

Close-to-convex Function Generates Remarkable Solution of 2nd order Complex Nonlinear Differential Equations

Shatha S.Alhily

Dept. of Mathematics, College of Sciences, Al- Mustansiriyah University
shathamaths@yahoo.co.uk.iq shathamaths@uomustansiriyah.edu

Recived : 28\9\2017

Revised : 19\10\2017

Accepted : 23\10\2017

Abstract.

Consider the complex nonlinear differential equation $f''(z) - \frac{2}{z} f'(z) - \frac{2}{z^2} f^3(z) = H(z)$, where $P(z) = \frac{-2}{z}, Q(z) = \frac{-2}{z^2}$ are complex coefficients, and $H(z)$ be a complex function performs non-homogeneous term of given equation.

In this paper, we investigated that $w(z) = \frac{zf'}{f}$ is a remarkable solution of given equation and belongs to hardy space H^2 ; with studying the growth of that solution by two ways; through the maximum modulus and Brennan's Conjecture and another by finding the supremum function of a volume of the surface area K_θ . Furthermore, we discussed the solution behaviour with meromorphic coefficients properties for given equation.

Keywords: Univalent Function, Positive Harmonic Functions, Growth of Solution.

Mathematics subject classification : 32-XX

Introduction

Consider a second order of complex non-linear differential equation

$$f'' + P(z)f' + Q(z)f^3 = H(z), \dots\dots\dots (1)$$

where $P(z), Q(z)$ are complex coefficients, and $H(z)$ be a complex function performs non-homogeneous term of given equation.

Let f be a close-to-convex function defined on the simply connected domain Ω onto unit disk D conformally, so there exist starlike function $\varphi(z)$ satisfies the condition $\Re\left(\frac{zf'}{\varphi}\right) > 0$, for more convenience we can suppose that $f(0) = 0$, that is to say; f is a starlike function satisfies the condition $\Re\left(\frac{zf'}{f}\right) > 0$, instead of φ .

In this paper, we consider a proper solution $w(z) = \frac{zf'}{f}$, which generated through the close-to-convex function f itself such that,

$$w'(z) = \frac{zf''f + f'f - zf'^2}{f^2}$$

$$w''(z) = \frac{2f''f - 3zf'f'' + zf'''f - 2f'^2}{f^2} + \frac{2zf'^3}{f^3}$$

$$w''(z) = \frac{2}{z} \left(\frac{zf''f}{f^2} - \frac{zf'^2}{f^2} + \frac{f'}{f} \right) + \frac{2}{z^2} \left(\frac{zf'}{f} \right)^3 + \left(\frac{2f'}{zf} - \frac{3zf'f''}{f^2} + \frac{zf'''}{f} \right),$$

$$w''(z) - \frac{2}{z} w'(z) - \frac{2}{z^2} w^3(z) = H(z),$$

it is easy to see that $w(z)$ satisfies the general equation (1) above

$$\text{where, } P(z) = \frac{-2}{z}; Q(z) = \frac{-2}{z^2}; \text{ and } H(z) = \frac{2f'}{zf} - \frac{3zf'f''}{f^2} + \frac{zf'''}{f}.$$

At this point, we have got 2^{nd} order complex differential equation of type (non-homogeneous) with coefficients of meromorphic function as follows:

$$f''(z) - \frac{2}{z} f'(z) - \frac{2}{z^2} f^3(z) = H(z) \dots \dots (2)$$

As a result has been reached to the proper solution for given equation to mark this type of coefficients.

Some research papers cover this type of topics which depending on how the solution of second order complex differential equations (linear –nonlinear) satisfying such condition to be in.

In [1] studied the same kind of complex differential equation with same conditions but when the equation's coefficients are polynomials, while the author in [4] considers such kind of complex differential equation and proved if the solution satisfies the condition $\log N\left(\rho, \frac{1}{w}\right) = O(\rho)$, which related to $\frac{1}{w} = g(z) * Exp(z)$, where * be the Hadamard convolution, and $g(z)$ is an entire function, hence it presented by the series of exponential form then should be the equation's coefficients constant.

