

## The superiority of fuzzy exponential family distributions in measuring the reliability of the machines

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### Abstract:

The study of the efficiency of the reliability of systems or productive systems in scientific life has a significant and important role in the scientific and technological development of these systems. This study deals with the superiority of mass fuzzy exponential distributions (exponential, wibble, kama, natural logarithmic) in measuring the reliability of the machines of the successive system.

The applied study included data taken from the Diesel Station north of Diwanayah, which is the time of failure and taken to a single block consisting of five machines operating respectively.

The most important results of the study are that the data follow the distributions of the mass fuzzy exponential and the best distribution is the natural distribution logarithmic as the owner of the highest reliability as it turns out that the machines decreased significantly and largely because of the lack of preventive maintenance of these machines because of lack of equipment backup materials as a result of the austerity policy as well as the poor fuel used in operation.

**Keywords:** Fuzzy reliability - Wibull distribution - fuzzy gama distribution - fuzzy groups

### Introduction

The Reliability has become an important aspect of the life and effectiveness of equipment and machinery. The interest increased during the Second World War and expanded in the recent years as a result of rapid developments and the use of electronic devices and complex systems. The time of this development has been increased interest in studying the causes of holidays that result in the cessation of machines and devices at different , the breakdown of these machines and devices leads to material losses as well as low production ,

The Parallel production systems consist of many production systems and the effectiveness of these systems can be determined by reliability. Validity suggests that the system is effective or ineffective.

The concept of reliability is the possibility of the ability of the machine or the device to complete the operations of non-failure (malfunction) either statistically, the reliability is the possibility that the machine or the device is working to accomplish a certain work for a period of time until the breakdown in this machine, Its occurrence is how long it will take to be repaired. Mistakes have an effect in the decision-making process to solve any problem. In practice, most of the problems faced by researchers may suffer from a lack of information or inaccuracy in the collection process. Adversely affect the solution of these problems.

The phenomenon that has the fuzzy character estimate the remaining time of the cancer patient misery of treatment for survival may determine the doctors a number of months and four months to stay alive and then will die life until we find that this patient may leave life before or after this time and hence the soul , The variable age of the patient is a misty variable, and other phenomena that have the fuzzy characteristics, the estimate of the operation period of the machine through the operating life of the machine and the expected time period in which the machine works without failure, such as working for two years, but this machine may fail before the period due to certain circumstances as high temperature or because of poor fuel and other factors that led to the breakdown of the machine ahead of time for breakdown, and here we find that estimate the function of the machine reliability and probably two years is hazy number.

**Research problem :**

The Diesel station north of Diwaniyah suffers from sudden stops at its engines, which have a negative impact on the production of electric power , as well as the lack of accuracy in times of breakdown and operation and we will study the fuzzy reliability .

**The Search's Goal:**

- 1- Calculating the reliability function of the system and the normal way of blurred failure times, which follow the exponential distributions of mass exponential and using the functions of belonging and non-trigonometric affiliation
- 2- Studying the distributions of mass fuzzy exponential and finding the preference of the exponential distributions of the Fuzzy groups in the calculation of the reliability of the successive system.

**The theoretical aspect :**

**1. Introduction:**

The reliability function [2] is defined as the probability that a vehicle will remain in a given system after time t, if our symbol of reliability function is:

$$R(t) = p(T > t) \quad t \geq 0$$

$$1 - \int_0^t f(t) dt \quad \dots\dots\dots (1)$$

As representing  $f(x)$  the probability density function (*p.d.f*) of the random variable reliability  $x_i$  any vehicle represents the vehicle's ability to continuity work to certain time periods without stop working before but in fact can be the vehicle to stop working before the time period identified here will show the type of uncertainty in Determination of the duration of the vehicle's suspension, which is due to the presence of fuzzy at the time of suspension of the vehicle and therefore the reliability to be estimated will be a fuzzy reliability.

This chapter deals with some of the basic concepts in the fuzzy groups and clarifies the concept of ambiguous reliability. It also includes the study of some distributions of fuzzy failure times (The Mass exponential )

**2 - Some Basic Concepts In Fuzzy Set**

The Fuzzy group theory is a generalization of classical group theory. The set group can include the classical group as a special case. The fuzzy group theory mathematical deals with form strict group theory to describe Fuzzy terminology in Fuzzy groups of linguistic modifiers to represent the disparity slightly in meaning, we find that the concept of the degrees of membership or concept of values of organic probabilistic we can get it in a simple as it can represent a membership of some elements in the overall groups and that this membership change from full membership to non-membership and either have full membership or ownership of membership or perhaps a partial membership, and thus any phrase is described as a mathematical function of a group of couples, each of which value .

There are some concepts of Fuzzy groups which we will discuss in detail as follows:

**2-1 Fuzzy Groups Fuzzy Set**

(( Zimmerman)) identified the Fuzzy groups as a set of elements that can be specific or non-specific so that each element that belongs to Group A and will be the degree of membership in one or non-specific to the group and the degree of membership of zero, and allows varying degrees between zero and one.

