{\alpha \pi}{2}, (z \in \mathbb{D})$$

and

$$\left| \arg\left(\frac{w g'(w)}{g(w)}\right) \right| < \frac{\alpha \pi}{2} \quad (w \in \mathbb{D}),$$

Where  $g$  equals  $f$  inverse, as defined in (1.7).

A function  $f \in \mathfrak{S}$  is classified as a bi-starlike function of order  $\alpha$  ( $0 \leq \alpha < 1$ ) if it satisfies the following criteria:

$$f \in \Sigma \text{ and } \operatorname{Re} \left( \frac{z f'(z)}{f(z)} \right) > \alpha \quad (z \in \mathfrak{D})$$

and

$$\operatorname{Re} \left( \frac{w g'(w)}{g(w)} \right) > \alpha \quad (w \in \mathfrak{D}),$$

where  $g$  equals  $f$  inverse, as defined in (1.7).

The bi-starlike and bi-convex function classes of order  $\alpha$ , denoted as  $\mathcal{S}_{\Sigma}^*(\alpha)$  and  $\mathcal{K}(\alpha)$  and corresponding to the classes, were similarly introduced. Non-sharp estimates for the first two Taylor–Maclaurin coefficients  $|a_2|$  and  $|a_3|$  were found for each of these classes (for further details, see [7, 8]). This investigation aims to provide two novel subclasses of the function class  $\Sigma$  and to derive coefficient estimates for  $|a_2|$  as well as  $|a_3|$  inside these subclasses, employing methodologies previously utilized in [9, 10].

**Definition 1.3** If the subsequent conditions are met, a function  $f(z)$  described as (1.1) is categorized within the class  $\mathcal{M}_{\Sigma}(\lambda, \beta, \alpha)$ :

$$f \in \Sigma \text{ and } \left| \arg \left( (f'(z))^{\lambda} + \beta z f''(z) \right) \right| < \frac{\alpha \pi}{2}$$

and

$$|\arg((g'(w))^{\lambda} + \beta z g''(w))| < \frac{\alpha \pi}{2}$$

Where ( $0 < \alpha \leq 1$ ,  $\beta \geq 1$ ,  $\lambda \geq 0$ ,  $z, w \in \mathfrak{D}$ ) with  $g = f^{-1}$  and given by (1.7).

**Definition 1.4** If the subsequent conditions are met, a function  $f(z)$  described as (1.1) is categorized within the class  $\mathcal{M}_{\Sigma}(\gamma, \lambda, \beta)$

$$f \in \Sigma \text{ and } \operatorname{Re} \left( (f'(z))^{\lambda} + \beta z f''(z) \right) > \gamma$$

and

$$\operatorname{Re}((g'(w))^{\lambda} + \beta z g''(w)) > \gamma$$

Where ( $0 \leq \gamma < 1$ ,  $\lambda \geq 0$ ,  $\beta \geq 1$ ,  $w, z \in \mathfrak{D}$ ) and  $g = f^{-1}$  and given by (1.7).

To demonstrate our theorem, we shall utilize the subsequent lemma:

**Lemma 1.5** [8]. If  $h$  belongs to the family  $H$ , where  $H$  represents all analytic functions in  $\mathfrak{D}$  and satisfy  $\operatorname{Re}(h(z)) > 0$ ,

$$h(z) = 1 + h_1 z + h_2 z^2 + \dots, \text{ then } |h_i| \text{ is bounded by 2 for all index } i.$$

## 2. Coefficients bounds for classes $\mathcal{M}_{\Sigma}(\lambda, \beta, \alpha)$ :and $\mathcal{M}_{\Sigma}(\gamma, \lambda, \beta)$

**Theorem 2.1** Suppose  $f(z)$  defined by (1.1) belong to the class  $\mathcal{M}_\Sigma(\lambda, \beta, \alpha)$ ,  $0 < \alpha \leq 1$ ,  $\beta \geq 1$ ,  $\lambda \geq 0$  with  $z, w \in \Sigma$ . Then

$$|a_2| \leq \frac{2\alpha}{\sqrt{6\alpha(\lambda + 2\beta) + 4\lambda(\lambda - 1) - 4(\alpha - 1)(\lambda + \beta)^2}}$$

and

$$|a_3| \leq \frac{2\alpha}{3(\lambda + 2\beta)} + \frac{\alpha^2}{(\lambda + \beta)^2} \quad (2.1)$$

**Proof** To prove the theorem, we begin by expressing the definition (1.3) in their equivalent forms:

$$(f'(z))^\lambda + \beta z f''(z) = [p(z)]^\alpha \quad (2.2)$$

$$(g'(w))^\lambda + \beta z g''(w) = [q(w)]^\alpha \quad (2.3)$$

In this context,  $p(z)$  as well as  $q(w)$  belong to the class  $H$  and adhere to the stipulations outlined in section 1.1.