In [9] has been showed a second order complex linear differential equation with coefficients of entire functions such that one order of them lower than the other, consequently; the author proved that there exist non- constant solution has infinite order.

Moreover, in [5] the author has considered the differential equation of meromorphic function with coefficient of periodic function which effected on the type of solution to be non-trivial subnormal solution.

For an exciting of these concepts we refer to some Preliminaries as an auxiliary to reach the target:

Definition (Positive Harmonic Functions). [7]

Positive harmonic functions represented as a subclass of the class S of univalent functions, in sense of ; let f be an analytic functions in the unit disk D , with condition $\Re\{f(z)\} > 0$, in D and $f(0) = 1$, have positive real part ; in order to consider

Positive harmonic function in D , normalized by the condition $u(0) = 1$.

Brennan's Conjecture. [8]

Brennan's Conjecture is formulated as an estimate for conformal mapping $f: \Omega \rightarrow D$,

$$\iint_{\Omega} |\psi'|^p dx dy < \infty, \text{ for } \frac{4}{3} < p < 4, \dots \dots \dots (3)$$

where $\psi = f^{-1}$ and $dx dy = dA$ is the area measure on the plane.

By changing the variables we will be able to rewrite (3) in terms of

$$\iint_D |f'|^{2-p} dx dy < \infty.$$

Definition (Maximum Modulus).[6]

The modulus of a function f analytic in a domain D , does not have weak local maximum in D unless f is constant. If f is an analytic in a bounded domain and continuous in the closure, then $|f(z)|$ must have maximum value on the boundary ∂D .

Theorem (Distortion Theorem).[6]

For each f in the class of univalent function S defined on unit disk D ,

$$\frac{1-r}{(1+r)^3} \leq |f'(z)| \leq \frac{1+r}{(1-r)^3}, \quad |z| = r < 1.$$

For each $z \in D, z \neq 0$, equality works if and only if f satisfies rotation of the koebe function $f(z) = \frac{z}{(1-z)^2}$.

Theorem (Prawitz's theorem). [6],[8].

If $f(z)$ belongs to the class of univalent functions S , so that $f: D \rightarrow \Omega$, then for $0 < p < \infty, \mathcal{M}_p^p(r, f) \leq p \int_0^r \frac{1}{\rho} \mathcal{M}_\infty^p(\rho, f) d\rho, 0 < r < 1$.

1. Problem Statement

In this section, we study the solution behaviour $w(z) = \frac{zf'}{f}$ for a second order complex non-linear differential equation (2) by letting $w(z) = w_1(z) + iw_2(z)$ as follows:

1. Examine the mean square solution $w(z)$ of given equation (2) belongs to Hardy space $H^2(\Omega)$ through the harmonicity property for the real part of the solution $\Re(w(z)) = w_1(z)$.

2. Examine the role of each of the maximum modulus and Brennan's Conjecture in the growth of solution $w(z)$ for given equation (2) depending on a fact that the theory of conformal maps considers as an identically relation between the boundaries of the image and of the pre-image .
3. Show that, the solution $w(z)$ of given equation (2) can to be bounded if its coefficients are bounded.

2. Main Theorems

Theorem (2.1). If the solution $w(z) = \frac{zf'}{f}$ of equation (2) whose a positive harmonic real part, then its solution $w(z)$ belongs to Hardy space $H^2(\Omega)$.

Proof .

Given $f: \Omega \rightarrow D$ conformally, that is f whose inverse function $f^{-1}: D \rightarrow \Omega$.

