

Proof of Stronger Goldbach Conjecture

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Abstract

The stronger (binary) Goldbach conjecture expresses that "every even integer greater than or equal to 4 can be written as a sum of two odd prime numbers". The introduce paper demonstrates this conjecture by proven that there exists a positive integer a for each integer number $n \geq 4$, such that $n - a$ and $n + a$ are simultaneously primes.

Keywords: Number theory; Primes and their distribution; Goldbach conjecture.

Mathematics Subject Classification : 11P32

1.Introduction.

The issue under thought had its beginning in a letter composed by Goldbach to Euler in 1742 [1]. He expounded on his thought to the celebrated mathematician Euler, who at initially addressed the letter with some neglect, viewing the outcome as minor. That wasn't very wise of Euler: the "Goldbach conjecture", as it's turned out to be known, remains unproven 'til today which has been verified and it is currently known to be align to $4 \cdot 10^{14}$. (see [2]). This guess suggests the guess that "all odd numbers greater than 7 are the sum of three odd primes", which is alluded to today differently as the odd Goldbach guess, or the ternary Goldbach guess. While the feeble Goldbach guess seems to have been at last demonstrated in 2013 by Helfgott [3], [4].

We should attempt with a few cases, for instance:

$$\begin{aligned}
 4 &= 2 + 2, \\
 6 &= 3 + 3, \\
 10 &= 3 + 7, \\
 12 &= 5 + 7, \\
 14 &= 7 + 7, \\
 16 &= 3 + 13, \\
 18 &= 7 + 11.
 \end{aligned}$$

See that the cases are not one of a kind, for example $14 = 3 + 11 = 7 + 7$.

2.The proof of stronger Goldbach conjecture.

The proof is very simple after we prove the following theorem :

Theorem (2.1) For each $n \in \mathbb{Z}, n \geq 4, \exists a \in \mathbb{Z}^+$, such that $(n - a)$ and $(n + a)$ are simultaneously primes.

Example (2.2) We first verify that there exists such a positive integer number a , for $4 \leq n \leq 71$, and the results were recorded as table 1.

<i>n</i>	<i>a</i>	<i>n</i>	<i>a</i>
4	1	38	9,15,21,23,33,35
5	2	39	2,8,20,22,28,32,34
6	1	40	3,21,27,33
7	4	41	12,18,30,38
8	3,5	42	1,5,11,19,25,29,31,37
9	2,4	43	24,30,36,40
10	3,7	44	3,15,27,39
11	6,8	45	2,8,14,16,22,26,28, 34,38
12	1,5,7	46	15,33,43
13	6,10	47	6,24,36,42
14	3,9	48	5,11,19,25,31,35,41
15	2,4,8	49	12,18,30
16	3,13	50	3,9,21,33,39,47
17	6,12,14	51	8,10,20,22,28,32,38, 46
18	1,5,11,13	52	9,15,21,45,49
19	12	53	6,30,36,48,50
20	3,9,17	54	7,13,17,25,35,43,47,51
21	2,8,10,16	55	12,18,24,42,48,52
22	9,15,19	56	3,15,27,33,45,51,53
23	6,18	57	4,10,14,16,26,40,44,46, 50,52
24	5,7,13,17,19	58	15,21,39,45,51,55
25	6,12,18,22	59	12,30,42,48,54
26	3,15	60	1,7,13,19,23,29,37,41,43, 47,49,53
27	4,10,14,16,20	61	18,42,48
28	9,15,25	62	9,21,39,45,51
29	12,18,24	63	10,16,20,26,34,40,44,46, 50
30	1,7,11,13,17,23	64	3,33,45
31	12,28	65	6,18,24,36,42,48
32	9,15,21,27,29	66	5,7,13,23,35,37,43,47,61
33	4,10,14,20,26,28	67	6,30,36,60,64
34	3,27	68	15,21,39,45,63
35	6,12,18,24,32	69	2,10,28,32,38,40,58,62
36	5,7,11,17,23,25,31	70	3,9,27,33,39,57,67
37	6,24,30,34	71	12,18,30,42,60,66,68

Table1. Checking conjecture 1, $4 \leq n \leq 71$.