Thus, the group theory is characterized by the presence of the function of affiliation so that each element of the total group is associated with a number in the range [0,1] represents the achievement of that element of the characteristic which is trying to sub-group to represent it mathematically If we had a space that includes all the elements of a comprehensive group ,

$$X = \{x_i\} \quad i = 1, 2, \dots, n$$

$$A = \{x_i, \mu_A(x_i), x_i \in X, \mu_A(x_i) \in (0,1)\} \quad \dots\dots\dots (2)$$

A is called a blurred group and is called a function of belonging. We note that a function  $\mu_A(x_i)$  that is associated with  $x_i$  that represents the degree of belonging of that element to group A [8].

**2-2  $\alpha$ -cut**

Defined  $\alpha$  as the lowest degree of belonging owned by any element of the cloud group A and the value is within the closed period [0,1] [7]

**3-2 Membership Functions :**

The main motivations for the formation of fuzzy clusters are to deal with concepts of a hazy nature that cannot be categorically determined. Each group A is defined in terms of a comprehensive set X defined by a function called the membership function  $x \in X$  and symbolized by  $(\mu_A, R)$  Each element  $x \in X$  indicating a value in the closed period [0,1] characterizes the degree of membership of element X in A. The most famous of these formulas:

- **Triangular-Shape Membership Function [13]**  
 It is a function of three parameters (a, b, c) and its general form

$$\mu_{A(x,a,b,c)} = \begin{cases} 0 & \text{if } x < a \\ \frac{x-a}{b-a} & \text{if } a \leq x \leq b \\ \frac{c-x}{c-b} & \text{if } b \leq x \leq c \\ 0 & \text{if } x > c \end{cases} \quad (3)$$

- **Trapezoidal Function [13]**

Is a function that has four parameters (a, b, c, d) and can be expressed as follows :

$$\mu_A(x,a,b,c,d) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ 1 & b \leq x \leq c \\ \frac{d-x}{d-c} & c \leq x \leq d \\ 0 & x > d \end{cases} \quad (4)$$

- **Bell function [13]** is a nonlinear exponential function that expresses the natural curve shape and its formula:

$$\mu(x) = ce^{-\frac{(x-a)^2}{b}} \quad -\infty < x < \infty \quad (5)$$

There are other functions, such as the function of oversize and other functions.

The functions used in this research are the functions of the triangular member ship function non-member, ship Function and symbolizes these functions with the symbol and the variance in the form of two forms : [11: pp352]:

$$\mu_{A^{\sim}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2} & a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

$$v_{A^{\sim}}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1} & a_1 \leq x \leq a_2 \\ \frac{x-a_2}{a_3-a_2} & a_2 \leq x \leq a_3 \\ 1 & \text{otherwise} \end{cases} \quad (7)$$

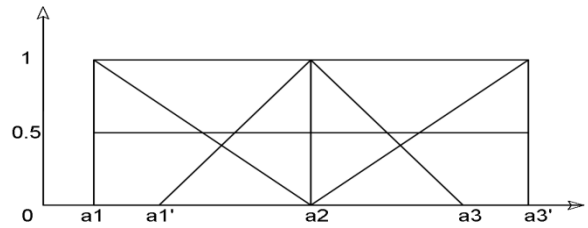


Figure (1): represents the function of belonging - not belonging to the fuzzy number (reference No. 11)

**3 -The Fuzzy Reliability**

The Fuzzy reliability is defined as follows:

The probability of working the vehicle with varying degrees of success and for a period of time t When calculating the reliability of any vehicle and for a specified period between t1 and t2, it is certain that the vehicle is working at time t1 but the vehicle may stop working before time t2 ie the time of stop t2 is something of uncertainty, For this reason, the value t2 is ambiguous and according to the group theory, the whole set of elements is fuzzy .

On this basis we will deal in this research with the concept of fuzzy reliability symbolized by the symbol  $\tilde{R}$  and expressed as follows:

Let  $\tilde{R}$  be part of a group A whereas A is a group of fuzzy event and if it expresses the performance of the vehicle to be operated and reflects the performance of a group of vehicles. From the definition of fuzzy conditional probability we have [15]:

$$P(A \cap A_i) = P(A) \cdot P(A_i | A) \quad (8)$$

By compensating equation (9) in equation (8) we get:

$$P(A) = R, \quad P(A \cap A_i) = R \quad (9)$$

And assuming that  $A_i^{\sim}$  it is the degree (R) in of belonging in and compensation  $\mu_{A^{\sim}}(R)$  in  $P(A_i^{\sim} | A)$  :

$$\tilde{R} = P(A_i^{\sim} | A) \cdot R \quad (10)$$

According to the definition of reliability we obtain:

$$\tilde{R} = R \cdot \mu_{A_i}(R) \quad (11)$$

In equation (8) we obtain:

$$R = \int_T^{\infty} f(t) dt \quad (12)$$

$$\tilde{R} = \int_T^{\infty} f(t) dt \cdot \mu_{A_i}(R) \quad (13)$$

We also assume that the random fuzzy variable represents the working time  $t$  of the system components. It is also assumed  $\mathcal{X}$  to have a distribution function  $f(x, \theta)$  and a cumulative distribution function  $F_x(t)$  since [4: pp1295-1298]

$$F_x(t) = P(X \leq t) \quad (14)$$

Depending on the natural cumulative distribution function, the mathematical uncertainty is defined as follows:

$$F_x(t) = P(T > t) = 1 - F_x(t) \quad (15)$$

$$= \{ [1 - F_{\max}(x)(\alpha)], [1 - F_{\min}(x)(\alpha)], [\mu_{F(x)(\alpha)}] \}$$

