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On DM- Compact Smarandache Topological Semigroups

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Abstract

In this present paper, we have introduced some new definitions On DM-compact Smarandache topological semigroup and DM-L. compact Smarandache topological semigroup for the compactness in topological spaces and groups. We obtained some results related to DM-compact Smarandache topological semigroup and DM-L. compact Smarandache topological semigroup, for example any infinite group can be a DM- compact Smarandache topological semigroup.

Keywords: semigroups, groups, Smarandache semigroup, DM-covering, topological semigroup, topological group, direct product, isomorphism.

Mathematics subject classification: 22-XX

1. Introduction

A topological group $(G,\tau,*)$ is said to be compact topological group, if a topological space (G,τ) is a compact space [1]. Also a group $(G,*)$ is said to be D-compact group if for every D-cover group of $(G,*)$, there exists a finite sub-D-cover group of $(G,*)$ [5].

Vasantha Kandasamy [6], introduced details on a Smarandache structure on a set **G** means a weak structure **W**on **G**, where there exists a proper subset **H** of **G** embedded with a strong structure **S**. Here, we investigated on DM-compact Smarandache topological semigroup and DM-L. compact Smarandache topological semigroup for the compaction in topological spaces and groups, we obtain some good results related to these concepts above. Al-Khafajy [7], introduced details on D-Compact Smarandache Groupoids. Al-Khafajy and Sadek [8], studied the D-Compact Topological Groups.

Motivated by this, we introduce and study the DM-compact Smarandache topological semigroup and DM-L. compact Smarandache topological semigroup.

2. Definitions

2.1 Definition

1- We say that the triple $(G,\tau,*)$ is a topological semigroup if (G,τ) is a topological space and $(G,*)$ is a semigroup, where $*: G \times G \to G$ is a continuous, (the set $G \times G$ has the product topology).

2- [1]We say that the triple $(G,\tau,*)$ is a topological monoid if (G,τ) is a topological space and (G,τ) is a semigroup with a unit element (monoid).

3- [2]The topological semigroup $(G,\tau,*)$ is called topological group and denoted by $(G,*,\tau)$ if $(G,*)$ is a group, such that, writing $p(x) = x^{-1}$ the inversion map $p : G \to G$ is continuous.

4- Let $(G,\tau,*)$ is a topological semigroup (monoid, group), the topological subsemigroup (submonoid, subgroup) $(H,\tau_H,*)$ is a subset H of G with the topological and semigroup (monoid, group) structures induced from G make H a topological semigroup (monoid, group), respectively, where $\tau_H = H \cap \tau$

2.2 Definition

Let $(G,\tau,*)$ be a topological semigroup, T is a non empty subset of G, and I be an indexed (I is a finite or an infinite set), we say that;

1- The family $\{A_i ;\ A_i \in \tau, \forall i \in I\}$ is a DM-covering set of T if $T \subseteq \bigcup_{i\in I} A_i$.

2-The set T is DM- compact Smarandache set if for any DM-covering set of T, there is a finite DM-subcovering set of T, $\{A_j\}_{j\in J}$, (J is a finite set), such that $T = \bigcup_{j\in J} A_j$ and $(A_j,*)$ is a group $\forall\, j \in J$ under the same operation $*$ on G.

3-The set T is DM-L. compact Smarandache set if for any DM-covering set of T , there is a countable DM-subcovering set of T, $\{A_s\}_{s\in S}$,(S is a countable set), such that $T = \bigcup_{s\in S} A_s$ and $(A_s,*)$ is a group $\forall\, s \in S$ under the same operation $*$ on G.

2.3 Definition

Let $(G,\tau,*)$ be a topological semigroup and I be an indexed (I is a finite or an infinite) set, we say that;

1- The family $\{G_i ;\ G_i \in \tau, \forall i \in I\}$ is DM-covering of $(G,\tau,*)$ if $G = \bigcup_{i\in I} G_i$.

2- The topological semigroup $(G,\tau,*)$ is DM-weakly compact Smarandache topological semigroup, if there is a finite DM-covering of $(G,\tau,*)$, such that $(G_i,*)$ is a group $\forall\, i \in I$ under the same operation $*$ on G .

