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Application of Modified Adomian Decomposition Method to (2+1)-dimensional Non-linear Wu-Zhang system

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Abstract

In this paper, the Domain decomposition method (ADM) with modified polynomials is applied for nonlinear (2+1) – dimensional Wu-Zhang system . we compared the solution of the system with MVIM, HPM and RDTM [17,13,15]. The numerical results obtained by this polynomial are very effective, convenient and quite accurate to system of partial differential equations. A comparative between the modifications method and the other methods is present from some examples to show the efficiency of each method.

Keywords: Modified Adomian decomposition method, Wu- Zhang system.

Mathematics Subject Classification: 65M99,35G50 .

1. Introduction

The nonlinear partial differential equations (NPDEs) are encountered in various disciplines, such as physics, mechanics, chemistry, biology, mathematics and engineering. Many efforts have been made on the study of NPDEs. Long wave in shallow water is a subject of broad interests and has along colorful history.

Wu and Zhang derived three sets of model equations for modeling nonlinear and dispersive long gravity waves travelling in two horizontal directions on shallow waters of uniform depth [16]. One of these equations, Wu-Zhang equation (which describes (2+1)-dimensional dispersive long wave), can be written as[16]

$$u_t + u\,u_x + v\,u_y + w_x = 0\,,$$
$$v_t + u\,v_x + v\,v_y + w_y = 0\,,\ (1)$$
$$w_t + (u\,w)_x + (v\,w)_y + \frac{1}{3}(u_{xxx} + u_{xyy} + u_{xxy} + u_{yyy}) = 0\,,$$

where w is the elevation of the water, u is the surface velocity of water along x-direction, and v is the surface velocity of water along y-direction. The explicit solutions of Eq. (1) are very helpful for costal and civil engineers to apply the nonlinear water wave model in harbor and coastal design. Recently introduced some new methods and are applied to nonlinear equations such as Variation Iteration Method [11], Homotopy Perturbation Method [10,13], the homogeneous balance method [9], the differential transform method (DTM) [5]. Among all of the methods, the reduced differential transform method (RDTM), employed for solving the nonlinear partial differential equations [12]. The RDTM is presented to overcome the demerit of complex calculation of DTM. This technique doesn't require any discretization, linearization or small perturbations and therefore it reduces significantly the numerical computation.

Ma Z.Y [13], used Homotopy perturbation method (HPM) for the Wu-Zhang equation in fluid dynamics. Extended tanh method and exp-function method and its application to (2+1)_dimensional dispersive long wave nonlinear equations [4]. Application to Reduced differential Transform method to the Wu-Zhang equation is presented in [15], and the solution of Wu-Zhang equations using the modified variational iteration method obtained by [17]. solitons and other solution Wu-Zhang system are discussed in [14].

At the beginning of the 1980, a so-called Adomian decomposition method (ADM) [1] has been used to solve effectively, easily, and accurately a large class of linear and nonlinear equations, solutions partial, deterministic or stochastic differential equations with approximates which converge rapidly.

Several studies have been proposed to modify the regular Adomian decomposition method for initial value problems in ordinary and partial differential equations [3,6]. Error analysis of Adomian series solution to a class of nonlinear differential equation is introduction in [7]. Alkresheh ,H. A. [2] , proposed two new adomian decomposition method, and [8] used the ADM to solve coupled system of nonlinear physical equations, coupled system of diffusion-reaction equation and integro-differential diffusion-reaction equation.

This paper is organized as follows. In section 2, the basic ideas of the modified Adomian decomposition method are described, in section 3, is devoted to solving a nonlinear (2+1)-dimensional Wu-Zhang system by MADM, the results and comparisons of

the numerical solutions are presented in section 4 and concluding remarks are given in section 5.

2. The modified Adomian decomposition method (MADM)

Let us consider the following equation

$Lu + Nu + Ru = f(x)$ (2)

where L is an invertible linear operator, N represents the nonlinear operator and R is the remaining linear part, from equation (2) we have

$Lu = f(x) - Nu - Ru,$ (3)

Now, applying the inverse operator L^{-1} to both sides of equation (3) then use the initial conditions we find

$u = g(x) - L^{-1} Nu - L^{-1} Ru,$ (4)

where $L^{-1} = \int_0^x (.)\ ds$, and g(x) represents the terms having from integrating the remaining term $f(x)$ and from using the given initial or boundary conditions. The ADM assumes that the nonlinear operator $N(u)$ can be decomposed by an infinite series of polynomials given by

$N(u) = \sum_{n=0}^{\infty} A_n(u_o, u_1, \ldots, u_n)$

where A_n are the Adomian's polynomials [1] defined as:

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{\infty}\lambda^i u_i\right)\right]_{\lambda=0}, \quad n = 0,1,2,\ldots \qquad (5)$$

El-Kalla [7] introduce a new formula for Adomian polynomials, he claimed that the Adomian solution using this new formula converges faster than using Adomian polynomials (5). Kalla polynomial given in the following form:

$$A_n = N(S_n) - \sum_{i=0}^{n-1} A_i(u_o, u_1, \ldots, u_{n-1}) \qquad (6)$$

Where $S_n = u_o + u_1 + \cdots + u_n$ and A_n can be given as :

$$A_0 = N(u_o)$$

$$A_1 = \frac{d}{dx}(N(u_o))u_1 + \frac{1}{2}\frac{d^2}{dx^2}(N(u_o))u_1^2 + \frac{1}{6}\frac{d^3}{dx^3}(N(u_o))u_1^3 + \frac{1}{24}\frac{d^4}{dx^4}(N(u_o))u_1^4 + \cdots$$

$$A_2 = \frac{d}{dx}(N(u_o))u_2 + \frac{1}{2}\frac{d^2}{dx^2}(N(u_o))[2u_1 u_2 + u_2^2] + \frac{1}{6}\frac{d^3}{dx^3}(N(u_o))\,[3u_1^2 u_2 + 3u_1 u_2^2 + u_2^3] + \cdots$$

$$\vdots$$

And so on. These formulas are easy to compute by using Maple 13 software.