$\mathcal{X}$  : Normal random variable

$F_{\max}(x)(\alpha)$ : Is the cumulative distribution function of the upper limit at the cutting level- $\alpha$

$F_{\min}(x)(\alpha)$ : Is the cumulative distribution function of the minimum at the cutting level- $\alpha$

$F_{\min}(x)(\alpha)$ : The degree of belonging to the cumulative distribution function at the level of cutting- $\alpha$

#### 4- Fuzzy Reliability calculation of systems :

The components of any functional system are generally related to several forms, such as sequential correlation and parallel correlation, and there are double-installation systems. Each of these forms has special mathematical treatment, the results of which are reflected in the results of the total system

#### 4-2 Series System [5: pp74]

It consists of a set of components linked together in a way that can be represented in **Figure (2-2)**, which shows the connection of the system respectively, so that failure of one of the components causes the collapse of the entire system and that the success of the system depends on the success of the work of its components in this model to obtain the highest reliability so that The number of components is less than possible. If it is the uncertainty of the system of components, the reliability of the system  $R$  is calculated as follows:

$N$  represents the number of system dependencies  
 $\tilde{R}_S = \prod_{i=1}^n \tilde{R}_i \quad (17)$



Figure (2): Sequential scheme scheme (pp74: source 5)

#### 4.2 Parallel System [5: pp74]

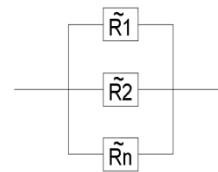
This system consists of a set of components that are related to each other so that the success of the system depends on the success of at least one component and lose the system function when the breakdown of all components at one time so the system's fuzzy reliability is:

$$\tilde{R}_P = 1 - \prod_{i=1}^n (1 - \tilde{R}_i) \quad (18)$$

As:

$\tilde{R}_i$  : Is the fuzzy reliability of the component  $i$

$n$  : The number of system dependencies The parallel system is represented as follows:



(16)

Figure 3: Parallel Link Scheme (PP75: Source 5)

#### 5 - Distributions of the exponential mass

This study included a study of a number of important distributions in measuring reliability, including:

#### 5.1 Fuzzy exponential Distributions

The exponential distribution is one of the most common distributions that is a special case of gamma distribution and is associated with some continuous distributions such as (Pareto Distribution). If the random variable  $\mathcal{X}$  has a Pareto distribution with a parameter  $\lambda$  the random variable  $z = \ln(x)$  is distributed in the exponential distribution  $\frac{1}{\lambda}$ . The risk function is a constant quantity, the probability density function for exponential distribution is as follows [16]:

$$f(t; \lambda) = \lambda e^{-\lambda t} \quad t > 0, \lambda > 0 \quad (19)$$

As  $\lambda$  it is the parameter of measurement for exponential distribution. If the distribution parameter is ambiguous then the probability density function will be defined as the following formula [10: pp75]:

$$f(t) = \tilde{\lambda} e^{-\tilde{\lambda} t} \quad , t > 0, \lambda > 0 \quad (20)$$

$$f(t; \tilde{\lambda}) = \{ f(t)[\alpha], \mu_{f(t)} | f(t)[\alpha], f_{\min}(t)[\alpha], f_{\max}(t)[\alpha], \mu_{f(t)} \} \quad (21)$$

$$f_{\min}(t)[\alpha] = \inf \{ f(t, \lambda)[\alpha] | \tilde{\lambda} \in \lambda[\alpha] \}$$

$$= \{ \lambda^L[\alpha] e^{-\lambda^L[\alpha](t)} \} \quad \dots \dots \quad (22)$$

$$f_{\max}(t)[\alpha] = \sup \{ f(t, \lambda)[\alpha] \mid \lambda \in \tilde{\lambda}[\alpha] \}$$

$$= \left\{ \lambda u[\alpha] e^{-\lambda u[\alpha](t)} \right\} \quad (23)$$

As:

$\tilde{\lambda}$  : Measurement parameter for exponential distribution

$f(\mathcal{X}, \tilde{\lambda})$  : Fuzzy probability density function

$\lambda^u[\alpha]$  : Measurement parameter for minimum exponential distribution

$\lambda^l[\alpha]$  : Measurement parameter for upper limit of exponential distribution

The average is:

$$E(t) = \left\{ \frac{1}{\lambda} \mid \lambda \in \tilde{\lambda}[\alpha] \right\} \quad (24)$$

The distribution variation is:

$$\text{var}(t) = \left\{ \frac{1}{\lambda^2} \mid \lambda \in \tilde{\lambda}[\alpha] \right\} \quad (25)$$

The aggregate function (C.D.F) is

$$F(t; \tilde{\lambda}) = \left\{ F(t)[\alpha], \mu_{F(t)} \mid F(t)[\alpha] = F_{\min}(t)[\alpha], F_{\max}(t)[\alpha], \mu_{F(t)} = \alpha \right\}$$

$$F_{\min}(t)[\alpha] = \inf \left\{ F(t, \lambda)[\alpha] \mid \lambda \in \lambda[\alpha] \right\}$$

$$= 1 - e^{-\lambda L[\alpha]} \quad (26)$$

$$F_{\max}(t)[\alpha] = \sup \left\{ F(t, \lambda)[\alpha] \mid \lambda \in \tilde{\lambda} \right\}$$

$$= 1 - e^{-\lambda^u[\alpha]} \quad (27)$$

From (25) and (26) we obtain :

$$F(t, \lambda) = \left[ e^{-\lambda L[\alpha]t}, e^{-\lambda^u[\alpha]t} \right] \quad (28)$$