These functions may be expressed as:

$$p(z) = 1 + p_1 z + p_2 z^2 + p_3 z^3 + \dots \text{ and } q(w) = 1 + q_1 w + q_2 w^2 + q_3 w^3 + \dots$$

By corresponding coefficients in equations such as (2.2) and (2.3), we obtain the following relations:

$$2(\lambda + \beta)a_2 = \alpha p_1, \quad (2.4)$$

$$3(\lambda + 2\beta)a_3 + 2\lambda(\lambda - 1)a_2^2 = \alpha p_2 + \frac{\alpha(\alpha - 1)}{2} p_1^2 \quad (2.5)$$

$$-2(\lambda + \beta)a_2 = \alpha q_1, \quad (2.6)$$

and

$$3(\lambda + 2\beta)(2a_2^2 - a_3) + 2\lambda(\lambda - 1)a_2^2 = \alpha q_2 + \frac{\alpha(\alpha - 1)}{2} q_1^2. \quad (2.7)$$

Utilizing equations (2.4) and (2.6), we derive:

$$p_1 = -q_1 \quad (2.8)$$

and

$$8(\lambda + \beta)^2 a_2^2 = \alpha^2 (p_1^2 + q_1^2). \quad (2.9)$$

Subsequently, combining equations (2.5), (2.7), and (2.9), we acquire:

$$\begin{aligned} [6(\lambda + 2\beta) + 4\lambda(\lambda - 1)]a_2^2 &= \alpha(q_2 + p_2) + \frac{\alpha(\alpha - 1)}{2}(p_1^2 + q_1^2) \\ &= \alpha(q_2 + p_2) + \frac{4(\alpha - 1)(\lambda + \beta)^2}{\alpha} a_2^2 \end{aligned} \quad (2.10)$$

Thus, we conclude:

$$a_2^2 = \frac{\alpha^2(q_2 + p_2)}{6\alpha(\lambda + 2\beta) + 4\lambda(\lambda - 1) - 4(\alpha - 1)(\lambda + \beta)^2} \quad (2.11)$$

Consequently, we ascertain:

$$|a_2| \leq \frac{2\alpha}{\sqrt{6\alpha(\lambda + 2\beta) + 4\lambda(\lambda - 1) - 4(\alpha - 1)(\lambda + \beta)^2}}. \quad (2.12)$$

This establishes the upper limit for  $|a_2|$  as per (2.1).

Subsequently, to create the constraint on  $|a_3|$ , we remove (2.5) from (2.7), resulting in

$$6(\lambda + 2\beta)a_3 - 6(\lambda + 2\beta)a_2^2 = \alpha(p_2 - q_2) + \frac{\alpha(\alpha - 1)}{2}(p_1^2 - q_1^2) \quad (2.13)$$

By substituting equation (2.4) into equation (2.9) outside equation (2.13), we derive:

$$6(\lambda + 2\beta)a_3 = \alpha(p_2 - q_2) + 6(\lambda + 2\beta)a_2^2 \quad (2.14)$$

Alternatively, it might be articulated as,

$$a_3 = \frac{\alpha(p_2 - q_2)}{6(\lambda + 2\beta)} + \frac{\alpha^2(p_1^2 + q_1^2)}{8(\lambda + \beta)^2} \quad (2.15)$$

We can determine the subsequent upper limit for  $|a_3|$ :

$$|a_3| \leq \frac{2\alpha}{3(\lambda + 2\beta)} + \frac{\alpha^2}{(\lambda + \beta)^2} \quad (2.16)$$

This concludes the proof.

**Theorem 2.2** Suppose  $f(z)$  defined by (1.1) belong to the class  $\mathcal{M}_\Sigma(\lambda, \beta, \alpha)$ , ( $0 \leq \gamma < 1$ ,  $\lambda \geq 0$ ,  $\beta \geq 1$  with  $w, z \in \mathbb{X}$ ).

