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Comparison of four non-Bayesian methods to estimate the scale parameter for Modified Weibull distribution by using the Simulation

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Abstract.,

In this paper, four methods were obtained to estimate the scale parameter of Modified Weibull distribution using complete data, which are the Modified Moments Estimator (MME), the Maximum Likelihood Estimator (MLE), White Estimator (WE) and Least Squares Estimator (LSE), Monte Carlo simulation is used to compare these four estimators with respect to the Mean Square Error criteria (MSE), and the results on simulated samples of the comparison showed that for all the varying sample size in this study, and in all cases for the four methods The MLE method is best followed by the OLSE method then the WE method and the MME method .

Key Words. Modified Weibull distribution, Modified Moments Estimator, Least Squares Estimator, White Estimator.

Introduction

The probability density function of any random variable t having a modified Weibull distribution (MWD) with scale parameter $\alpha > 0$ and both shape parameters $\beta \geq 0$ and $\lambda > 0$ is given by

$$f(t;\alpha,\beta,\lambda)=\begin{cases}\alpha(\beta+\lambda t)t^{\beta-1}\exp(\lambda t-\alpha t^{\beta}e^{\lambda t}) & , t\geq 0 \\ 0 & , o.w\end{cases} \quad (1)$$

If $\beta = 0$, the resulting distribution is called the type 1 extreme-value which is also known as a log-gamma distribution when $\lambda = 0$ then MWD reduces to the two-parameter Weibull distribution. Also when both $\beta = 2$ and $\lambda = 0$ then MWD reduces to one-parameter Rayleigh distribution. The modified Weibull model was developed by Xie et al.(2003) [1] this lifetime distribution is an important feature for reliability analysis.

Vasile et al. [2] studied the method of Bayes to estimate the parameters of the MWD and Upadhyaya and Gupta [3] using Markov chain Monte Carlo simulation to study the Bayes analysis of the MWD. Ateya [4] study the estimation problem of the censored sample of order statistics generalized from MWD.

The cumulative distribution function and reliability function respectively are

$$F(t;\alpha,\beta,\lambda)=1-\exp(-\alpha t^{\beta}e^{\lambda t}) \quad (2)$$

$$R(t)=\exp(-\alpha t^{\beta}e^{\lambda t}) \quad (3)$$

We review four methods which are the Modified Moments Estimator (MME), the Maximum Likelihood Estimator (MLE), White Estimator (WE) and Least Squares Estimator (LSE), These methods are compared in Section 6, using the mean square error (MSE) criteria, all these four methods which are using to estimate the scale parameter for Modified Weibull are non-Bayesian methods .

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2. Modified Moments Estimator (MME)

(Whitten and Cohen), in (1982), proposed a new modification on moment method [5], using the following equation

$$E\left(\hat{F}(t_{(i)})\right)=F(t_{(i)}) \qquad i=1,2,...,n \quad (4)$$

$$t_{(1)}\le t_{(2)}\le . \quad . \quad .\le t_{(n)}$$

Where $t_{(i)}$ is the i's order random variable, $\hat{F}(t_{(i)})$ is estimated unbiased for function distribution $F(t_{(i)})$ and by replacement $F(t_{(i)})$ by the plotting position formula

$$P_i=\frac{i}{n+1}, i=1,2,...,n \qquad (5)$$

$$F(t_{(i)})=\frac{i}{n+1}, i=1,2,...,n \qquad (6)$$

Then

$$F(t_{(1)})=\frac{1}{n+1} \qquad (7)$$

From equations (2) and (7), we get

$$\exp(-\alpha t_{(1)}{}^{\beta}e^{\lambda t_{(1)}})=1-\frac{1}{n+1} \qquad (8)$$

By taking the natural logarithm of equation (8), we get

$$\hat{\alpha}_{MME}=\frac{-Ln\left(1-\frac{1}{n+1}\right)}{t_{(1)}{}^{\beta}e^{\lambda t_{(1)}}} \qquad (9)$$

Where the symbol $\hat{\alpha}_{MME}$ indicates the estimate of the scale parameter α by using MM method.

