Journal of AL-Qadisiyah for computer science and mathematics Vol.10 No.2 Year 2018
ISSN (Print): 2074 – 0204 ISSN (Online): 2521 – 3504

Intertwining approximation in space $L_{\psi,p}(I)$, $0<p<1$

Nada Zuhair Abd AL-Sada

Department of Mathematics, College of education
Al-Qadisiyah University
E-mail:Nadawee70@yahoo.com

Recived : 17\1\2018 **Revised : 4\2\2018** **Accepted : 26\2\2018**

Available online : 3 /4/2018

DOI: 10.29304/jqcm.2018.10.2.376

Abstract: In this paper, we find the degree of best approximation between a pair of a nearly intertwining polynomials, and a pair of a nearly intertwining splines to a non-negative function $f \in L_{\psi,p}(I) \cap \Delta^0(\mathbb{j}_{\mathfrak{s}})$, in "$L_{\psi,p}$, $0<p<1$", we find the order of best a nearly intertwining approximation in the above terms.

Keywords: approximation, nearly intertwining, spline, modulus of smoothness.

Mathematics Subject Classification: 41A10,41A50 .

1.Introduction: The weighted quasi normed space $L_{\psi,p}(I), 0<p<1$ have form([6]) :

$$"L_{\psi,p}(I)=\{f, f: I\subset\Re\to\Re : \left(\int_I \left|\frac{f(x)}{\psi(x)}\right|^p dx\right)^{\frac{1}{p}}<\infty, \quad 0<p<1\}"$$

.

And the (quasi) norm $(\|f\|_{L_{\psi,p}(I)}<\infty)$, where as always,

$$(\|f\|_{L_{\psi,p}(I)}=\left(\int_I \left|\frac{f(x)}{\psi(x)}\right|^p dx\right)^{\frac{1}{p}}) , x\in I$$

Let $\mathfrak{s}\geq 0$, so that $-b=\jmath_{\mathfrak{s}+1}<\jmath_{\mathfrak{s}}<\cdots<\jmath_1<\jmath_0=b$ for $\jmath_{\mathfrak{s}}\in\jmath_{\mathfrak{s}}$.

And we suppose $\Delta(\mathbb{j}_{\mathfrak{s}})$, are all set of non-negative functions f on I=$[-b,b]$, and we will write a function f which belongs to the same class $\Delta^0(\mathbb{j}_{\mathfrak{s}})$,is said to be copositive. Ration estimates of the approximation of the restriction are given in terms of $(\Delta_n(x)\omega_k^{\varphi}(f',\Delta_n(x))_p)$, in some inequality in this paper where $\Delta_n(x)=n^{-1}\sqrt{1-x^2}+n^{-2}$,and $c_1\Delta_n(x)\leq h_{\jmath}\leq c_2\Delta_n(x)$, ([6]). For $x\in\mathfrak{t}_{\jmath}=[\mathbb{X}_{\jmath+1},\mathbb{X}_{\jmath}]$,and c_1,c_2 are constants and $h_{\jmath}=|\mathfrak{t}_{\jmath}.|$ "Nearly intertwining approximation", in which intertwining points are allowed to shift by an amount no larger than $\Delta_n(\jmath_i)$, (using $\mathbb{j}_{\mathfrak{s}}^*$,instead of $(\mathbb{j}_{\mathfrak{s}})$, improves to the order of $(n^{-1}\omega_k^{\varphi}(f',n^{-1})_{\psi,p})$, for f .

2. Notations and Definitions:

Let $(\delta = min|\mathrm{j}_{i+1} - \mathrm{j}_i|, 0 \le i \le s)$ where $\mathrm{j}_0 = -b$ and $\mathrm{j}_{s+1} = b$, ([6]).

And let $\mathrm{I}_i = [\mathrm{j}_i^{(v)}, \mathrm{j}_i^{(\mathcal{K}-1)}]$, and $\mathbb{J}_i = \left[\frac{\mathrm{j}_i+\mathrm{j}_i^{(v)}}{\mathcal{K}-1}, \frac{\mathrm{j}_i+\mathrm{j}_i^{(\mathcal{K}-1)}}{\mathcal{K}-1}\right]$.