Let $w(z) = w_1(z) + iw_2(z)$, with

$$w(z) = \frac{zf'}{f}, \dots \dots \dots (4)$$

Then

$$f^{-1}(w) = \frac{z(f^{-1})'}{f^{-1}} \dots \dots \dots (5) .$$

One can rewrite equation (5) as the form

$$\frac{1}{f(w)} = \frac{z(f^{-1})'}{f^{-1}} .$$

As known that, $f(z)$ is a starlike function , so its having a positive real part, that is ;

$$\Re\left(\frac{zf'}{f}\right) > 0, \quad \frac{1}{w_1(z)} = \frac{z(f^{-1})'}{f^{-1}} .$$

Hence, we obtain $w_1(z) = \frac{f^{-1}}{z(f^{-1})'} > 0$.

Obviously, f^{-1} is an analytic function on D with $\Re(f^{-1}) > 0$.

As a result , $w_1(z)$ is a positive harmonic function that can be formulated by Poisson integral with an unique positive unit measure $d\mu(t)$ as follows

$$w_1(re^{i\theta}) = \int_0^{2\pi} \frac{e^{i\theta} + z}{e^{i\theta} - z} d\mu(\theta);$$

where $\mu(\theta) \geq 0$ and $\int d\mu(\theta) = 1$.

Now , would be take modulus for both of sides with suppose that $z = re^{i\theta}$ to obtain

$$\begin{aligned} |w_1(re^{i\theta})| &= \left| \int_0^{2\pi} \frac{e^{i\theta} + z}{e^{i\theta} - z} d\mu(\theta) \right| \\ &= \frac{1+r}{1-r} \left| \int_0^{2\pi} d\mu(\theta) \right| \\ &= \frac{1+r}{1-r} \mu(2\pi) \\ &= \frac{2\mu(2\pi)}{1-r} < \frac{C}{1-r} ; C \text{ is a constant.} \end{aligned}$$

Normally we need to do some calculations for the integral mean of the positive harmonic real function $w_1(re^{i\theta})$ in order to check the

possibility of the solution existence and its finite.

$$\begin{aligned} &\int_0^1 \int_0^{2\pi} |w_1(re^{i\theta})|^2 r dr d\theta \\ &= \int_0^1 \int_0^{2\pi} |w_1(re^{i\theta})| |w_1(re^{i\theta})| r dr d\theta, \\ &< \int_0^1 \int_0^{2\pi} \frac{C}{(1-r)} |w_1(re^{i\theta})| r dr d\theta \\ &= \int_0^1 (1-r)^{-1} dr \cdot \int_0^{2\pi} |w_1(re^{i\theta})| ds , \\ &\text{where , } s = r\theta \Rightarrow ds = rd\theta; \\ &\int_0^{2\pi} |w_1(re^{i\theta})| ds < K, \text{ since } w_1(re^{i\theta}) \in H^1(D), \end{aligned}$$

which implies to

$$\begin{aligned} &\int_0^1 \int_0^{2\pi} |w_1(re^{i\theta})|^2 r dr d\theta \\ &< K \int_0^1 (1-r)^{-1} dr \end{aligned}$$

Consequently, for all values of p ; we obtain $\int_0^1 \int_0^{2\pi} |w_1(re^{i\theta})|^2 r dr d\theta < \infty$.

Now, by doing a short simplifications which including Brennan's conjecture, we obtain

$$\iint_{\Omega} |w(z)|^2 dx dy < \infty .$$

Finally, we conclude that $w(z) \in H^2(\Omega)$. ■

Growing of Solution [8].

In fact, a growth of solution, it was and still an important objective for a researchers in this aspect and for along years. It's obvious that the growth of any solution belongs to given domain, it is controlled by a growing the coefficients of given equation, so there is a match between the solution and those certain coefficients of given equation.

Here, we shall concentrate in terms of the maximum modulus as a main tool to study the growth of solution especially for a harmonic solution which is a positive real part of holomorphic function f .