3. Maximum Likelihood Estimator (MLE)

The likelihood function for three-parameter Modified Weibull distribution (1) is [1].

$$L(\alpha,\beta,\lambda;t_1,t_2,...,t_n)=\alpha^n\prod_{i=1}^{n}(\beta+\lambda t_i)\prod_{i=1}^{n}t_i{}^{\beta-1} \exp\sum_{i=1}^{n}(\lambda t_i-\alpha t_i{}^{\beta}e^{\lambda t_i}) \qquad (10)$$

Taking the logarithm of the likelihood function, so we get the function

$$\ln L=n\ln\alpha+\sum_{i=1}^{n}\ln(\beta+\lambda t_i)+(\beta-1)\sum_{i=1}^{n}\ln t_i+\sum_{i=1}^{n}(\lambda t_i-\alpha t_i{}^{\beta}e^{\lambda t_i})) \qquad (11)$$

The partial derivative of the log-likelihood function with respect to unknown parameters α is

$$\frac{\partial\ln L}{\partial\alpha}=\frac{n}{\alpha}-\sum_{i=1}^{n}t_i^{\beta}e^{\lambda t_i} \qquad (12)$$

We place the partial derivative for log-likelihood with respect to α to zero as follows

$$\frac{n}{\hat{\alpha}}-\sum_{i=1}^{n}t_i^{\beta}e^{\lambda t_i}=0 \qquad (13)$$

$$\hat{\alpha}_{MLE}=\frac{n}{\sum_{i=1}^{n}t_i^{\beta}e^{\lambda t_i}} \qquad (14)$$

Where the symbol $\hat{\alpha}_{MLE}$ indicates the estimate of the scale parameter α by using MLE method.

4. Least Squares Estimator (LSE)

The idea of this method is to minimize the sum of squared differences between observed sample values and the estimate expected values by linear approximation [6].

$$\varepsilon_i=y_i-\beta_0-\beta_1x_1-\beta_2x_2 \qquad (15)$$

$$\sum_{i=1}^{n}\varepsilon_i^2=\sum_{i=1}^{n}[y_i-\hat{y}_i]^2 \qquad (16)$$

$$\sum_{i=1}^{n}\varepsilon_i^2=\sum_{i=1}^{n}[y_i-\hat{\beta}_0-\hat{\beta}_1x_1-\hat{\beta}_2x_2]^2 \qquad (17)$$

By using the CDF of modified Weibull distribution (2) which are as follows

$$1-[F(t_i)]=\exp(-\alpha t_i^{\beta}e^{\lambda t_i}) \qquad (18)$$

By taking the double logarithm of above equation getting

$$\ln(-\ln[1-\{F(t_i)\}])=\ln\alpha+\beta\ln t_i+\lambda t_i \qquad (19)$$

Comparing the above equation with the simple linear model

$$Y=\beta_0+\beta_1x_1+\beta_2x_2+\varepsilon \qquad (20)$$

We get

$$Y=\ln(-\ln[1-\{F(t_i)\}]), x_1=\ln t_i, x_2=t_i,$$

$$\beta_0=\ln\alpha, \beta_1=\beta, \beta_2=\lambda \quad .$$

$$\varepsilon=\ln(-\ln[1-\{F(t_i)\}])-\ln\alpha-\beta\ln t_i-\lambda t_i \qquad (21)$$

By taking the sum square of above equation for the two sides to reach

$$\sum_{i=1}^{n}\varepsilon_i^2=\sum_{i=1}^{n}[\ln(-\ln[1-\{F(t_i)\}])-\ln\alpha-\beta\ln t_i-\lambda t_i]^2 \qquad (22)$$

$$\frac{\partial(\sum_{i=1}^{n}\varepsilon_i^2)}{\partial\alpha}=-\frac{2}{\alpha}\sum_{i=1}^{n}[\ln(-\ln[1-\{F(t_i)\}])-\ln\alpha-\beta\ln t_i-\lambda t_i] \qquad (23)$$

