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Non-polynomial spline finite difference method for solving second order boundary value problem

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Abstract

Second -order two-point boundary value problems (BVPs) were solved based on a of non-polynomial spline general functions with finite difference method.In this paper we discusses a method depended on using special finite-difference approximations for derivatives and formatting a formula that can be deal with endpoints that exceed the usual finite-difference formula for derivatives. Convergence analysis of the method is discussed. The numerical description of the method is shown by four examples. So the results obtained by our method are very encouraging over other existing methods.

Keywords: Non-polynomial spline method, Finite difference, Boundary value problem, Truncation error, Exact solution.

Mathematics Classification 65L1265M0641A15.

1. Introduction

In this paper our concern is approximating the solution of the second order linear BVP by using non-polynomial spline function[5-9,11-13],we discussed the numerical solution for two cases of the problem using non-polynomial spline function.
consider the linear second order BVP of the form [7,10]

$$y''(x)+r(x)y'+s(x)y=g(x) \quad ,x\in[a,b] \qquad (1)$$

subject to the boundary conditions (B.Cs)

$$(A) \quad y(a)=\alpha_1 \quad ,y(b)=\beta_1$$

$$(B) \quad y'(a)=\alpha_2 \quad ,y'(b)=\beta_2 .$$

Where $r(x)$, $s(x)$ and $g(x)$ are continuous function on $[a,b]$, $\alpha_1, \beta_1, \alpha_2$ and β_2 are real constant .The main objective of our research is to introduce a new spline method to approximate the second order BVP as in (1).

This paper is organized as follows. In section2, derive of the method with (B.Cs)-(A). Analysis of the method in section 3.In section4, derive of the method with (B.Cs)-(B). In section 5, non-polynomial spline solution.Section 6 convergence analysis .Section 7 some numerical examples to show the performance of the proposed method and for comparison purposes with another numerically methods. Finally, the conclusion is given in section 8.
The main objective of our research is to introduce a new non-polynomial spline method to approximate the second order boundary value problem as in (1).

2 . Derive of the method with B.Cs -(A)

We introduced a finite set of grid point x_i by dividing the interval $[a,b]$ into n equal parts where $x_i = a + ib$,i=0,1,2,...,n, $x_0 = a$ $x_n = b$ and $h = \frac{b-a}{n+1}$.

Let $y(x)$ be the exact solution of problem (1) and y_i be the approximation value to $y(x_i)$ value obtained by the spline function $\varphi_i(x)$ passing through the point (x_i, y_i) and (x_{i+1}, y_{i+1}).
Each non-polynomial spline segment has the form [4]

$$\varphi_i(x) = a_i + b_i(x - x_i) + c_i \sinh(k(x - x_i)) + d_i \cosh(k(x - x_i)) \quad (2)$$
$$, i = 0,1,2,\dots,n-1$$

Where a_i, b_i, c_i, d_i are constants and k is free parameter to be determined later. Our non-polynomial spline is now defined by the relations
(i)

$$y_i(x) = \varphi_i(x) \quad , x \in [x_i, x_{i+1}] \quad i = 1,2,\dots,n$$
.
(ii)

$$\varphi_i(x) \in C^{\infty}[x_i, x_{i+1}].$$

(3)
First, we developd expressions for the four coefficients a_i, b_i, c_i, d_i of (2) in terms of $y_i, y_{i+1}, \psi_i, \psi_{i+1}$,where

$$(i)\varphi_i(x_i) = y_i \qquad (ii)\varphi_i(x_{i+1}) = y_{i+1}$$
$$(i)\varphi_i''(x_i) = \psi_i \qquad (ii)\varphi_i''(x_{i+1}) = \psi_{i+1}$$
(4)

From Eq. (2) and (4), we obtained via a straight forward calculation the following expression:

$$a_i = y_i - \frac{h^2}{\theta^2}\psi_i \ , \ b_i = \frac{1}{h}(y_{i+1} - y_i) + \frac{h}{\theta^2}(\psi_i - \psi_{i+1}) \ ,$$
$$d_i = \frac{h^2}{\theta^2}\psi_i \quad and \quad c_i = \frac{h^2}{\theta^2 \sinh\theta}(\psi_{i+1} - \psi_i \cosh\theta)$$
$$where \quad i = 0,1,2,\dots,n-1 \quad and \quad \theta = kh. \qquad (5)$$

