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Some Properties of Fuzzy AB-ideal of AB-algebras

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Abstract:

In this research we have introduced the concept of fuzzy AB-ideal of AB-algebra and also we proved some relevant characteristics and theories. We also studied the fuzzy relations on AB-algebras and fuzzy derivations AB-ideals. We presented the characteristics and theories that illustrat the two concepts that is promted us to study Cartesian Product of fuzzy derivations AB-ideals.

Keywords:

Fuzzy derivation AB-ideal of AB-algebras, Cartesian Product of Fuzzy Derivations AB-ideals .

Mathematics Subject Classification: 06F35, 03G25, 03B52.

Introduction:

The notion of BCC-algebras was proposed by W.A. Dudek in ([9],[10],[11]). S.S. Ahn and H.S. Kim have introduced the notion of QS-algebras. A.T. Hameed and et al ([7],[8]) introduced the notions of QS-ideal and fuzzy QS-ideal of QS-algebra. A.T. Hameed, and et al have introduced the notion of fuzzy QS-ideal of QS-algebra in [1]. A.T. Hameed introduce new of abstract algebras: is called KUS-algebras and define its ideals as KUS-algebras in ([5]). In 2017, A.T. Hameed and B.N. Abbas. introduced the notion of AB-ideals in AB-algebras, described connections between such ideal and congruences and some properties of AB-algebra in ([2],[3]). A.T. Hameed and B.N. Abbas, considered the fuzzification of AB-ideals in AB-algebras [4]. In this paper, we introduce the notion of fuzzy derivation on AB-algebras and obtain some of related properties and we characterized Cartesian Product of Fuzzy Derivations AB-ideals .

1.Preliminaries*:*

We review some definitions and properties that will be useful in our results.

Definition 1.1.([2],[3]) Let X be a set with a binary operation "$*$" and a constant 0.Then (X, $*$, 0) is called **an AB-algebra** if the following axioms satisfied: for all x, y, z ∈X:

(i) $((x * y) * (z * y)) * (x * z) = 0,$

(ii) $0 * x = 0,$

(iii) $x * 0 = x,$

Note that: Define a binary relation (≤) on X by $x * y = 0$ if and only if, $x \leq y$.
Then(X, ≤) is a partially ordered set.

Proposition 1.2.([2],[3]) In any AB-algebra X, for all x, y, z ∈ X, the following properties hold:

(1) $(x * y) * x = 0.$

(2) $x \leq y$ *implies* $x * z \leq y * z.$

(3) $x \leq y$ *implies* $z * y \leq z * x.$

Remark 1.3.([2],[3]) An AB-algebra is satisfies for all x, y, z ∈X

(1) $(x * y) * z = (x * z) * y$

(2) $(x * (x * y)) * y = 0.$

Definition 1.4.([2],[3]) Let X be an AB-algebra and I ⊆ X. I is called **an AB-ideal of X** if it satisfies the following conditions:

(i) 0 ∈ I,

(ii) $(x * y) * z$ ∈ I and y ∈I imply $x * z$ ∈I.

Definition 1.5.([2],[3]) For an AB-algebra X, we denote $x \wedge y = y * (y * x)$, for all x, y ∈X,
$x \wedge y \leq x, y$.

Definition 1.6.([2],[3]) An AB-algebra is said to be **commutative** if and only if, satisfies for all x, y ∈ X, $x * (x * y) = y * (y * x)$, i.e, $x \wedge y = y \wedge x$.

Definition 1.7.([2],[3]) Let X be an AB-algebra. A mapping d : X → X is a left-right derivation (briefly,(l,r)-derivation) of X,
if it satisfies the identity $d(x * y) = (d(x) * y) \wedge (x * d(y))$, for all x, y ∈ X,
if d satisfies the identity $d(x * y) = (x * d(y)) \wedge (d(x) * y)$, for all x, y ∈ X.
Then d is a right-left derivation (briefly, (r, l)-derivation) of X.
Moreover, if d is both a (l,r) and (r,l)-derivation, then d is a derivation of X.

