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On the(G. n)_ Tupled fixed point theorems in fuzzy metric spacea

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Abstract

The porpose of this paper is to introduce a new concepts of (G. n)-tupled fixed point and (G. n)- tupled coincidence point. And, to study the existence of tupled fixed (coincidence) point for any type of mappings. We will also establish some convergence theorems to a unique (G. n) – tupled fixed (coincidence) point in the complete fuzzy metric spaces.

Keywords: fuzzy metric spaces, continuous t – norm , fixed point, upper semi – continuous, equicontinuous.

Subject classification: 46S40 .

1. Introduction

In [1] , Kamosil and Mchalek introduced the concept of fuzzy metric spaces(F. M. S). the existence of fixed points for mappings in fuzzy metric spaces studied by Gregri and Sapea
[2], Mihet [3].The Fixed point theory for contractive mappings in fuzzy metric spaces is associated the fixed point theory for the same type of mappings in probabilistic metric space of menger type see, Qlu and Hong[4], Hong and Peng[5], Mohudine and Alotibietal [6], Wang [7], Hong [8], Sadtietal [9] ,[10] and many others. Zhand and Xiao[11] and Hu[12] introduced a coupled fixed point theorem for. In this paper, we introduce the concepts of (G. n) – tupled fixed (coincidence) point and we establish the (G. n)– tupled fixed (coincidence) point theorems in fuzzy metric spac
es .

Now, we recall the following:

Definition (1.1) [3]

A binary operation $\aleph : [0,1]^2 \rightarrow [0,1]$ is called a continuous t– norm if the following conditions are satisfy:

i. $\aleph$ is a associative and commutative.
ii. $a\aleph 1 = a \quad \forall a \in [0,1]$.
iii. $a\aleph b \le c\aleph d$ whenever $a \le c$ & $b \le d$, $\forall\, a, b, c, d \in [0,1]$.
iv. $\aleph$ is continuous.

And denoted by (c. t. n)

Definition (1.2)[4]

A triple $(F, \mathcal{G}, \aleph)$ is called fuzzy metric space (F. M. S) if $X \neq \emptyset$, $\aleph$ is continuous t - norm and $\mathcal{G}: F \times F \times (0, \infty) \rightarrow$ [0,1] is a fuzzy set the satisfying the following conditions.

i. $\mathcal{G}_{(x,y,t)} > 0$
ii. $\mathcal{G}_{(x,y,t)} = 1 \ iff \ x = y$
iii. $\mathcal{G}_{(x,y,t)} = \mathcal{G}_{(y,x,t)}$
iv. $\mathcal{G}_{(x,y,.)}: (0, \infty) \rightarrow [0,1]$ is continuous.
v. $\mathcal{G}_{(x,z,t+s)} \ge \mathcal{G}_{(x,y,t)} \aleph \mathcal{G}_{(y,z,s)} \ \forall\, t, s > 0$."

Now, we will add the condition
$\lim_{t\rightarrow\infty} \mathcal{G}_{(x,y,t)} = 1 \quad \forall x, y \in F$.

Lemma (1.3) [3]

In any fuzzy metric space $(F, \mathcal{G}, \aleph)$, where $\aleph$ is (c. t. n)If there exits Δ∈ ℃ such that $\mathcal{G}_{(x,y,\emptyset(t))} \le \mathcal{G}_{(x,y,t)}$, $\forall\, t > 0$ then $x = y$.

Definition(1.4) [9]

For any $v \in [0,1]$, the sequence $< \aleph^n v >^{\infty}{}_{n=1}$ be defined by:
$\aleph^1 v = v \ and \ \aleph^n v = (\aleph^{n-1} v)\aleph v$. Then a t – norm $\aleph$ is said to be (c. t. n)of $\mathcal{H} - \mathcal{T}$ype if the sequence $< \aleph^n v >^{\infty}{}_{n=1}$ is equicontinuous at $v = 1$.

Definition (1.5) [13]

Let $(F, \mathcal{G}, \aleph)$ be a(F. M. S) then

i. A sequence in (v_n) in X is said to be convergent to a point $v \in X$ if $\lim_{t\rightarrow\infty} \mathcal{G}_{(v_n,v,t)} = 1$ for all $t > 0$.
ii. A sequence in (v_n) in X is called a Cauchy sequence if for each $0 < \varepsilon < 1 \ and \ t > 0$,there exists a positive integer n_0 such that $\mathcal{G}_{x_n,v_m,t)} > 1 - \varepsilon$ for each $n, m \ge n_0$."

Now we will give the concept of $(G. n)$_ tupled fixed(coincidence) point.

Definition (1.6)

Let $Z_1, Z_2, \dots \dots, Z_n: F^n \rightarrow F$ are mappings. Any element $(x_1, x_2, \dots \dots, x_n) \in F^n$ is called a $(G. n)$_ tupled fixed point of this mappings if

$$Z_1\left(\left(Z_2(\dots\dots(Z_{n(\ x_1, x_2, \dots, x_n\ )})\dots..)\right)\right) = x_1$$
$$Z_1\left(\left(Z_2(\dots\dots(Z_{n\ (x_2, x_3, \dots, x_1)}\ )\dots..)\right)\right) = x_2$$
$$\vdots$$
$$Z_1\left(\left(Z_2(\dots\dots(Z_{n(\ \ x_n, x_1, \dots, x_{n-1}\ )})\dots..)\right)\right) = x_n.$$

Definition (1.7)

Let $Z_1, Z_2, \dots \dots, Z_n: F^n \rightarrow F$ and $E_1, E_2, \dots \dots$, $E_n: F \rightarrow F$ are mappings. Any element $(x_1, x_2, \dots \dots, x_n) \in F^n$ is called $(G. n)$ – tupled coincidence point of this mapping if

$$Z_1\left(Z_2\left(\dots\dots\left(Z_{n(x_1,x_2,\dots,x_n)}\right)\dots..\right)\right) = E_1\left(E_2\left(\dots\dots\left(E_{n(x_1)}\right)\dots..\right)\right)$$
$$Z_1\left(Z_2\left(\dots\dots\left(Z_{n(x_2,x_3,\dots,x_1)}\right)\dots..\right)\right) = E_1\left(E_2\left(\dots\dots\left(E_{n(x_2)}\right)\dots..\right)\right)$$
$$\vdots$$
$$\mathcal{F}_1\left(\mathcal{F}_2\left(\dots\dots\left(\mathcal{F}_{n(x_n,x_1,\dots,x_{n-1})}\right)\dots..\right)\right) = \mathcal{P}_1\left(\mathcal{P}_2\left(\dots\dots\left(\mathcal{P}_{n(x_n)}\right)\dots..\right)\right).$$

In this paper ,we consider ℃ is the set of all functions $\Delta: [0,\infty) \to [0,\infty)$ such that:
Δ is increasing function.
Δ is upper semi – continuous .
$\sum_{n=0}^{\infty} \Delta^n{}_{(t)} < \infty$; $\forall t > 0$ where $\Delta^{n+1}{}_{(t)} = \Delta(\Delta^n{}_{(t)}),\ n \in N$.