Now using the continuity of the first derivatives at the point (x_i, y_i).that is $\varphi_{i-1}'(x_i) = \varphi_i'(x_i)$ yield the following relations:

$$y_{i-1} - 2y_i + y_{i+1} = h^2(\xi\psi_{i-1} + 2\eta\psi_i + \xi\psi_{i+1}) \qquad (6)$$

where

$$\psi_i = -r(x_i)y' - s(x_i)y_i + g_i(x_i),$$
$$\xi = \left(\frac{h}{\theta^2} - \frac{h}{\theta\sinh\theta}\right) \ and \ \eta = \left(\frac{-2}{\theta^2} + \frac{2\cosh\theta}{\theta\sinh\theta}\right) \ .$$

The truncation errors,t_i $, i = 1,2,\dots,n$ associated with the scheme (6) can be obtained as follows: first we re-write the scheme (6) in the form

$$y_{i-1} - 2y_i + y_{i+1} = h^2[\xi y_{i-1}'' + 2\eta y_i'' + \xi y_{i+1}''] + t_i$$
$$for\ i = 1,2,3,\dots,n \qquad (7)$$

The terms y_{i-1}, y_i, y_{i+1}. in Eq.(6)are expanded around the point x_i using the Taylor series and the expansion for t_i i=1,2,3,….n, can be obtained.

$$t_i = \begin{cases} (1-(2\xi+2\eta)h^2 y_i'' + \frac{h^4}{12}(1-12\xi)y_i^{(4)} + \\ \frac{24}{360}(1-30\xi)h^6 y_i^{(6)} + O(h^8) \\ i = 1,2,3,\dots,n. \end{cases} \qquad (8)$$

The scheme (6) gives rise to family of method of different order as follows:
1-second order methods

$$As \quad k \to 0 \quad , \xi = \frac{1}{6} \ and \ \eta = \frac{1}{3}$$ .Then

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local truncation error is

$$t_i = -\frac{1}{12}h^4 y_i^{(4)} + O(h^6) \qquad i = 1,2,\dots,n \qquad (9)$$

2-Forth order methods

For $\xi = \frac{1}{12}$ *and* $\eta = \frac{5}{12}$ .Then the truncation errors in Eq.(6) is

$$t_i = -\frac{h^6}{240}h^6 y_i^{(6)} + O(h^8) \qquad i = 1,2,\dots n$$

(10)

3 .Analysis of the method with B.Cs.(A)

To illustrate the application of the spline method developed in the previous section we considered the linear boundary value problem that is given in Eq. (1)at grid point (x_i, y_i) . we choose an integer $n > 0$ and divide the interval $[a,b]$ in n equal subintervals .Where the step size $h = \frac{b-a}{n+1}$,also approximate forth derivative in Eq.(1) by using non-polynomial spline by substituting $\varphi_i = y_i''$ in Eq.(1) we get the following Eqs.

$$\psi_{i-1} = -r(x_{i-1})y'_{i-1} - s(x_{i-1})y_{i-1} + g(x_{i-1}) \quad (11)$$

$$\psi_i = -r(x_i)y'_i - s(x_i)y_i + g(x_i) \quad (12)$$

$$\psi_{i+1} = -r(x_{i+1})y'_{i+1} - s(x_{i+1})y_{i+1} + g(x_{i+1}) \quad (13)$$

The first derivative approximate by using finite difference as follow[4]:

$$y'_i = \frac{y_{i+1} - y_{i-1}}{2h} \quad , \quad y'_{i-1} = \frac{-3y_{i-1} + 4y_i - y_{i+1}}{2h}$$

$$y'_{i+1} = \frac{y_{i-1} - 4y_i + 3y_{i+1}}{2h} \qquad (14)$$

So Eqs.(11)-(13) and using (14) are become in the form:

$$\psi_{i-1} = \frac{-r(x_{i-1})(-3y_{i-1} + 4y_i - y_{i+1})}{2h} - s(x_{i-1})y_{i-1} + g(x_{i-1}).$$

$$\psi_i = \frac{-r(x_i)(y_{i+1} - y_{i-1})}{2h} - s(x_i)y_i + g(x_i).$$

$$\psi_{i+1} = \frac{-r(x_{i+1})(y_{i-1} - 4y_i + 3y_{i+1})}{2h} - s(x_{i+1})y_{i+1} + g(x_{i+1}). \qquad (15)$$