The distributive reliability function of the distribution is calculated as follows [10: pp76]:

$$R_{\min}(t)[\alpha] = 1 - \sup \left\{ F(t, \lambda)[\alpha] \mid \lambda \in \tilde{\lambda}[\alpha] \right\}$$

$$= e^{-\lambda^u[\alpha]t} \quad (29)$$

$$R_{\max}(t)[\alpha] = 1 - \inf \left\{ F(t, \lambda)[\alpha] \mid \lambda \in \tilde{\lambda}[\alpha] \right\}$$

$$= e^{-\lambda L[\alpha]t} \quad (30)$$

From (29) and (30) we get:

$$R(t, \tilde{\lambda}) = \left[ e^{-\lambda L[\alpha]t}, e^{-\lambda^u[\alpha]t} \right] \quad (31)$$

## 5.2 – Fuzzy Log-Normal Distribution<sup>[6][1]</sup>

This distribution is characterized by its close relationship to one of the most important statistical distributions and widespread, which is the normal distribution and can clarify the relationship as follows : If a random variable  $X$  has a normal distribution with parameters  $X \sim N(\mu, \sigma)$ , a random variable  $\chi$  has the natural logarithmic distribution with the same parameters. Note that  $y = e^{-x}$  the new parameters do not represent the mean and variance of the variable. The logarithmic distribution has the following probability density function:

$$f(t; \mu, \sigma^2) = \frac{1}{t\sqrt{2\pi\sigma^2}} e^{-\left(\frac{\ln(t)-\mu}{\sigma}\right)^2}; t > 0, -\infty < \mu < \infty, \sigma^2 > 0 \quad (32)$$

If the time of life follows the lognormal distribution of the two parameters  $\mu$  and  $\sigma$ , then exponential function potential is known :

$$f(x, \tilde{\mu}, \tilde{\sigma}) = \{ f(x)[\alpha], \mu f(x) \mid f(x)[\alpha], [f_{\min}(x)[\alpha], f_{\max}(x)[\alpha], \mu f(x) = \alpha] \}$$

Throughout this function we could get (C.D.F)

$$F(t, \tilde{\mu}, \tilde{\sigma}) = \{ F(t)[\alpha], \mu F(t) \mid F(t)[\alpha] = [F_{\min}(t)[\alpha], F_{\max}(t)[\alpha], \mu F(t) = \alpha] \}$$

$$= \left\{ \Phi\left(\frac{\log(t) - \mu^u[\alpha]}{\sigma^u[\alpha]}\right), \Phi\left(\frac{\log(t) - \mu^L[\alpha]}{\sigma^L[\alpha]}\right) \right\} \quad (33)$$

$\mu^u$  = The expected minimum for failure time .

$\mu^L$  = The expected maximum for failure time.

$\sigma^u$  = The standard deviation to the minimum for failure time .

$\sigma^L$  = The standard deviation to the maximum for failure time

By using function ( C.D.F) fuzzy we could get the fuzzy reliability as follows :

$$R(t)[\alpha] = 1 - F(t, \mu, \sigma)$$

$$= \left\{ 1 - \Phi\left(\frac{\log(t) - \mu^L}{\sigma^L}\right), 1 - \Phi\left(\frac{\log(t) - \mu^u}{\sigma^u}\right) \right\} \quad (34)$$

And its average

$$E(t) = \left\{ e^{\mu + \frac{1}{2}\sigma^2} / \mu \in \mu[\alpha], \sigma^2 \in \sigma[\alpha] \right\} \quad (35)$$

And contrast:

$$\text{var}(T) = \left\{ [e^{\sigma^2} - 1] [e^{2\mu + \sigma^2}] / \mu \in \bar{\mu}[\alpha], \sigma^2 \in \bar{\sigma}[\alpha] \right\}$$

Using the CDF function. We can calculate the reliability function of fuzzy coli:

$$\tilde{R}(t)[\alpha] = \left\{ 1 - \tilde{F}(t, \mu, \sigma) \right\} = \left\{ 1 - \phi\left(\frac{\log t - \mu^L}{\sigma^L}\right), 1 - \phi\left(\frac{\log t - \mu^U}{\sigma^U}\right) \right\}$$

### 5-3(fuzzy Weibull distribution)

It is the most useful distribution of reliability analyzes by controlling the distribution parameters that enable us to make a fit for the distribution of( times) or (ages) . The Weibull distribution of 1951 was used by the researcher Wallodi Weibull for the experimental demonstration of changes in expansion Iron and also used to express the period of service spent by radio personnel. This distribution can be shortened to the exponential distribution when the shape parameter is equal to one and Wipple's distribution has an increasing failure rate when the shape parameter is greater than one and has a decreasing failure rate when the shape parameter is less than one.