2.Main Results

Now, we establish the convergence theorems to a unique $(G.n)$ – tupled fixed point as follows :

Theorem (2.1): Let $Z_1, Z_2, \dots\dots, Z_n: F^n \to F$ and let $(F, \mathcal{G}, \aleph)$ be a complete (F.M.S) such that $\aleph$ (c.t.n) of $\mathcal{H}-\mathcal{T}$ype Suppose that $\Delta \in$ ℃ satisfying:

$$\mathcal{G}[Z_1((Z_2(\dots\dots(Z)\dots..))),\ Z_1((Z_2(\dots\dots(Z)\dots..))), \Delta_{(t)}] \geq \mathcal{G}[x_1, y_1, t]\aleph\mathcal{G}[x_2, y_2, t]\aleph\dots\dots\aleph\mathcal{G}[x_n, y_n, t] \quad (2.1)$$

where $t > 0$ and $x_i, y_i \in F$, $\forall i = 1,2,\dots\dots,n$

If F containing $Z_1((Z_2(\dots\dots(Z_{n(x^n)})\dots..)))$.
Then there exists a unique $(G.n)$ – tupled fixed point of compose these mappings.

Proof:

Suppose that $x_0{}^1, x_0{}^2, \dots\dots, x_1{}^n \in F$, since F containing $Z_1((Z_2(\dots\dots(Z_{n(x^n)})\dots..)))$, that there exists $x_1{}^1, x_1{}^2, \dots\dots, x_1{}^n \in F$ such that

$$x_1{}^1 = Z_1\left(\left(Z_2\left(\dots\dots\left(Z_{n(x_0{}^1, x_0{}^2, \dots, x_0{}^n)}\right)\dots..\right)\right)\right),$$

$$x_1{}^2 = Z_1\left(\left(Z_2\left(\dots\dots\left(Z_{n(x_0{}^2, x_0{}^3, \dots, x_0{}^n, x_0{}^1)}\right)\dots..\right)\right)\right)$$

$$\vdots$$

$$x_1{}^n = Z_1\left(\left(Z_2\left(\dots\dots\left(Z_{n(x_0{}^n, x_0{}^1, \dots, x_0{}^{n-1})}\right)\dots..\right)\right)\right)$$

Also, $x_2{}^1 = Z_1\left(\left(Z_2\left(\dots\dots\left(Z_{n(x_1{}^1, x_1{}^2, \dots, x_1{}^n)}\right)\dots..\right)\right)\right)$,

$$x_2{}^2 = Z_1\left(\left(Z_2\left(\dots\dots\left(Z_{n(x_1{}^2, x_1{}^3, \dots, x_1{}^n, x_1{}^1)}\right)\dots..\right)\right)\right)$$

$$\vdots$$

$$x_2{}^n = Z_1\left(Z_2\left(\left(\dots\dots\left(Z_{n(x_1{}^n, x_1{}^1, \dots, x_1{}^{n-1})}\right)\right)\dots..\right)\right)$$

In general, we can construct the sequences, $< x_k{}^1 >, < x_k{}^2 >, \dots$, and $< x_k{}^n >$ as

$$x_k{}^1 = Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^1, x_{k-1}{}^2, \dots, x_{k-1}{}^n)}\right)..\right)\right)\right),$$

$$x_k{}^2 = Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^2, x_{k-1}{}^3, \dots, x_1{}^n, x_{k-1}{}^1)}\right)\dots\right)\right)\right)$$

$$\vdots$$

$$x_k{}^n = Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^n, x_{k-1}{}^1, \dots, x_{k-1}{}^{n-1})}\right)\dots\right)\right)\right)$$

We want to show that the above sequences are Cauchy sequences in $(F, \mathcal{G}, \aleph)$. Since $\aleph$ is (c.t.n) of $\mathcal{H}-\mathcal{T}$ype then we have, $\forall \lambda > 0\ \exists\ \mu > 0$ such that:
(1-μ) ℵ (1-μ) ℵ … ℵ (1-μ) $\geq 1 - \lambda$, $\forall n \in N$.
On other hand, for all $x, y \in F$, $\mathcal{G}(x, y, .)$ is continuous and $\lim_{t\to\infty}(x, y, t) = 1$ then there exists $t_\circ > 0$ such that:

$$\mathcal{G}[x_0{}^1, x_1{}^1, t_0] \geq 1 - \mu, \mathcal{G}[x_0{}^2, x_1{}^2, t_0] \geq 1 - \mu, \dots\dots, \mathcal{G}[x_0{}^n, x_1{}^n, t_0] \geq 1\text{-}\mu \quad (2.2)$$

By using (2.1), we get:

- $\mathcal{G}[x_1{}^1, x_2{}^1, \Delta_{(t_0)}] =$

$$\mathcal{G}\left[\begin{array}{c} Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_0{}^1, x_0{}^2, \dots, x_0{}^n)}\right)\dots\right)\right)\right), \\ Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_1{}^1, x_1{}^2, \dots, x_1{}^n)}\right)\dots\right)\right)\right), \Delta_{(t_0)} \end{array}\right]$$

$$\geq \mathcal{G}[x_0{}^1, x_1{}^1, t_0]\aleph\mathcal{G}[x_0{}^2, x_1{}^2, t_0]\aleph\dots\dots\aleph\mathcal{G}[x_0{}^n, x_1{}^n, t_0]$$

Also,

- $\mathcal{G}[x_1{}^2, x_2{}^2, \Delta_{(t_0)}] =$

$$\mathcal{G}\left[\begin{array}{c} Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_0{}^2, x_0{}^3, \dots, x_0{}^n, x_0{}^1)}\right)\dots\right)\right)\right), \\ Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_1{}^2, x_1{}^3, \dots, x_1{}^n, x_0{}^1)}\right)\dots\right)\right)\right), \\ \Delta_{(t_0)} \end{array}\right]$$

$$\geq \mathcal{G}[x_0{}^2, x_1{}^2, t_0]\aleph\mathcal{G}[x_0{}^3, x_1{}^3, t_0]\aleph\dots\dots * \mathcal{G}[x_0{}^n, x_1{}^n, t_0]$$

we continue this process in the same way

- $\mathcal{G}[x_1{}^n, x_2{}^n, \Delta_{(t_0)}] =$

$$\mathcal{G}\left[\begin{array}{c} Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_0{}^n, x_0{}^1, \dots, x_0{}^{n-1})}\right)\dots\right)\right)\right), \\ Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_1{}^n, x_1{}^1, \dots, x_1{}^{n-1})}\right)\dots\right)\right)\right), t_0 \end{array}\right]$$

$$\geq \mathcal{G}[x_0{}^n, x_1{}^n, t_0]\aleph\mathcal{G}[x_0{}^1, x_1{}^1, t_0]\aleph\dots\dots\aleph\mathcal{G}[x_0{}^{n-1}, x_1{}^{n-1}, t_0]$$

As the sameway
and by using above inequalities

- $\mathcal{G}[x_2{}^1, x_3{}^1, \Delta^2{}_{(t_0)}] = \mathcal{G}\left[\begin{matrix} Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_1{}^1,x_1{}^2,\dots,x_1{}^n)}\right)\dots\right)\right)\right), \\ Z_1\left(Z_2\left(\dots\left(Z_{n(x_2{}^1,x_2{}^2,\dots,x_2{}^n)}\right)\dots\right)\right), \Delta^2{}_{(t_0)}\end{matrix}\right]$