Example 1.8. Let X = {0,1,2,3} be an AB-algebra in which the operation (*) is defined as follows:

*	0	1	2	3
0	0	0	0	0
1	1	0	1	0
2	2	2	0	0
3	3	3	1	0

Define a map d : G → G by

$$d(x) = \begin{cases} 0 & if\ x = 0,1,3, \\ 2 & if\ x = 2. \end{cases}$$

And define a map d^* : G → G

by$d^*(x) \begin{cases} 0 & if\ x = 0,1, \\ 2 & if\ x = 2,3. \end{cases}$

Then it is easily checked that d is both a (l,r) and (r,l)-derivation of G and d^* is a(r,l)-derivation but not a (l,r)-derivation of G.

Definition1.9.([2],[3]) A derivation of an AB-algebra is said to be **regular** if d(0) = 0.

Definition 1.10.([4]) A fuzzy subset μ of AB-algebra X is called **a fuzzy AB-subalgebra of X**
if $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$, for all x, y ∈ X.

Definition 1.11.([4]) A fuzzy subset μ of AB-algebra X is called **a fuzzy AB-ideal of X** if it satisfies :

FAB_1) $\mu(0) \geq \mu(x)$;

FAB_2) $\mu(x * z) \geq \min\{\mu(x * (y * z)), \mu(y)\}$,

for all x, y, z ∈ X.

***Definition 1.12.* [6].** A fuzzy μ is called a fuzzy relation on any set S, if μ is a fuzzy subset

μ: X× X→ [0,1].

***Definition 1.13.* [6].**If μ is a fuzzy relation on a set S and is a fuzzy subset of X, then μ is a fuzzy relation on β if $\mu\{x, y\} \leq \min(\beta(x), \beta(y))$, ∀x, y ∈ X.

***Definition 1.14* [6].**Let μ and β be a fuzzy subset of a set X, the Cartesian product of μ and β is defined by $(\mu \times \beta)(x, y) = \min\{\mu(x), \beta(y)\}$, ∀ x, y∈ X.

***Lemma 1.15* [6].** Let μ and β be a fuzzy subset of a set X, then

(i) $\mu \times \beta$ a fuzzy relation on X,

(ii) $(\mu \times \beta)_t = \mu_t \times \beta_t$, ∀ t∈[0,1].

2.(Fuzzy) Derivations AB-Ideals on AB-Algebras:

We review the definition of fuzzy derivations AB-algebra and study some properties of it.

Definition 2.1. Let $(X, *, 0)$ be an AB-algebra. and $d : X \to X$ be a self map. A non-empty subset A of an AB-algebra X is called **left derivations AB-ideal of X,** If it satisfies the following conditions:

1) $0 \in A$,
2) $d(x) * (y * z)$ ∈A, and $d(y)$ ∈A implies $d(x * z)$ ∈A.

Definition2.2. Let $(X, *, 0)$ be an AB-algebra. and $d : X \to X$ be a self map. A non-empty subset A of an AB- algebra X is called **right derivations AB-ideal of X,** If it satisfies the following conditions:

1) 0 ∈ A,
2) $x * d(y * z)$ ∈A, and $d(y)$ ∈A implies $d(x * z)$ ∈A.

Definition 2.3. Let(X ;*,0)be an AB-algebra. and $d : X \to X$ be a self map. A non-empty subset A of an AB- algebra X is called derivations AB-ideal of If it satisfies the following conditions:

1) 0∈A,
2) $d((x * y) * z)$ ∈A, and $d(y)$ ∈A implies $d(x * z)$ ∈A.

Definition 2.4. Let $(X, *, 0)$ be an AB-algebra. and $d : X \to X$ be a self map. A non-empty subset A of an AB-algebra X is called **a fuzzy derivations AB-ideal of X** If it satisfies the following conditions:

1) $\mu(0) \geq \mu(x)$;
2) $\mu(d(x*z)) \geq \min\{\mu(d(x*(y*z)), \mu(d(y))\}$" , for all x, y, z ∈ X

Example 2.5. Let X={0,1,2,3} be an AB-algebra in which the operation (*) is defined as follows:

*	0	1	2	3
0	0	0	0	0
1	1	0	1	0
2	2	2	0	0
3	3	3	1	0

Define a map d : X→X by

$$d(x) = \begin{cases} 0 & if\ x = 0,1,3, \\ 2 & if\ x = 2. \end{cases}$$

Define a fuzzy setμ: X → [0,1] by $\mu(d(0)) = t_0$, $\mu(d(1)) = t_1$, $\mu(d(2)) = \mu(d(3)) = t_2$

Where $t_0, t_1, t_2 \in [0,1]$, with

$t_0 > t_1 > t_2$. Routine calculations give that $\mu$is not fuzzy left (right)-derivations AB-ideal of AB-algebra.