Such that $\acute{\mathrm{j}}_i < \acute{\acute{\mathrm{j}}}_t < \cdots < \mathrm{j}_i^{(\mathcal{K}-1)}$, $v = 1, \ldots, \mathcal{K} - 2$.

$(c_1 \hbar_j \le |\mathrm{I}_i| = (\mathcal{K}-1)|\mathbb{J}_i| \le c_2 \hbar_j)$, where $c_i, i = 1,2$ positive number.

$$\mathbb{j}_s^* = \left\{\mathrm{j}_i^{(v)}, v = 1, \ldots, (\mathcal{K}-1) : \mathbb{X}_{j(i)+1} = \acute{\mathrm{j}}_t < \cdots < \mathrm{j}_i^{(\mathcal{K}-1)} = \mathbb{X}_{j(i)}, 0 < i \le s, j = 1, \ldots, n\right\}$$

$\Delta^0(\mathbb{j}_s^*)$,setall functions $(f), \ni (-1)^{s-i} f(x) \ge 0$.We denote

$$(\mathrm{I}_i \setminus \mathbb{j}_i = \left[\mathrm{j}_i^{(v)}, \frac{\mathrm{j}_i + \mathrm{j}_i^{(v)}}{\mathcal{K}-1}\right) \cup \left(\frac{\mathrm{j}_i + \mathrm{j}_i^{(\mathcal{K}-1)}}{\mathcal{K}-1}, \mathrm{j}_i^{(\mathcal{K}-1)}\right]).$$

Now we will write some important definitions in our work

$$\left((alm\Delta)^0(\mathbb{j}_s^*)\right) = \left\{f : \left((-1)^{s-i} f(x)\right) \ge 0, x \in \mathrm{I}_i \setminus \mathbb{j}_i\right\}).$$

Let

$$\Delta_h^{\mathcal{K}}(f, x, I)_\psi = \Delta_h^{\mathcal{K}}((f, x)_\psi = \begin{cases} \sum_{i=0}^{\mathcal{K}} \binom{\mathcal{K}}{i} (-1)^{\mathcal{K}-i} \dfrac{f(x - \frac{\mathcal{K}h}{2} + ih))}{\psi\left(x + \frac{\mathcal{K}h}{2}\right)} & , \quad x \pm \dfrac{(\mathcal{K}h)}{2} \in I \\ 0 & , \quad o.w. \end{cases}.$$

The "Ditzian-Totik modulus of smoothness" of (f)([6]):

$$\omega_{\mathcal{K}}^{\varphi}(f, , \delta, I)_{\psi,p} = \sup_{0 < h \le \delta} \left\| \Delta_{h\varphi(.)}^{k}(f, .) \right\|_{L_{\psi,p}(I)}.$$

The degree of "almost intertwining polynomial" of (f)([6]):

$$\widetilde{E_n}(f, alm\mathbb{j}_s)_{\psi,p} = inf\left\{\|p - f\|_{L_{\psi,p,}} + \|f - q\|_{L_{\psi,p,}} : p, q \in \prod_n, (-1)^{s-i}(p(x) - f(x)) \ge 0, (-1)^{s-i}((f(x) - q(x) \ge 0)\right\}.$$

The degree of "nearly intertwining polynomial approximation"of(f), with respect to$\mathbb{j}_s$([6]):

$$\widetilde{E_n}(f, nearly\mathbb{j}_s)_{\psi,p} = inf\left\{(\|P - Q\|_{L_{\psi,p,}}) : P, Q \in \prod_n, P(x) - f(x) \in \Delta^0(\mathbb{j}_s^*), f(x) - Q(x)\Delta^0(\mathbb{j}_s^*)\right\}.$$

From the definitions above we get for $f \in L_{\psi,p}(I) \cap \Delta^0(\mathbb{j}_s)$([6]):

:

$$\widetilde{E_n}(f, alm\mathbb{j}_s)_{\psi,p} \le \widetilde{E_n}(f, nearly\mathbb{j}_s)_{\psi,p} \le \widetilde{E_n}(f, \mathbb{j}_s)_{\psi,p,}. \quad \ldots(1$$