$\geq \mathcal{G}[x_1{}^1, x_2{}^1, \Delta_{(t_0)}] \aleph \mathcal{G}[x_1{}^2, x_2{}^2, \Delta_{(t_0)}] \aleph \dots \aleph \mathcal{G}[x_1{}^n, x_2{}^n, \Delta_{(t_0)}]$
$\geq \mathcal{G}[x_0{}^1, x_1{}^1, t_0]^n \aleph \mathcal{G}[x_0{}^2, x_1{}^2, t_0]^n \aleph \dots\dots \aleph \mathcal{G}[x_0{}^n, x_1{}^n, t_0]^n$

And,

- $\mathcal{G}[x_2{}^2, x_3{}^2, \Delta^2{}_{(t_0)}] = \mathcal{G}\left[\begin{matrix} Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_1{}^2,x_1{}^3,\dots,x_1{}^n,x_1{}^1)}\right)\dots\right)\right)\right), \\ Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_2{}^2,x_2{}^3,\dots,x_2{}^n,x_1{}^1)}\right)\dots\right)\right)\right), \Delta^2{}_{(t_0)}\end{matrix}\right]$

$\geq \mathcal{G}[x_1{}^2, x_2{}^2, \Delta_{(t_0)}] \aleph \mathcal{G}[x_1{}^3, x_2{}^3, \Delta_{(t_0)}] \aleph \dots \aleph \mathcal{G}[x_1{}^n, x_2{}^n, \Delta_{(t_0)}]$
$\geq \mathcal{G}[x_0{}^n, x_1{}^n, t_0]^n \aleph \mathcal{G}[x_0{}^1, x_1{}^1, t_0]^n \aleph \dots\dots \aleph \mathcal{G}[x_0{}^{n-1}, x_1{}^{n-1}, t_0]^n$

Continue this process, we get

- $\mathcal{G}[x_2{}^n, x_3{}^n, \Delta^2{}_{(t_0)}] = \mathcal{G}\left[\begin{matrix} Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_2{}^n,x_2{}^1,\dots,x_2{}^{n-1})}\right)\dots\right)\right)\right) \\ , Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_3{}^n,x_3{}^1,\dots,x_3{}^n)}\right)\dots\right)\right)\right), \Delta^2{}_{(t_0)}\end{matrix}\right]$

$\geq \mathcal{G}[x_2{}^n, x_3{}^n, \Delta_{(t_0)}] \aleph \mathcal{G}[x_2{}^1, x_3{}^1, \Delta_{(t_0)}] \aleph \dots \aleph \mathcal{G}[x_2{}^{n-1}, x_3{}^{n-1}, \Delta_{(t_0)}]$
$\geq \mathcal{G}[x_0{}^1, x_1{}^1, t_0]^n \aleph \mathcal{G}[x_0{}^2, x_1{}^2, t_0]^n \aleph \dots\dots \aleph \mathcal{G}[x_0{}^n, x_1{}^n, t_0]^n$

Similarly

- $\mathcal{G}[x_k{}^1, x_{k+1}{}^1, \Delta^k{}_{(t_0)}] = \mathcal{G}\left[\begin{matrix} Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^1,x_{k-1}{}^2,\dots,x_{k-1}{}^n)}\right)\dots\right)\right)\right), \\ Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_k{}^1,x_k{}^2,\dots,x_k{}^n)}\right)\dots\right)\right)\right), \Delta^k{}_{(t_0)}\end{matrix}\right]$

$\geq \mathcal{G}[x_{k-1}{}^1, x_k{}^1, \Delta^{k-1}{}_{(t_0)}] \aleph \mathcal{G}[x_{k-1}{}^2, x_k{}^2, \Delta^{k-1}{}_{(t_0)}]$
$\aleph \quad \dots \quad \aleph\, \mathcal{G}[x_{k-1}{}^n, x_k{}^n, \Delta^{k-1}{}_{(t_0)}]$
$\vdots$
$\geq \mathcal{G}[x_0{}^1, x_1{}^1, t_0]^{n^{k-1}} \aleph \mathcal{G}[x_0{}^2, x_1{}^2, t_0]^{n^{k-1}} \aleph \dots\dots \aleph\, \mathcal{G}[x_0{}^n, x_1{}^n, t_0]^{n^{k-1}}$

Also,

- $\mathcal{G}[x_k{}^2, x_{k+1}{}^2, \Delta^k{}_{(t_0)}] = \mathcal{G}[Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^2,x_{k-1}{}^3,\dots,x_{k-1}{}^n,x_{k-1}{}^1)}\right)\dots\right)\right)\right), Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_k{}^2,x_k{}^3,\dots,x_k{}^n,x_k{}^1)}\right)\dots\right)\right)\right), \Delta^k{}_{(t_0)}]$

$\geq \mathcal{G}[x_{k-1}{}^2, x_k{}^2, \Delta^{k-1}{}_{(t_0)}] * \mathcal{G}[x_{k-1}{}^3, x_k{}^3, \Delta^{k-1}{}_{(t_0)}] * \dots * \mathcal{G}[x_{k-1}{}^1, x_k{}^1, \Delta^{k-1}{}_{(t_0)}]$
$\vdots$
$\geq \mathcal{G}[x_0{}^2, x_1{}^2, t_0]^{n^{k-1}} \aleph \mathcal{G}[x_0{}^3, x_1{}^3, t_0]^{n^{k-1}} \aleph \dots\dots \aleph\, \mathcal{G}[x_0{}^1, x_1{}^1, t_0]^{n^{k-1}}$

Continue this process, as the same way we get

$\mathcal{G}[x_k{}^n, x_{k+1}{}^n, \Delta^k{}_{(t_0)}] \geq \mathcal{G}[x_0{}^n, x_1{}^n, t_0]^{n^{k-1}} \aleph\, \mathcal{G}[x_0{}^1, x_1{}^1, t_0]^{n^{k-1}} \aleph \dots\dots \aleph\, \mathcal{G}[x_0{}^{n-1}, x_1{}^{n-1}, t_0]^{n^{k-1}}$

Now, by using above inequalities and for each $n_0 \leq n < m$, we have

$\mathcal{G}\left[x_k{}^n, x_m{}^n, \sum_{k=n_0}^{\infty} \Delta^k{}_{(t_0)}\right] \geq \mathcal{G}\left[x_k{}^n, x_m{}^n, \sum_{k=n_0}^{m-1} \Delta^{k+1}{}_{(t_0)}\right] \geq$
$\mathcal{G}[x_k{}^n, x_{k+1}{}^n, \Delta^k{}_{(t_0)}] * \mathcal{G}[x_{k+1}{}^n, x_{k+2}{}^n, \Delta^{k+1}{}_{(t_0)}] * \dots * \mathcal{G}[x_{m-1}{}^n, x_m{}^n, \Delta^{m-1}{}_{(t_0)}]$
$\vdots$
$\geq \mathcal{G}[x_0{}^n, x_1{}^n, t_0]^{n^{k-1}} \aleph \mathcal{G}[x_0{}^1, x_1{}^1, t_0]^{n^{k-1}} \aleph \dots\dots \aleph\, \mathcal{G}[x_0{}^{n-1}, x_1{}^{n-1}, t_0]^{n^{k-1}} \aleph$
$\mathcal{G}[x_0{}^n, x_1{}^n, t_0]^{n^k} \aleph \mathcal{G}[x_0{}^1, x_1{}^1, t_0]^{n^k} \aleph \dots\dots \aleph\, \mathcal{G}[x_0{}^{n-1}, x_1{}^{n-1}, t_0]^{n^k} \aleph \dots \aleph$
$\mathcal{G}[x_0{}^n, x_1{}^n, t_0]^{n^{m-2}} \aleph \mathcal{G}[x_0{}^1, x_1{}^1, t_0]^{n^{m-2}} \aleph \dots\dots \aleph\, \mathcal{G}[x_0{}^{n-1}, x_1{}^{n-1}, t_0]^{n^{m-2}}$

