Journal of AL-Qadisiyah for computer science and mathematics Vol.9 No.2 Year 2017
ISSN (Print): 2074 – 0204 ISSN (Online): 2521 – 3504

Integration of the Al-Tememe Transformation To find the Inverse of Transformation And Solving Some LODEs With (I.C)

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Recived : 20\4\2015 Revised : 14\3\2017 Accepted : 14\9\2017

Abstract:

Our aim in this paper is to find the integration of Al-Tememe ($\mathcal{T}$) transformation to help us in finding ($\mathcal{T}^{-1}$) for some functions without using of partition method and solve linear ordinary differential equations (LODEs) with variable coefficients by using (I.C) .

Mathematical subject classification:

Introduction:

We will use the new idea exists in [3] to find the integration of Al-Tememe transformation so it will give us the ability to find ($\mathcal{T}^{-1}$) by new method for some functions.

Basic definitions and concepts :

Definition 1: [1]

Let f is defind function at a period (a, b) then the integral transformation for f whose it's symbol $F(p)$ is defined as :

$$F(p) = \int_a^b k(p, x) f(x) dx,$$

Where k is a fixed function of two variables x and p , called the kernel of the transformation, and a, b are real numbers or $\mp\infty$, such that the integral above converges.

Definition 2: [2]

Al-Tememe transformation for the function $f(x)\,; x > 1$ is defined by the following integral :

$$\mathcal{T}[f(x)] = \int_1^\infty x^{-p} f(x) dx = F(p)$$

Such that this integral is convergent , p is positive constant.

Property 1: [2]

Al-Tememe transformation is characterized by the linear property, that is:

$$\mathcal{T}[Af(x) + Bg(x)] = A\mathcal{T}[f(x)] + B\mathcal{T}[g(x)] ,$$

Where A , B are constants ,the functions $f(x)$, $g(x)$ are defined when $x > 1$.

Al-Tememe transform of some fundamental functions are given in table(1) [2] :

From Al-Tememe definition and the above table, we get:

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Theorem1:

If $\mathcal{T}[f(x)] = F(p)$ and a is constant, then $\mathcal{T}[x^{-a}f(x)] = F(p+a)$ .see [2]

Definition 3: [2]

Let $f(x)$ be a function where $(x > 1)$ and $\mathcal{T}[f(x)] = F(p), f(x)$ is said to be an inverse for the Al-Tememe transformation and written as $\mathcal{T}^{-1}[F(p)] = f(x)$, where $\mathcal{T}^{-1}$ returns the transformation to the original function.

Property 2: [2]

If $\mathcal{T}^{-1}[F_1(p)] = f_1(x)$, $\mathcal{T}^{-1}[F_2(p)] = f_2(x)$,…, $\mathcal{T}^{-1}[F_n(p)] = f_n(x)$ and $a_1, a_2, \dots, a_n$ are constants then,

$$\mathcal{T}^{-1}[a_1F_1(p) + a_2F_2(p) + \cdots + a_nF_n(p)] = a_1f_1(x) + a_2f_2(x) + \cdots + a_nf_n(x)$$

Theorem 2: [2]

If the function f(x) is defined for $x > 1$ and its derivatives $f^{(1)}(x), f^{(2)}(x), \dots, f^{(n)}(x)$ are exist then:

$$\mathcal{T}\left[x^n f^{(n)}(x)\right] = -f^{(n-1)}(1) - (p-n)f^{(n-2)}(1) - \cdots - (p-n)\big(p-(n-1)\big)\dots(p-2)f(1) + (p-n)!\,F(p)$$

Definition 4 : [4]

A function $f(x)$ is piecewise continuous on an interval $[a,b]$ if the interval can be partitioned by a finite number of points $a = x_0 < x_1 < \cdots < x_n = b$ such that:

1. $f(x)$ is continuous on each subinterval (x_i, x_{i+1}), for $i = 0,1,2,\dots,n-1$
2. The function f has jump discontinuity at x_i , thus

$$\left|\lim_{x\to x_i^+} f(x)\right| < \infty, \; i = 0,1,2,\dots,n-1 \;;\; \left|\lim_{x\to x_i^-} f(x)\right| < \infty, \quad i = 0,1,2,\dots,n$$

Note: A function is piecewise continuous on $[0,\infty)$ if it is piecewise continuous in $[0,A]$ for all $A > 0$

Integration of the Laplace Transform [3]

If f is piecewise continuous function on $[0,\infty]$ of exponential order $\alpha \geq 0$, and :

$g(x) = \int_0^x f(u)du \quad \Rightarrow \quad \mathcal{L}[g(x)] = \frac{1}{p}\mathcal{L}[f(x)]$; p positive number.

$$\Rightarrow g(x) = \int_0^x f(u)du = \mathcal{L}^{-1}\left[\frac{F(p)}{p}\right]$$

Integration of Al-Tememe transforms:

In ordinary differential equations it is also necessary to compute Al-Tememe transform of an integral.

By the same method we can derive similar law to find a $\mathcal{T}^{-1}$ for some functions:

Consider, $g(x) = \int_1^x f(u)du$; $g'(x) = f(x)$

To find $\mathcal{T}[g(x)]$

$$\because \mathcal{T}[g(x)] = \int_1^\infty x^{-p}g(x)dx$$

$$\therefore \mathcal{T}[g(x)] = \frac{x^{-p+1}}{-p+1}\cdot g(x)\Big|_1^\infty - \frac{1}{-p+1}\int_1^\infty x^{-p+1}\cdot f(x)dx$$

$$= 0 + \frac{1}{p-1}\int_1^\infty x\cdot x^{-p}f(x)dx$$

$$\Rightarrow \mathcal{T}[g(x)] = \frac{1}{p-1}\mathcal{T}[x\cdot f(x)] \;\; ;p>1$$

By take $\mathcal{T}^{-1}$ to both sides we get :

$$g(x) = \mathcal{T}^{-1}\left[\frac{1}{p-1}\cdot\mathcal{T}[x\cdot f(x)]\right]$$

$$\therefore \mathcal{T}^{-1}\left[\frac{\mathcal{T}[x\cdot f(x)]}{p-1}\right] = \int_1^x f(u)\,du = g(x) \;\blacksquare.$$

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Example 1: To find

$$\mathcal{T}^{-1}\left[\frac{1}{(p-1)(p-5)}\right]$$

We note that,

$$\mathcal{T}^{-1}\left[\frac{1}{(p-1)(p-5)}\right]=\mathcal{T}^{-1}\left[\frac{1/(p-5)}{(p-1)}\right]$$

$$=\mathcal{T}^{-1}\left[\frac{\mathcal{T}(x^4)}{(p-1)}\right]=\mathcal{T}^{-1}\left[\frac{\mathcal{T}(x\cdot x^3)}{(p-1)}\right]$$

Applying the previous relation we get :

$$\mathcal{T}^{-1}\left[\frac{\mathcal{T}(x\cdot x^3)}{(p-1)}\right]=\int_1^x f(u)du$$

$$=\int_1^x u^3du=\frac{u^4}{4}\Big|_1^x=\frac{x^4}{4}-\frac{1}{4}$$

Example 2: To find

$$\mathcal{T}^{-1}\left[\frac{1}{(p-1)(p+2)(p-3)}\right]$$

We note that,

$$\mathcal{T}^{-1}\left[\frac{1}{(p-1)(p+2)(p-3)}\right]$$

$$=\mathcal{T}^{-1}\left[\frac{1/(p+2)(p-3)}{(p-1)}\right]$$

$$\because \frac{1}{(p+2)(p-3)}=\frac{-1/5}{(p+2)}+\frac{1/5}{(p-3)}$$

$$\therefore \mathcal{T}^{-1}\left[\frac{1/(p+2)(p-3)}{(p-1)}\right]$$

$$=\mathcal{T}^{-1}\left[\frac{\mathcal{T}\left(-1/5\,x^{-3}+1/5\,x^2\right)}{(p-1)}\right]$$