Theorem (2.2). If $w(z)$ is a solution of equation (2), with

$$\int \int_{|w|<1} |\psi'(w)|^2 dx dy = \int \int_{\Omega} dx dy . \text{ then}$$

Proof. For more convenience, let $f^{-1} = \psi$.
 Applying Second Green Identity as follow:

$$\frac{1}{2\pi} \int_0^{2\pi} \frac{\partial |\psi|^{-2}}{\partial r} r d\theta$$

$$= \frac{1}{2\pi} \int \int_{|z|<1} \Delta(|\psi|^{-2}) dx dy,$$

We notice that Laplacian operator in L_p -space defined as $\Delta(|\psi|^p) = p^2 |\psi|^{p-2} |\psi'|^2$, such that when $p = -2$ implies to

$$\frac{1}{2\pi} \int_0^{2\pi} \frac{\partial |\psi|^{-2}}{\partial r} d\theta$$

$$= \frac{2}{\pi r} \int \int_{|w|<1} (|\psi|^{-4}) |\psi'|^2 dx dy,$$

where

$$\int \int_{|w|<1} |\psi'|^2 dx dy = \int \int_{\Omega} dx dy.$$

As a result ψ has a maximum modulus

$$\mathcal{M}_{\infty}(r, \psi) = \max_{r<1} |\psi(z)|,$$

so that

$$|\psi(z)| \leq \max_{r<1} |\psi(z)|,$$

this act allowed us to apply Prowitz's theorem and letting $\psi(z) = z$ which gives already $|\psi(z)| = |z| = r$ in order to obtain,

$$\frac{d}{dr} \left(\frac{1}{2\pi} \int_0^{2\pi} |\psi|^{-2} d\theta \right)$$

$$= \frac{2}{\pi r} \int \int_{|w|<1} |z|^{-4} dA(z),$$

$$\leq \frac{2}{\pi r} \int \int_{|w|<1} \max |z|^{-4} dA(z),$$

$$= \frac{2}{\pi r} \int_0^{2\pi} \int_0^{M(r)} \rho^{-4} \rho d\rho d\theta,$$

$$< \frac{-2}{r} \cdot \frac{r^{-2}}{(1-r)^{-4}}.$$

Now, we have to take the integral with respect to r for both of sides as follows:

$$\frac{1}{2\pi} \int_0^{2\pi} |\psi|^{-2} d\theta \leq -2 \int_0^r \frac{\rho^{-2}}{\rho(1-\rho)^{-4}} d\rho,$$

$$\leq -2 \underbrace{\left(\int_0^r \frac{\rho^{-2}}{\rho} d\rho \right)}_{(1)} \underbrace{\left(\int_0^r \frac{d\rho}{(1-\rho)^{-4}} \right)}_{(2)}.$$

The integral (1) over Ω could be shrink over unit disk where $p = -2$, and $\psi = f^{-1}$ that means f is defined conformally from Ω onto unit disk D , we let $q = -p$ to obtain

$$\int_0^r \frac{\rho^{-2}}{\rho} d\rho = \int_{\partial D = \{z: 0 < |z| = \rho < r\}} \rho^{-2} d\rho \leq \infty.$$

The integral (2) over Ω with a short calculations we obtain

$$\int_0^r \frac{d\rho}{(1-\rho)^{-4}} = \frac{-(1-\rho)^5}{5} \Big|_0^r$$

$$= \frac{1}{5} \left[1 - \frac{1}{(1-r)^{-5}} \right]$$

$$= \frac{1}{5} \left[\frac{(1-r)^{-5} - 1}{(1-r)^{-5}} \right] \leq \frac{C_{-2}}{(1-r)^{-5}},$$

where $C_p = C_{-2}$ is a constant.