3. Application MADM to the Wu-Zhang system

This section is devoted to study the analytical solution of the Wu-Zhang system (1) using MADM, we make a comparison for the corresponding absolution error between the using of the proposed polynomial in MADM and the methods in [13,15,17].

let us consider the standard form of equation (1) in an operator form:

$$L_t u + u L_x u + v L_y u + L_x w = 0,$$
$$L_t v + u L_x v + v L_y v + L_y w = 0, \quad (7)$$
$$L_t w + u L_x u + w L_x u + v L_y w + w L_y v + \frac{1}{3}\left(L_{xxx} u + L_{xyy} u + L_{xxy} v + L_{yyy} v\right) = 0,$$

$u(x,y,o) = g_1(x,y),\ v(x,y,o) = g_2(x,y),\ w(x,y,o) = g_3(x,y),$

Where the notations $L_t = \frac{\partial}{\partial_t}, L_x = \frac{\partial}{\partial_x}, L_y = \frac{\partial}{\partial_y}, L_{xxx} = \frac{\partial^3}{\partial_{x^3}}, L_{xyy} = \frac{\partial^3}{\partial_x \partial_{y^2}},\ L_{xxy} = \frac{\partial^3}{\partial_{x^2}\partial_y}, and\ L_{yyy} = \frac{\partial^3}{\partial y^3},$ Symbolize the linear differential operator.

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Assuming l_t^{-1} the inverse of the operator l_t exists and can conveniently by taken as definite integrate with respect to (t):

Define $l_t^{-1} = \int_0^t (.)dt$ than the system (7) becomes :

$$u(x,y,t) = g_1(x,y) - l_t^{-1}\big(\vartheta_1(u) + \vartheta_2(u,v) + l_x\, w\big),$$
$$v(x,y,t) = g_2(x,y) - l_t^{-1}\big(\vartheta_3(u,v) + \vartheta_4(v) + l_y\, w\big),$$
$$w(x,y,t) = g_3(x,y) - l_t^{-1}\Big(\vartheta_5(u,w) + \vartheta_6(u,w) + \vartheta_7(v,w) + \vartheta_8(v,w) + \tfrac{1}{3}(L_{xxx}\, u + L_{xyy}\, u + L_{xxy}\, v + L_{yyy}\, v)\Big),$$

Where

$\vartheta_1(u) = u\, u_x$, $\vartheta_2(u,v) = v\, u_y$, $\vartheta_3(u,v) = u v_x$
$\vartheta_4(v) = v\, v_x$, $\vartheta_5(u,w) = u\, w_x$, $\vartheta_6(u,w) = w\, u_x$
$\vartheta_7(v,w) = v\, w_y$, $\vartheta_8(u,w) = w\, v_y$

The MADM assumes an infinite series for the unknown functions $u(x,y,t), v(x,y,t)$ *and* $w(x,y,t)$ *in the form*

$$u(x,y,t) = \sum_{n=0}^{\infty} u_n(x,y,t)$$
$$v(x,y,t) = \sum_{n=0}^{\infty} v_n(x,y,t)$$
$$w(x,y,t) = \sum_{n=0}^{\infty} w_n(x,y,t)$$

We can write $\vartheta_1, \vartheta_2 \ldots\ldots\ldots, \vartheta_8$ by an infinite series of Adomian polynomial in the form

$\vartheta_1(u) = \sum_{n=0}^{\infty} A_n$, $\vartheta_2(u,v) = \sum_{n=0}^{\infty} B_n$, $\vartheta_3(u,v) = \sum_{n=0}^{\infty} C_n$
$\vartheta_4(v) = \sum_{n=0}^{\infty} D_n$, $\vartheta_5(u,w) = \sum_{n=0}^{\infty} E_n$, $\vartheta_6(u,w) = \sum_{n=0}^{\infty} F_n$
$\vartheta_7(v,w) = \sum_{n=0}^{\infty} G_n$, $\vartheta_8(v,w) = \sum_{n=0}^{\infty} H_n$

Where $A_n, B_n, C_n, D_n \ldots\ldots.., H_n$ are the appropriate modified Adomian polynomials

$$A_n(u_0,u_1,\ldots\ldots\ldots.u_n) = \frac{1}{n!}\frac{d^n}{d\rho^n}\Big[\varphi_1\Big(\sum_{k=0}^{\infty}\rho^k u_k\Big)\Big]_{\rho=0},\ n \ge 0$$
$$B_n(u_0,u_1,\ldots,u_n,v_0,v_1,\ldots,v_n) = \frac{1}{n!}\frac{d^n}{d\rho^n}\left[\varphi_2\left(\sum_{k=0}^{\infty}\rho^k u_k, \sum_{k=0}^{\infty}\rho^k v_k\right)\right]_{\rho=0},\ n \ge 0$$
$$\vdots \qquad\qquad (8)$$
$$H_n(v_0,v_1,\ldots,v_n,w_0,w_1,\ldots,w_n) = \frac{1}{n!}\frac{d^n}{d\rho^n}\Big[\varphi_8\Big(\sum_{k=0}^{\infty}\rho^k v_k, \sum_{k=0}^{\infty}\rho^k w_k\Big)\Big]_{\rho=0},\ n \ge 0$$