Let $l = \max\{n^{k-1}, n^k, n^{m-2}\}$

$\geq \mathcal{G}[x_0{}^n, x_1{}^n, t_0]^l \aleph \mathcal{G}[x_0{}^1, x_1{}^1, t_0]^l \aleph \dots\dots \aleph\, \mathcal{G}[x_0{}^{n-1}, x_1{}^{n-1}, t_0]^l \aleph$
$> \mathcal{G}[x_0{}^n, x_1{}^n, t_0]^{ml} \aleph \mathcal{G}[x_0{}^1, x_1{}^1, t_0]^{ml} \aleph \dots\dots \aleph\, \mathcal{G}[x_0{}^{n-1}, x_1{}^{n-1}, t_0]^{ml}$
$\geq (1-\mu) \aleph \; (1-\mu) \aleph \dots\dots \aleph \; (1-\mu) \geq (1-\lambda)$

And hence, $\mathcal{G}[(x_k{}^n, x_m{}^n, t)] > (1-\lambda)$

So, $< x_k{}^n >$ is Cauchy sequence. As the same way, we get $< x_k{}^1 >, < x_k{}^2 >$ and $< x_k{}^{n-1} >$ are Cauchy sequences .Since F is complete then there exists $a_1, a_2, \dots\dots, a_n \in X$ such that $\lim_{k\to\infty} x_k{}^1 = \lim_{k\to\infty} Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^1,\dots,x_{k-1}{}^n)}\right)\dots\right)\right)\right) \to a_1$

$\lim_{k\to\infty} x_k{}^2 = \lim_{k\to\infty} Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^2,\dots,x_{k-1}{}^1)}\right)\dots\right)\right)\right) \to a_2$

$\lim_{k\to\infty} x_k{}^n =$

$\lim_{k\to\infty} Z_1\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^n,\dots,x_{k-1}{}^{n-1})}\right)\dots\right)\right) \longrightarrow a_n$

$$\mathcal{G}\begin{bmatrix} Z_1\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^1,x_{k-1}{}^2,\dots,x_{k-1}{}^n)}\right)\dots\right)\right) \\ ,Z_1\left(Z_2\left(\dots\left(Z_{n(a_1,a_2,\dots,a_n)}\right)\dots\right)\right),\Delta_{(t)} \end{bmatrix}$$

$\geq \mathcal{G}[x_{k-1}{}^1, a_1, t] \aleph \mathcal{G}[x_{k-1}{}^2, a_2, t] \aleph \dots \aleph \mathcal{G}[x_{k-1}{}^n, a_n, t]$

As $n \to \infty$ and by continuity of $\mathcal{G}$, we get

$$\mathcal{G}\left[a_1, Z_1\left(\left(Z_2\left(\dots\left(Z_{n(a_1,a_2,\dots,a_n)}\right)\dots\right)\right)\right),\Delta_{(t)}\right] = 1$$

Also,

$$\mathcal{G}\begin{bmatrix} Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^2,x_{k-1}{}^3,\dots,x_{k-1}{}^1)}\right)\dots\right)\right)\right) \\ ,Z_1\left(\left(Z_2\left(\dots\left(Z_{n(a_1,a_2,\dots,a_n)}\right)\dots\right)\right)\right),\Delta_{(t)} \end{bmatrix}$$

$\geq \mathcal{G}[x_{k-1}{}^2, a_2, t] \aleph \mathcal{G}[x_{k-1}{}^3, a_3, t] \aleph \dots \aleph \mathcal{G}[x_{k-1}{}^1, a_1, t]$

As, $n \to \infty$

$$\mathcal{G}\left[a_2, Z_1\left(Z_2\left(\dots\left(Z_{n(a_2,a_3,\dots,a_n)}\right)\dots\right)\right),\Delta_{(t)}\right] = 1$$

Continuity

$$\mathcal{G}\begin{bmatrix} (x_{k-1}{}^n, x_{k-1}{}^1, \dots, x_{k-1}{}^{n-1}), \\ Z_1\left(\left(Z_2\left(\dots\left(Z_{n(a_n,a_1,\dots,a_{n-1})}\right)\dots\right)\right)\right),\Delta_{(t)} \end{bmatrix}$$

$\geq \mathcal{G}[x_{k-1}{}^n, a_n, t] \aleph \mathcal{G}[x_{k-1}{}^1, a_1, t] \aleph \dots \aleph$
$\mathcal{G}[x_{k-1}{}^{n-1}, a_{n-1}, t]$

As $n \to \infty$,we get

$$\mathcal{G}\left[a_n, Z_1\left(\left(\left(Z_2\left(\dots\left(Z_{n(a_n,a_1,\dots,a_{n-1})}\right)\dots\right)\right)\right)\right),\Delta_{(t)}\right]$$

$= 1$. And hens,

$a_1 = Z_1\left(\left(Z_2\left(\dots\left(Z_{n(a_1,a_2,\dots,a_n)}\right)\dots\right)\right)\right)$,

$a_2 = Z_1\left(\left(Z_2\left(\dots\left(Z_{n(a_2,a_3,\dots,a_1)}\right)\dots\right)\right)\right)$, …… ,

$a_n = Z_1\left(Z_2\left(\dots\left(Z_{n(a_n,\dots,a_{n-1})}\right)\dots\right)\right)$

Therefore,

$(a_1, a_2, \dots\dots, a_n)$ is $(G.n)$ – tupled fixed point of compose the mappings of $Z_1, Z_2, \dots\dots, Z_n$.

Corollary(2.2)

Let $(X, \mathcal{G}, *)$ be a(F.M.S) .Under the same assumptions of theorem(2.1) but

$$\mathcal{G}\begin{bmatrix} Z_1\left(\left(Z_2\left(\dots\dots\left(Z_{n(x_1,x_2,\dots,x_n)}\right)\dots\dots\right)\right)\right), \\ Z_1\left(\left(Z_2\left(\dots\dots\left(Z_{n(y_1,y_2,\dots,y_n)}\right)\dots\dots\right)\right)\right),kt \end{bmatrix}$$

$\geq \mathcal{G}[x_1, y_1, t] * \mathcal{G}[x_2, y_2, t] * \dots\dots * \mathcal{G}[x_n, y_n, t]$

where $k \in (0,1)$, $t > 0$ and $x_i, y_i \in F$, $\forall\, i = 1,2,\dots\dots,n$.Then there exists a unique $(G.n)$ – tupled fixed point of compose the mappings $Z_1, Z_2, \dots\dots, Z_n$.