3. Auxiliary Result:

In the following theorems we show that a"nearly intertwining approximation"by $\{p_1, p_2\} \subset \Pi_r$, a nearly intertwining polynomials has an order,$C|\mathrm{I}_i|\omega_{\mathcal{K}}^{\varphi}(\acute{f}, |\mathrm{I}_i|, \mathrm{I}_i)_{\psi,p}$ and the generalization to $f \in L_{\psi,p} \cap \Delta^0(\mathbb{j}_s)$, which has an order $Cn^{-1}\omega_{\mathcal{K}}^{\varphi}(\acute{f}, n^{-1})_{\psi,p}$, also a nearly intertwining approximation by $\{S_1, S_2\}$, from the order (r), on the knot sequence$\{\mathbb{X}_j\}_{j=0}^{n}$, a "nearly intertwining splines" has an order $C|\mathbb{j}_{\mathcal{K}}|\omega_{\mathcal{K}}^{\varphi}(\acute{f}, |\mathbb{j}_{\mathcal{K}}|, \mathbb{j}_{\mathcal{K}})_{\psi,p}$, where$C$, both of the above dependent on (p, κ).

Theorem (1):
Let $f \in L_{\psi,p}(\mathrm{I}_i) \cap \Delta^0(\mathbb{j}_s^*)$, and$\mathbb{j}_i \subset \mathrm{I}_i$, be two sub intervals of I .Then there exist $\mathrm{j}_1 \in \mathbb{j}_1$, and two polynomial p_1, p_2 of degree k $\ni \{p_1, p_2\}$a nearly intertwining pair for f $\ni p_2(x) \le f(x) \le p_1(x)$ onI_i, with respect to $\{\mathrm{j}_1\}$, that satisfies:

$$\widetilde{E_n}(f, nearly\mathbb{j}_s)_{\psi,p} \le C|\mathrm{I}_i|\omega_{\mathcal{K}}^{\varphi}(\acute{f}, |\mathrm{I}_i|\mathrm{I}_i)_{\psi,p,}. \quad \ldots(2$$

Where C, dependent on (p, κ).

Proof: It is known:

"Any inferable functions (h) on $(-\infty, \infty)$,

$$|\{\chi : M(h(\chi)) > \tau\}| \le c\tau^{-1} \int_{-\infty}^{\infty} |h(\chi)|, \tau > 0,$$

where $M(h(\chi)) = \sup|J|^{-1}\int_J |h(\chi)|$, is the "$Hardy - Little\ wood\ maximal\ operator$" [4])).

Let

$$F(x) = \begin{cases} \left(\frac{f}{\psi}\right)(\mathrm{j}_i^{(v)}) & ; \quad x < \mathrm{j}_i^{(v)} \\ \frac{1}{2}\left(\frac{f}{\psi}\right)(x) & ; \quad x \in \mathrm{I}_i \\ \left(\frac{f}{\psi}\right)(\mathrm{j}_i^{(\mathcal{K}-1)}) & ; \quad x > \mathrm{j}_i^{(\mathcal{K}-1)} \end{cases},$$

And $t = 2c|\mathbb{j}_i|^{-1}|\mathrm{I}_i|\int_{\mathrm{I}_i}|\acute{f}|$.

By using the above operator to F', we get: $|\{x: M(F(x))\acute{} > t\}| \le Ct^{-1}\int_{-\infty}^{\infty}|F(x)|$

$$= Ct^{-1}\int_{\mathrm{I}_i}|\acute{F}(x)| = \frac{C}{4}t^{-1}\int_{\mathrm{I}_i}\left|\left(\frac{f}{\psi}\right)'(x)\right| = \frac{1}{2}\frac{|\mathbb{j}_i|}{|\mathrm{I}_i|} \; ; \; 0 < p < 1 .$$

Thus there exists $\mathrm{j}_i \in \mathbb{j}_i$, such that $M(F(\mathrm{j}_1))\acute{} \le t$.