For examples, the first polynomials using formulas (8) are computed be:

$$A_o = u_o\frac{\partial u_o}{\partial x}$$
$$A_1 = u_o\frac{\partial u_1}{\partial x} + u_1\frac{\partial u_o}{\partial x} + u_1\frac{\partial u_1}{\partial x}$$
$$A_2 = u_o\frac{\partial u_2}{\partial x} + u_2\frac{\partial u_o}{\partial x} + u_2\frac{\partial u_1}{\partial x} + u_1\frac{\partial u_2}{\partial x} + u_2\frac{\partial u_2}{\partial x}$$
$$A_3 = u_3\frac{\partial u_o}{\partial x} + u_o\frac{\partial u_3}{\partial x} + u_3\frac{\partial u_1}{\partial x} + u_1\frac{\partial u_3}{\partial x} + \cdots$$
$$A_4 = u_4\frac{\partial u_o}{\partial x} + u_o\frac{\partial u_4}{\partial x} + \cdots$$
$$\vdots$$
$$B_o = v_o\frac{\partial u_o}{\partial y}$$
$$B_1 = v_o\frac{\partial u_1}{\partial y} + v_1\frac{\partial u_o}{\partial y} + v_1\frac{\partial u_1}{\partial y}$$
$$B_2 = v_o\frac{\partial u_2}{\partial y} + v_2\frac{\partial u_o}{\partial y} + v_2\frac{\partial u_1}{\partial y} + v_1\frac{\partial u_2}{\partial y} + v_2\frac{\partial u_2}{\partial y}$$
$$B_3 = v_3\frac{\partial u_o}{\partial y} + v_o\frac{\partial u_3}{\partial y} + v_3\frac{\partial u_1}{\partial y} + v_1\frac{\partial u_3}{\partial y} + \cdots$$
$$\vdots$$
$$C_o = u_o\frac{\partial v_o}{\partial x}$$
$$C_1 = u_o\frac{\partial v_1}{\partial x} + u_1\frac{\partial v_o}{\partial x} + u_1\frac{\partial v_1}{\partial x}$$
$$C_2 = u_o\frac{\partial v_2}{\partial x} + u_2\frac{\partial v_o}{\partial x} + u_2\frac{\partial v_1}{\partial x} + u_1\frac{\partial v_2}{\partial x} + u_2\frac{\partial v_2}{\partial x}$$
$$C_3 = u_3\frac{\partial v_o}{\partial x} + u_o\frac{\partial v_3}{\partial x} + u_3\frac{\partial v_1}{\partial x} + u_1\frac{\partial v_3}{\partial x} + \cdots$$

⋮

$$D_o = v_o \frac{\partial v_o}{\partial y}$$

$$D_1 = v_o \frac{\partial v_1}{\partial y} + v_1 \frac{\partial v_o}{\partial y} + v_1 \frac{\partial v_1}{\partial y}$$

$$D_2 = v_o \frac{\partial v_2}{\partial y} + v_2 \frac{\partial v_o}{\partial y} + v_2 \frac{\partial v_1}{\partial y} + v_1 \frac{\partial v_2}{\partial y} + v_2 \frac{\partial v_2}{\partial y}$$

$$D_3 = v_3 \frac{\partial v_o}{\partial y} + v_o \frac{\partial v_3}{\partial y} + v_3 \frac{\partial v_1}{\partial y} + v_1 \frac{\partial v_3}{\partial y} + \cdots$$

⋮

$$E_o = u_o \frac{\partial w_o}{\partial x}$$

$$E_1 = u_o \frac{\partial w_1}{\partial x} + u_1 \frac{\partial w_o}{\partial x} + u_1 \frac{\partial w_1}{\partial x}$$

$$E_2 = u_o \frac{\partial w_2}{\partial x} + u_2 \frac{\partial w_o}{\partial x} + u_2 \frac{\partial w_1}{\partial x} + u_1 \frac{\partial w_2}{\partial x} + u_2 \frac{\partial w_2}{\partial x}$$

$$E_3 = u_3 \frac{\partial w_o}{\partial x} + u_o \frac{\partial w_3}{\partial x} + u_3 \frac{\partial w_1}{\partial x} + u_1 \frac{\partial w_3}{\partial x} + \cdots$$

⋮

$$F_o = w_o \frac{\partial u_o}{\partial x}$$

$$F_1 = w_o \frac{\partial u_1}{\partial x} + w_1 \frac{\partial u_o}{\partial x} + w_1 \frac{\partial u_1}{\partial x}$$

$$F_2 = w_o \frac{\partial u_2}{\partial x} + w_2 \frac{\partial u_o}{\partial x} + w_2 \frac{\partial u_1}{\partial x} + w_1 \frac{\partial u_2}{\partial x} + w_2 \frac{\partial u_2}{\partial x}$$

$$F_3 = w_3 \frac{\partial u_o}{\partial x} + w_o \frac{\partial u_3}{\partial x} + w_3 \frac{\partial u_1}{\partial x} + w_1 \frac{\partial u_3}{\partial x} + \cdots$$