The Variable Weibull  $\omega = \alpha, \beta$  and in which field  $0 < x < \infty$  its measurement parameter  $\theta > 0$  an form  $\beta > 0$  Therefore, the probability density function p.d.f is as follows [3: pp200]:

$$f(t) = (\beta t^{\beta-1} / \theta^\beta) e^{-\left(\frac{t}{\theta}\right)^\beta} \quad t > 0; \beta, \theta > 0 \quad (38)$$

In case the distribution parameters are fuzzy then the probability density function will be defined as [9: pp82]:

$$f(x, \tilde{\theta}) = \left\{ \frac{\beta}{\theta[\alpha]} \left(\frac{x}{\theta[\alpha]}\right)^{\beta-1} e^{-\left(\frac{x}{\theta[\alpha]}\right)^\beta} [\alpha], \mu f(x) \mid f(x)[\alpha] \right\} \quad (39)$$

Using this function we can obtain the aggregate function C.D.F:

$$F(t, \tilde{\theta}) = \{ F(t)[\alpha], \mu_{f(t)} \mid F(x)[\alpha] = [F_{\min}(t)[\alpha], F_{\max}(t)[\alpha]], \mu F(t) = \alpha \}$$

$$= \left\{ 1 - e^{-\left(\frac{t}{\theta^L[\alpha]}\right)^\beta}, 1 - e^{-\left(\frac{t}{\theta^U[\alpha]}\right)^\beta} \right\} \quad (40)$$

Depending on the CD.F mist function, we calculate the uncertainty reliability function for the failure times on the following formula:

$$\tilde{R}(t)[\alpha] = \left\{ e^{-\left(\frac{t}{\theta^L[\alpha]}\right)^\beta}, e^{-\left(\frac{t}{\theta^U[\alpha]}\right)^\beta} \right\} \quad (41)$$

and its average

$$\left| (\theta \Gamma(1 + \beta^{-1}) \mid \theta \in \tilde{\theta}[\alpha]) \right. \quad (37) \quad (42)$$

Contrast :

$$\left| \theta^2 [\Gamma(\beta+1) / \beta] - [\Gamma\{(\beta+1) / \beta\}]^2 \mid \theta \in \tilde{\theta}[\alpha] \right. \quad (43)$$

### 5-4 (Fuzzy Gama distribution)

One of the most important Gamma distributions used in the field of reliability which is often used as a model for the distribution of failure times in electrical, mechanical, and electromechanical systems. This distribution has the following probability density function [2]:

$$f(t, \lambda, r) = \frac{\lambda^r t^{(r-1)} e^{-\lambda t}}{\Gamma(r)}, \quad t \geq 0 \quad (44)$$

If the distribution parameters are fuzzy , then the density function p.d.f will be defined as follows

$$\tilde{f}(t, \lambda, r) = \frac{\lambda^r t^{(r-1)} e^{-\lambda t}}{\Gamma(r)}, \quad t \geq 0, \lambda \in \tilde{\lambda}[\alpha], r \in \tilde{r}[\alpha] \quad (45)$$

As :

$\lambda$  Shape parameter

r Represents the measurement parameter

$\Gamma(r)$  gamma function

It could be identify the Gama Function as follows :

$$\Gamma(r) = \int_0^\infty t^{(r-1)} e^{-t} dt \quad (46)$$

This function is characterized by [12]:

1- Each  $r > 1$  is  $\Gamma(r) = (r-1) \cdot \Gamma(r-1)$

2-for each correct number(n) is  $\Gamma(n) = (n-1)!$

3-  $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$



For the blended aggregate function of the Cama D.F distribution, it is defined as:

$$F(t, \bar{r}, \bar{\lambda}) = \left\{ \frac{1}{\Gamma(r)} (r, \lambda t) \mid r \in \bar{r}[\alpha], \lambda \in \bar{\lambda}[\alpha] \right\} \quad (47)$$

And distributional mean :

$$E(t) = \left\{ \frac{r}{\lambda} \mid \lambda \in \bar{\lambda}[\alpha], r \in \bar{r}[\alpha] \right\} \quad (48)$$

The contrast

$$\text{var}(t) = \left\{ \frac{r}{\lambda^2} \mid \lambda \in \bar{\lambda}[\alpha], r \in \bar{r}[\alpha] \right\} \quad (49)$$

The fuzzy reliability function of the fuzzy Gama distribution [4]:

$$\bar{R}(t) = \frac{1}{\Gamma(r)} \Gamma(r, \lambda t) \mid \lambda \in \bar{\lambda}[\alpha], r \in \bar{r}[\alpha] \quad (50)$$

It is defined at the  $\alpha$ -level :

$$\bar{R}(t)[\alpha] = [R_1[\alpha], R_2[\alpha]] \quad (51)$$

as :

$$R_1[\alpha] = \min \frac{1}{\Gamma(r)} \Gamma(r, \lambda t) \mid \lambda \in \bar{\lambda}[\alpha], r \in \bar{r}[\alpha] \quad (52)$$

$$R_2[\alpha] = \max \frac{1}{\Gamma(r)} \Gamma(r, \lambda t) \mid \lambda \in \bar{\lambda}[\alpha], r \in \bar{r}[\alpha] \quad (53)$$

Applied side :  
 1. Introduction :

Due to the importance of the location and population density of the Middle Euphrates region and the urgent need for electric power, a diesel station was established in Diwanayah in 2012 with a design card of 196MW. It consists of 48 engines (generating unit with a capacity of 4.02 MW) distributed over 8 blocks and each block consists of 6 motors connected in parallel with auxiliary devices Fuel purifiers, oil purifiers, water treatment unit, oil heaters, boilers, air compressors and fuel tanks of both types (Diesel and HFo oil). It was initially operating on the fuel oil (Diesel) and then the automatic

conversion of the operation of black oil for the low costs and economic feasibility.