We place the partial derivative $\sum_{i=1}^{n}\varepsilon_i^2$ to zero as follows

$$-\frac{2}{\alpha}\sum_{i=1}^{n}[\ln(-\ln[1-\{F(t_i)\}])-\ln\alpha-\beta\ln t_i-\lambda t_i]=0$$

$$\hat{\alpha}_{LSE}=\exp\left(\frac{\sum_{i=1}^{n}\ln(-\ln[1-\{F(t_i)\}])-\beta\sum_{i=1}^{n}\ln t_i-\lambda\sum_{i=1}^{n}t_i}{n}\right) \quad (24)$$

Where the symbol $\hat{\alpha}_{LSE}$ indicates the estimate of the scale parameter α by using LSE Method and $F(t_i)$ is Empirical Distribution Functions which is [7].

$$F(t_i)=\frac{i-0.5}{n} \quad (25)$$

5. White Estimator (WE)

This method is mainly based on the reliability function of the distribution whose parameters are to be estimated and the formula of the function converted to a formula similar to the Linear Regression Equation, and its characteristics are to use its estimators as primary estimators for other estimation methods.

This method is applied to the modified Weibull distribution by taking the natural logarithm of both sides of the formula (3) we get the following formula

$$-\ln R(x)=\alpha t^{\beta}e^{\lambda t} \quad (26)$$

Comparing formula (26) with the following linear regression formula [8].

$$z=\phi y+\varphi \quad (27)$$

$$z=-\ln R(t),\ \phi=\alpha,\ y=(t^{\beta}e^{\lambda t}),\ \varphi=0 \quad (28)$$

$$\hat{\phi}=\frac{\sum_{i=1}^{n}(y_i-\bar{y})(z_i-\bar{z})}{\sum_{i=1}^{n}(y_i-\bar{y})^2} \quad (29)$$

Where

$$\bar{z}=\frac{\sum_{i=1}^{n}z_i}{n}=\frac{\sum_{i=1}^{n}-\ln R(t_i)}{n} \quad (30)$$

And

$$R(t_i)=1-F(t_i)=1-\frac{i-0.5}{n} \quad (31)$$

Set $(\hat{\phi})$ in form (28) we obtain

$$\hat{\alpha}_{WE}=\hat{\phi} \quad (32)$$

Where the symbol $\hat{\alpha}_{WE}$ indicates the estimate of the scale parameter α by using WE Method.

6. Monte Carlo Simulation

The Monte Carlo simulation is using to compare the MSE for the scale parameter modified Weibull distributions with respect to MME, MLE, WE and OLSE methods, in this study we used the three models. The first model is α=0.1,β=0.2,λ=2,the second model is α=0.05, β=0.2,λ=0.1 and the third model is α=0.3, β=2,λ=0.2.the probability density functions with this models are illustrated in Fig. 1

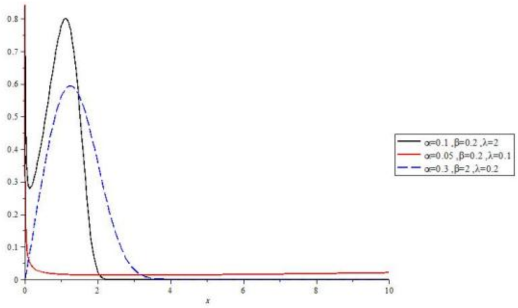

Fig.1. Different models shapes of pdf versus time

The MSE for any model in this Simulation is calculated using 24,800 simulated samples. All computations in this simulation are performed using MATLAB R2014a. We consider sample sizes n=10, 25, 100 and 200. We can generate random numbers from the modified Weibull distribution by using the inversion of the cumulative distribution function. And we replicate the data of experiment N times (N=200,500,1000,2000,2500) with sample size n, the results of simulation presented in the following Tables.