⋮

$$G_o = v_o \frac{\partial w_o}{\partial y}$$

$$G_1 = v_o \frac{\partial w_1}{\partial y} + v_1 \frac{\partial w_o}{\partial y} + v_1 \frac{\partial w_1}{\partial y}$$

$$G_2 = v_o \frac{\partial w_2}{\partial y} + v_2 \frac{\partial w_o}{\partial y} + v_2 \frac{\partial w_1}{\partial y} + v_1 \frac{\partial w_2}{\partial y} + v_2 \frac{\partial w_2}{\partial y}$$

$$G_3 = v_3 \frac{\partial w_o}{\partial y} + v_o \frac{\partial w_3}{\partial y} + v_3 \frac{\partial w_1}{\partial y} + v_1 \frac{\partial w_3}{\partial y} + \cdots$$

⋮

$$H_o = w_o \frac{\partial v_o}{\partial y}$$

$$H_1 = w_o \frac{\partial v_1}{\partial y} + w_1 \frac{\partial v_o}{\partial y} + w_1 \frac{\partial v_1}{\partial y}$$

$$H_2 = w_o \frac{\partial v_2}{\partial y} + w_2 \frac{\partial v_o}{\partial y} + w_2 \frac{\partial v_1}{\partial y} + w_1 \frac{\partial v_2}{\partial y} + w_2 \frac{\partial v_2}{\partial y}$$

$$H_3 = w_3 \frac{\partial v_o}{\partial y} + w_o \frac{\partial v_3}{\partial y} + w_3 \frac{\partial v_1}{\partial y} + w_1 \frac{\partial v_3}{\partial y} + \cdots$$

⋮

And so on, the nonlinear system (7) is constructed as:

$$u_0(x,y,t) = g_1(x,y),\ v_0(x,y,t) = g_2(x,y), w_0(x,y,t) = g_3(x,y),$$

$$u_{n+1}(x,y,t) = l_t^{-1}[l_x\, w + A_n + B_n]; \quad n \geq 1,$$

$$v_{n+1}(x,y,t) = l_t^{-1}[l_y\, w + C_n + D_n]; \quad n \geq 1, \quad (9)$$

$$w_{n+1}(x,y,t) = l_t^{-1}\left[\frac{1}{3}\left(L_{xxx}\, u + L_{xyy}\, u + L_{xxy}\, v + L_{yyy}\, v\right) + E_n + F_n + G_n + H_n\right]; \quad n \geq 1,$$

Where The functions $g_1(x,y), g_2(x,y), and\ g_3(x,y)$,Are initial conditions. We construct the solution $u(x,y,t)$,$v(x,y,t)$ And $w(x,y,t)$ as follow:

$\lim_{n\to\infty} \varphi_n = u(x,y,t)$, $\lim_{n\to\infty} \widetilde{\varphi_n} = v(x,y,t)$ and $\lim_{n\to\infty} \widehat{\varphi_n} = w(x,y,t)$

$\varphi_n(x,y,t) = \sum_{k=0}^{\infty} u_k(x,y,t)$, $\widetilde{\varphi_n}(x,y,t) = \sum_{k=0}^{\infty} v_k(x,y,t)$, $\widehat{\varphi_n}(x,y,t) = \sum_{k=0}^{\infty} w_k(x,y,t)$

And the recurrence relation is given as in (9).

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4. Applications

We consider the Solutions of the system (9) with the initial and conditions [13,15,17].

$$u_o(x,y,t) = -\frac{K_3+K_2bb_0}{K_1} + \frac{2}{3}\sqrt{3}k_1\, tanh(k_1x+k_2y),$$
$$v_o(x,y,t) = bb_0 + \frac{2}{3}\sqrt{3}k_2\, tanh(k_1x+k_2y), \quad (10)$$

$$w_o(x,y,t) = \frac{2}{3}(k_1^2+k_2^2)\, sech(k_1x+k_2y)^2$$

Where b_0, k_1, k_2 *and* k_3 *are arbitrary constants*.
To calculate the terms of the MADN for $u(x,y,t), v(x,y,t)$ and $w(x,y,t)$, we substitute the initial conditions (10) into the system (9) and using Maple 13 language , the solutions of the system (9) can be obtained as follows:

$$u_1(x,y,t) = \frac{2}{3}\frac{K_1t\sqrt{3}K_3}{cosh(K_1x+K_2y)^2}$$
$$v_1(x,y,t) = \frac{2}{3}\frac{K_2t\sqrt{3}K_3}{cosh(K_1x+K_2y)^2}$$
$$w_1(x,y,t) = -\frac{4}{3}\frac{t\sinh(K_1x+K_2y)K_3(K_1^2x+K_2^2y)}{cosh(K_1x+K_2y)^3}$$

$$u_2(x,y,t) = -\frac{2}{9}\frac{1}{cosh(K_1x+K_2y)^5}(k_1k_3^2t^2\ \sinh(k_1x+k_2y)^{-4K_1^2t-4tK_2^2+3\ \sqrt{3}\cosh(k_1x+k_2y)^2}))$$

$$v_2(x,y,t) = -\frac{2}{9}\frac{1}{cosh(K_1x+K_2y)^5}(k_1k_3^2t^2\ \sinh(k_1x+k_2y)^{-4K_1^2t-4tK_2^2+3\ \sqrt{3}\cosh(k_1x+k_2y)^2}))$$