2. Data collection stage :

Data on working times were collected from the planning and maintenance department of one block which consists of six motors connected in parallel for a period of five months from 1/1/2015 to 1/6/2015. All non mechanical and non-electrical stops were excluded. In some cases, the machine may be disrupted and then return to work on the same day before issuing the order for that fault, thus producing inaccuracies. In running the holidays, we are told that the holidays are maintenance times.

The following table shows the times of holidays and one block.

|   | Eng.1 | Eng.2 | Eng.3 | Eng.4 | Eng.5 |
|---|-------|-------|-------|-------|-------|
| 1 | 32~   | 56~   | 26~   | 17~   | 29~   |
| 2 | 105~  | 13~   | 42~   | 22~   | 26~   |
| 3 | 42~   | 83~   | 62~   | 42~   | 22~   |
| 4 | 85~   | 61~   | 67~   | 170~  | 52~   |
| 5 | 56~   | 326~  | 13~   | 88~   | 87~   |

Table (1) shows the breakdown times for the five machines

By collecting data and information on the production machines at the diesel station north of Diwanayah, the stopping hours were obtained. As shown in Table (4-1) we arranged these data upward and then calculated the function of belonging to these times as shown below and according to the formulas (6, 7)

13, 22, 26, 29, 32, 45, 52, 56, 61, 62, 67, 85, 87, 88, 105, 170, 326

$$\mu_{\Gamma_3}(t) = \left\{ \begin{array}{ll} \frac{t-3}{13-3} & 3 \leq t \leq 13 \\ \frac{23-t}{23-13} & 13 \leq t \leq 23 \end{array} \right\}$$

$$\nu_{\Gamma_3}(t) = \left\{ \begin{array}{ll} \frac{13-t}{13-3} & 3 \leq t \leq 13 \\ \frac{t-13}{23-13} & 13 \leq t \leq 23 \end{array} \right\}$$

In order to obtain the fuzzy reliability is extracted and for fuzzy numbers as shown below:

$$13\alpha = 3 + 10\alpha \quad , \quad 23 - 10\alpha \quad \quad 13\beta = 13 - 10\beta \quad , \quad 13 + 10\beta$$

$$22\alpha = 12 + 10\alpha \quad , \quad 32 - 10\alpha \quad \quad 22\beta = 22 - 10\beta \quad , \quad 13 + 10\beta$$

$$26\alpha = 16 + 10\alpha + 36 - 10\alpha \quad \quad 26\beta = 26 - 10\beta \quad , \quad 13 + 10\beta$$

Thus, the function of fuzzy reliability is measured by using the trigonometric function and the non-affiliation function, and by the type of distribution the data takes

\* Measurement of the fuzzy reliability of the failure times that follow the distal exponential mass distributions

• If the failure times are followed for the exponential distribution, we apply the formula (41) mentioned in the theoretical side and using the function of belonging and non-trigonometric affiliation

• If follow the best time to distribute and apply the formula (56) mentioned in the theoretical part

Using the function of belonging and non-trigonometry

• If the failure times are followed for the distribution of Gama, the formula (65) mentioned in the theoretical aspect is applied

Using the function of belonging and non-trigonometry

• If the failure times are followed for natural logarithmic distribution, the formula (47) mentioned in the theoretical side shall be applied using the function of belonging and non-trigonometric affiliation. In applying the above formulas,

1- The reliability of the system is significantly reduced as it does not return again because of the lack of preventive maintenance on these machines in order to increase their production capacity

2 – The fluctuation of the values of reliability uncertainty at different levels of the values of  $\alpha$  as the value of the value increases the value  $\alpha$  of the system

3 - It has been observed that the natural logarithmic distribution is the best in the measurement of the fuzzy dependence of the system because it is the owner of the highest priority.

**Table (2): The values of uncertainty for the failure times that follow the exponential distribution and using the trigonometric function**

| $\alpha$ | 0.1       |           | 0.2       |           | 0.3       |           | 0.4       |           | 0.5       |           |
|----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
|          | MIN       | MAX       | MIN       | MAX       | MIN       | MAX       | MIN       | MAX       | MIN       | MAX       |
| 1        | 0.9953251 | 0.9999356 | 0.9959653 | 0.9998836 | 0.9965431 | 0.9998085 | 0.9970616 | 0.9997053 | 0.9975239 | 0.9995686 |
| 2        | 0.9720468 | 0.9928779 | 0.9742092 | 0.9916701 | 0.9762698 | 0.99034   | 0.9782294 | 0.9888837 | 0.9800885 | 0.9872979 |
| 3        | 0.9505615 | 0.9795584 | 0.9535541 | 0.977261  | 0.9564473 | 0.9748222 | 0.9592405 | 0.9722414 | 0.9619331 | 0.9695182 |
| 4        | 0.8335587 | 0.8674438 | 0.8387506 | 0.8616576 | 0.8438936 | 0.8557929 | 0.8489854 | 0.8498538 | 0.8540237 | 0.8438443 |
| 5        | 0.5208156 | 0.5193613 | 0.5274864 | 0.5111798 | 0.5342103 | 0.5030882 | 0.5409864 | 0.4950874 | 0.5478137 | 0.4871786 |