Cr.	MSE				N
Meth. / n	MME	MLE	WE	LSE	
10	2.383729236800643	0.001438063444104	0.002239672831175	0.002669346419856	200
	33.648011790890713	0.001765433113177	0.003885925331741	0.003269032572172	500
	21.252011566744255	0.001655171755509	0.004702689841331	0.002667558871810	1000
	31.479130706661955	0.001602443785124	0.004870285091825	0.002507155344652	2000
	23.075840594685047	0.001561226500369	0.006141199825043	0.002628589177462	2500
25	6.033424239420221	0.000616354782930	0.000841312007086	0.001453861618977	200
	4.507147322274402	0.000545467405829	0.001085909024811	0.000897824828307	500
	19.206956103971507	0.000475412286613	0.001391672857426	0.000806102267597	1000
	6.086150606366833	0.000473905174187	0.001444151975628	0.000862067567216	2000
	19.350973297827583	0.000490254983197	0.001973022131112	0.000777758467862	2500
100	3.210931476253547	0.000071962327029	0.000227684493831	0.000130169924409	200
	1.454521076404451	0.000102951778319	0.000275130880454	0.000179175351622	500
	10.616429106555705	0.000093776771464	0.000316419393898	0.000162374177441	1000
	24.339393251617938	0.000098610585834	0.000342047813986	0.000159220442238	2000
	99.336813474673278	0.000104208303217	0.000468051412648	0.000170623720589	2500
200	2.214592525395552	0.000054626588130	0.000096813656970	0.000087189024627	200
	5.975713459530112	0.000046840887403	0.000126351090577	0.000081744652007	500
	20.413847579365985	0.000051966435075	0.000162230952909	0.000087498830131	1000
	12.057602293630540	0.000050726119845	0.000175632920196	0.000084772316627	2000
	9.972721383806443	0.000050804962080	0.000239815413910	0.000083709803411	2500

Table-1. MSE of scale parameter α for the first model

Cr.	MSE				N
Meth. / n	MME	MLE	WE	LSE	
10	0.600528542955062	0.000362103793363	0.000682870997850	0.000657123460946	200
	8.648731876098426	0.000574184732385	0.000912874105925	0.001075636703398	500
	30.235692929472133	0.000729490743855	0.001093849682803	0.001255720751272	1000
	23.147495988702524	0.000739364551927	0.001179560068047	0.001235487059464	2000
	97.329825822911943	0.001036060095879	0.001675616166989	0.001677181513686	2500
25	0.560890317361752.	٠.٠٠٠١٢٧٧١٦٥٣٧٦٦٠	0.000952598840050	0.000240620231339	200
	1.296706222958869	0.000173725581724	0.000247168907412	0.000296538842959	500
	5.190199546287034	0.000202547441944	0.000318077071852	0.000342345089545	1000
	24.519165732042783	0.000223687261805	0.000352924856757	0.000384973562539	2000
	16.470487995151860	0.000121449654185	0.000487054189287	0.000208214775373	2500
100	14.133282096560173	0.000025527706921	0.000042877140125	0.000039033745534	200
	6.999907208554048	0.000035479752785	0.000065512298778	0.000057606322895	500
	3.966748072195498	0.000045260998636	0.000079070454860	0.000073347671188	1000
	438.7659200985143	0.00004785263240	0.000088558994225	0.000077234908500	2000
	359.1210829682811	0.00006449266160	0.000118367166696	0.000103925437500	2500
200	24.097332481215879	0.000013911561315	0.000024075051749	0.000023895975042	200
	26.759433362973123	0.000018244664363	0.000036646460792	0.000031587280336	500
	17.992704050140315	0.000020490848670	0.000039537033232	0.000034800882159	1000
	531.3921693138716	0.00002270771970	0.000044770081252	0.000038863877600	2000
	430.5269491736749	0.00003088326490	0.000060449409516	0.000051601324300	2500