Now the substituting of the Eq. (15) in (5) we get the following n linear algebraic Eqs. with n unknown

$$\left[-1 + \xi h\left(\frac{3r(x_{i-1})}{2h} - \frac{r(x_{i+1})}{2} - h^2 s(x_{i-1})\right) + \eta h r(x_i)\right] y_{i-1}$$

$$+\left[2 - 2\xi h(r(x_{i-1}) - r(x_{i+1})) - 2\eta h^2 s(x_i)\right] y_i$$

$$\left[-1 + \xi h\left(\frac{r(x_{i-1})}{h} - \frac{3r(x_{i+1})}{2}\right) - h^2 s(x_{i+1})) - \eta h r(x_i)\right] y_{i+1}$$

$$= -h^2 \xi (r(x_{i-1}) + r(x_{i+1})) - 2h^2 \eta r(x_i) \qquad for\ i = 1, 2, \dots, n$$

(16)

4. Derive the method with B.Cs-(B)

Proceeding like in section(2),but we needed to **derive special formula when** $i = 1, n$ **using the boundary conditions** , finite difference (14) and the step size $h = \frac{b-a}{n+1}$ as following:

$$\left[-\frac{2}{3} + \xi h^2\left(\frac{-4r(x_2)}{3h} + \frac{4s(x_0)}{3}\right) + 2\eta h^2\left(\frac{2r(x_1)}{3} + s(x_1)\right)\right] y_1$$

$$+\left[\frac{2}{3} + \xi h^2\left(\frac{4r(x_2)}{3h} - \frac{s(x_0)}{3} + s(x_2)\right) + 4\eta h^2 \frac{r(x_1)}{3}\right] y_2$$

$$= \left[\frac{2h}{3} + \xi h^2\left(\frac{r(x_2)}{3} - r(x_0) + \frac{2hs(x_0)}{3}\right) - \eta h^2 \frac{r(x_1)}{3}\right] \alpha_3$$

$$+ h^2 \xi (r(x_0) + r(x_2)) + 2h^2 \eta r(x_1) \qquad for\ i = 1$$

(17)

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$$\left[\frac{2}{3}+\xi h^2\left(\frac{-4r(x_{n-1})}{3h}-\frac{s(x_0)}{3}+s(x_{n-1})\right)-2\eta h^2\frac{r(x_n)}{3}\right]y_{n-1}$$
$$+\left[\frac{-2}{3}+\xi h^2\left(\frac{4r(x_{n-1})}{3h}-\frac{4s(x_{n+1})}{3}\right)+2\eta h^2\left(\frac{2r(x_n)}{3h}+s(x_n)\right)\right]y_n$$
$$=\left[\frac{-2h}{3}+\xi h^2\left(\frac{r(x_{n-1})}{3}-r(x_{n+1})-\frac{2hs(x_{n+1})}{3}\right)-\eta h^2\frac{r(x_n)}{3}\right]\alpha_4$$
$$+h^2\xi(r(x_{n-1})+r(x_{n+1}))+2h^2\eta r(x_n) \qquad for\ i=n.$$
(18)

5. Non-polynomial spline solutions

The scheme (16) gives rise to a linear system of order $(n\times n)$ and may be written in the matrix form as

$$Ay+h^2DG=Q$$
(19)