Now when $\mathcal{K} > 1, v = 1, \dots, \mathcal{K}-2$, that is:

$$L_1(x) = f(\mathrm{j}_1) + t\frac{(\mathrm{j}_i^{(\mathcal{K}-1)} - \mathrm{j}_i^{(v)})}{\mathcal{K}-1},$$

$$L_2(x) = f(\mathrm{j}_1) - t\frac{(\mathrm{j}_i^{(\mathcal{K}-1)} - \mathrm{j}_i^{(v)})}{\mathcal{K}-1}, \quad \mathcal{K} > 1, v = 1, \dots, \mathcal{K}-2$$

They will form an "intertwining pairs" of f on I_i with respect to $\{\mathrm{j}_i\}$. We have from $L_1(x)$, $\mathrm{j}_i \le x \le \mathrm{j}_i^{(\mathcal{K}-1)}$ it is easy from modified Chebyshev partition ([6]),and since $M\left(\acute{F}(\mathrm{j}_1)\right) \le t$, hence

$$\frac{(\mathrm{j}_i^{(\mathcal{K}-1)} - \mathrm{j}_i^{(v)})}{\mathcal{K}-1} M(\acute{F})(\mathrm{j}_i) \le t\frac{(\mathrm{j}_i^{(\mathcal{K}-1)} - \mathrm{j}_i^{(v)})}{\mathcal{K}-1}, \text{ and}$$

$$f(\mathrm{j}_1) + \frac{(\mathrm{j}_i^{(\mathcal{K}-1)} - \mathrm{j}_i^{(v)})}{\mathcal{K}-1} M((\acute{F})(\mathrm{j}_1)) \le f(\mathrm{j}_1) + t\frac{(\mathrm{j}_i^{(\mathcal{K}-1)} - \mathrm{j}_i^{(v)})}{\mathcal{K}-1},$$

Hence

$$f(\mathrm{j}_1) + \frac{(\mathrm{j}_i^{(\mathcal{K}-1)} - \mathrm{j}_i^{(v)})}{\mathcal{K}-1} M((\acute{F})(\mathrm{j}_1)) \le L_1(x),$$

For an error estimate of L_1 and L_2 we first note from I_i, $L_2(x) \le f(x) \le L_1(x)$

$$|L_1(x) - L_2(x)| = \left| f(\mathrm{j}_1) + t\frac{(\mathrm{j}_i^{(\mathcal{K}-1)} - \mathrm{j}_i^{(v)})}{\mathcal{K}-1} - f(\mathrm{j}_1) + t\frac{(\mathrm{j}_i^{(\mathcal{K}-1)} - \mathrm{j}_i^{(v)})}{\mathcal{K}-1} \right|$$

$$= 2t\left|\frac{(\mathrm{j}_i^{(\mathcal{K}-1)} - \mathrm{j}_i^{(v)})}{\mathcal{K}-1}\right|$$

$$= 2\left|\frac{(\mathrm{j}_i^{(\mathcal{K}-1)} - \mathrm{j}_i^{(v)})}{\mathcal{K}-1}\right|\left(\frac{2C}{4}|\mathrm{j}_i|^{-1}|\mathrm{I}_i|\int_{\mathrm{I}_i}\left|\left(\frac{f}{\psi}\right)'(x)\right|\right)$$

$$\le C|\mathrm{I}_i|\int_{\mathrm{I}_i}\left|\left(\frac{f}{\psi}\right)'(x)\right|, \text{ where } C$$

dependents on $p, \mathcal{K}$.

By (Theorem (1.2.1),[6]):

$$\int_{\mathrm{I}_i}|L_1(x) - L_2(x)|^p \le C^p|\mathrm{I}_i|^p\int_{\mathrm{I}_i}\left|\left(\frac{f}{\psi}\right)'(x)\right|^p$$

And since $|\mathrm{I}_i| \le Ch_j$ and $h_j \approx \Delta_n(x)$, then we get from the above that :

$$\|L_1 - L_2\|_{L_{\psi,p}(\mathrm{I}_i)} \le C\Delta_n(x)\|\acute{f}\|_{L_{\psi,p}(\mathrm{I}_i)}, \quad \dots (3$$

Let P' be a "best polynomial approximation" to (f') on I_i, of degree $r-1$, $P = \int_{\mathrm{j}_i}^{x}\acute{P}(t)dt$, Since $\int_{\mathrm{j}_i}^{x}\acute{P}(t)dt > 0$, this implies that $P \in \Delta^0(\mathbb{j}_s^*)$, to prove (2) apply (3) to $f - P \in \Delta^0(\mathbb{j}_s^*)$, then $\{L_1, L_2\}$, a nearly intertwining pair for $f - \mathrm{P}$.