Applying the previous relation we get :

$$=\mathcal{T}^{-1}\left[\frac{\mathcal{T}\left[x\cdot\left(-1/5\,x^{-4}+1/5\,x\right)\right]}{(p-1)}\right]=\int_1^x f(u)du$$

$$=\int_1^x\left(-1/5\,u^{-4}+1/5\,u\right)du$$

$$=\left(1/15\,u^{-3}+1/10\,u^2\right)\Big|_1^x=1/15\,x^{-3}+1/10\,x^2-\frac{1}{6}$$

Example 3: To find

$$\mathcal{T}^{-1}\left[\frac{1}{(p-1)[(p-2)^2+4]}\right]$$

We note that,

$$\mathcal{T}^{-1}\left[\frac{1/2\cdot\frac{2}{(p-2)^2+4}}{(p-1)}\right]=1/2\,\mathcal{T}^{-1}\left[\frac{\frac{2}{(p-2)^2+4}}{(p-1)}\right]$$

$$=1/2\,\mathcal{T}^{-1}\left[\frac{\mathcal{T}[x\cdot\sin(2\ln x)]}{(p-1)}\right]$$

Applying the previous relation we get :

$$1/2\,\mathcal{T}^{-1}\left[\frac{\mathcal{T}[x\cdot\sin(2\ln x)]}{(p-1)}\right]=1/2\int_1^x f(u)du$$

$$=1/2\int_1^x \sin(2\ln u)\,du$$

$$\int_1^x \sin(2\ln u)\,du$$

$$=u\sin(2\ln u)\Big|_1^x$$

$$-2\int_1^x \cos(2\ln u)\,du$$

$$=x\sin(2\ln x)-0$$

$$-2\left(u\cos(2\ln u)\Big|_1^x\right.$$

$$\left.+2\int_1^x \sin(2\ln u)\,du\right)$$

$$=x\sin(2\ln x)-2[x\cos(2\ln x)-1]$$

$$-4\int_1^x \sin(2\ln u)\,du$$

$$\Rightarrow\ 5\int_1^x \sin(2\ln u)\,du$$

$$=x\sin(2\ln x)-2x\cos(2\ln x)$$

$$+2$$

$$\therefore\int_1^x \sin(2\ln u)\,du$$

$$=\frac{1}{5}[x\sin(2\ln x)-2x\cos(2\ln x)$$

$$+2]$$

$$so,\ \ 1/2\int_1^x \sin(2\ln u)\,du$$

$$=\frac{1}{10}[x\sin(2\ln x)$$

$$-2x\cos(2\ln x)+2]$$

$$\therefore \mathcal{T}^{-1}\left[\frac{1}{(p-1)[(p-2)^2+4]}\right]$$

$$=\frac{1}{10}[x\sin(2\ln x)$$

$$-2x\cos(2\ln x)+2]$$

Example 4 : To find

$$\mathcal{T}^{-1}\left[\frac{p+2}{(p+1)^3-8}\right]$$

We note that,

$$\mathcal{T}^{-1}\left[\frac{p+2}{(p+1)^3-8}\right]$$

$$=\mathcal{T}^{-1}\left[\frac{p+2}{[(p+1)-2][(p+1)^2+2(p+1)+4]}\right]$$

$$=\mathcal{T}^{-1}\left[\frac{(p+2)}{(p-1)(p^2+4p+7)}\right]$$

$$=\mathcal{T}^{-1}\left[\frac{p+2}{(p-1)\left[(p+2)^2+\left(\sqrt{3}\right)^2\right]}\right]$$

$$=\mathcal{T}^{-1}\left[\frac{\frac{p+2}{(p+2)^2+\left(\sqrt{3}\right)^2}}{(p-1)}\right]$$