As a result we get

$$\frac{1}{2\pi} \int_0^{2\pi} |\psi|^{-2} d\theta \leq \frac{C_{-2}}{(1-r)^{-5}}.$$

Applying distortion theorem in order to obtain

$$\frac{1}{2\pi} \int_0^{2\pi} |\psi'|^{-2} d\theta \leq \left[\frac{(1+r)^{-2}}{r^{-2}(1-r)^{-2}} \right]$$

$$* \left(\frac{1}{2\pi} \int_0^{2\pi} |\psi|^{-2} d\theta \right),$$

$$\leq \frac{C'_{-2}}{(1-r)^{-7}},$$

$$\frac{1}{2\pi} \int_0^{2\pi} |\psi|^{-2} d\theta \leq K(r). \blacksquare$$

Theorem (2.3). Let $w(z)$ be a solution of equation (2) in Ω , let $C_i(z)$; $i = 0,1,2$, be the set of all non-identically zero coefficients, such that $z = re^{i\theta}$; If $0 \leq \theta \leq 2\pi$. there exist a supremum function of a volume of the surface area K_{θ} defined on $D_{\rho} \subset \Omega$, then

$$|w(re^{i\theta})|$$

$$< \left| \exp \left(\int_{u_0}^{\rho} \max_{\substack{0 \leq r < 1 \\ 0 \leq \theta \leq 2\pi}} |C_i(\rho e^{i\theta})|_{i=0,1,2} dr \right. \right.$$

$$\left. \left. + \delta_0 \right) \right|$$

Proof. Let $r \in (0,1)$ in D , hence we have to suppose $\rho = \frac{1+r}{2}$; that is, $\rho \in \left[\frac{1}{2}, 1 \right]$.

Set,

$$J(r e^{i\theta}) = \max_{\substack{0 \leq \rho < 1 \\ 0 \leq \theta \leq 2\pi}} |C_i(\rho e^{i\theta})|_{i=0,1,2} \text{ on } \left[0, \frac{1}{2} \right], \dots \dots \dots (6)$$

which presents the largest value of the function $C_i(\rho e^{i\theta})$; $i = 0,1,2$ along the edge of given domain $D \subset \Omega$ as well as we shall divide the interval into n parts between of

$r = 0$, and $\rho = \frac{1}{2}$ as follows:

$$\mathcal{H} = \{u_0, u_1, \dots, u_{n-1}, u_n = \rho\} \in \left[0, \frac{1}{2} \right],$$

which represents the interior area of $D \subset \Omega$, so that $J(r e^{i\theta})$ and \mathcal{H} both of them show the volume of the surface area $D \subset \Omega$, that is why, one can define a relation $V(J(r e^{i\theta}), \mathcal{H})$ such that

$$V(J(r e^{i\theta}), \mathcal{H})$$

$$- \int_{u_0}^{\rho} \max_{\substack{0 \leq r < 1 \\ 0 \leq \theta \leq 2\pi}} |C_i(\rho e^{i\theta})|_{i=0,1,2} dr < \delta_0,$$

where δ_0 is a constant.

Define such kind of function

$$K_\theta = 3 \sup J(r e^{i\theta}), \quad \text{for all } r \in [u_0, \rho].$$

Hence, we already have

$$K_\theta \geq 3 \cdot J(r e^{i\theta}) \text{ for all } r \in [u_0, \rho] \dots \dots \dots (7)$$

So when an inequality (7) satisfies the equality with taking the integral on the interval $[u_0, \rho]$ then

$$\frac{1}{3} \int_{u_0}^{\rho} K_\theta dr = V(J(r e^{i\theta}), \mathcal{H})$$

Consequently,

$$\frac{1}{3} \int_{u_0}^{\rho} K_\theta dr < \int_{u_0}^{\rho} \max_{\substack{0 \leq r < 1 \\ 0 \leq \theta \leq 2\pi}} |C_i(\rho e^{i\theta})|_{i=0,1,2} dr + \delta_0,$$

then

$$\exp\left(\frac{1}{3} \int_{u_0}^{\rho} K_\theta dr\right) < \exp\left(\int_{u_0}^{\rho} \max_{\substack{0 \leq r < 1 \\ 0 \leq \theta \leq 2\pi}} |C_i(\rho e^{i\theta})|_{i=0,1,2} dr + \delta_0\right), \dots \dots \dots (8)$$

Look at the left – hand side of the statement (8) which shows a growth estimate the solution of equation (2), that is

$$w(z) = \exp\left(\frac{1}{3} \int_{u_0}^{\rho} K_\theta dr\right)$$

depending on the volume of the surface area $D \subset \Omega$, that generated by coefficients $C_i(z) = \left\{P(z) = \frac{-2}{z}; Q(z) = \frac{-2}{z^2}\right\}$.