Define $p_i = L_i(x) + \mathrm{P}, i = 1,2$, obviously $p_1 - f, p_2 - f \in \Delta^0(\mathbb{j}_s^*)$, and $|\mathrm{j}_i^{(v)} - \mathrm{j}_i| \le \Delta_n(\mathrm{j}_i)$, and $\{p_1, p_2\}$, a "nearly intertwining pair" of polynomials of degree r for f on I_i, and

$$\|p_1 - p_2\|_{L_{\psi,p}(\mathrm{I}_i)} = \|L_1 - L_2\|_{L_{\psi,p}(\mathrm{I}_i)} \le C\Delta_n(x)\|\acute{f} - \acute{P}\|_{L_{\psi,p}(\mathrm{I}_i)},$$

by using (Theorem 2.3.2[6]) we get $\|p_1 - p_2\|_{L_{\psi,p}(\mathrm{I}_i)} \le C\Delta_n(x)\omega_{\mathcal{K}}^{\varphi}(\acute{f}, |\mathrm{I}_i|, \mathrm{I}_i)_{\psi,p}$, hence

$$\|p_1 - p_2\|_{L_{\psi,p}(\mathrm{I}_i)} \le C|\mathrm{I}_i|\omega_{\mathcal{K}}^{\varphi}(\acute{f}, |\mathrm{I}_i|, \mathrm{I}_i)_{\psi,p}, \quad \dots (4$$

Now ,by using the definition of best nearly intertwining approximation by a pair $\{p_1, p_2\}$, we get

$$\widetilde{E_n}(f, nearly\mathbb{j}_s^*)_{\psi,p} \le\le C|\mathrm{I}_i|\omega_{\mathcal{K}}^{\varphi}(\acute{f}, |\mathrm{I}_i|, \mathrm{I}_i)_{\psi,p}, \quad \dots (5$$

The result (5) can be generalized by using (Theorem 2.1.2[6]) when: $f \in L_{\psi,p}(I) \cap \Delta^0(\mathbb{j}_s)$, $I = [-b, b]$, specifically when the function is derived by a pair $\{p_1, p_2\}$, this implies that

$$\widetilde{E_n}(f, nearly\mathbb{j}_s)_{\psi,p} \le Cn^{-1}\omega_{\mathcal{K}}^{\varphi}(\acute{f}, n^{-1})_{\psi,p}, \quad \dots (6$$

Also there exist a nearly intertwining pair of polynomial $\{p_1, p_2\} \subset \Pi_n$, satisfy

$|p_1(x) - p_2(x)| = |L_1(x) - L_2(x)| \le C|I_i| \|\acute{f}\|_{L_{\psi,p}(I_i)}$,

and there for

$|p_1(x) - p_2(x)| \le C\Delta_n(x)\omega_{\mathcal{K}}^{\varphi}(\acute{f}, \Delta_n(x))_{\psi,p}$.

Corollary: Suppose $f \in L_{\psi,p}(I) \cap \Delta^0(\mathbb{j}_s)$, $0 < p < 1, k > 1$, there is a pair of " a nearly intertwining polynomials " $\{p_1, p_2\}$, of degree r to f with respect to $\{j_1\}$, satisfies:

i) $\widetilde{E_n}(f, nearly\mathbb{j}_s)_{\psi,p} \le Cn^{-1}\omega_{\mathcal{K}}(\acute{f}, n^{-1})_{\psi,p}$.

ii) $\widetilde{E_n}(f, almy\mathbb{j}_s)_{\psi,p} \le Cn^{-1}\omega_{\mathcal{K}}(\acute{f}, n^{-1})_{\psi,p}$.

Where C, dependents on p, $\mathcal{K}$.

Proof:

i) By using (Theorem 1.6.3 [6]), and the result (5) then the prove is complete.

ii) By the relationship (1) and the result (i) of this corollary we get the result.

Theorem(2): Suppose $f \in L_{\psi,p}(I) \cap \Delta^0(\mathbb{j}_s)$, $\mathbb{j}_s = \{j_i, i = 1, \dots, s : -b = j_0 < j_1 < \cdots < j_s < j_{s+1} = b\}$. Let $\{\mathbb{X}_j\}_{j=0}^{n}$, is a single knot sequence, there is a pair of "a nearly intertwining spline" $\{S_1, S_2\}$, of degree $r, r \ge 2$ on $\{\mathbb{X}_j\}_{j=0}^{n}$, for f with respect to $\mathbb{j}_s$, satisfy :

$\|S_1 - S_2\|_{L_{\psi,p}(\mathbb{j}_\kappa)} \le C|\mathbb{j}_\kappa|\omega_{\mathcal{K}}^{\varphi}(\acute{f}, |\mathbb{j}_\kappa|, \mathbb{j}_\kappa)_{\psi,p}$.