Table-2. MSE of scale parameter α for the second model

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Cr. \ Meth. n	MSE				N
	MME	MLE	WE	LSE	
10	92.783635257522718	0.021375659212246	0.024583355922621	0.033511280002231	200
	900.8764969978296	0.026903098460800	0.032863467813329	0.041021218490000	500
	4032.351352442018	0.026240629047000	0.039378588580908	0.042246888000000	1000
	1249.939960709109	0.026092726897000	0.042464162449683	0.047452379245000	2000
	9905.778740941878	0.034568533836000	0.060322182011600	0.061281843026000	2500
25	56.574535102180100	0.003309256212745	0.007029355824195	0.006213497462386	200
	469.6859933465812	0.004983251232800	0.008898080666838	0.089833525741000	500
	291.0073221553217	0.007531189153100	0.011450774586706	0.013178561010400	1000
	898.4793785900660	0.007865722885000	0.012705294843266	0.013474168028600	2000
	1279.291462242854	0.010626346911000	0.017533950814360	0.018584726738000	2500
100	15.632592952906153	0.000977621525935	0.001686915251769	0.001557690728678	200
	27.297885386289721	0.001348569043347	0.002495441162018	0.002265631504168	500
	40.967280881829538	0.001650572281862	0.003081280689277	0.002644477362376	1000
	15945.93289070901	0.001736131620000	0.003215367422850	0.002805816320000	2000
	13172.30320540781	0.002322145640000	0.004331281405101	0.003760887680000	2500
200	1500.818106616001	0.000511532437000	0.000851435380450	0.000873104024000	200
	6312.182944410619	0.000602154237100	0.001080440933797	0.001045570960600	500
	527.8395062493505	0.000724162434800	0.001370912560041	0.001256029543400	1000
	19061.14861102400	0.000813865600000	0.001592103002813	0.001388968840000	2000
	15444.74049969535	0.001104891550000	٠.002153481173084	0.001837551020000	2500

Table-3. MSE of scale parameter α for the third model

7. The conclusion

Note that we can make the following comments for the results in the above tables :

(1)The results of simulation presented in Table 1 is following conclusions may be summarized: the MLE is the best method and comes after LSE except when n=10,25 with N=200 according to the MSE criterion.

(2)The results of simulation presented in Table 2 is following conclusions may be summarized: the MLE is the best method and comes after LSE except when n=10 with N=500,1000,2000,2500 and when n=25 with N=500,1000,2000 according to the MSE criterion.

(3)The results of simulation presented in Table 3 is following conclusions may be summarized: the MLE is the best method and comes after LSE except when n=10 and when n=25 with N=500,1000,2000,2500 and when n=200 with N=200, according to the MSE criterion.

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مقارنة بين أربع طرق لابيزية لتقدير معلمة القياس لتوزيع ويبل المعدل باستخدام المحاكاة

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المستخلص

في هذه البحث ، تم الحصول على أربع طرق لتقدير معلمة المقياس لتوزيع ويبل المعدل باستخدام البيانات الكاملة، وهي مقدر العزوم المعدلة (MME) ومقدر الامكان الاعظم (MLE) و مقدر وايت (WE) ومقدر المربعات الصغر (LSE).وقد استخدمت محاكاة مونت كارلو للمقارنة بين هذه المقدرات الأربعة وفقا لمعيار متوسط مربعات الخطأ (MSE)، وقد أظهرت النتائج على عينات المحاكاة للمقارنة أن لجميع حجوم العينات المتفاوتة في هذه الدراسة ، وفي جميع الحالات عن الطرق الأربعة بان طريقة MLE هي الأفضل يتبعها طريقة OLSE ثم طريقة WE وطريقة MME .