Finally,

$$\left|w(re^{i\theta})\right| < \left|\exp\left(\int_{u_0}^{\rho} \max_{\substack{0 \leq r < 1 \\ 0 \leq \theta \leq 2\pi}} |C_i(\rho e^{i\theta})|_{i=0,1,2} dr + \delta_0\right)\right|.$$

The proof is complete. ■

Solution Behaviour with Meromorphic Coefficients. [3]

One can study the behaviour of solution which depending on the kind of given equation such (linear- nonlinear) or its coefficients.[2]

We back here into a second order of complex non-linear differential equation (non-homogenous $f''(z) - \frac{2}{z} f'(z) - \frac{2}{z^2} f^3(z) = H(z)$; where it's necessary to refer to the role of the coefficients $P(z) = \frac{-2}{z}; Q(z) = \frac{-2}{z^2}$; which possess the irregular singularity point at origin. As known in theory of meromorphic differential equations; one can define a special sector around a singularity point 0 of meromorphic coefficient $Q(z) = \frac{-2}{z^2}$

must be bounded by stokes ray, which presents through the argument of z that takes arbitrary real values connected with solution to be in Ω rather than in $\mathbb{C} \setminus \{0\}$.

Let $Q(z) = \frac{-2}{z^2}$ be meromorphic function in D and let the values which $Q(z)$ assumes in D lie in a domain Ω (simply connected domain) whose boundary γ has positive logarithmic energy $T(r, Q)$ is bounded, then $Q(z)$ is of bounded in $D \subset \Omega$ by coefficients properties.

Theorem (2.4). If the coefficients of equation (2) be meromorphic functions with pole at origin have a property of bounded, then $w(z)$ already is bounded solution.

Proof.

Given $f''(z) - \frac{2}{z} f'(z) - \frac{2}{z^2} f^3(z) = H(z)$ be a second order non-homogeneous complex differential equation with $P(z), Q(z)$ are a meromorphic functions in Ω , having a pole in origin point with some properties as follows: Let $P(z) = \frac{-2}{z} \Rightarrow P'(z) = \frac{-2}{z^2}$; which implies to $P'(z) = Q(z)$.

On the other word, consider $P(z) - 2(\log z)'$ such that, $P'(z) = 2(\log z)'' = Q(z)$.

Set,

$$J(r e^{i\theta}) = \max_{\substack{0 \leq \rho < 1 \\ 0 \leq \theta \leq 2\pi}} |Q(\rho e^{i\theta})|,$$

By maximal inequality we obtain

$$\int_0^{2\pi} \sqrt{J(r e^{i\theta})} d\theta \leq C \int_0^{2\pi} \sqrt{|Q(\rho e^{i\theta})|} d\theta, \dots \dots \dots (9)$$

(cf. [7]) the solution $w(z)$ of equation (2) has been able to grow in as shown in previous theorem (2.2), one can rewrite $w(z)$ in more detailed as follows

$w(z) \rightarrow w(z e^{i\theta})$; where $z = r e^{i\theta}$. for any periodic solution to obtain

$$\int_0^{2\pi} \log|w(r e^{i\theta})| d\theta \leq K + \int_0^{2\pi} \sqrt{J(r e^{i\theta})} d\theta,$$

by inequality (9) we obtain

$$\int_0^{2\pi} \log|w(r e^{i\theta})| d\theta \leq K + C \int_0^{2\pi} \sqrt{|Q(\rho e^{i\theta})|} d\theta.$$

Now, we already have

$$|Q(\rho e^{i\theta})| = |2(\log z)''| = \frac{2}{\rho^2}.$$

Obviously;

$$\int_0^{2\pi} \sqrt{|Q(z)|} d\theta = 2\sqrt{2}\pi \rho.$$

It is so clear to notice that , as $\rho \rightarrow 1 - 0$,

$$\int_0^1 \int_0^{2\pi} \sqrt{|Q(z)|} d\theta d\rho = \sqrt{2} \pi \dots \dots \dots (10)$$

In sense $w(z)$ already is a bounded solution by properties of the equation's coefficients because the function $Q(z)$ is meromorphic function at the origin point and the integral (9) is a bounded. ■

Conclusions.