Example 2.6. Consider the set X = {0, 1, 2, 3, 4, 5} with the operation defined by the following table:

*	0	1	2	3	4	5
0	0	0	0	0	0	0
1	1	0	0	0	0	1
2	2	2	0	0	1	1
3	3	2	1	0	1	1
4	4	4	4	4	0	1
5	5	5	5	5	5	0

define d(x): X → X by $d(x) = \begin{cases} 0 & \text{if } x = 0,1,2,3,4 \\ 5 & \text{if } x = 5 \end{cases}$

define a fuzzy set μ: X → [0,1] by μ(d (0))= t_0, μ(d (1))= t_1,
μ(d (1)) = μ(d (2)) = t_2, μ(d (3))= μ(d (4)) = t_3.

Where $t_0, t_1, t_2, t_3 \in[0,1]$,with $t_0 > t_1 > t_2 > t_3$. Routine calculations give that μ is a fuzzy derivations AB-ideal of AB-algebra.

Theorem 2.7. Let μ be a fuzzy derivations AB-ideal of AB-algebra X.

(1) If $x \le d(y)$, then $\mu(d(x)) \ge \mu(d(y))$,

(2) If $x * y \le d(y)$, then $\mu(d(x * y)) \ge \mu(d(y))$,

(3) If $(x * y) * (z * y) \le d(x * z)$, then $\mu(d((x * y) * (z * y))) \ge \mu(d(z * x))$,

(4) If $\mu(d(x * y)) = \mu(d(0))$, then $\mu(d(x)) \ge \mu(d(y))$

Proof :

(1) Let $x \le d(y)$, and since $d(y) \le y$, hence $x \le y$, i.e. $x * y = 0$, then

$\mu(d(x)) = \mu(d(x * 0))$

$\ge min\{\mu(d((x * y) * 0), \mu(d(y))\}$

(from Definition (2.4(2)))

$= \min\{\mu(d(x * y)), \mu(d(y))\}$

$= min\{\mu(d(0)), \mu(d(y))\} = \mu(d(y))$

(2) Let $x * y \le d(y)$, Then by Theorem (2.7(1)) we get, $\mu(d(x * y)) \ge \mu(d(x))$

(3) Let $(x * y) * (z * y) \le d(x * z)$, by (Theorem (2.7(1)) we get,

$\mu(d((x * y) * (z * y))) \ge \mu(d(z * x))$

(4) Let $\mu(d(x * y)) = \mu(d(0))$, then $\mu(d(x)) = \mu(d(x * 0)) \ge min\{\mu(d((x * y) * 0), \mu(d(y))\} = min\{\mu(d(x * y)), \mu(d(y))\}$

$= min\{\mu(0), \mu(d(y))\} = \mu(d(y))$.

Proposition 2.8. The intersection of any set of fuzzy derivations AB-ideals of AB-algebra X is also fuzzy left derivations AB-ideal.

Proof:

Let $\{\mu_i\}$ be a family of fuzzy derivations AB-ideals of AB-algebra X, then

∀x, y, z ∈X$(\cap\mu_i)(0)$ = inf $(\cap\mu_i(0))$

$\ge$ inf $(\mu_i(d(x)) = (\cap\mu_i)(d(x))$ and

$(\cap\mu_i)(d(x * z)) = inf(\mu_i(d(x * z))$

$\ge inf(min\{\mu_i(d((x * y) * z)), \mu_i(d(y))\}$

$= min\{inf(\mu_i(d((x * y) * z)), \mu_i(d(y))\}$

$= min\{(\cap\mu_i)(d((x * y) * z), (\cap\mu_i)(d(y))\}$.

Theorem 2.9. Let μ be a fuzzy set in X, then μ is a fuzzy derivations AB-ideal of X if and only if it satisfies : ∀ α∈[0, 1]), U(μ , α) ≠ φ implies U(μ ,α) is AB-ideal of X, where U (μ , α) = {x ∈ X / μ $(d(x)) \ge \alpha\}$.