3- The topological semigroup $(G,\tau,*)$ is DM-compact Smarandache topological semigroup if for every DM-covering of $(G,\tau,*)$ there is a finite sub-DM-covering $\{G_j\}_{j\in J}$, (J is a finite set), such that $G = \bigcup_{j\in J} G_j$ and $(G_j,*)$ is a group $\forall\, j \in J$ under the same operation $*$ on G.

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[1] See [6]

[2] See [2]

4- The topological semigroup $(G,\tau,*)$ is DM-weakly L. compact Smarandache topological semigroup, if there is a countable DM-covering of $(G,\tau,*)$, such that $(G_j,*)$ is a group $\forall\, j \in J$ under the same operation $*$ on G .

5- The topological semigroup $(G,\tau,*)$ is DM-L. compact Smarandache topological semigroup if for every DM-covering of $(G,\tau,*)$ there is a countable sub-DM-covering $\{G_s\}_{s\in S}$, (S is a countable set), such that $G = \bigcup_{s\in S} G_s$ and $(G_s,*)$ is a group $\forall\, s \in S$ under the same operation $*$ on G .

2.4 Definition

[3]Let $(G,\tau,*)$ and $(\bar{G},\bar{\tau},\bar{*})$ be two topological semigroups, we say that;

1- $f:(G,\tau,*) \to (\bar{G},\bar{\tau},\bar{*})$ is a *homomorphism* if $f:(G,\tau) \to (\bar{G},\bar{\tau})$ is a continuous and $f(x*y)= f(x)\,\bar{*}\, f(y) \quad \forall x, y \in G$.

2- $f:(G,\tau,*) \to (\bar{G},\bar{\tau},\bar{*})$ is an *isomorphism* if it is a topological homeomorphism and $f(x*y)= f(x)\,\bar{*}\, f(y) \quad \forall x, y \in G$.

3 Main Results

The prove of the following lemma is direct, hence is omitted.

3.1 Lemma

Any DM- compact Smarandache topological semigroup is DM- weakly (DM-L.) compact Smarandache topological semigroup.

3.2 Theorem

Any infinite group can be a DM- compact Smarandache topological semigroup.

Proof

Let $(G,*)$ is any an infinite group, I is a set (finite or infinite), defined

$\tau = \{\, A_i \subseteq G\,;\, A_i^c$ is a finite set, $(A_i,*)$ group $\forall i \in I\ \&\ A_{i_1} \subseteq A_{i_2}$ for $i_1 \le i_2 \,\} \cup \emptyset$.

It is clear that $\tau \ne \emptyset$, since every finite group G, $(o(G) \ge 4)$, has nontrivial subgroups unless it is cyclic of prime order, but G is an infinite so G has nontrivial subgroups, [3].

It is easy to prove that $(G,*)$ is a topological space;

1- $\emptyset \in \tau$ and $G^c = \emptyset$ is a finite $\Longrightarrow\ G \in \tau$.

2- Let $A_1, A_2 \in \tau$ so A_1^c, A_2^c are finite, but $(A_1 \cap A_2)^c = A_1^c \cup A_2^c \Longrightarrow (A_1 \cap A_2)^c$ is finite and we know that $(A_1 \cap A_2,*)$ is a group $\Longrightarrow A_1 \cap A_2 \in \tau$.

3- Let$A_s \in \tau$, $\forall s \in S \Longrightarrow A_s^c$ is a finite $\forall s \in S$ $\Longrightarrow \bigcap_{s\in S} A_s^c$ is a finite and $(\bigcup_{s\in S} A_s)^c = \bigcap_{s\in S} A_s^c$, and we know that $\bigcup_{s\in S} A_s = A_t$ for some t where $s \le t\ \ \forall s \in S$ so $(\bigcup_{s\in S} A_s,*)$ is a group and hence $\bigcup_{s\in S} A_i \in \tau$.

Therefore $(G,*)$ is a topological space. And hence $(G,\tau,*)$ is a topological group, which is also a topological semigroup.