Where $A=C+hBr-h^2Bs$,

$$,C=\begin{bmatrix}2 & -1 & & & \\ -1 & 2 & -1 & & \\ & -1 & 2 & -1 & \\ & & & \ddots & \\ & & & -1 & 2\end{bmatrix},$$

and $Br=Z_{ij}$, $Bs=U_{ij}$ are define as

$$M_{ij}=\begin{cases}-2\xi(r(x_0)-r(x_2)) & i=1,\ j=1,\\ \xi\left(\frac{3r(x_{i-1})}{2}-\frac{r(x_{i+1})}{2}\right)-h\eta r(x_i) & i>j,\\ -2\xi(r(x_{i-1})-r(x_{i+1})) & i=j,\\ \xi\left(\frac{r(x_{i-1})}{2}-\frac{r(x_{i+1})}{2}\right)-h\eta r(x_i) & i<j,\\ -2\xi(r(x_{n-2})-r(x_n)) & i=j=n-1,\end{cases}$$

$$N_{ij}=\begin{cases}2\eta s(x_i), & i=j=1,2,\dots,n-1\\ \xi s(x_{i-1}), & i>j,\\ \xi s(x_{i+1}), & i<j\end{cases}$$

$$G=(g(x_1),g(x_2),\dots,g(x_n))^T,$$
$$Y=(y_1 y_2,\dots,y_n)^T$$

$$D=\begin{bmatrix}2\eta & \xi & & & & \\ \xi & 2\eta & \xi & & & \\ & \xi & 2\eta & \xi & & \\ & & & \ddots & & \\ & & & & \xi & 2\eta\end{bmatrix}$$

and $Q=(q_1,q_2,\dots,q_n)^T$

where

$$q_1=-h^2\xi r(x_0)-(-1+h\xi(\frac{3r(x_0)}{2}+\frac{r(x_0)}{2})-hs(x_0))+h\eta r(x_1))y_0$$

$$q_i=0 \quad ,i=2,3,\dots,n-2.$$

$$q_{n-1}=-h^2\xi r(x_n)-(-1+h\xi(\frac{r(x_{n-2})}{2}-\frac{3r(x_n)}{2})-hs(x_{n-2}))+h\eta r(x_{n-1}))y_n$$

we assume that

$$\bar{Y}=(y(x_1),y(x_2),\dots,y(x_{n-1}))^T.$$

Be the exact solution of the given boundary value problem (1) at nodal point x_i For $i=1,2,\dots,n$ .then we have

$$A\bar{Y}+h^2DG=T(h)+Q, \qquad (20)$$
$$A(\bar{Y}-Y)=AE=T(h)$$
(21)

6. Convergence analysis

In this section we discuss the convergence property of (6),we now turn back to the error eq. in (18) and rewrite it in the form:

$$E=A^{-1}T=[C+hBr-h^2Bs]^{-1}T=[I+C^{-1}(hBr-h^2Bs)]^{-1}C^{-1}T$$
$$\|E\|_\infty\le\|[I+C^{-1}(hBr-h^2Bs)\|_\infty\|C^{-1}\|_\infty\|T\|_\infty$$
(22)

In order to derive the bound on $\|E\|_\infty$,the following two lemmas are needed.

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Lemma(1) :The matrix ($C + hBr - h^2Bs$) is nonsingular if

$$\|r\|_\infty < \frac{8h\varepsilon}{(a-b)^2(8\xi+2\eta)} \; and \; \|s\|_\infty < \frac{8(1-\varepsilon)}{(a-b)^2}$$

$$where \; 0<\varepsilon<1.$$

Proof:

Since ,

$$A = C + hBr - h^2Bs = [I + C^{-1}(hBr - h^2Bs)]C$$

and the matrix N is nonsingular ,so to prove A nonsingular it's sufficient to show

$[I + C^{-1}(hBr - h^2Bs)]$nonsingular.

Since

$$\|C^{-1}(hBr - h^2Bs)\|_\infty \le \|C^{-1}\|_\infty \|hBr - h^2Bs\|_\infty \le \|C^{-1}\|_\infty (\|hBr\|_\infty + \|h^2Bs\|_\infty) \quad (23)$$

Moreover,

$$\|C^{-1}\|_\infty \le \frac{(a-b)^2}{8h^2} \quad [4, 10]$$

$$\|hBr\|_\infty \le h(8\xi+2\eta)\|r\|_\infty, \quad and \quad \|h^2Bs\|_\infty \le h^2\|s\|_\infty$$