Then

$$|a_2| \leq \sqrt{\frac{2(1 - \gamma)}{3(\lambda + 2\beta) + 2\lambda(\lambda - 1)'}}$$

and

$$|a_3| \leq \frac{(1 - \gamma)^2}{(\lambda + \beta)^2} + \frac{2(1 - \gamma)}{3(\lambda + 2\beta)}. \quad (2.17)$$

**Proof** It can be inferred from definition (1.4), there exist  $p(z)$  and  $q(w) \in H$  such that.

$$(f'(z))^\lambda + \beta z f''(z) = \gamma + (1 - \gamma) p(z) \quad (2.18)$$

and

$$(g'(w))^\lambda + \beta z g''(w) = \gamma + (1 - \gamma) q(w) \quad (2.19)$$

By equating the coefficients in equations (2.18) as well as (2.19), we arrive at:

$$2(\lambda + \beta)a_2 = (1 - \gamma)p_1, \quad (2.20)$$

$$3(\lambda + 2\beta)a_3 + 2\lambda(\lambda - 1)a_2^2 = (1 - \gamma)p_2, \quad (2.21)$$

$$-2(\lambda + \beta)a_2 = (1 - \gamma)q_1, \quad (2.22)$$

and

$$3(\lambda + 2\beta)(2a_2^2 - a_3) + 2\lambda(\lambda - 1)a_2^2 = (1 - \gamma)q_2. \quad (2.23)$$

By employing calculations (2.20) along with (2.22), we derive the subsequent conclusions:

$$p_1 = -q_1 \quad (2.24)$$

and

$$8(\lambda + \beta)^2 a_2^2 = (1 - \gamma)^2 (p_1^2 + q_1^2) \quad (2.25)$$

From equations (2.21) and (2.23), it can be concluded that:

$$6(\lambda + 2\beta)a_2^2 + 4\lambda(\lambda - 1)a_2^2 = (1 - \gamma)(p_2 + q_2)$$

Consequently, we obtain the following:

$$a_2 = \sqrt{\frac{(1 - \gamma)(p_2 + q_2)}{6(\lambda + 2\beta) + 4\lambda(\lambda - 1)}} \quad (2.26)$$

This establishes the top limit for  $|a_2|$ , in agreement with (2.17):

$$|a_2| \leq \sqrt{\frac{2(1 - \gamma)}{3(\lambda + 2\beta) + 2\lambda(\lambda - 1)}} \quad (2.27)$$

Subsequently, to establish the constraint on  $|a_3|$ , we subtract (2.21) both (2.23) and (2.24), yielding

$$6(\lambda + 2\beta)a_3 - 6(\lambda + 2\beta)a_2^2 = (1 - \gamma)(p_2 - q_2), \quad (2.28)$$

Alternatively, it can be expressed as:

$$a_3 = a_2^2 + \frac{(1 - \gamma)(p_2 - q_2)}{6(\lambda + 2\beta)}, \quad (2.29)$$

Applying equation (2.24) within equation (2.43), we obtain:

$$a_3 = \frac{(1 - \gamma)^2 (p_1^2 + q_1^2)}{8(\lambda + \beta)^2} + \frac{(1 - \gamma)(p_2 - q_2)}{6(\lambda + 2\beta)}, \quad (2.30)$$

We can determine the subsequent upper limit for  $|a_3|$ :

$$|a_3| \leq \frac{(1 - \gamma)^2}{(\lambda + \beta)^2} + \frac{2(1 - \gamma)}{3(\lambda + 2\beta)} \quad (2.31)$$

This finishes the proof.

### 3. Corollaries and consequences

By entering  $\lambda = 1$  into Theorem (2.1) along with Theorem (2.2), we derive the respective corollaries.

**Corollary 3.1.** Suppose  $f(z)$  specified in (1.1) belong to the class  $\mathcal{M}_\Sigma(1, \beta, \alpha)$ ,  $0 < \alpha \leq 1$ ,  $\beta \geq 1$ , with  $z, w \in \mathbb{X}$ . Then

$$|a_2| \leq \frac{2\alpha}{\sqrt{6\alpha(1+2\beta) - 4(\alpha-1)(1+\beta)^2}}$$

and

$$|a_3| \leq \frac{2\alpha}{3(1+2\beta)} + \frac{\alpha^2}{(1+\beta)^2}$$

**Corollary 3.2** Assume  $f(z)$  specified in (1.1) belong to the class  $\mathcal{M}_\Sigma(\gamma, \lambda, \beta)$ ,  $(0 \leq \gamma < 1, \lambda \geq 0, \beta \geq 1$  with  $w, z \in \mathbb{X}$ . Then

$$|a_2| \leq \sqrt{\frac{2(1-\gamma)}{3(1+2\beta)}},$$

and

$$|a_3| \leq \frac{(1-\gamma)^2}{(1+\beta)^2} + \frac{2(1-\gamma)}{3(1+2\beta)}.$$

Through substituting  $\lambda = 0$  in Theorem (2.1) along with Theorem (2.2), we obtain the subsequent ramifications, which are as follows:

**Corollary 3.3.** Assume  $f(z)$  specified in (1.1) belong to the class  $\mathcal{M}_\Sigma(\lambda, \beta, \alpha)$ ,  $0 < \alpha \leq 1$ ,  $\beta \geq 1, \lambda \geq 0$  and  $z, w \in \mathbb{X}$ .