Let $\{A_\lambda\,;\, A_\lambda \in \tau, \lambda \in \Lambda\}$, indexed by Λ, be any DM-covering of $(G,\tau,*)$, that is $G = \bigcup_{\lambda\in\Lambda} A_\lambda$. Let $A_\circ \in \{A_\lambda\}_{\lambda\in\Lambda} \Longrightarrow (A_\circ,*)$ is a group and $A_\circ^c$ is a finite set, suppose that $A_\circ^c = \{a_1, a_2, \ldots, a_n\}$, where $a_j \in G\ \ \forall j \in J$. For

[3] See [2]

each $j \in J$ there is $A_{\lambda_j} \in \{A_\lambda\}_{\lambda \in \Lambda}$ such that $a_j \in A_{\lambda_j} \Longrightarrow A_\circ^c = \bigcup_{j \in J} A_{\lambda_j}$. But $G = A_\circ \cup A_\circ^c$ $\Longrightarrow G = A_\circ \cup (\bigcup_{j \in J} A_{\lambda_j})$,so that there is a finite sub-DM-covering $\{A_\circ, A_{\lambda_1}, A_{\lambda_2}, \ldots, A_{\lambda_n}\}$ which is $(A_\circ,*)$ and $(A_{\lambda_j},*)$ are groups for each $j \in J$, and therefore $(G,\tau,*)$ is a DM- compact Smarandache topological semigroup.

By Lemma 3.1 and Theorem 3.2 we can prove the following, any infinite group can be a DM- weak (DM-L.) compact Smarandache topological semigroup.

We can prove directly, by order the group and Lemma 3.1, the following theorem,

3.3 Theorem

Let $(G,\tau,*)$ be a topological semigroup, such that G is a finite set. Then the following are equivalents;

1-$(G,\tau,*)$ is a DM- compact Smarandache topological semigroup.

2-$(G,\tau,*)$ is a DM-L. compact Smarandache topological semigroup.

3.4 Theorem

Let $(G,\tau,*)$ be a topological semigroup and $A,\ B \subseteq G$, if $A,\ B$ are DM- compact Smarandache set. Then $A \cup B$ is a DM- compact Smarandache set.

Proof

Let $\{U_i\}_{i \in I}$ be a DM-covering set of $A \cup B$ where $U_i \in \tau, \forall i \in I$, so that, $A \subseteq \bigcup_{i \in I} U_i$ and $B \subseteq \bigcup_{i \in I} U_i$ but A and B are DM- compact Smarandache sets, then there are finite subsets $J_1 . J_2 \subseteq I$ such that $A \subseteq \bigcup_{s \in J_1} U_s$ and $B \subseteq \bigcup_{t \in J_2} U_t$ where $(U_s,*)$ and $(U_t,*)$ are groups for each $s \in J_1, t \in J_2$, hence $A \cup B \subseteq \left(\bigcup_{s \in J_1} U_s\right) \cup \left(\bigcup_{t \in J_2} U_t\right) = \bigcup_{j \in J_1 \cup J_2} U_j$ where $J_1 \cup J_2$ is a finite set and $(U_j,*)$ is a group for each $j \in J_1 \cup J_2$. Therefore $A \cup B$ is a DM- compact Smarandache set.

The prove of the following corollary is direct from Theorem 3.4, hence is omitted.

3.5 Corollary

Let $(G,\tau,*)$ be a topological semigroup and $A\,.\,B \in \tau$ such that $A \cup B$ is a DM- compact Smarandache set, if $(B,*)$ is group. Then A is a DM- compact Smarandache set.

3.6 Theorem

Let $(G,\tau,*)$ be a topological semigroup and $A, B \subseteq G$, if

1- $A \cup B$ is a DM- compact Smarandache set,

2- A and B are disjoint open sets,

3- $(A,*)$, $(B,*)$are groups,

Then A and B are DM- compact Smarandache sets.

Proof

Let $\{U_i\}_{i \in I}$ be a DM-covering set of A where $U_i \in \tau\,.\forall i \in I \Longrightarrow A \cup B \subseteq (\bigcup_{i \in I} U_i) \cup B$ but $A \cup B$ is a DM-Smarandache compact set, so that, there is a finite subset of I such that $A \cup B \subseteq (\bigcup_{j \in J} U_j) \cup B$ where $(U_j,*)$ are group $\forall j \in J \Longrightarrow (A \cup B) \cap A \subseteq \left[\left(\bigcup_{j \in J} U_j\right) \cup B\right] \cap A \Longrightarrow A \subseteq A \cap (\bigcup_{j \in J} U_j) \Longrightarrow A \subseteq \bigcup_{j \in J} U_j$, hence A is a DM- compact Smarandache set.

By similarity can we prove that B is a DM- compact Smarandache set.