Where C, dependents on p, $\mathcal{K}$.

Proof: Let there exist a polynomials $\{p_1, p_2\}$, of degree r, and interpolate f at k points at $\mathbb{j}_c = [b - \mu|I|, b], 0 < \mu < \frac{1}{2}$, (Theorem(2.3.4):[6]). By using differentiated in (theorem(2.1.2)[6]) in case $b - \mu|I| < b - \frac{1}{2}\mu|I|$, and by the result (4) then there exist "a nearly intertwining pair of a polynomials " $\{p_1, p_2\} \subset \Pi_r$, for f at $\mathbb{j}_c \ni$ " $p_2(x) \le f(x) \le p_1(x)$", $\forall x \in \mathbb{j}_c$, hence

$\|p_1 - p_2\|_{L_{\psi,p}(I_i)} \le C|\mathbb{j}_c|\omega_{\mathcal{K}}^{\varphi}(\acute{f}, |\mathbb{j}_c|, \mathbb{j}_c)_{\psi,p}$.

Now, we define $\overline{p_1}$ and $\overline{p_2}$ on $\mathbb{j}_c$, by $\overline{p_1} = p_1$ and $\overline{p_2} = p_2$ if $(-1)^{s-i} > 0$, $i = 1, \dots, s$, and $\overline{p_1} = p_2$ and $\overline{p_2} = p_1$ if $(-1)^{s-i} < 0$. Hence "$((-1)^{s-i}(\overline{p_1(x)} - f(x)) \ge 0$, $(-1)^{s-i}(\overline{p_1(x)} - f(x)) \le 0$", and

$\|\bar{p}_1 - \bar{p}_2\|_{L_{\psi,p}(\mathbb{j}_c)} \le C|\mathbb{j}_c|\omega_{\mathcal{K}}^{\varphi}(\acute{f}, |\mathbb{j}_c|, \mathbb{j}_c)_{\psi,p}$.… (3.7

Near thepoint $(-b + \mu|I|)$, we will construct different local polynomial. Specifically, we will approximate (f'), at $\mathbb{j}_{\mathcal{A}} = [-b + \mu|I|, b - \mu|I|]$. From the above also there exist a pair of polynomial $\{P, Q\}$ of degree $< r - 1$, such that $Q(x) \le f'(x) \le P(x), \forall x \in \mathbb{j}_{\mathcal{A}}$, then

$\|P - Q\|_{L_{\psi,p}(\mathbb{j}_{\mathcal{A}})} \le C|\mathbb{j}_{\mathcal{A}}|\omega_{\mathcal{K}}^{\varphi}(\acute{f}, |\mathbb{j}_{\mathcal{A}}|, \mathbb{j}_{\mathcal{A}})_{\psi,p}$.

Let "$P^* = P$ and $Q^* = Q$", if $((-1)^{s-i} > 0)$, and "$P^* = Q$ and $Q^* = P$" if $((-1)^{s-i} \le 0)$. Now to check that

$\overline{P^*(x)} = \int_{-b+\mu|I|}^{x} P^*(t)dt + f(-b + \mu|I|)$, and

$\overline{Q^*(x)} = \int_{-b+\mu|I|}^{x} Q^*(t)dt + f(-b + \mu|I|)$,

satisfy the inequalities $((-1)^{s-i}(P^*(x) - f(x))\operatorname{sgn}(x - (-b + \mu|I|)) \ge 0$, $((-1)^{s-i}(Q^*(x) - f(x))\operatorname{sgn}(x - (-b + \mu|I|)) \le 0$, hence

$$\|\bar{P}^* - \bar{Q}^*\|_{L_{\psi,p}(\mathbb{j}_{\mathcal{A}})} = \left\| \int_{-b+\mu|I|}^{x} (P^*(t) - Q^*(t))dt \right\|_{L_{\psi,p}(\mathbb{j}_{\mathcal{A}})},$$

$$= \left\| \int_{-b+\mu|I|}^{x} (P(t) - Q(t))dt \right\|_{L_{\psi,p}(\mathbb{j}_{\mathcal{A}})}$$

$$\le \left\| \int_{\mathbb{j}_{\mathcal{A}}} (P(t) - Q(t)) \right\|_{L_{\psi,p}(\mathbb{j}_{\mathcal{A}})},$$