**Table (3): The values of the uncertainty of the failure times that follow the exponential distribution and the non-trigonometric function**

| a | 0.1        |            | 0.2        |            | 0.3        |            | 0.4        |            | 0.5        |            |
|---|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
|   | MIN        | MAX        | MIN        | MAX        | MIN        | MAX        | MIN        | MAX        | MIN        | MAX        |
| 1 | 0.1741521  | 0.1732936  | 0.1639001  | 0.1868645  | 0.1542517  | 0.2014982  | 0.1451712  | 0.2172779  | 0.1366252  | 0.2342933  |
| 2 | 0.05908045 | 0.04874455 | 0.0556025  | 0.05256183 | 0.0523293  | 0.05667803 | 0.04924878 | 0.06111659 | 0.0463496  | 0.06590274 |
| 3 | 0.02890183 | 0.02038604 | 0.02720044 | 0.02198251 | 0.0255992  | 0.023704   | 0.02409223 | 0.0255603  | 0.02267397 | 0.02756197 |
| 4 | 0.00347958 | 0.00161294 | 0.00327474 | 0.00173925 | 0.00308197 | 0.00187546 | 0.00290054 | 0.00202233 | 0.00272979 | 0.0021807  |
| 5 | 4.5165E-05 | 1.1069E-05 | 4.2506E-05 | 1.1935E-05 | 4.0004E-05 | 1.287E-05  | 3.7649E-05 | 1.3878E-05 | 3.5433E-05 | 1.4965E-05 |

**( Table 5)The values of uncertainty are shown for the failure times that follow the Weppel distribution and using the function of trigonometry**

| a | 0.1       |           | 0.2       |           | 0.3       |           | 0.4       |           | 0.5       |           |
|---|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
|   | MIN       | MAX       | MIN       | MAX       | MIN       | MAX       | MIN       | MAX       | MIN       | MAX       |
| 1 | 0.9999223 | 0.9999997 | 0.9999482 | 0.9999992 | 0.9999662 | 0.9999998 | 0.9999785 | 0.9999957 | 0.9999913 | 0.9999867 |
| 2 | 0.9964003 | 0.9994885 | 0.9971235 | 0.9992819 | 0.997726  | 0.9990131 | 0.9982225 | 0.9996695 | 0.9982376 | 0.9996273 |
| 3 | 0.9825484 | 0.9949086 | 0.9849966 | 0.9937068 | 0.9871884 | 0.9923095 | 0.9891377 | 0.9907004 | 0.9888638 | 0.9908594 |
| 4 | 0.7956014 | 0.8603317 | 0.8057959 | 0.8512236 | 0.8158759 | 0.8419315 | 0.8258164 | 0.8324728 | 0.835604  | 0.8228651 |
| 5 | 0.2607925 | 0.3729729 | 0.2715716 | 0.3608502 | 0.2823301 | 0.3490501 | 0.2956722 | 0.3375728 | 0.305401  | 0.3264178 |

(Table 6) shows the values of the fuzzy reliability of the failure times that follow the Weppel distribution and the use of the trigonometric function

| n | 0.1       |           | 0.2       |           | 0.3       |           | 0.4       |           | 0.5       |           |
|---|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
|   | MIN       | MAX       | MIN       | MAX       | MIN       | MAX       | MIN       | MAX       | MIN       | MAX       |
| 1 | 0.9998868 | 0.9999993 | 0.9999184 | 0.9999983 | 0.999942  | 0.9999964 | 0.9999595 | 0.9999929 | 0.999987  | 0.9999722 |
| 2 | 0.9967806 | 0.9994327 | 0.9973647 | 0.9992366 | 0.9978583 | 0.9998889 | 0.9982726 | 0.99868   | 0.9982996 | 0.9986177 |
| 3 | 0.9846364 | 0.9949633 | 0.9866952 | 0.9938836 | 0.9885369 | 0.9926373 | 0.9901759 | 0.9912094 | 0.9898849 | 0.9916269 |
| 4 | 0.8099397 | 0.8697895 | 0.8204215 | 0.8606205 | 0.8307096 | 0.8511961 | 0.8407819 | 0.8415339 | 0.8316524 | 0.8506171 |
| 5 | 0.2404458 | 0.3683794 | 0.2531115 | 0.3540432 | 0.2663218 | 0.2663218 | 0.2800744 | 0.3265693 | 0.2943642 | 0.3134465 |

Table (7): The values for the uncertainty of the failure times that follow the distribution of Gama and the use of the trigonometric function

| n | 0.1       |           | 0.2       |           | 0.3       |           | 0.4       |           | 0.5       |           |
|---|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
|   | MIN       | MAX       | MIN       | MAX       | MIN       | MAX       | MIN       | MAX       | MIN       | MAX       |
| 1 | 0.9999714 | 0.9999976 | 0.9999765 | 0.9999962 | 0.999982  | 0.999994  | 0.9999871 | 0.9999909 | 0.9999913 | 0.9999867 |
| 2 | 0.9958357 | 0.9995185 | 0.9965732 | 0.9993603 | 0.9972157 | 0.999163  | 0.9976834 | 0.9999208 | 0.9982376 | 0.9986273 |
| 3 | 0.9810243 | 0.995294  | 0.9831962 | 0.9943688 | 0.9852301 | 0.9933243 | 0.9871205 | 0.9921557 | 0.9888638 | 0.9908594 |
| 4 | 0.7971126 | 0.8632029 | 0.8032841 | 0.8561954 | 0.8096437 | 0.8492439 | 0.8161762 | 0.8423724 | 0.8228651 | 0.8356604 |
| 5 | 0.2876003 | 0.3559262 | 0.2968061 | 0.3411782 | 0.3063416 | 0.3288409 | 0.316211  | 0.3169154 | 0.3054178 | 0.326401  |