3.7 Theorem

Let $(G,\tau,*)$ be a topological semigroup and $A \subseteq H \subseteq G$, if $(H,*)$ is a group and A is a DM-compact Smarandache set in $(G,\tau,*)$. Then A is a DM- compact Smarandache set in $(H,\tau_H,*_H)$.

Proof

Let $\{H_i\}_{i\in I}$ be any DM-covering set of A in $(H,\tau_H,*_H)$, (where $\tau_H = H \cap \tau$), that is $A \subseteq \bigcup_{i\in I} H_i$ and $H_i = G_i \cap H$. $G_i \in \tau$, $\forall i \in I \implies A \subseteq \bigcup_{i\in I}(G_i \cap H) = (\bigcup_{i\in I} G_i) \cap H$ $\implies A \subseteq \bigcup_{i\in I} G_i$ but A is a DM- compact Smarandache set in $(G,\tau,*)$, so there is a finite subset $J \subseteq I$ such that $A = \bigcup_{j\in J} G_j$ and $(G_j,*)$ is a group $\forall j \in J \implies \implies \mathrm{A} = (\bigcup_{j\in J} \mathrm{G_j}) \cap \mathrm{H}$ $= \bigcup_{j\in J}(\mathrm{G_j} \cap \mathrm{H})$, where $(G_j \cap H,*)$ is a group $\forall j \in J$.

Therefore A is a DM- compact Smarandache set in $(H,\tau_H,*_H)$.

The prove of the following theorem is direct, hence is omitted.

3.8 Theorem

Let $(G,\tau,*)$ be a DM- compact Smarandache topological semigroup and $H \subseteq G$, if $(H,*)$ is a subgroup of $(G,*)$. Then $(H,\tau_H,*)$ is a DM-compact Smarandache topological semigroup.

3.9 Corollary

Let $(G,\tau,*)$ is a DM- compact Smarandache topological semigroup and $H_i \subseteq G$, indexed by I, be any family of subset of G such that $(H_i,*)$ is a subgroup of $(G,*)$ for each $i \in I$. Then $(\bigcap_{i\in I} H_i,\mathcal{T},*)$ is a DM- compact Smarandache topological semigroup, (where $\mathcal{T} = (\bigcap_{i\in I} H_i) \cap \tau$).

3.10 Theorem

Let $(G,\tau,*)$ and $(\bar{G},\bar{\tau},\bar{*})$ are two topological semigroups, if $(G.*)$ is a group and $(\bar{G},\bar{\tau},\bar{*})$ is a DM- compact Smarandache topological semigroup. Then $(G \times \bar{G},\tau \times \bar{\tau},\otimes)$ is a DM-compact Smarandache topological semigroup.

Proof

Let $\{(G \times \bar{G}_i ,\otimes); \bar{G}_i \in \bar{\tau}, \forall i \in I\}$ be any DM-covering of $G \times \bar{G} \implies G \times \bar{G} = \bigcup_{i\in I}(G \times \bar{G}_i)$ $= G \times (\bigcup_{i\in I} \bar{G}_i) \implies \bar{G} = \bigcup_{i\in I} \bar{G}_i$ but $(\bar{G},\bar{\tau},\bar{*})$ is a DM- compact Smarandache topological semigroup, so there is a finite subset $J \subseteq I$ such that $\bar{G} = \bigcup_{j\in J} \bar{G}_j$ and $(\bar{G}_j ,\bar{*})$ is a group $\forall j \in J$

$\implies G \times \bar{G} = G \times (\bigcup_{j\in J} \bar{G}_j) = \bigcup_{j\in J}(G \times \bar{G}_j)$ where $G \times \bar{G}_j \in \tau \times \bar{\tau}$ and $(G \times \bar{G}_j,\otimes)$ is a group for each $j \in J$. Therefore $(G \times \bar{G},\tau \times \bar{\tau},\otimes)$ is a DM- compact Smarandache topological semigroup.

3.11 Theorem

Let $(G,\tau,*)$ and $(\bar{G},\bar{\tau},\bar{*})$ be two DM-compact Smarandache topological semigroups. Then $(G \times \bar{G},\tau \times \bar{\tau},\otimes)$ is a DM- compact Smarandache topological semigroup.