$\le C|\mathbb{j}_{\mathcal{A}}| \|P - Q\|_{L_{\psi,p}(\mathbb{j}_{\mathcal{A}})}$,

$\le C|\mathbb{j}_{\mathcal{A}}|^2\omega_{\mathcal{K}}^{\varphi}(\acute{f}, |\mathbb{j}_{\mathcal{A}}|, \mathbb{j}_{\mathcal{A}})_{\psi,p}$, that is

$\|\bar{P}^* - \bar{Q}^*\|_{L_{\psi,p}(\mathbb{j}_c)} \le C|\mathbb{j}_{\mathcal{A}}|^2\omega_{\mathcal{K}}^{\varphi}(\acute{f}, |\mathbb{j}_{\mathcal{A}}|, \mathbb{j}_{\mathcal{A}})_{\psi,p}$.

After the interlocking polynomials have been configured "intertwining",with the function f , which has a right approximation order, now we will merge these polynomials together in the form of "smooth spline approximants"(S_1 ,S_2) , on $\{\mathbb{X}_j\}_{j=0}^{n}$. If both $\mathfrak{j}_{\mathcal{B}}$ $=[b-\mu|I|,b-\frac{1}{2}\mu|I|]$, and $\mathfrak{j}_c$, are non-contaminated, then P_3 and P_2, overlap on$\mathfrak{j}_{\mathcal{B}}$, which contains (m) interior knots from $\{\mathbb{X}_j\}_{j=0}^{n}$. By "Beatsons Lemma "[4]), ∃ a splines $\bar{S}_i$, has order r at $\mathfrak{j}_c$,on $\{\mathbb{X}_j\}_{j=0}^{n}$,which are associated with a polynomials P_3 , P_2, in technique at points" $b-\mu|I|,b-\frac{1}{2}\mu|I|$ ", countinuously.

Furthermore, the draw of splines $\bar{S}_i$, it is located between the polynomials" P_3 , P_2 " , accordingly:

" $sgn\,(P_3(x)-f(x)) = sgn\,(P_2(x)-f(x)) = sgn\,(\bar{S}_i(x)-f(x))$ " , $x\in \mathfrak{j}_c$.

By the same method, taking in to account the overlapping polynomials " P_1 , P_4 "

" $sgn\,(P_4(x)-f(x)) = sgn\,(P_1(x)-f(x)) = sgn\,(S_i(x)-f(x))$ " ,

$x\in \mathfrak{j}_{\mathcal{T}}=[-b+\mu|I|,b-\frac{1}{2}\mu|I|]$.Then we get by ([1])that:

$$\int_{\mathfrak{j}_{\mathcal{T}}}|\bar{S}_i-S_i|^p \le 2^p\left(\int_{\mathfrak{j}_{\mathcal{T}}}|P_3-P_4|^p + \int_{\mathfrak{j}_{\mathcal{T}}}|P_1-P_2|^p\right) ,$$

by the inequality (7) on an interval $\mathfrak{j}_{\mathcal{T}}$, then:

$\|\bar{S}_i-S_i\|_{L_{\psi,p}(\mathfrak{j}_{\mathcal{T}})} \le C|\mathfrak{j}_{\mathcal{T}}|\omega_{\mathcal{K}}^{\varphi}(\acute{f},|\mathfrak{j}_{\mathcal{T}}|,\mathfrak{j}_{\mathcal{T}})_{\psi,p}$.… (8

In the same way, we can overlapping a polynomial pieces which fall within the periods contaminated intervals. The Spline pieces $\bar{S}_i$,S_i , check the same guess above with a slightly larger interval of $\mathfrak{j}_{\mathcal{T}}$, on the right-hand side.