Corollary(2.3)

Let $(F, \mathcal{G}, \aleph)$ be a $(F.M.S)$Under the same assumptions of theorem(2.1) but

$$M\begin{bmatrix} Z_1\left(\left(Z_2\left(\dots\dots\left(Z_{n(x_1,x_2,\dots,x_n)}\right)\dots\dots\right)\right)\right), \\ Z_1\left(\left(Z_2\left(\dots\dots\left(Z_{n(y_1,y_2,\dots,y_n)}\right)\dots\dots\right)\right)\right),\Delta(t) \end{bmatrix}$$

$\geq \mathcal{G}[x_1, y_1, t]^{a_1} \aleph$
$\mathcal{G}[x_2, y_2, t]^{a_2} \aleph \dots\dots \aleph\, \mathcal{G}[x_n, y_n, t]^{a_n}$

where $\sum_{i=1}^{n} a_i \leq 1$, $t > 0$ and $x_i, y_i \in F$ $\forall\, i = 1,2,\dots\dots,n$.Then there exists a unique $(G.n)$ – tupled fixed point of compose the mappings $Z_1, Z_2, \dots\dots, Z_n$.

Corollary(2.4)

Let $(F, \mathcal{G}, \aleph)$ be a $(F.M.S)$.Under the same assumptions of theorem(2.1) but

$$\mathcal{G}\begin{bmatrix} Z_1\left(\left(Z_2\left(\dots\dots\left(Z_{n(x_1,x_2,\dots,x_n)}\right)\dots\dots\right)\right)\right), \\ Z_1\left(\left(Z_2\left(\dots\dots\left(Z_{n(y_1,y_2,\dots,y_n)}\right)\dots\dots\right)\right)\right),kt \end{bmatrix}$$

$\geq \mathcal{G}[x_1, y_1, t]^{a_1} \aleph \mathcal{G}[x_2, y_2, t]^{a_2} \aleph \dots\dots \aleph \mathcal{G}[x_n, y_n, t]^{a_n}$

where $\sum_{i=1}^{n} a_i \leq 1$, $k \in (0,1)$, and $x_i, y_i \in F$, $\forall\, i = 1,2,\dots\dots,n$.Then there exists a unique $(G.n)$ – tupled fixed of compose the mappings $Z_1, Z_2, \dots\dots, Z_n$.

Theorem (2.5)

Let $(F, \mathcal{G}, \aleph)$ be a fuzzy metric space and A, B are two families of mappings such that $A = \{Z_1, Z_2, \dots\dots, Z_n : X^n \to X\}$, $B = \{E_1, E_2, \dots\dots, E_n : F \to F\}$. Suppose that $\Delta \in$ ℃ satisfying

$\mathcal{G}[Z_1\left(Z_2\left(\dots\dots\left(Z_{n(x_1,x_2,\dots,x_n)}\right)\dots\dots\right)\right)$,
$Z_1\left(Z_2\left(\dots\dots\left(Z_{n(y_1,y_2,\dots,y_n)}\right)\dots\dots\right)\right),\Delta_{(t)}]$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_1)}\right)\dots\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(y_1)}\right)\dots\dots\right)\right),t \end{bmatrix} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_2)}\right)\dots\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(y_2)}\right)\dots\dots\right)\right),t \end{bmatrix} \aleph \dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_n)}\right)\dots\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(y_n)}\right)\dots\dots\right)\right),t \end{bmatrix}$$

(2.3)

where $t > 0$ and $x_i, y_i \in F$ $\forall\, i = 1,2,\dots\dots,n$

If $E_1\left(E_2\left(\dots\dots\left(E_{n(X)}\right)\dots\dots\right)\right)$ is complete subspace of F containing $Z_1(Z_2(\dots\dots(Z)\dots\dots))$.Then there exists a unique $(G.n)$ – tupled coincidence fixed point of compose the mappings of $A$and B.

Proof:

Suppose that, $x_0{}^1, x_0{}^2, \dots\dots, x_1{}^n \in F$, since $E_1\left(E_2(\dots\dots(E_{n(x)})\dots..)\right)$ containing $Z_1\left(Z_2(\dots\dots(Z_{n(x^n)})\dots..)\right)$, that there exists $x_1{}^1, x_1{}^2, \dots\dots, x_1{}^n \in F$ such that

$$E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots..\right)\right) = Z_1\left(Z_2\left(\dots\dots\left(Z_{n(x_0{}^1,x_0{}^2,\dots,x_0{}^n)}\right)\dots..\right)\right)$$

$$E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^2)}\right)\dots..\right)\right) = Z_1\left(Z_2\left(\dots\dots\left(Z_{n(x_0{}^2,x_0{}^3,\dots,x_0{}^n,x_0{}^1)}\right)\dots..\right)\right)$$

$$\vdots$$

$$E_1\left(E_2(\dots\dots(E_{n(x_1{}^n)})\dots..)\right) = Z_1\left(Z_2\left(\dots\dots\left(Z_{n(x_0{}^n,x_0{}^1,\dots,x_0{}^{n-1})}\right)\dots..\right)\right)$$

Also, $E_1\left(E_2\left(\dots\dots\left(E_{n(x_2{}^1)}\right)\dots..\right)\right) = Z_1\left(Z_2\left(\dots\dots\left(Z_{n(x_1{}^1,x_1{}^2,\dots,x_1{}^n)}\right)\dots..\right)\right)$

$$E_1\left(E_2\left(\dots\dots\left(E_{n(x_2{}^2)}\right)\dots..\right)\right) = Z_1\left(Z_2\left(\dots\dots\left(Z_{n(x_1{}^2,x_1{}^3,\dots,x_1{}^n,x_1{}^1)}\right)\dots..\right)\right)$$

$$\vdots$$

$$E_1\left(E_2(\dots\dots(E_{n(x_2{}^n)})\dots..)\right) = Z_1\left(Z_2\left(\dots\dots\left(Z_{n(x_1{}^n,x_1{}^1,\dots,x_1{}^{n-1})}\right)\dots..\right)\right)$$

In general, we can construct the sequences, $< E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^1)}\right)\dots..\right)\right) >$, $< E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^2)}\right)\dots..\right)\right) >, \dots,$ and $< E_1\left(E_2(\dots\dots(E_{n(x_k{}^n)})\dots..)\right) >$ as follows

$$E_1\left(E_2\left(\dots\left(E_{n(x_k{}^1)}\right)\dots\right)\right) = Z_1\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^1,x_{k-1}{}^2,\dots,x_{k-1}{}^n)}\right)..\right)\right)$$

$$E_1\left(E_2\left(\dots\left(E_{n(x_k{}^2)}\right)\dots\right)\right) = Z_1\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^2,x_{k-1}{}^3,\dots,x_1{}^n,x_{k-1}{}^1)}\right)\dots\right)\right)$$

$$\vdots$$

$$E_1\left(E_2(\dots(E_{n(x_k{}^n)})\dots)\right) = Z_1\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^n,x_{k-1}{}^1,\dots,x_{k-1}{}^{n-1})}\right)\dots\right)\right)$$

Now, we want to show that the above sequences are Cauchy sequences in $(F, M, \aleph)$, since $\aleph$ is t – norm of H – type, this implies $\forall\ ⋌ > 0\ \exists\ \mu > 0$ such that $(1-\mu)\aleph(1-\mu)\aleph\dots\aleph(1-\mu) \geq 1 - ⋌$, $\forall n \in N$. on other hand. For all $x, y \in X$, $M(x, y, .)$ is continuous and $\lim_{t\to\infty}(x, y, t) = 1$ then there exists $t_\circ > 0$ such that.