Proof

Let $(G,\tau,*)$ and $(\bar{G},\bar{\tau},\bar{*})$ are two DM- compact Smarandache topological semigroup $\implies$ there exists a DM-covering groups $\{G_a\}_{a\in A}$ and $\{\bar{G}_b\}_{b\in B}$of G and $\bar{G}$, respectively,, [4]$\implies \mathrm{G} \times \bar{\mathrm{G}} =$ $(\bigcup_{a\in A} \mathrm{G_a}) \times (\bigcup_{b\in B} \bar{\mathrm{G}}_b) = \bigcup_{a\in A, b\in B}(\mathrm{G_a} \times \bar{\mathrm{G}}_b)$ $\implies \{\mathrm{G_a} \times \bar{\mathrm{G}}_b\}_{a\in A, b\in B}$ is a DM-covering of $(G \times \bar{G},\tau \times \bar{\tau},\otimes)$.

[4] See [4]

Let $\{\mathcal{W}_i\}_{i\in I}$ be any DM-covering of $(G \times \bar{G},\tau \times \bar{\tau},\otimes) \Longrightarrow G \times \bar{G} = \bigcup_{i\in I} \mathcal{W}_i$, such that $\mathcal{W}_i = \mathcal{U}_i \times \mathcal{V}_i$, where $\mathcal{U}_i \in \tau$, $\mathcal{V}_i \in \bar{\tau}$ for each $i \in I$. But $(G,\tau,*)$ is a DM- compact Smarandache topological semigroup, so there is a finite subset of I such that $G = \bigcup_{j\in J} \mathcal{U}_j$ and $(\mathcal{U}_j,*)$ is a group for each $j \in J$.

Let $\mathcal{U}_{j_1} \in \{\mathcal{U}_j\}_{j\in J} \Longrightarrow \{\mathcal{U}_{j_1} \times \mathcal{V}_i\}_{i\in I}$ is a DM-covering of $(\mathcal{U}_{j_1} \times \bar{G},\otimes) \Longrightarrow \mathcal{U}_{j_1} \times \bar{G} = \bigcup_{i\in I}(\mathcal{U}_{j_1} \times \mathcal{V}_i)$, but $\mathcal{U}_{j_1} \times \bar{G}$ is a DM-compact Smarandache topological semigroup from Theorem 3.11 since $(\mathcal{U}_{j_1},*)$ is a group and $(\bar{G},\bar{*})$ is a DM- compact Smarandache topological semigroup, so there is a finite set $S \subseteq I$ such that $\{\mathcal{U}_{j_1} \times \mathcal{V}_s\}_{s\in S}$ is a group $\forall s \in S$ and $U_{j_1} \times \bar{G} = \bigcup_{s\in S}(\mathcal{U}_{j_1} \times \mathcal{V}_s) = \mathcal{U}_{j_1} \times (\bigcup_{s\in S} \mathcal{V}_s) \Longrightarrow \bigcup_{j_1\in J}(\mathcal{U}_{j_1} \times (\bigcup_{s\in S} \mathcal{V}_s))$
$= (\bigcup_{j\in J} U_j) \times (\bigcup_{s\in S} V_s) = G \times \bar{G}$ [5] $\Longrightarrow G \times \bar{G}$
$(\bigcup_{j\in J} U_j) \times (\bigcup_{s\in S} V_s) = \bigcup_{j\in J. s\in S}(U_j \times V_s)$,
where $(\mathcal{U}_j \times \mathcal{V}_s,\otimes)$ are groups for each $j \in J$. $s \in S$. Therefore $(G \times \bar{G},\tau \times \bar{\tau},\otimes)$ is a DM-compact Smarandache topological semigroup.

The prove of the following corollary is direct, hence is omitted.

3.12 Corollary

Let $(G,\tau,*)$ is a DM- compact Smarandache topological semigroup and $H.S$ are two subsets of G. Then $H \times S$ is a DM- compact Smarandache set in $(G \times G,\tau \times \tau,\otimes)$.

3.13 [6]Theorem

Let $\{G_i : i \in I\}$ be a family of topological groups. Then the direct $G = \prod_{i\in I} G_i$, equipped with the product topology is a topological group.