$$=\mathcal{T}^{-1}\left[\frac{\mathcal{T}\left[x^{-3}cos\left(\sqrt{3}ln\,x\right)\right]}{(p-1)}\right]$$

$$=\mathcal{T}^{-1}\left[\frac{\mathcal{T}\left[x\cdot x^{-4}cos\left(\sqrt{3}ln\,x\right)\right]}{(p-1)}\right]$$

$$=\int_1^x f(u)du$$

By applying the previous relation we get :

$$\mathcal{T}^{-1}\left[\frac{\mathcal{T}\left[x\cdot x^{-4}cos\left(\sqrt{3}ln\,x\right)\right]}{(p-1)}\right]$$

$$=\int_1^x u^{-4}\,cos\left(\sqrt{3}ln\,u\right)du$$

$$=-1/3\,u^{-3}cos\left(\sqrt{3}ln\,u\right)\Big|_1^x$$

$$-\frac{1}{\sqrt{3}}\int_1^x u^{-4}\,sin\left(\sqrt{3}ln\,u\right)du$$

$$=-1/3\,x^{-3}cos\left(\sqrt{3}ln\,x\right)$$

$$+1/3-\frac{1}{\sqrt{3}}\int_1^x u^{-4}\,sin\left(\sqrt{3}\,ln\,u\right)du$$

$$=-1/3\,x^{-3}cos\left(\sqrt{3}ln\,x\right)+1/3$$

$$-\frac{1}{\sqrt{3}}\left[\frac{-u^{-3}}{3}sin\left(\sqrt{3}ln\,u\right)\Big|_1^x+\frac{1}{\sqrt{3}}\int_1^x u^{-4}\,cos\left(\sqrt{3}ln\,u\right)du\right]$$

$$=-1/3\,x^{-3}cos\left(\sqrt{3}ln\,x\right)$$

$$+1/3+\frac{1}{3\sqrt{3}}x^{-3}sin\left(\sqrt{3}ln\,x\right)$$

$$-\frac{1}{3}\int_1^x u^{-4}cos\left(\sqrt{3}ln\,u\right)du$$

$$\Rightarrow\frac{4}{3}\int_1^x u^{-4}cos\left(\sqrt{3}ln\,u\right)du$$

$$=-1/3\,x^{-3}cos\left(\sqrt{3}ln\,x\right)$$

$$+1/3+\frac{1}{3\sqrt{3}}x^{-3}sin\left(\sqrt{3}ln\,x\right)$$

$$\int_1^x u^{-4}cos\left(\sqrt{3}ln\,u\right)du$$

$$=-\frac{1}{4}x^{-3}cos\left(\sqrt{3}ln\,x\right)+\frac{1}{4}$$

$$+\frac{1}{4\sqrt{3}}x^{-3}sin\left(\sqrt{3}ln\,x\right)$$

$$=\frac{1}{4}\left[1+\frac{1}{\sqrt{3}}x^{-3}sin\left(\sqrt{3}ln\,x\right)-x^{-3}cos\left(\sqrt{3}ln\,x\right)\right]$$

Example 5: To solve the differential equation:

$$xy'-2y=ln\,x\quad;y(1)=0$$

We take Al-Tememe transformation to both sides of above by Th.2 and Table(1) :

$$-y(1)+(p-1)\mathcal{T}(y)-2\mathcal{T}(y)=\frac{1}{(p-1)^2}$$

$$\mathcal{T}(y)=\frac{1}{(p-3)(p-1)^2}=\frac{{}^1/_{(p-3)(p-1)}}{(p-1)}$$

$$=\frac{{}^1/_{[(p-2)^2-1]}}{(p-1)}$$

So,

$$\mathcal{T}(y) = \frac{\mathcal{T}[x.sinh(lnx)]}{(p-1)}$$

After taking $\mathcal{T}^{-1}$ to both sides we get:

$$y = \mathcal{T}^{-1}\left[\frac{\mathcal{T}[x.sinh(lnx)]}{(p-1)}\right]$$

$$= \int_1^x f(u)du = \int_1^x sinh(lnu)\,du$$

$$\because sinh(lnu) = \frac{e^{lnu} - e^{-lnu}}{2} = \frac{1}{2}(u - u^{-1})$$

$$\therefore \int_1^x sinh(lnu)\,du = \frac{1}{2}\left(\frac{u^2}{2} - ln\,u\right)\Big|_1^x$$

$$= {}^1/_4\,x^2 - {}^1/_2\,ln\,x - {}^1/_4$$

$$\Rightarrow y = {}^1/_4\,x^2 - {}^1/_2\,ln\,x - {}^1/_4$$

Example 6: To solve the differential equation:

$$x^2y'' - xy' = x^2 \;;\; y'(1) = 0\,, y(1) = 2$$

We take Al-Tememe transformation to both sides of above :

$$\mathcal{T}(x^2y'') - \mathcal{T}(xy') = \mathcal{T}(x^2)$$

$$-y(1) - y'(1)(p-2) + (p-2)(p-1)\mathcal{T}(y) + y(1)$$

$$-(p-1)\mathcal{T}(y) = \frac{1}{(p-3)}$$

$$(p-3)(p-1)\mathcal{T}(y) = \frac{1}{(p-3)}$$

$$\mathcal{T}(y) = \frac{1}{(p-1)(p-3)^2} = \frac{{}^1/_{(p-3)^2}}{(p-1)}$$

After taking $\mathcal{T}^{-1}$ to both sides we get:

$$y = \mathcal{T}^{-1}\left[\frac{{}^1/_{(p-3)^2}}{(p-1)}\right] = \mathcal{T}^{-1}\left[\frac{\mathcal{T}(x.x\,lnx)}{(p-1)}\right]$$

$$= \int_1^x f(u)du = \int_1^x u\,ln\,u\,du$$

$$\int_1^x u\,ln\,u\,du = \left(\frac{1}{2}u^2\,ln\,u - \frac{1}{4}u^2\right)\Big|_1^x$$

$$= \frac{1}{2}x^2\,ln\,x - \frac{1}{4}x^2 + \frac{1}{4}$$

$$\Rightarrow y = \frac{1}{2}x^2\,ln\,x - \frac{1}{4}x^2 + \frac{1}{4}$$

ID	Function ,$f(x)$	$F(p) = \int_1^\infty x^{-p}f(x)dx = \mathcal{T}[f(x)]$	Regional of convergence
1	$k; k = constant$	$\frac{k}{p-1}$	$p > 1$
2	x^n , $n \in R$	$\frac{1}{p-(n+1)}$	$p > n+1$
3	$ln\,x$	$\frac{1}{(p-1)^2}$	$p > 1$
4	$x^n\,ln\,x\,, n \in R$	$\frac{1}{[p-(n+1)]^2}$	$p > n+1$
5	$sin\,(a\,ln\,x)$	$\frac{a}{(p-1)^2 + a^2}$	$p > 1$
6	$cos\,(a\,ln\,x)$	$\frac{p-1}{(p-1)^2 + a^2}$	$p > 1$
7	$sinh\,(a\,ln\,x)$	$\frac{a}{(p-1)^2 - a^2}$	$\lvert p-1\rvert > a$
8	$cosh\,(a\,ln\,x)$	$\frac{p-1}{(p-1)^2 - a^2}$	$\lvert p-1\rvert > a$

Table **(1) [2].**

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تكامل تحويل التميمي لايجاد معكوس التحويل وحل بعض المعادلات الخطية الاعتيادية الخاضعة لشروط ابتدائية

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المستخلص :

هدفنا في هذا البحث هو ايجاد معكوس تحويل التميمي بأستخدام التكامل وايجاد حل بعض المعادلات التفاضلية الاعتيادية الخطية ذات المعاملات المتغيرة الخاضعة لشروط ابتدائية .