Then

$$|a_2| \leq \frac{2\alpha}{\sqrt{12\alpha\beta - 4(\alpha-1)\beta^2}}$$

and

$$|a_3| \leq \frac{2\alpha}{6\beta} + \frac{\alpha^2}{\beta^2}$$

**Corollary 3.4** Assume  $f(z)$  specified in (1.1) belong to the class  $\mathcal{M}_\Sigma(\gamma, \lambda, \beta)$ ,  $(0 \leq \gamma < 1, \lambda \geq 0, \beta \geq 1$  and  $w, z \in \mathbb{X}$ .

Then

$$|a_2| \leq \sqrt{\frac{2(1-\gamma)}{6\beta}},$$

and

$$|a_3| \leq \frac{(1-\gamma)^2}{\beta^2} + \frac{2(1-\gamma)}{6\beta}.$$

Through substituting  $\lambda = 1$  with  $\beta = 1$  in Theorem (2.1) along with Theorem (2.2), we obtain the subsequent ramifications, which are as follows:

**Corollary 3.5** Assume  $f(z)$  specified in (1.1) belong to the class  $\mathcal{M}_{\Sigma}(1,1,\alpha)$ ,  $0 < \alpha \leq 1$ , and  $z, w \in \mathbb{D}$ . Then

$$|a_2| \leq \frac{2\alpha}{\sqrt{18\alpha - 16(\alpha - 1)}} \text{ and } |a_3| \leq \frac{2\alpha}{9} + \frac{\alpha^2}{4}$$

**Corollary 3.6** Assume  $f(z)$  specified in (1.1) belong to the class  $\mathcal{M}_{\Sigma}(\gamma, 1, 1)$ ,  $(0 \leq \gamma < 1)$ , with  $w, z \in \mathbb{D}$ . Then

$$|a_2| \leq \sqrt{\frac{2(1-\gamma)}{9}}, \text{ and } |a_3| \leq \frac{(1-\gamma)^2}{4} + \frac{2(1-\gamma)}{9}$$

#### Remark 2.18

- i) The estimates for coefficients  $|a_2|$  and  $|a_3|$  in Corollary 3.5 & 3.6 obtained by [11].
- ii) The estimates for coefficients  $|a_2|$  and  $|a_3|$  in Corollary 3.1 & 3.2 obtained by [11].

## 4. Hilbert Space Framework and a Basic Example

The Hilbert space framework can further enhance the study of analytic functions by providing a complete inner product structure that allows functions to be represented through square-summable series. This enables the use of functional analytic tools in estimating Taylor coefficients and studying geometric properties of analytic classes. In particular, the Hardy space  $H^{2D}$  serves as a fundamental example in this context, where analytic functions can be viewed as infinite-dimensional vectors, allowing the application of projection techniques and linear operator theory to investigate growth, distortion, and convexity properties (Duren, 1970). Moreover, this representation plays a significant role in the study of univalent and bi-univalent functions by connecting Taylor coefficients with norm inequalities in Hilbert spaces, which is essential in coefficient estimation problems for modern subclasses of analytic functions (Pommerenke, 1975). In addition, reproducing kernel Hilbert spaces (RKHS) provide a stronger analytical framework for studying convolution-type operators such as the Hadamard product, which naturally arises in geometric function theory and operator-based function classes (Sarangapani & Gupta, 2018).

A simple example of a Hilbert space is the space  $l^2$  of all square-summable sequences. A sequence  $a_n$  belongs to  $l^2$  if

$$\sum_{n=1}^{\infty} |a_n|^2 < \infty.$$

The inner product in  $l^2$  is defined by

$$\langle a, b \rangle = \sum_{n=1}^{\infty} a_n \overline{b_n},$$

and the corresponding norm is

$$|a| = \left( \sum_{n=1}^{\infty} |a_n|^2 \right)^{\frac{1}{2}}.$$

Now consider an analytic function

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad z \in u.$$

If the coefficient sequence  $(1, a_2, a_3, \dots)$  belongs to  $l^2$ , then  $f$  can be treated as a vector in a Hilbert space.

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