From Theorem 3.11 and Theorem 3.13, respectively, and by induction we can prove the following theorem;

3.14 Theorem

If $(G,\tau,*)$ is a DM- compact Smarandache topological semigroup, then $(G^n,\tau^n,\otimes)$ is a DM- compact Smarandache topological semigroup, where $(\boldsymbol{x} \otimes \boldsymbol{y}) = (x_1 * y_1,..., x_n * y_n)$ for each $x_i, y_i \in G$, $i = 1, 2, . . ., n$.

3.15 Theorem

The product of any finite collection of DM-compact Smarandache topological semigroups is a DM- compact Smarandache topological semigroup.

The following corollary is direct from Corollary 3.12 and Theorem 3.15;

3.16 Corollary

Suppose I is non-empty set and $(G_i,\tau_i,*_i)$ is a DM- compact Smarandache topological semigroups for each $i \in I$, if H_i is a subset of G_i , $\forall i \in I$. Then $\prod_{i\in I} H_i$ is a DM- compact Smarandache set in $\prod_{i\in I} G_i,\mathcal{S},\otimes)$, where $\mathcal{S} = \tau_{\prod_{i\in I} G_i}$ the usual product topology.

3.17 Theorem

Let $(G,\tau,*)$ and $(\bar{G},\bar{\tau},\bar{*})$ be two topological semigroups and $f : (G,\tau,*) \rightarrow (\bar{G},\bar{\tau},\bar{*})$ is an isomorphism. Then

[5] See [4]

[6] proposition 3.3.4., p.18, [1].

1- If A is a DM- compact Smarandache set in $(G,\tau,*)$ $\Longrightarrow$ $f(A)$ is a DM- compact Smarandache set in $(\bar{G},\bar{\tau},\bar{*})$.

2- If B is a DM- compact Smarandache set in $(\bar{G},\bar{\tau},\bar{*})$ and f is an open map $\Longrightarrow$ $f^{-1}(B)$ is a DM- compact Smarandache set in $(G,\tau,*)$.

Proof

1- Let $\{\bar{G}_i\}_{i\in I}$ be any DM-covering set of $f(A)$ in $(\bar{G},\bar{\tau},\bar{*})$ that is $f(A) \subseteq \bigcup_{i\in I}\bar{G}_i$ [7]$\Longrightarrow$ $A \subseteq f^{-1}(\bigcup_{i\in I}\bar{G}_i) = \bigcup_{i\in I} f^{-1}(\bar{G}_i)$, it is clear that $f^{-1}(\bar{G}_i) \in \tau$.$\forall i \in I$ since $\bar{G}_i \in \bar{\tau}$ for each $i \in I$ and f is continuous , but A is a DM- compact Smarandache set in $(G, \tau, *)$, so there is a finite subset of I such that $A = \bigcup_{j\in J} f^{-1}(\bar{G}_i)$ and $(f^{-1}(\bar{G}_i), *)$ is a group $\forall j \in J$ $\Longrightarrow A = f^{-1}\left(\bigcup_{j\in J}\bar{G}_j\right)$ $\Longrightarrow$ $f(A) = f\left(f^{-1}\left(\bigcup_{j\in J}\bar{G}_j\right)\right) = \bigcup_{j\in J}\bar{G}_j$ where $(\bar{G}_j,\bar{*})$ is a group $\forall j \in J$ since f is an isomorphism $\Longrightarrow$ $f(A)$ is a DM- compact Smarandache set in $(\bar{G},\bar{\tau},\bar{*})$.

2- Let $\{G_i\}_{i\in I}$be any DM-covering set of $f^{-1}(B)$ in$(G,\tau,*)$$\Longrightarrow f^{-1}(B) \subseteq \bigcup_{i\in I} G_i$,$(G_i \in \tau.\ \forall i \in I)$ $\Longrightarrow B \subseteq f(\bigcup_{i\in I} G_i) = \bigcup_{i\in I} f(G_i)$, it is clear that $f(G_i) \in \bar{\tau}.\ \forall i \in I$ since f is an open map , but B is a DM- compact Smarandache set in $(\bar{G},\bar{\tau},\bar{*})$, so there is a finite subset $J \subseteq I$ such that $B = \bigcup_{j\in J} f(G_j)$ where $\left(f\left(G_j\right),\bar{*}\right)$ is a group $\forall j \in J \Longrightarrow B = f(\bigcup_{j\in J} G_j) \Longrightarrow f^{-1}(B) = \bigcup_{j\in J} G_j$,
where $(G_j,*)$ is a group $\forall j \in J$ since f is an isomorphism $\Longrightarrow f^{-1}(B)$ is a DM- compact Smarandache set in $(G,\tau,*)$.