Where

$$\|r\|_\infty = \max_{a\le x_i\le b}|r(x_i)|, \quad and \quad \|s\|_\infty = \max_{a\le x_i\le b}|s(x_i)|$$

substituting $\|hBr\|_\infty, \|C\|_\infty, \; and \; \|hBs\|_\infty$ in Eq. (23) we get,

$$\|C^{-1}(hBr - h^2Bs)\|_\infty \le \frac{(b-a)^2}{8h}(8\xi+2\eta)\|r\|_\infty + \frac{(b-a)^2}{8}\|s\|_\infty$$

since

$$\|r\|_\infty < \frac{8h\varepsilon}{(b-a)^2(8\xi+2\eta)} \quad and \quad \|s\|_\infty < \frac{8(1-\varepsilon)}{(b-a)^2} \quad (24)$$

Eq. (23) lead to $\|C^{-1}(hBr - h^2Bs)\|_\infty \le 1.$

from lemma (1), show that the matrix A is nonsingular.

Since $\|C^{-1}(hBr - h^2Bs)\|_\infty \le 1.$so using lemma (1) and Eq. (22) we obtine

$$\|E\|_\infty \le \frac{\|C^{-1}\|_\infty \|T\|_\infty}{1 - \|C^{-1}\|_\infty \|(hBr - h^2Bs)\|}$$

From Eq. (6) we have:

$$\|T_i\|_\infty = \frac{h^4}{12}F_4, F_4 = \max_{a\le x\le b}|y^{(4)}(x_i)|.$$

then

$$\|E\|_\infty \le \frac{\|C^{-1}\|_\infty \|T\|_\infty}{1 - \|C^{-1}\|_\infty \|(hBr - h^2Bs)\|} \cong O(h^2) \quad (25)$$

Also from eq. (9) we have:

$$\|T_i\|_\infty = \frac{h^6}{240}F_6, F_6 = \max_{a\le x\le b}|y^{(6)}(x_i)|.$$

then

$$\|E\|_\infty \le \frac{\|C^{-1}\|_\infty \|T\|_\infty}{1 - \|C^{-1}\|_\infty \|(hBr - h^2Bs)\|} \cong O(h^4) \quad (26)$$

Theorem (1) [3]:

Let y(x) is the exact solution of the continuous boundary value problem (1) with the B.Cs-(A) and (B) and let $y(x_i), i = 1,2,\dots,n-1$,satisfies the discrete boundary value problem (18) in further ,if $e_i = y(x_i) - y_i$ then

1- $\|E\|_\infty \cong O(h^2)$ for second order convergent method.

2- $\|E\|_\infty \cong O(h^4)$ for forth order convergent method.

Whose $\|E\|_\infty$ given by(25)and(26),respectively neglecting all errors due to round off.

7. Numerical examples

We now consider four numerical examples to illustrate the comparative performance of non-polynomial spline finite difference method in scheme (6) .All calculation are implemented by maple 18. In example (1) andexample (2),we the scheme (6) is being applied to solve this problem for n=16 and compared with exact solution in tables (1) and(2)respectively ,moreover for n=8,16,32,64 and 128.We are compute solutions at grid point, the observed maximum absolute errors $L_\infty = |y_i - y(x_i)|$ where y_i is the numerical solution and $y(x_i)$ is exact solution all are tabulated in tables (3) , (4) respectively, In table (5) our results with the results given in [1]are compared .

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We deduce that our result are more accurate, The figures (1),(2) and(3) illustrate the comparison of the real solution with numerical solution.

In example (3) and example (4) ,We applied the scheme (6)with Eq.(17)and Eq. (18) to solve this problem for different values n and compared with exact solution in tables (6) and (7) ,moreover n=8,16,32,64 and 128,the observed maximumabsolute errors are tabulated in tables (8) and (9) ,the figures (3) and (4) illustrate the comparison of the real solution with numerical solution.

Example 1 [1]:Consider the boundary value problem

$$y'' - (x+1)y' - 2y = (1-x^2)e^{-x} \qquad 0 \le x \le 1$$

$$y(0) = -1, y(1) = 0.$$

With exact solution

$$y(x) = (x-1)e^{-x}$$

Example 2 [1]:Consider the boundary value problem

$$y'' - y' = -e^{x-1} - 1 \qquad 0 \le x \le 1$$

$$y(0) = 0, y(1) = 0.$$

With exact solution $y(x) = x(1 - e^{x-1})$.