$$w_2(x,y,t) = -\frac{2}{27}\frac{1}{cosh(K_1x+K_2y)^5}(K_3^2t^2(-20k_1^4\, t\sqrt{3} - 40k_2^2\, k_1^2\, t\sqrt{3} - 20k_2^4\, t\sqrt{3} + 27k_1^2\, cosh(k_1x+k_2y)^2 + 27k_2^2\, cosh(k_1x+k_2y)^2 - 18\, k_1^2 cosh(k_1x+k_2y)^4 - 18\, k_2^2 cosh(k_1x+k_2y)^4 + 16\sqrt{3}\, t\, k_1^4 cosh(k_1x+k_2y)^2 + 16\sqrt{3}\, t\, k_2^4 cosh(k_1x+k_2y)^2 + 32\sqrt{3}\, t\, k_1^2k_1^2 cosh(k_1x+k_2y)^2))$$

$$u(x,y,t) = u_o(x,y,t) + u_1(x,y,t) + u_2(x,y,t) + \cdots = \frac{-1}{2835}\frac{1}{k_1\cosh(k_1x+k_2y)^{11}}(2835\cosh(k_1x+k_2y)^{11}k_2bb_0 \ldots\ldots$$

$$v(x,y,t) = v_o(x,y,t) + v_1(x,y,t) + v_2(x,y,t) + \cdots = \frac{1}{2835}\frac{1}{\cosh(k_1x+k_2y)^{11}}(2835\, bb_0\cosh(k_1x+k_2y)^{11} - 1890 \ldots$$

$$w(x,y,t) = w_o(x,y,t) + w_1(x,y,t) + w_2(x,y,t) + \cdots = \frac{2}{8505}\frac{1}{\cosh(k_1x+k_2y)^{12}}(55040k_3^4t^7\ \sqrt{3}k_2^2k_1^6\cosh(k_1x+k_2y)^2 - 20480 \ldots$$

The approximate solutions of the system (1) are obtained as follows

$$u(x,y,t) = u_0(x,y,t) + u_1(x,y,t) + u_2(x,y,t) + \cdots$$
$$v(x,y,t) = v_0(x,y,t) + v_1(x,y,t) + v_2(x,y,t) + \cdots$$
$$w(x,y,t) = w_0(x,y,t) + w_1(x,y,t) + w_2(x,y,t) + \cdots$$

This solution is convergent to the exact solution [13]

$$u(x,y,t) = -\frac{K_3+K_2b_0}{K_1} + \frac{2\sqrt{3}}{3}k_1\, tanh(k_1x+k_2y+k_3t),$$

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$v(x,y,t) = b_0 + \frac{2\sqrt{3}}{3} k_2 \tanh(k_1 x + k_2 y + k_3 t),$

$w(x,y,t) = \frac{2}{3}(k_1^2 + k_2^2)\, sech^2(k_1 x + k_2 y + k_3 t),$

We comparisons the absolute errors and mean square error for the MADN results for $u(x,y,t), v(x,y,t) and\ w(x,y,t)$,for the first three approximations with HPM [13] ,RDTM [15], and MVIM [17], when $b_0 = k_1 = 0.1, k_2 = k_3 = 0.01$, $t = 5, y = 20$ $for\ the\ solution\ of\ the\ Wu\ Zhang\ system\ (1)$ $with\ the\ initial\ conditions\ (10).$

Table 1: Comparison of absolute errors using u_3 for various values of x in Wu-Zhang system between MADM and MVIM [17] when $b_0 = k_1 = 0.1 , k_2 = k_3 = 0.01 , t = 5 , and\ y = 20$

x	u_{exact}	u_{MADM}	$\lvert u_{exact} - u_{MVIM} \rvert$	$\lvert u_{exact} - u_{MADM} \rvert$
-50	0.225452768	0.225452769	1.6451825 E-6	10.39242830 E-10
-40	0.225342395	0.225342395	1.2148126 E-5	4.330797530 E-10
-30	0.224530095	0.224530099	8.9315718 E-5	3.594419403 E-9
-20	0.218700684	0.218700708	6.3605735 E-4	2.448538578 E-8
-10	0.183340683	0.183340786	3.6024380 E-3	1.023749896 E-7
0	0.0817192288	0.0817189240	6.4447985 E-3	3.048061683 E-7
10	0.0120486423	0.120487416	1.9107842 E-4	8.922341521 E-3
20	0.002932728	0.002932733	2.9196656 E-4	5.192973059 E-9
30	0.005123370	0.005123372	4.0172038 E-5	1.372972068 E-9
40	0.005423074	0.005423074	5.44887493 E-6	2.078596792 E-10
50	0.005463694	0.005463694	7.37648418 E-7	5.196177359 E-11
MSE			5.00446211 E-6	1.018121171 E-14

Table 2: Comparison of absolute errors using v_3 for various values of x in Wu-Zhang system between MADM and MVIM [17] when