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), t_0 \end{bmatrix} \geq 1-\mu$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^2)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^2)}\right)\dots\right)\right), t_0 \end{bmatrix} \geq 1-\mu$$

$$\vdots \tag{2.4}$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2(\dots\dots(E_{n(x_0{}^n)})\dots)\right), \\ E_1\left(E_2(\dots\dots(E_{n(x_1{}^n)})\dots)\right), t_0 \end{bmatrix} \geq 1-\mu$$

By using (2.3), we get

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_2{}^1)}\right)\dots\right)\right), \Delta_{(t_0)} \end{bmatrix} = \mathcal{G}\begin{bmatrix} Z_1\left(Z_2\left(\dots\left(Z_{n(x_0{}^1,x_0{}^2,\dots,x_0{}^n)}\right)\dots\right)\right), \\ Z_1\left(Z_2\left(\dots\left(Z_{n(x_1{}^1,x_1{}^2,\dots,x_1{}^n)}\right)\dots\right)\right), \Delta_{(t_0)} \end{bmatrix}$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), t_0 \end{bmatrix} * \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^2)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^2)}\right)\dots\right)\right), t_0 \end{bmatrix} \aleph\dots\dots\aleph\ \mathcal{G}\begin{bmatrix} E_1\left(E_2(\dots\dots(E_{n(x_0{}^n)})\dots)\right), \\ E_1\left(E_2(\dots\dots(E_{n(x_1{}^n)})\dots)\right), t_0 \end{bmatrix}$$

Also,

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^2)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_2{}^2)}\right)\dots\right)\right), \Delta_{(t_0)} \end{bmatrix} = \mathcal{G}\begin{bmatrix} Z_1\left(Z_2\left(\dots\left(Z_{n(x_0{}^2,x_0{}^3,\dots,x_0{}^n,x_0{}^1)}\right)\dots\right)\right) \\ , Z_1\left(Z_2\left(\dots\left(Z_{n(x_1{}^2,x_1{}^3,\dots,x_1{}^n,x_0{}^1)}\right)\dots\right)\right), \aleph_{(t_0)} \end{bmatrix}$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^2)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^2)}\right)\dots\right)\right), t_0 \end{bmatrix} \aleph\ \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^3)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^3)}\right)\dots\right)\right), t_0 \end{bmatrix} \aleph\dots\dots\aleph\ \mathcal{G}\begin{bmatrix} E_1\left(E_2(\dots\dots(E_{n(x_0{}^n)})\dots)\right), \\ E_1\left(E_2(\dots\dots(E_{n(x_1{}^n)})\dots)\right), t_0 \end{bmatrix}$$

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we continue this process in the same way

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_1{}^n)}\right)\ldots\right)\right), \\ E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_2{}^n)}\right)\ldots\right)\right), \Delta_{(t_0)}\end{bmatrix} = \mathcal{G}\begin{bmatrix} Z_1\left(Z_2\left(\ldots\left(Z_{n(x_0{}^n, x_0{}^1,\ldots,x_0{}^{n-1})}\right)\ldots\right)\right), \\ Z_1\left(Z_2\left(\ldots\left(Z_{n(x_1{}^n, x_1{}^1,\ldots,x_1{}^{n-1})}\right)\ldots\right)\right), t_0\end{bmatrix}$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_0{}^n)}\right)\ldots\right)\right), \\ E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_1{}^n)}\right)\ldots\right)\right), t_0\end{bmatrix} \aleph\, \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_0{}^1)}\right)\ldots\right)\right), \\ E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_1{}^1)}\right)\ldots\right)\right), t_0\end{bmatrix} \aleph\ldots\ldots\aleph\, \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_0{}^{n-1})}\right)\ldots\right)\right), \\ E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_1{}^{n-1})}\right)\ldots\right)\right), t_0\end{bmatrix}$$

As the same way and by using above inequalities,

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_2{}^1)}\right)\ldots\right)\right), \\ E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_3{}^1)}\right)\ldots\right)\right), \Delta^2{}_{(t_0)}\end{bmatrix} = \mathcal{G}\begin{bmatrix} Z_1\left(Z_2\left(\ldots\left(Z_{n(x_1{}^1, x_1{}^2,\ldots,x_1{}^n)}\right)\ldots\right)\right), \\ Z_1\left(Z_2\left(\ldots\left(Z_{n(x_2{}^1, x_2{}^2,\ldots,x_2{}^n)}\right)\ldots\right)\right), \Delta^2{}_{(t_0)}\end{bmatrix}$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_1{}^1)}\right)\ldots\right)\right), \\ E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_2{}^1)}\right)\ldots\right)\right), \Delta_{(t_0)}\end{bmatrix} \aleph\, \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_1{}^2)}\right)\ldots\right)\right), \\ E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_2{}^2)}\right)\ldots\right)\right), \Delta_{(t_0)}\end{bmatrix} \aleph\ldots\aleph\, \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_1{}^n)}\right)\ldots\right)\right), \\ E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_2{}^n)}\right)\ldots\right)\right), \emptyset_{(t_0)}\end{bmatrix}$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_0{}^1)}\right)\ldots\right)\right), \\ E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_1{}^1)}\right)\ldots\right)\right), t_0\end{bmatrix}^n \aleph\, \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_0{}^2)}\right)\ldots\right)\right), \\ E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_1{}^2)}\right)\ldots\right)\right), t_0\end{bmatrix}^n \aleph\ldots\ldots\aleph\, \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_0{}^n)}\right)\ldots\right)\right), \\ E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_1{}^n)}\right)\ldots\right)\right), t_0\end{bmatrix}^n$$

- $$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_2{}^2)}\right)\ldots\right)\right), \\ E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_3{}^2)}\right)\ldots\right)\right), \Delta^2{}_{(t_0)}\end{bmatrix} = \mathcal{G}\begin{bmatrix} Z_1\left(Z_2\left(\ldots\left(Z_{n(x_1{}^2, x_1{}^3,\ldots,x_1{}^n, x_1{}^1)}\right)\ldots\right)\right), \\ Z_1\left(Z_2\left(\ldots\left(Z_{n(x_2{}^2, x_2{}^3,\ldots,x_2{}^n, x_1{}^1)}\right)\ldots\right)\right), \Delta^2{}_{(t_0)}\end{bmatrix}$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_1{}^2)}\right)\ldots\right)\right), \\ E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_2{}^2)}\right)\ldots\right)\right), \Delta_{(t_0)}\end{bmatrix} \aleph\, \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_1{}^3)}\right)\ldots\right)\right), \\ E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_2{}^3)}\right)\ldots\right)\right), \Delta_{(t_0)}\end{bmatrix} \aleph\ldots\aleph\, \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_1{}^n)}\right)\ldots\right)\right), \\ E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_2{}^n)}\right)\ldots\right)\right), \Delta_{(t_0)}\end{bmatrix}$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_0{}^n)}\right)\ldots\right)\right), \\ E_1\left(E\left(\ldots\ldots\left(E_{n(x_1{}^n)}\right)\ldots\right)\right), t_0\end{bmatrix}^n \aleph\, \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_0{}^1)}\right)\ldots\right)\right), \\ E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_1{}^1)}\right)\ldots\right)\right), t_0\end{bmatrix}^n \aleph\ldots\ldots\aleph\, \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_0{}^{n-1})}\right)\ldots\right)\right), \\ E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_1{}^{n-1})}\right)\ldots\right)\right), t_0\end{bmatrix}^n$$