In this research paper, we concluded that : The harmonicity property of real part of the solution $\Re\left(w(z) = \frac{zf'}{f}\right)$ for a second order complex non-linear differential equation (2) had a role in :

1. Determine the space which contains such kind of solution.
2. Examine the growth of solution $w(z)$ for given equation (2).
3. Examine that the equation (2) must have a bounded solution $w(z)$ if its coefficients already are bounded.

References

[1] Anja. B, **Exceptional Solutions of n-th Order Periodic Linear Differential Equations**, complex variables, theory and application : an international journal . Vol.34 . No.1-2. (2007), pp. 7-17.

[2] Alexander. V. and Peter Y., **Mean type of functions of bounded and Martin functions in Denjoy domains**, J. Advances in Mathematics, Vol.290,No.28,(2016).

[3] Arscott.F.M., **Heun's Differential Equations**, Oxford University Press (2000), pp.(131-132).

[4] Li- peng Xiao, **Oscillation theorem for Higher Order Linear Differential Equations with Periodic Coefficients**. J. Math. Anal. Appl.376 (2011), pp.713-724.

[5] Nan. li, **Growth of solution to second order complex differential equations** , Electronic Journal of Differential Equations, Vol. 20, No. 51,(2014),p p.(1-12).

[6] Peter .L. Duren, **Univalent Function**, Grundlehren der mathematischen Wissenschaften, Springer Verlag, (1983).

[7] Peter .L. Duren, **Theory of H^p spaces**, Acadmic Press, New York and London, INC (1970).

[8] Steven. G. Krantz, **Complex Analysis: The Geometric Viewpoint.**, The Mathematical Assoc. of America, United States of America, (1990).

[9] Simon H., Joseph M. and John R., **On the Growth of Solutions of $f'' + gf' + hf = 0$.**, J. transactions of the american mathematical society, Vol. 324, No.2, April (1991).

الداله القريبة الى التحذب تولد حلاً ملحوظاً للمعادلات العقدية اللاخطية التفاضلية ذات المرتبة الثانية

شذى سامي سجاد الحلي
قسم الرياضيات – كلية العلوم – الجامعة المستنصرية
shathamaths@uomustansiriyah.edu.iq
shathamaths@yahoo.co.uk

المستخلص:

في هذه الورقة البحثية تناولنا معادلات اللاخطية التفاضلية العقدية ذات المرتبة الثانية من النوع الغير متجانس

$$f''(z) - \frac{2}{z} f'(z) - \frac{2}{z^2} f^3(z) = H(z),$$

حيث $P(z) = \frac{-2}{z}$, $Q(z) = \frac{-2}{z^2}$ ثوابت عقدية ، و $H(z)$ دالة عقدية تمثل الحد الغير متجانس من المعادلة المعطاة.

تم التحقق من أن $w(z) = \frac{zf'}{f}$ هو حل ملحوظ للمعادلة المعطاة وينتمي الى فضاء هاردي H^2 (hardy space H^2) ،

مع دراسة نمو الحل $w(z) = \frac{zf'}{f}$ بطريقتين : الأولى من خلال معامل الحد الأقصى maximum modulus

وتخمين برينان Brennan's Conjecture وأخرى عن طريق إيجاد الحد الأعلى لحجم المساحة السطحية θ K . علاوة على ذلك ، ناقشنا سلوك الحل مع خصائص معاملات المعادلة المعطاة (من نوع دوال ميرومورفيك).