Proof: Assume that μ is a fuzzy derivations AB-ideal of X, let $\alpha\in$ 0,1 be such that U $(\mu, \alpha) \neq \varphi$ and x, y ∈X, such that x ∈U (μ,α) then μ $(d(x)) \ge \alpha$, and so by (fl_2) $\mu(d(0)) = \mu(d(0 * y)) \ge min\{\mu(d(0 * x) * y), \mu(d(x))\} = min\{\mu(d(0) * y), \mu(d(x))\} = min\{\mu(0 * y), \mu(d(x))\} = min\{\mu(0), \mu(d(x))\} = \alpha$, hence 0∈U(μ , α).

Let $d(x * (y * z))$ ∈U(μ , α) and $d(y) \in$ U(μ , α), by (FL_2) $\mu(d(x * z)) \ge min\{\mu(d((x * y) * z), \mu(d(y))\} = \alpha$, so that x*z ∈ U(μ , α) .

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Hence U(μ , α) is AB-ideal of X.
Conversely, Let U(μ , α) is AB-ideal of X, let x, y, z ∈X be such that μ(d(x*z))>min{ μ(d(x*y)*z), μ(d(y))}, taking $\beta_0 = \frac{1}{2}\{\mu(d(x*z)) + \min\{\mu(d(x*y)*z), \mu(d(y))\}\}$, we have $\beta_0 \in[0,1]$, $\mu(d(x*z)) < \beta_0 < min\{\mu(d(x*y)*z), \mu(d(y))\}\}$, it follows that $d(x*y)*z \in$ U(μ , β_0) and y ∈ U(μ , β_0), this this is a contradiction and therefore μ is a fuzzy derivations AB-ideal of X.

3. *Cartesian Product of Fuzzy Derivations AB-ideals:*

The following definitions introduce the notion of Cartesian Product of Fuzzy Derivations AB-ideals and some Properties of it.

Definition 3.1. Let μ and β be a fuzzy derivations subset of a set X,**the Cartesian product of μ and β is** defined by

$(\mu \times \beta)(d(x,y)) = min\{\mu(d(x)), \beta(d(y))\}$, ∀x, y ∈ X.

Definition 3.2. If β is a fuzzy derivations subset of a set X, the strongest fuzzy relation on X, that is a **fuzzy derivations relation on β is μ_β** given by

$\mu_\beta(d(x,y)) = min\{\beta(d(x)), \beta(d(y))\}$, ∀ x, y ∈ X.

Proposition 3.3. For a given fuzzy derivations subset β of AB-algebra X, let μ_β be the strongest fuzzy derivations relation on X. If μ_β is a fuzzy derivations AB-ideal of X ×X then $\beta(d(x)) \leq \beta(d(0)) = \beta(0)$,∀x∈X.

Proof:

Since $\mu_\beta$is a fuzzy derivation AB-ideal of X ×X it follows from (F_1) that

μ_β(x,x) = min{ $\beta(d(x)), \beta(d(x))\} \leq \beta(d(0,0))$

= min{ $\beta(d(0), \beta(d(0)\}$ where (0,0)∈X ×X then $\beta(d(x)) \leq \beta(d(0)) = \beta(0)$.

Remark 3.4. Let X and Y be AB-algebras, we define $*$ on X ×Y by

$(x,y)*(u,v)(x*u, y*v), \forall(x,y),(u,v) \in$ X×Y then clearly (X ×Y,* ,(0,0)) is an AB-algebra.

Theorem 3.5. Let μ and β be a fuzzy derivations AB-ideals of AB-algebra X, then μ × β is a fuzzy derivations AB-ideal of X ×X.

Proof:

(1) $(\mu \times \beta)$(d (0,0)) = min{ $\mu(0), \beta(0)$
≥ min{ $\mu(d(x)), \beta(d(x))\}$= μ × β(d(x,y)) ∀x, y ∈X ×X.

(2) Let $(x_1, x_2),(y_1, y_2),(z_1, z_2) \in$X×X, then

μ × $\beta(x_1 * z_1, x_2 * z_2)$ = min{μ(d(x_1, z_1) ∗ β (d(x_2, z_2))
≥ min{ $\mu(d(x_1 * (y_1 * z_1)),\mu(d(y_1)),$
$\beta(d(x_2 * (y_2 * z_2)),\beta(d(y_2))\}$
= min{min{ $\mu(d(x_1 * (y_1 * z_1)),$
$\beta(d(x_2 * (y_2 * z_2))\}$,min{ $\mu(d(y_1)),\beta(d(y_2))\}$
= min{μ × $\beta((d(x_1 * (y_1 * z_1)),$
$(d(x_2 * (y_2 * z_2))),$μ × $\beta(d(y_1)), (d(y_2)))\}$
Hence μ × β is a fuzzy derivations AB-ideal of X×X .