3.18 Theorem

Let $(G,\tau,*)$ and $(\bar{G},\bar{\tau},\bar{*})$ be two topological semigroups and $f : (G,\tau,*) \to (\bar{G},\bar{\tau},\bar{*})$ is an isomorphism. Then the following are equivalents;

1-$(G,\tau,*)$ is a DM-compact Smarandache topological semigroup,

2-$(\bar{G},\bar{\tau},\bar{*})$ is a DM- compact Smarandache topological semigroup.

Proof

$(\Longrightarrow)$ Suppose that $(G,\tau,*)$ is a DM- compact Smarandache topological semigroup , let $\{\bar{G}_i\ ;\ \bar{G}_i \in \bar{\tau}\ , \forall i \in I\}$ be any *DM-covering* of $(\bar{G},\bar{\tau},\bar{*})$ $\Longrightarrow$ $\bar{G} = \bigcup_{i\in I}\bar{G}_i$ $\Longrightarrow$ $G = f^{-1}(\bar{G}) = f^{-1}(\bigcup_{i\in I}\bar{G}_i) \Longrightarrow G = \bigcup_{i\in I} f^{-1}(\bar{G}_i)$, but $(G,\tau,*)$ is a DM-compact Smarandache topological semigroup, so there is a finite subset of I such that $G = \bigcup_{j\in J} f^{-1}(\bar{G}_i)$ and $(f^{-1}(\bar{G}_i).*)$ is a group $\forall j \in J$ $\Longrightarrow$ $G = f^{-1}\left(\bigcup_{j\in J}\bar{G}_j\right)$ $\Longrightarrow$ $\bar{G} = f(G) = f(f^{-1}\left(\bigcup_{j\in J}\bar{G}_j\right)) = \bigcup_{j\in J}\bar{G}_j$, where $\left(\bar{G}_j\ .\bar{*}\right)$ is a group $\forall j \in J$. Therefore $(\bar{G},\bar{\tau},\bar{*})$ is a DM- compact Smarandache topological semigroup.

$(\Longleftarrow)$ Suppose that $(\bar{G},\bar{\tau},\bar{*})$ is a DM- compact Smarandache topological semigroup, let $\{G_i\ ;\ G_i \in \tau,\ \forall i \in I\}$ be any DM-covering of $(G,\tau,*)$ $\Longrightarrow$ $G = \bigcup_{i\in I} G_i$ $\Longrightarrow$ $\bar{G} = f(G) = f(\bigcup_{i\in I} G_i) \Longrightarrow \bar{G} = \bigcup_{i\in I} f(G_i)$, but $(\bar{G},\bar{\tau},\bar{*})$ is a DM-compact Smarandache topological semigroup, so there is a finite subset of I such

[7] See [4]

that $\bar{G} = \bigcup_{j \in J} f(G_j)$ and $(f(G_j), \bar{*})$ is a group $\forall j \in J \Longrightarrow \bar{G} = f(\bigcup_{j \in J} G_j) \Longrightarrow G = f^{-1}(\bar{G}) = f^{-1}\left(f\left(\bigcup_{j \in J} G_j\right)\right) = \bigcup_{j \in J} G_j$, where $(G_j, *)$ is a group $\forall j \in J$.

Therefore $(G, \tau, *)$ is a DM- compact Smarandache topological semigroup.

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حول تراص سمارانداش لشيه الزمر التوبولوجية من نوع - DM

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المستخلص :

قدمنا في هذا البحث بعض التعاريف الجديدة حول تراص سمارانداش لشيه الزمر التوبولوجية من نوع - DM و حول تراص سمارانداش لشيه الزمر التوبولوجية من نوع .DM-L وهي تربط الفضاءات التوبولوجية ونظرية الزمر. حصلنا على بعض النتائج تتعلق بتراص سمارانداش لشيه الزمر التوبولوجية من نوع - DM و تراص سمارانداش لشيه الزمر التوبولوجية من نوع .DM-L ، منها كل زمرة غير منتهية ممكن تكون تراص سمارانداش لشيه زمرة توبولوجية من نوع- DM .