Example 3 [11]:Consider the boundary value problem

$$y'' + y = -1 \qquad 0 \le x \le 1$$

$$y'(0) = \frac{1-\cos(1)}{\sin(1)} = -y'(1).$$

With exact solution

$$y(x) = \cos x \frac{1-\cos(1)}{\sin(1)} \sin x - 1.$$

Example 4 [9]:Consider the boundary value problem

$$-y'' = (2 - 4x^2)y \qquad 0 \le x \le 1$$

$$y'(0) = 0, y'(1) = \frac{-2}{e^1}.$$

With exact solution $y(x) = e^{-x^2}$.

Table 1: Comparison numerical solution of non-polynomial spline finite difference method for (n=16) with exact solution of example (1).

x	numerical solution		exact solution
	Second order method(ξ=1/6 , η=1/3)	Forth order method (ξ=1/12 , η=5/12)	
0.0625	-0.880669135	-0.880668474	-0.880699746
0.1250	-0.772131021	-0.772128861	-0.77218479
0.1875	-0.673515692	-0.673511558	-0.673586159
0.2500	-0.584018988	-0.584012695	-0.584100587
0.3125	-0.50289777	-0.502889373	-0.502985745
0.3750	-0.429465486	-0.429455222	-0.429555799
0.4375	-0.363088043	-0.363076287	-0.363177296
0.5000	-0.303179969	-0.303167199	-0.30326533
0.5625	-0.249200851	-0.249187614	-0.249279986
0.6250	-0.200652029	-0.200638912	-0.200723036
0.6875	-0.157073513	-0.157061126	-0.157134868
0.7500	-0.118041136	-0.118030087	-0.118091638
0.8125	-0.083163896	-0.083154783	-0.083202621
0.8750	-0.052081495	-0.052074892	-0.052107752
0.9375	-0.024462057	-0.024458503	-0.024475352

Table 2: Comparison numerical solution of non-polynomial spline finite difference method for (n=16) with exact solution of example (2).

x	numerical solution		exact solution
	Second order method(ξ=1/6 , η=1/3)	Forth order method (ξ=1/12 , η=5/12)	
0.0625	0.038043273	0.038039019	0.038024648
0.1250	0.072928609	0.072920342	0.072892248
0.1875	0.104350364	0.104338372	0.104297379
0.2500	0.131976602	0.131961227	0.131908362
0.3125	0.155449676	0.155428619	0.155365132
0.3750	0.174370442	0.174349572	0.174276964
0.4375	0.188322801	0.188299959	0.188220014
0.5000	0.196844046	0.196819852	0.19673467
0.5625	0.199435511	0.199410674	0.199322704
0.6250	19555679100	0.195532119	0.195444201
0.6875	0.184622441	0.184598846	0.184514255
0.7500	0.165998192	0.165976919	0.165899413
0.8125	0.138998192	0.13897997	0.138913842
0.8750	0.102878735	0.102865077	0.10281521
0.9375	0.056835959	0.056828319	0.056800254

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Table 3: The maximum absolute errors ofnon-polynomial spline finite difference method for example (1).

n	max norm(L)	
	Second order method(ξ=1/6 , η=1/3)	Forth order method (ξ=1/12 , η=5/12)
8	3.63E-04	4.04E-04
16	9.03129E-05	1.01E-04
32	2.25E-05	2.53E-05
64	5.57478E-06	6.26843E-06
128	9.87E-07	1.2426E-06

Table 4:The maximum absolute errors of non-polynomial spline finite difference method for example (2).

n	max norm(L)	
	Second order method(ξ=1/6 , η=1/3)	Forth order method (ξ=1/12 , η=5/12)
8	0.00045128	0.000352401
16	1.12E-04	8.79708E-05
32	2.83E-05	2.20701E-05
64	7.06974E-06	5.51684E-06
128	1.76738E-06	1.38E-06