Continue this process, we get

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_2{}^n)}\right)\ldots\right)\right), \\ E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_3{}^n)}\right)\ldots\right)\right), \Delta^2{}_{(t_0)}\end{bmatrix} = \mathcal{G}\begin{bmatrix} Z_1\left(Z_2\left(\ldots\left(Z_{n(x_2{}^n, x_2{}^1,\ldots,x_2{}^{n-1})}\right)\ldots\right)\right), \\ Z_1\left(Z_2\left(\ldots\left(Z_{n(x_3{}^n, x_3{}^1,\ldots,x_3{}^n)}\right)\ldots\right)\right), \aleph^2{}_{(t_0)}\end{bmatrix}$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_2{}^n)}\right)\ldots\right)\right), \\ E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_3{}^n)}\right)\ldots\right)\right), \Delta_{(t_0)}\end{bmatrix} \aleph\, \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_2{}^1)}\right)\ldots\right)\right), \\ E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_3{}^1)}\right)\ldots\right)\right), \Delta_{(t_0)}\end{bmatrix} \aleph\ldots\aleph\, \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_2{}^{n-1})}\right)\ldots\right)\right), \\ E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_3{}^{n-1})}\right)\ldots\right)\right), \Delta_{(t_0)}\end{bmatrix}$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_0{}^1)}\right)\ldots\right)\right), \\ E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_1{}^1)}\right)\ldots\right)\right), t_0\end{bmatrix}^n \aleph\, \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_0{}^2)}\right)\ldots\right)\right), \\ E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_1{}^2)}\right)\ldots\right)\right), t_0\end{bmatrix}^n \aleph\ldots\ldots\aleph\, \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_0{}^n)}\right)\ldots\right)\right), \\ E_1\left(E_2\left(\ldots\ldots\left(E_{n(x_1{}^n)}\right)\ldots\right)\right), t_0\end{bmatrix}^n$$

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Similarly

- $$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k+1}{}^1)}\right)\dots\right)\right), \Delta^k{}_{(t_0)} \end{bmatrix} =$$

$$\mathcal{G}\begin{bmatrix} Z_1\left(Z\left(\dots\left(Z_{n(x_{k-1}{}^1, x_{k-1}{}^2, \dots, x_{k-1}{}^n)}\right)\dots\right)\right), \\ Z_1(Z_2(\dots(Z)\dots)), \Delta^k{}_{(t_0)} \end{bmatrix}$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k-1}{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^1)}\right)\dots\right)\right), \Delta^{k-1}{}_{(t_0)} \end{bmatrix} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k-1}{}^2)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^2)}\right)\dots\right)\right), \Delta^{k-1}{}_{(t_0)} \end{bmatrix} \aleph \dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k-1}{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^n)}\right)\dots\right)\right), \Delta^{k-1}{}_{(t_0)} \end{bmatrix}$$

$$\vdots$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), t_0 \end{bmatrix}^{n^{k-1}} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^2)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^2)}\right)\dots\right)\right), t_0 \end{bmatrix}^{n^{k-1}} \aleph \dots\dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^n)}\right)\dots\right)\right), t_0 \end{bmatrix}^{n^{k-1}}$$

Also

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^2)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k+1}{}^2)}\right)\dots\right)\right), \Delta^k{}_{(t_0)} \end{bmatrix} =$$

$$\mathcal{G}[Z_1\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^2, x_{k-1}{}^3, \dots, x_{k-1}{}^n, x_{k-1}{}^1)}\right)\dots\right)\right),$$
$$Z_1\left(Z_2\left(\dots\left(Z_{n(x_k{}^2, x_k{}^3, \dots, x_k{}^n, x_k{}^1)}\right)\dots\right)\right), \Delta^k{}_{(t_0)}]$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k-1}{}^2)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^2)}\right)\dots\right)\right), \Delta^{k-1}{}_{(t_0)} \end{bmatrix}$$

$$* \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k-1}{}^3)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^3)}\right)\dots\right)\right), \\ \Delta^{k-1}{}_{(t_0)} \end{bmatrix} * \dots *$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k-1}{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^1)}\right)\dots\right)\right), \Delta^{k-1}{}_{(t_0)} \end{bmatrix}$$

$$\vdots$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^2)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^2)}\right)\dots\right)\right), t_0 \end{bmatrix}^{n^{k-1}} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^3)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^3)}\right)\dots\right)\right), t_0 \end{bmatrix}^{n^{k-1}} \aleph \dots\dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), t_0 \end{bmatrix}^{n^{k-1}}$$

Continue this process, as the same way we get

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k+1}{}^n)}\right)\dots\right)\right), \Delta^k{}_{(t_0)} \end{bmatrix}$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^n)}\right)\dots\right)\right), t_0 \end{bmatrix}^{n^{k-1}} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), t_0 \end{bmatrix}^{n^{k-1}} \aleph \dots\dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^{n-1})}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^{n-1})}\right)\dots\right)\right), t_0 \end{bmatrix}^{n^{k-1}}$$

Now, by using above inequalities and for each $n_0 \leq n < m$, we have

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_m{}^n)}\right)\dots\right)\right), \sum_{k=n_0}^{\infty} \Delta^k{}_{(t_0)} \end{bmatrix}$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_m{}^n)}\right)\dots\right)\right), \sum_{k=n_0}^{m-1} \Delta^{k+1}{}_{(t_0)} \end{bmatrix}$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k+1}{}^n)}\right)\dots\right)\right), \Delta^k{}_{(t_0)} \end{bmatrix} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k+1}{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k+2}{}^n)}\right)\dots\right)\right), \aleph^{k+1}{}_{(t_0)} \end{bmatrix} \aleph \dots$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{m-1}{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_m{}^n)}\right)\dots\right)\right), \emptyset^{m-1}{}_{(t_0)} \end{bmatrix}$$

$$\vdots$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^n)}\right)\dots\right)\right), t_0 \end{bmatrix}^{n^{k-1}} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), t_0\end{bmatrix}^{n^{k-1}} \aleph \dots\dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^{n-1})}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^{n-1})}\right)\dots\right)\right), t_0\end{bmatrix}^{n^{k-1}} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^n)}\right)\dots\right)\right), t_0\end{bmatrix}^{n^{k}} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E\left(\dots\dots\left(E_{n(x_0{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), t_0\end{bmatrix}^{n^{k}} \aleph \dots\dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^{n-1})}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^{n-1})}\right)\dots\right)\right), t_0\end{bmatrix}^{n^{k}} \aleph \dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^n)}\right)\dots\right)\right), t_0\end{bmatrix}^{n^{m-2}} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), t_0\end{bmatrix}^{n^{m-2}} \aleph \dots\dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^{n-1})}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^{n-1})}\right)\dots\right)\right), t_0\end{bmatrix}^{n^{m-2}}$$

Let $l = \max\{n^{k-1}, n^k, n^{m-2}\}$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^n)}\right)\dots\right)\right), t_0\end{bmatrix}^{l} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), t_0\end{bmatrix}^{l} \aleph \dots\dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^{n-1})}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^{n-1})}\right)\dots\right)\right), t_0\end{bmatrix}^{l} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^n)}\right)\dots\right)\right), t_0\end{bmatrix}^{l} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), t_0\end{bmatrix}^{l} \aleph \dots\dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^n)}\right)\dots\right)\right), t_0\end{bmatrix}^{l} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), t_0\end{bmatrix}^{l} \aleph \dots\dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^{n-1})}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^{n-1})}\right)\dots\right)\right), t_0\end{bmatrix}^{l}$$

$$> \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^n)}\right)\dots\right)\right), t_0\end{bmatrix}^{ml} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), t_0\end{bmatrix}^{ml} \aleph \dots\dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^{n-1})}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^{n-1})}\right)\dots\right)\right), t_0\end{bmatrix}^{ml}$$

$\geq (1-\mu)\aleph(1-\mu)\aleph \dots\dots \aleph(1-\mu) \geq (1-\lambda)$

And hence,

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_m{}^n)}\right)\dots\right), t\right)\end{bmatrix} > (1-\lambda)$$

So, $< E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^n)}\right)\dots\right)\right) >$ is Cauchy sequence.