**Table (8): The values of the uncertainty of the failure times that follow the natural distribution of logarithmic and using the trigonometric function.**

| p | 0.1       |           | 0.2       |           | 0.3       |           | 0.4       |           | 0.5       |           |
|---|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
|   | MIN       | MAX       | MIN       | MAX       | MIN       | MAX       | MIN       | MAX       | MIN       | MAX       |
| 1 | 0.9999900 | 0.9999904 | 0.9999907 | 0.9999900 | 0.9999924 | 0.9999979 | 0.9999946 | 0.9999964 | 0.9999966 | 0.9999940 |
| 2 | 0.9929952 | 0.9994304 | 0.9942914 | 0.9992063 | 0.9954255 | 0.999919  | 0.9964015 | 0.9985582 | 0.9972264 | 0.9981138 |
| 3 | 0.9703485 | 0.993583  | 0.9737067 | 0.9922001 | 0.9768976 | 0.9906374 | 0.9799025 | 0.9888919 | 0.9827052 | 0.9869634 |
| 4 | 0.7543248 | 0.8259274 | 0.7604414 | 0.8181854 | 0.7668029 | 0.8105954 | 0.7734009 | 0.8031789 | 0.7802255 | 0.7959561 |
| 5 | 0.3525427 | 0.3813772 | 0.3597718 | 0.37001   | 0.3672213 | 0.3605663 | 0.3748959 | 0.3506424 | 0.3828002 | 0.341034  |

**Table (9): The values of fuzzy reliability of the failure times that follow the natural logarithmic distribution and using the trigonometric function**

| p | 0.1       |           | 0.2       |           | 0.3       |           | 0.4       |           | 0.5       |           |
|---|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
|   | MIN       | MAX       | MIN       | MAX       | MIN       | MAX       | MIN       | MAX       | MIN       | MAX       |
| 1 | 0.9999432 | 1         | 0.9999659 | 0.999999  | 0.9999004 | 0.999996  | 0.9998991 | 0.9999988 | 0.9999943 | 0.9999966 |
| 2 | 0.994669  | 0.9993216 | 0.9957954 | 0.99991   | 0.9967304 | 0.9985471 | 0.997496  | 0.9979669 | 0.9981138 | 0.9972264 |
| 3 | 0.9748288 | 0.9924294 | 0.9783675 | 0.9904815 | 0.9815617 | 0.9882218 | 0.9844225 | 0.9856342 | 0.9869634 | 0.9827052 |
| 4 | 0.7536585 | 0.8194801 | 0.764285  | 0.8097317 | 0.7748882 | 0.7999293 | 0.7854512 | 0.7900889 | 0.7859561 | 0.7802255 |
| 5 | 0.3038869 | 0.4186169 | 0.3125287 | 0.4093174 | 0.3217419 | 0.4002505 | 0.331242  | 0.3914127 | 0.341034  | 0.3828002 |

**Conclusions :**

1- The results obtained in the applied side in the measurement of fuzzy reliability showed that the logarithmic distribution is the best distribution among distributions. The exponential mass is the distribution with the highest reliability

2 - Due to the results obtained in the application side, the function of fuzzy reliability decreases with time as well as the holidays which increase with time due to the lack of spare materials and the equipment needed to maintain these engines for its durability.

3- In this study, the study of the applied side showed the fluctuation of the value of the reliability function and the mean time by the difference of the value of alpha .

4 - It has been concluded that the production machines suffer from frequent delays by observing the rate of holidays during the five months as a result of the lack of preventive maintenance of these engines because of the lack of equipment backup materials, which leads to a shortage of capacity of these machines .

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## افضلية توزيعات العائلة الاسية الضبابية في قياس معولية المكانن

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المستخلص :

يدرس البحث قياس المعولية الضبابية كما تناول ايضا دراسة افضلية توزيعات العائلة الاسية الضبابية (الاسي، ويبيل، كاما، الطبيعي اللوغارتمي) في قياس معولية النظام المتتالي.

وقد اشتملت الدراسة التطبيقية على بيانات تم اخذها من محطة ديزل شمال الديوانية وهي عبارة عن اوقات الفشل والمأخوذة لبلوك واحد مكون من خمس مكانن تعمل على التوالي.

وتمثلت اهم نتائج الدراسة ان التوزيع الطبيعي اللوغارتمي هو افضل توزيع من بين توزيعات العائلة الاسية الضبابية، اذ يُعد التوزيع هو التوزيع صاحب اعلى معولية. وان الماكينات انخفضت معوليتها وبشكل كبير مع تزايد معدل العطل بسبب عدم اجراء ادامة لهذه الماكينات بسبب قلة التجهيزات بالمواد الاحتياطية نتيجة لسياسة التقشف المتبعة بالإضافة الى رداءة الوقود المستعمل في التشغيل.