Now, we define the final spline S_1 over $\mathfrak{j}_c$ as follows:

If there is only one polynomial P_1 over $\mathfrak{j}_c$,then we set S_1 to P_1. If there are two polynomials overlapping on $\mathfrak{j}_c$,must be a combination spline $\bar{S}_i$, set S_1 to $\bar{S}_i$. We get from the above $S_1-f\in\Delta^0(\mathfrak{j}_s)$, on an interval $I=[-b,b]$.By the same method we set $S_2-f\in\Delta^0(\mathfrak{j}_s)$.Since the intervals " ($\mathfrak{j}_i$, $\left[\frac{(\mathrm{j}_i+\mathrm{j}_i^{(\mathcal{K}-1)})}{\mathcal{K}-1},\tilde{a}\right]$, $\left[\tilde{b},\frac{\mathrm{j}_i+\mathrm{j}_i^{(v)})}{\mathcal{K}-1}\right]$) " , in the partition o (I_i), where" $\tilde{a}=\frac{\mathrm{j}_i+\mathrm{j}_i^{(\mathcal{K}-1)}}{\mathcal{K}-1}+\mu|\mathfrak{j}_i|$ " and" $\tilde{b}=\frac{\mathrm{j}_i+\mathrm{j}_i^{(v)}}{\mathcal{K}-1}-\mu|\mathfrak{j}_i|$ ", (see Lemma (2.3.1)[6]),can be compared to size. And each interval $\mathfrak{j}_\kappa=[-b+\mu|I|,b]$,denote contain more than (m), such interval ((the value of (m) depends on the length of the original interval)).Therefore we can get the result from (7)and (8), that is

$\|S_1-S_2\|_{L_{\psi,p}(\mathfrak{j}_\kappa)} \le C|\mathfrak{j}_\kappa|\omega_{\mathcal{K}}^{\varphi}(\acute{f},|\mathfrak{j}_\kappa|,\mathfrak{j}_\kappa)_{\psi,p}.$

Where C , dependents on p ,$\mathcal{K}$.

References:

[1] Y.K.HU,K.A.Kopotun and X.M.YU,"Constrained approximation in Sobolev Spaces", Canada.Math.Bull.Vol.49 (1),1997 pp. 74-99.

[2] K.A. Kopotun, On k-monotone polynomial and spline approximation in $L_p(0<p<\infty)$ (quasi)norm,Approximation Theory VIII,World Scientific Publishing Co.,(eds.C.Chui and L.Schumaker),(1995),295-302.

[3] R. A. DeVore and G. G. Lorentz, ``Constructive Approximation," Springer-Verlag, Berlin,1993.

[4] R.K. Beatson, Restricted range approximation by splines and variational inequalities,SIAMJ.Numer.Anal.19(1982),372-380.

[5] R.A.DeVore,D.Leviatan and X.M.Yu, polynomial approximation in $L_p(0<p<1)$ space,Constr.Approx.8(1992),187-201.

[6] N.Z.Abd Al-Sada (2015):" On Positive and Copositive Approximation in $L_{\psi,p}(I)$ Spaces $0<p<1$"Ph.D dissertation, Al-Mustansiriya University, College of education.

[7] Y.K.Hu,K.Kopotun and X.M.Yu,On positive and copositive polynomial and spline approximation in $L_p[-1,1],0<p<\infty$,J.Approx.Theory 86(1996),320-334.

[8] Z.Ditzian and V.Totik,Moduli of Smoothness,Berlin,SPRINGER-verlag,1987.

Journal of AL-Qadisiyah for computer science and mathematics Vol.10 No.2 Year 2018
ISSN (Print): 2074 – 0204 ISSN (Online): 2521 – 3504

التقريب المتشابك في الفضاء $L_{\psi,p}(I)$, $0 < p < 1$

ندى زهير عبد السادة

قسم الرياضيات، كلية التربية جامعة القادسية
E-mail:Nadawee70@yahoo.com

المستخلص:

في هذا البحث تم ايجاد درجة افضل تقريب لزوج من متعددات الحدود المتشابكة بين زوج من شرائح متعددات الحدود المتشابكة تقريبا للدالة المقيدة, $f \in L_{\psi,p}(I) \cap \Delta^0(\mathfrak{j}_s)$ ، في الفضاء $0 < p < 1$, $L_{\psi,p}$, هذا يعني ايجاد رتبة افضل تقريب متشابك تقريبا للمصطلحات اعلاه .