As the same way, we get

$< E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^1)}\right)\dots\right)\right) >$,

$< E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^2)}\right)\dots\right)\right) >$ and

$< E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^{n-1})}\right)\dots\right)\right) >$ are Cauchy sequences

Now, to prove that the mappings in A and B have $(G.n)$ – tuplet coincidence fixed point.

Since $E_1\left(E_2\left(\dots\dots\left(E_{n(X)}\right)\dots\right)\right)$ is complete subspace of X then there exists $x_1, x_2, \dots\dots, x_n \in E_1\left(E_2\left(\dots\dots\left(E_{n(X)}\right)\dots\right)\right)$ and $a_1, a_2, \dots\dots, a_n \in X$ such that

$$\lim_{k\to\infty} E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^1)}\right)\dots\right)\right) = \lim_{k\to\infty} Z_1\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^1,\dots,x_{k-1}{}^n)}\right)\dots\right)\right) \longrightarrow E_1\left(E_2\left(\dots\dots\left(E_{n(a_1)}\right)\dots\right)\right) = x_1$$

$$\lim_{k\to\infty} E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^2)}\right)\dots\right)\right) = \lim_{k\to\infty} Z_1\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^2,\dots,x_{k-1}{}^1)}\right)\dots\right)\right) \to E_1\left(E_2\left(\dots\dots\left(E_{n(a_2)}\right)\dots\right)\right) = x_2$$

$\vdots$

$$\lim_{k\to\infty} E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^n)}\right)\dots\right)\right)$$
$$= \lim_{k\to\infty} Z_1\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^n,\dots,x_{k-1}{}^{n-1})}\right)\dots\right)\right) \to E\left(E_2\left(\dots\left(g_{n(a_n)}\right)\dots\right)\right) = x_n$$

$$\mathcal{G}\begin{bmatrix} Z_1\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^1,x_{k-1}{}^2,\dots,x_{k-1}{}^n)}\right)\dots\right)\right), \\ Z_1\left(Z_2\left(\dots\left(Z_{n(a_1,a_2,\dots,a_n)}\right)\dots\right)\right), \Delta_{(t)} \end{bmatrix}$$
$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k-1}{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(a_1)}\right)\dots\right)\right), t \end{bmatrix} \aleph$$
$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k-1}{}^2)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(a_2)}\right)\dots\right)\right), t \end{bmatrix} \aleph \dots$$
$$\aleph\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k-1}{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(a_n)}\right)\dots\right)\right), t \end{bmatrix}$$

As $n\to\infty$ and by continuity of M, we get $\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(a_1)}\right)\dots\right)\right), \\ Z_1\left(Z_2\left(\dots\left(Z_{n(a_1,a_2,\dots,a_n)}\right)\dots\right)\right), \Delta_{(t)} \end{bmatrix} =$
1

Also,

$$\mathcal{G}\begin{bmatrix} Z_1\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^2,x_{k-1}{}^3,\dots,x_{k-1}{}^1)}\right)\dots\right)\right), \\ Z_1\left(Z_2\left(\dots\left(Z_{n(a_1,a_2,\dots,a_n)}\right)\dots\right)\right), \Delta_{(t)} \end{bmatrix}$$
$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k-1}{}^2)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(a_2)}\right)\dots\right)\right), t \end{bmatrix} \aleph$$
$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k-1}{}^3)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(a_3)}\right)\dots\right)\right), t \end{bmatrix} \aleph \dots \aleph$$
$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k-1}{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(a_1)}\right)\dots\right)\right), t \end{bmatrix}$$

As $n\to\infty$,

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(a_2)}\right)\dots\right)\right), \\ Z_1\left(Z_2\left(\dots\left(Z_{n(a_2,a_3,\dots,a_n)}\right)\dots\right)\right), \Delta_{(t)} \end{bmatrix} = 1$$

Continuity

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k-1}{}^n,x_{k-1}{}^1,\dots,x_{k-1}{}^{n-1})}\right)\dots\right)\right), \\ Z_1\left(Z_2\left(\dots\left(Z_{n(a_n,a_1,\dots,a_{n-1})}\right)\dots\right)\right), \Delta_{(t)} \end{bmatrix}$$
$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k-1}{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(a_n)}\right)\dots\right)\right), t \end{bmatrix} \aleph$$
$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k-1}{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(a_1)}\right)\dots\right)\right), t \end{bmatrix} \aleph \dots \aleph$$
$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k-1}{}^{n-1})}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(a_{n-1})}\right)\dots\right)\right), t \end{bmatrix}$$

As $n\to\infty$, we get.

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(a_n)}\right)\dots\right)\right), \\ Z_1\left(Z\left(\dots\left(Z_{n(a_n,a_1,\dots,a_{n-1})}\right)\dots\right)\right), \emptyset_{(t)} \end{bmatrix} = 1$$
$$\Rightarrow E_1\left(E_2\left(\dots\dots\left(E_{n(a_1)}\right)\dots\right)\right)$$
$$= Z_1\left(Z_2\left(\dots\left(Z_{n(a_1,a_2,\dots,a_n)}\right)\dots\right)\right) = x_1$$
$$E_1\left(E_2\left(\dots\dots\left(E_{n(a_2)}\right)\dots\right)\right)$$
$$= Z_1\left(Z_2\left(\dots\left(Z_{n(a_2,a_3,\dots,a_1)}\right)\dots\right)\right) = x_2$$
$$\vdots$$
$$E_1\left(E_2\left(\dots\dots\left(E_{n(a_n)}\right)\dots\right)\right)$$
$$= Z_1\left(Z_2\left(\dots\left(Z_{n(a_n,\dots,a_{n-1})}\right)\dots\right)\right) = x_n$$

Therefore, $(a_1, a_2, \dots\dots, a_n)$ is $(G.n)$ – tupled coincidence point of compose the mappings of A and B.

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حول نظريات النقطة الصامدة الثلاثية نوع $(G.n)$ في فضاءات مترية ضبابية

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المستخلص :
الهدف من هذا البحث هو لتقديم مفهومين جديدين هما النقطة الصامدة الثلاثية والنقطة المتطابقة الثلاثية نوع$(G.n)$ ولدراسة الوجود للنقطة الصامدة (المتطابقة) الثلاثية لاي نوع من التطبيقات. ايضا سننشئ نظريات التقارب الى نقطة صامدة (متطابقة) ثلاثية نوع$(G.n)$ وحيدة في الفضاءات المترية الضبابية الكاملة .