Theorem 3.6. Let β be a fuzzy derivations subset of AB-algebra X and let μ_β be the strongest fuzzy derivations relation on then βis a fuzzy derivations AB-ideal of X if and only if $\mu_\beta$is a fuzzy derivations AB-ideal of X ×X.

Proof:

Let β be a fuzzy derivations AB-ideal of X .

(1) From (F_1), we get $\mu_\beta(0,0)$
=min{ $\beta(d(0)), \beta(d(0))\}$= min{ $\beta(0), \beta(0)\}$
≥ min{ $\beta(d(x)), \beta(d(y))\}=\mu_\beta(d(x)),d(y)),$
∀x, y ∈X ×X.

(2) $\forall(x_1, x_2),(y_1, y_2),(z_1, z_2) \in$X×X, we have from ($F_2$)

$\mu_\beta(x_1 * z_1, x_2 * z_2) = \min\{\beta(d(x_1 * z_1), \beta(d(x_2 * z_2))\}$
≥ min{$\beta(d(x_1 * (y_1 * z_1)),\beta(d(y_1)),$
$\beta(d(x_2 * (y_2 * z_2)),\beta(d(y_2))\}$
= min{min{$\beta(d(x_1 * (y_1 * z_1)),$
$\beta(d(x_2 * (y_2 * z_2))\}$,min{ $\beta(d(y_1)),\beta(d(y_2))\}\}$
= min{$\mu_\beta((d(x_1 * (y_1 * z_1)),$
$(d(x_2 * (y_2 * z_2))),\mu_\beta(d(y_1)), (d(y_2)))\}$

Hence μ_β is a fuzzy derivations AB-ideal of X×X .

Conversely, let μ_β be a fuzzy derivations AB-ideal of X ×X,

(1) ∀(x, y)∈X×X, we have,

min{ $\beta(0)$, $\beta(0)$}= $\mu_\beta(x,y)$= min{ β(x), β(y)},

It follows that $\beta(0) \geq \beta$(x) ∀x∈X,

(2) Let $(x_1,x_2),(y_1,y_2),(z_1,z_2)$ ∈X×X, then

min{ $\beta(d(x_1 * z_1)$, β $(d(x_2 * z_2))$}

= $\mu_\beta(d(x_1 * z_1), d(x_2 * z_2))$

≥

min{$\mu_\beta(d(x_1,x_2)*((y_1,y_2)*(z_1,z_2)))$,$\mu_\beta(d(y_1), d(y_2))$}

= min{$\mu_\beta(d(x_1 * (y_1 * z_1)),d(x_2 * (y_2 * z_2))$,$\mu_\beta(d(y_1), d(y_2))$}

= min{min{$\beta(d(x_1 * (y_1 * z_1))$, $\beta(d(x_2 * (y_2 * z_2))$},min{ $\beta(d(y_1))$,$\beta(d(y_2))$}}

= min{$\beta(d(x_1 * (y_1 * z_1))$,$\beta(d(y_1))$, $\beta(d(x_2 * (y_2 * z_2))$,$\beta(d(y_2))$}

In particular, if we take$x_2,y_2,z_2 = 0$, then

$\beta(d(x_1 * z_1)$ ≥min{$\beta(d(x_1 * (y_1 * z_1))$,$\beta(d(y_1))$}.

Hence β be a fuzzy derivations AB-ideal of X.

Conclusions and discussion

1) we presented some properties related to fuzzy AB-ideal of AB- algebra, among these properties, derivations AB-ideals, Cartesian Product of Fuzzy Derivations AB-ideals.

2) We recommend finding other properties for this type of algebra.

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بعض خصائص المثاليات الضبابية الى الجبر -AB

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المستخلص :

في هذا البحث قدمنا مفهوم المثالي الضبابي الى الجبر -AB وكذلك برهنا بعض الخصائص والنظريات ذات الصلة بالموضوع وقدمنا الخصائص والنظريات التي توضح المفهومين -AB وكذلك درسنا العلاقات الضبابية واشتقاقات المثاليات الضبابية الى الجبر -AB مما دفعنا الى دراسة الضرب الديكارتي الى اشتقاقات المثاليات الضبابية الى الجبر