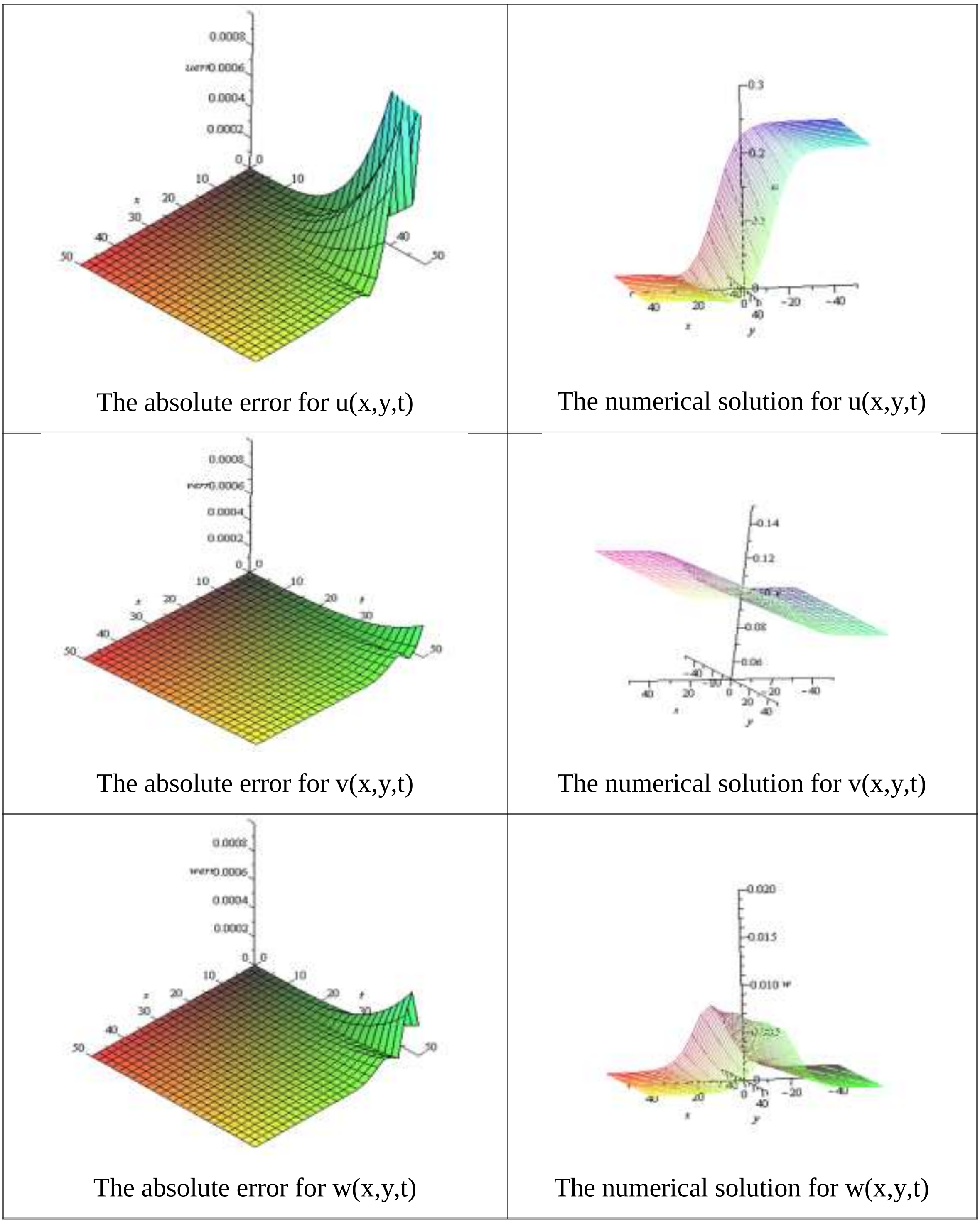
$$b_0 = k_1 = 0.1 , k_2 = k_3 = 0.01 , t = 5 , and\ y = 20$$

x	v_{exact}	v_{MADM}	$\lvert v_{exact} - v_{MVIM} \rvert$	$\lvert v_{exact} - v_{MADM} \rvert$
-50	0.0884547231	0.884547231	8.08571164 E-9	1.039242830 E-11
-40	0.0884657604	0.0884657604	5.96388798 E-8	6.330797530 E-11
-30	0.0885469904	0.0885469900	4.34887678 E-7	3.394419403 E-10
-20	0.0891299315	0.0891299291	2.91626141 E-6	2.438538578 E-9
-10	0.0926659316	0.0926659213	1.07971370 E-5	1.027749896 E-8
0	0.102828077	0.102828107	5.875I9681 E-6	3.050061683 E-8
10	0.109795135	0.109795126	7.18185473 E-6	8.962341521 E-9
20	0.111293272	0.111293273	1.30807366 E-6	5.392973059 E-10
30	0.111512337	0.111512337	1.84431598 E-7	1.072972068 E-10
40	0.111542307	0.111542307	2.50992772 E-8	2.078596792 E-11
50	0.111546369	0.111546369	3.39937593 E-9	5.196177359 E-12
MSE			1.93743805 E-11	1.018378476 E-16