Table 5:Comparison the maximum absolute errors ofnon-polynomial spline finite difference method for example (1) and example (2) for (n=32) with the maximum absolute errors of B-spline method[1].

maximum error			
	non-polynomial spline finite difference method		
	ξ=1/6 , η=1/3	ξ=1/12 , η=5/12	B-spline method [1]
example 1	2.25E-05	2.53E-05	5.70E-04
example 2	2.83E-05	2.27E-05	6.88E-04

Table 6: Comparison numerical solution of non-polynomial spline finite difference method for (n=16) with exact solution of example (3).

x	numerical solution		exact solution
	Second order method(ξ=1/6 , η=1/3)	Forth order method (ξ=1/12 , η=5/12)	
0.0625	0.031299775	0.030943688	0.032169192
0.1250	0.05940484	0.059048874	0.060307784
0.1875	0.083374298	0.083018496	0.084305898
0.2500	0.103114577	0.102758954	0.104069821
0.3125	0.118548619	0.118193162	0.119522374
0.3750	0.129616173	0.129260849	0.130603217
0.4375	0.136274035	0.159187958	0.137269078
0.5000	0.138496213	0.138141004	0.139493927
0.5625	0.136274035	0.135918796	0.137269078
0.6250	0.129616173	0.129260849	0.130603217
0.6875	0.118548619	0.118193162	0.119522374
0.7500	0.103114577	0.102758954	0.104069821
0.8125	0.083372975	0.083018496	0.084305898
0.8750	0.05940484	0.059048874	0.060307785
0.9375	0.031299775	0.030943688	0.032169192

Table 7: Comparison numerical solution of non-polynomial spline finite difference method for (n=16) with exact solution of example (4).

x	numerical solution		exact solution
	Second order method(ξ=1/6 , η=1/3)	Forth order method (ξ=1/12 , η=5/12)	
0.0625	0.994624959	0.993898323	0.99610137
0.1250	0.983104909	0.982364394	0.984496437
0.1875	0.964172404	0.963409595	0.965454552
0.2500	0.938262193	0.937469997	0.939413063
0.3125	0.905959649	0.905132689	0.906960618
0.3750	0.867978854	0.867113733	0.868815056
0.4375	0.825136448	0.824231864	0.82579704
0.5000	0.778322587	0.777379302	0.778800783
0.5625	0.728470452	0.72749111	0.72876333
0.6250	0.676525748	0.675514562	0.676633846
0.6875	0.623417583	0.622379908	0.623344309
0.7500	0.570031932	0.568973763	0.569782825
0.8125	0.517188709	0.51611612	0.516770583
0.8750	0.465623173	0.464541742	0.465043188
0.9375	0.415972143	0.414886376	0.415236829

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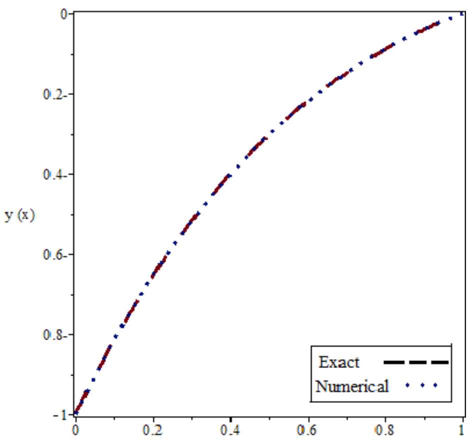
Table 8: The maximum absolute errors of non-polynomial spline finite difference method for example (3).

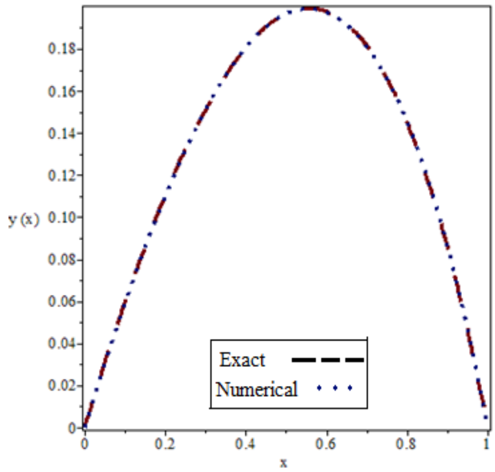
N	Maximum error	
	Second order method(ξ=1/6 , η=1/3)	Forth order method (ξ=1/12 , η=5/12)
8	3.45E-03	4.87E-03
16	9.97E-04	1.35E-03
32	2.66E-04	3.55E-04
64	6.82946E-05	9.08673E-05
128	1.86E-05	2.27E-05

Table 9: The maximum absolute errors of non-polynomial spline finite difference method for example (4).