Table 3: Comparison of absolute errors using w_3 for various values of x in Wu-Zhang system between MADM and MVIM [17] when

$$b_0 = k_1 = 0.1 , k_2 = k_3 = 0.01 , t = 5 , and\ y = 20$$

x	w_{exact}	w_{MADM}	$\lvert w_{exact} - w_{MVIM} \rvert$	$\lvert w_{exact} - w_{MADM} \rvert$
-50	2.01570751 E-6	2.01569976 E-6	3.705249242 E-7	7.749880486 E-12
-40	1.448799414 E-5	1.48798843 E-5	2.73I8146111 E-6	5.708491389 E-11
-30	1.09176198 E-4	1.09175786 E-4	1.986047907 E-5	4.118114838 E-10
-20	7.66334776 E-4	7.66332142 E-4	1.302774181 E-4	2.634558628 E-9
-10	4.01701111 E-3	4.01701120 E-3	4.145730832 E-4	8.8787905 E-11
0	6.32943331 E-3	6.32939492 E-3	1.153652836 E-4	3.839847338 E-8
10	1.88812677 E-3	1.88814161 E-3	2.265746793 E-5	1.484359887 E-8
20	2.92663787 E-4	2.92663846 E-4	6.972254850 E-7	5.892963187 E-11
30	4.03711466 E-5	4.03709936 E-5	1.991078199 E-7	1.529342660 E-10
40	5.47785204 E-6	5.47782786 E-6	2.877 10754 E-8	2.418224902 E-11
50	7.41607526 E-7	7.41604188 E-7	3.9511 0044 E-9	3.337771675 E-12
MSE			1.84607383 E-8	1.547236655 E-16



The absolute error for u(x,y,t) | The numerical solution for u(x,y,t)

The absolute error for v(x,y,t) | The numerical solution for v(x,y,t)

The absolute error for w(x,y,t) | The numerical solution for w(x,y,t)

Figure 1: The absolute error for the first three approximation between MADM and the exact solution, for various values of x and y when

$$b_0 = 0.1\,, k_1 = 0.1,\; k_2 = 0.01\,, k_3 = 0.01\,,\; and\; t = 5.$$

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Table 4: Comparison of absolute errors and mean square error using u_3 for Wu-Zhang system between MADM with HPM [13] and RDTM [15], for various values of x and y when $b_0 = 0.1\,, k_1 = 0.05,\; k_2 = 0.1\,, k_3 = 0.3\,,\; and\; t = 2.5$

(x,y)	u_{exact}	u_{MADM}	$\lvert u_{exact} - u_{HPM}\rvert$ $\lvert u_{exact} - u_{MVIM}\rvert$	$\lvert u_{exact} - u_{MADM}\rvert$
(0.001,-40)	6.25756167 E-0	6.25757298 E-0	1.13049917 E-5	1.13094127 E-5
(0.002,-30)	6.25646611 E-0	6.25654557 E-0	7.92908229 E-5	7.94573088 E-5
(0.003,-20)	6.24897325 E-0	6.24936663 E-0	3.85696522 E-4	3.93378218 E-4
(0.004,-10)	6.21412953 E-0	6.21299676 E-0	1.26708625 E-3	1.13277326 E-3
(0.005,0)	6.16332105 E-0	6.16444944 E-0	1.49035435 E-3	1.12839583 E-3
(0.006,10)	6.14564769 E-0	6.14581428 E-0	5.73725768 E-5	1.66595217 E-4
(0.007,20)	6.14273462 E-0	6.14247428 E-0	2.65974103 E-4	2.60340360 E-4
(0.008,30)	6.14232875 E-0	6.14228437 E-0	4.45040837 E-5	4.43855200 E-5
(0.009,40)	6.14227361 E-0	6.14226742 E-0	6.19002050 E-6	6.18845786 E-6
(0.01,50)	6.14226614 E-0	6.14226530 E-0	8.41789424 E-7	8.41763456 E-7
MSE			4.057899493 E-7	2.815176595 E-7

Table 5: Comparison of absolute errors and mean square error using v_3 for Wu-Zhang system between MADM with HPM [13] and RDTM [15], for various values of x and y when $b_0 = 0.1\,, k_1 = 0.05,\; k_2 = 0.1\,, k_3 = 0.3\,,\; and\; t = 2.5$

(x,y)	v_{exact}	v_{MADM}	$\lvert v_{exact} - v_{HPM}\rvert$ $\lvert v_{exact} - v_{MVIM}\rvert$	$\lvert v_{exact} - v_{MADM}\rvert$
(0.001,-40)	1.51233361 E-2	1.51459522 E-2	2.26099488 E-5	2.2616027 E-5
(0.002,-30)	1.29322262 E-2	1.30911405 E-2	1.58587648 E-4	1.58914298 E-4
(0.003,-20)	2.05349994 E-3	1.26674403 E-3	7.71401056 E-4	7.86755912 E-4
(0.004,-10)	7.17409386 E-2	7.40064832 E-2	2.53417253 E-3	2.26554453 E-3
(0.005,0)	1.73357903 E-1	1.71101111 E-1	2.98071071 E-3	2.25679205 E-3
(0.006,10)	2.08704626 E-1	2.08371441 E-1	1.14747188 E-4	3.33185019 E-4
(0.007,20)	2.14530751 E-1	2.15051429 E-1	5.31948231 E-4	5.20677749 E-4
(0.008,30)	2.15342497 E-1	2.15431265 E-1	8.90041228 E-5	8.87677480 E-5
(0.009,40)	2.15452784 E-1	2.15465160 E-1	1.23800410 E-5	1.23756930 E-5
(0.01,50)	2.15467717 E-1	2.15469398 E-1	1.68148224 E-6	1.68139959 E-6
MSE			1.62316013 E-6	1.12607062 E-6

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Table 6: Comparison of absolute errors and mean square error using v_3 for Wu-Zhang system between MADM with HPM [13] and RDTM [15], for various values of x and y when $b_0 = 0.1$, $k_1 = 0.05$, $k_2 = 0.1$, $k_3 = 0.3$, $and\ t = 2.5$