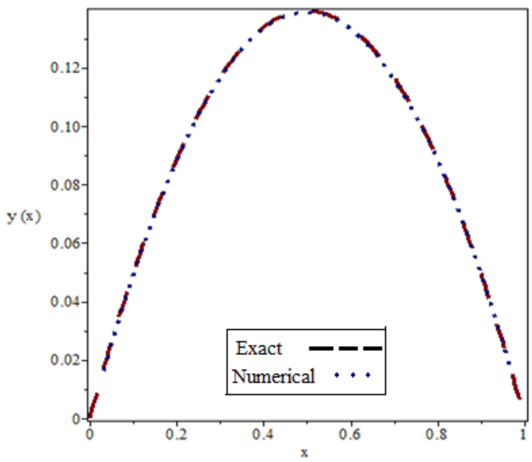
n	Second order method(ξ=1/6 , η=1/3)	Forth order method (ξ=1/12 , η=5/12)
8	7.15E-03	7.15E-03
16	1.48E-03	2.20E-03
32	4.20E-04	6.03E-04
64	1.11E-04	3.93E-05
128	2.68E-05	3.93377E-05



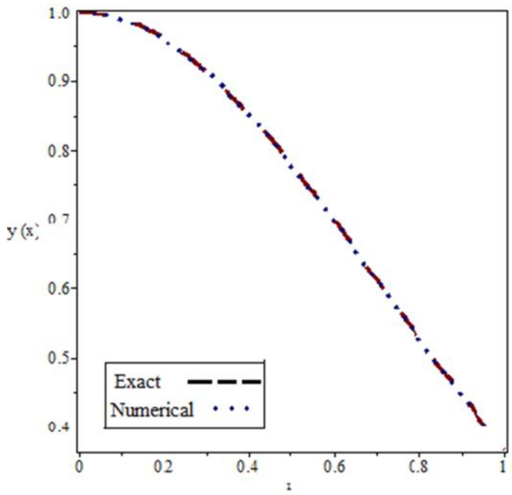
Figure(1): Comparison the exact and numerical solution (h=1/32) for



Figure(2): Gomparison the exact and numerical solution (h=1/32) for example(2).



Figure(3): Comparison the exact and numerical solution (h=1/32) for example (3).



Figure(4): Comparison the exact and numerical solution (h=1/32) for example (4) .

8. Conclusion

In this paper, the non-polynomial spline finite difference method was used to solve the general linear of the second boundary value problem, and show that this method is better in terms of accuracy and application, these have been verified by maximum absolute errors.

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طريقة الغير متعددات الحدود السبلاين مع االفروقات المحددة لحل مسائل قيم حدودية من الرتبة الثانية

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المستخلص :
تم حل مسائل قيم حدودية (BVPs) من الرتبة الثانية بطريقة غير متعددة الحدود السبلاين مع طريقة الفروقات المحددة. في هذا البحث نناقش طريقة تعتمد على استخدام التقريب الخاص للفروقات المحددة للمشتقات وتنسيق صيغة يمكن أن تتعامل مع نقاط النهاية التي تتجاوز صيغة الفروقات المحددة للمشتقات.تتم مناقشة تحليل التقارب لهذه الصيغة. يظهر الوصف العددي للطريقة بأربعة أمثلة. وبالتالي فإن النتائج التي تم الحصول عليها من خلال أسلوبنا مشجعة للغاية بالمقارنة مع الطرق الأخرى .

الكلمات المفتاحية: طريقة الغير متعددة الحدود السبلاين ،الفروقات المحددة، مسائل القيم الحدودية، خطأ القطع، الحل الحقيقي.