(x, y)	w_{exact}	w_{MADM}	$\lvert w_{exact} - w_{HPM} \rvert$ $\lvert w_{exact} - w_{MVIM} \rvert$	$\lvert w_{exact} - w_{MADM} \rvert$
(0.001,-40)	4.99692720 E-5	4.67304813 E-5	3.23703114 E-6	3.23879066 E-6
(0.002,-30)	3.62278514 E-4	3.40688032 E-4	2.14972816 E-5	2.15904817 E-5
(0.003,-20)	2.33738529 E-3	2.28043656 E-3	5.29010050 E-5	5.69487325 E-5
(0.004,-10)	7.83422424 E-3	8.54191973 E-3	7.31802585 E-4	7.07695496 E-4
(0.005,0)	4.96996963 E-3	3.63168325 E-3	1.32491691 E-3	1.33828638 E-3
(0.006,10)	9.47898572 E-4	1.37068310 E-3	3.99459208 E-4	4.22784533 E-4
(0.007,20)	1.35025275 E-4	8.08732586 E-5	5.71504440 E-5	5.41520164 E-5
(0.008,30)	1.84010586 E-5	6.03244902 E-6	1.24360974 E-5	1.23686096 E-5
(0.009,40)	2.49244365 E-6	7.14467502 E-7	1.77924640 E-6	1.77797617 E-6
(0.01,50)	3.37325458 E-7	9.47861898 E-8	2.42562615 E-7	2.42539268 E-7
MSE			2.457202663 E-7	2.477398551 E-7

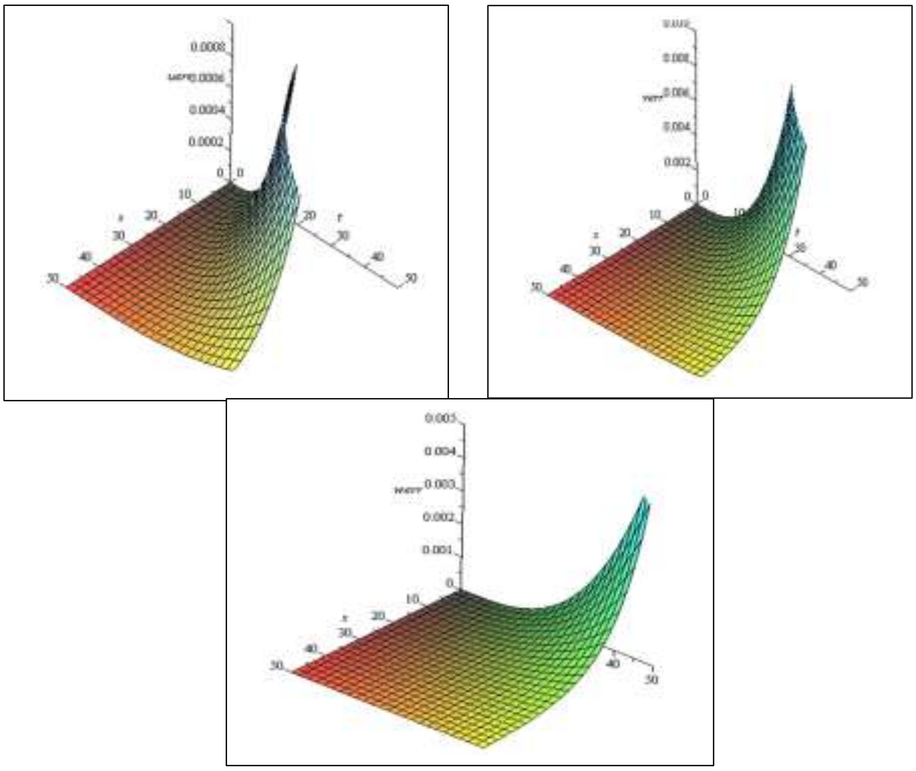

Figure 2: The absolute error for the first three approximation between MADM and the exact solution, for various values of x and t when

$b_0 = 0.1$, $k_1 = 0.05$, $k_2 = 0.1$, $k_3 = 0.3$, $and\ y = 50$

5. conclusions

The powerful modified adomian decomposition method was employed for analytic treatment of the nonlinear (2+1)- dimensional Wu-Zhang system , the MADN can be used directly and in a straight forward manner with rapid convergence successive approximations without any restrictive assumptions that may change in the physical behavior of the problem. Comparison at the MADM with several other methods that have been advanced for solving this system, shows that the new technique is reliable , powerful ,and promising as shown in the tables (1)-(6). The MADM provides analytic, verifiable, rapidly convergent approximation that yields insight into the character and the behavior of the solution just as in the closed form solution as shown in the figures (1) and (2). All the computations were carried out with the aid of maple 13 software.

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تطبيق طريقة تحليل ادوميان المحسنة لحل نظام Wu-Zhang غير الخطي ببعدين

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المستخلص

في هذا البحث، طريقة ادرمين المحسنة (MADM) استخدمت لحل نظام WU-Zhang غير الخطي ببعدين. نحن نقارب الحل للنظام مع طرائق MVIM و HPM وRDTM [15, 13, 17]، النتائج العددية التي حصلنا عليها هي ذات كفاءة عالية وملائمة ودقة كبيرة لحل انظمة المعادلات التفاضلية الجزئية. كذلك تمت مقارنة النتائج بين الطريقة المقترحة والطرائق الأخرى في بعض الأمثلة لإظهار كفاءة كل طريقة.