Remark (2.3) From table 1, we observe that notes:

- i. If n is even (odd) then a is odd (even). This is always true since we want $n + a$ odd prime number.
- ii. Since we want $n - a$ prime number, we have $n - a > 0$. Therefore, always we have $a < n$.
- iii. For every even integer no. n :
 a can take the values $n - 1, n - 3, n - 5, \dots, n - (n - 1)$.
 Also for every odd integer no. n :
 a can take the values $n - 1, n - 3, n - 5, \dots, n - (n - 2)$.
- iv. we get for every integer $n \geq 4$, $a = n - p$, where p is prime number, $3 \leq p \leq (n - 1)$ when n is even, and $3 \leq p \leq (n - 2)$ when n is odd, $n - a = n - (n - p) = p$.
- v. Let $S = \{1, 3, 5, 7, \dots, n - 1\}$.
 And $T = \{2, 4, 6, 8, \dots, n - 1\}$.
 It is easy to show that two cases:
 1. Let $a \in S$. For every positive even integer n , $n + a$ is odd number such that $2n - (n - 1) \leq n + a \leq 2n - 1$.
 2. Let $a \in T$. For every positive odd integer n , $n + a$ is odd number
 Such that $2n - (n - 2) \leq n + a \leq 2n - 1$,
 in both cases there exists a such that $n + a$ is prime.
- vi. From the above notes, we observe that for every integer $n \geq 4$, there exists positive integers a such that $n + a$ is prime and $n - a$ is prime.
 Now we are looking for an integer a makes $n + a$ and $n - a$ are Simultaneously primes.
 If $n - a = p$ then
 $n + a = n + n - p = 2n - p$, where p is prime number such that $3 \leq p \leq (n - 1)$, if n is even and $3 \leq p \leq (n - 2)$, if n is odd.
 Therefore we want to show that $2n - p$ is prime for some p .

Example (2.4)

Consider the following example, when n odd number. We define a function $f(x): A \rightarrow B$, such that

$$A = \{1, 3, 5, \dots, n - 2\},$$

$$B = \{2n - 1, 2n - 3, 2n - 5, \dots, n + 2\}$$

by $f(x) = 2n - x$ one to one and onto function, see for example when $n = 19$,

$$19 + 2 = 21 \leftrightarrow 19 - 2 = 17$$

$$19 + 4 = 23 \leftrightarrow 19 - 4 = 15$$

$$19 + 6 = 25 \leftrightarrow 19 - 6 = 13$$

$$19 + 8 = 27 \leftrightarrow 19 - 8 = 11$$

$$19 + 10 = 29 \leftrightarrow 19 - 10 = 9$$

$$19 + 12 = \mathbf{31} \leftrightarrow 19 - 12 = 7$$

$$19 + 14 = 33 \leftrightarrow 19 - 14 = 5$$

$$19 + 16 = 35 \leftrightarrow 19 - 16 = 3$$

$$19 + 18 = 37 \leftrightarrow 19 - 18 = 1$$

$$A = \{1, 3, 5, \dots, 17\}, \quad B = \{37, 35, 33, \dots, 21\},$$

$$f: A \rightarrow B \text{ defined by } f(x) = 2n - x.$$

If $a = 12$ then $19 + 12$ and $19 - 12$ are simultaneously primes. Similarly when n even number, we can define the one to one and onto function from the set

$$C = \{1, 3, 5, \dots, n - 1\}$$

to the set $D = \{2n - 1, 2n - 3, 2n - 5, \dots, n + 1\}$ by $f(x) = 2n - x$.

Proof of theorem (2.1)

For every even integer $n \geq 4$, there exists a positive integer $a = n - p$ where p is a prime number such that $3 \leq p \leq (n - 1)$, makes $n - a$ is prime for all p , as well

$n + a = n + n - p = 2n - p$. We want to show that $2n - p$ is prime for some p , such that $3 \leq p \leq (n - 1)$. Now let

$C = \{1, 3, 5, \dots, n - 1\}$, the set of all possible $n - a$,
 $D = \{2n - 1, 2n - 3, 2n - 5, \dots, n + 1\}$, the set of all possible $n + a$, and $f: C \rightarrow D$ defined by $f(x) = 2n - x$. In the both sets C and D , there are primes numbers, we show by contradiction that

$2n - x$ can be prime number for some prime no. x . Assume that $2n - x$ is not prime number $\forall x = p$. That is $2n - p \neq \bar{p}$, $\forall (\bar{p}$ prime number in the set D).

Therefore $f^{-1}(2n - p) \neq f^{-1}(\bar{p}), \forall \bar{p}$.
 So that $p \neq 2n - \bar{p}, \forall \bar{p}$. Similarly for odd integer. ■

Theorem (2.5)

The necessary and sufficient condition of stronger Goldbach conjecture is theorem (2.1).

Proof.

Let $n \geq 2$, $2n = p + q$ (p and q are prime numbers).

We want to show that: $\exists a \in \mathbb{Z}^+$, $s.t.$, $(n - a)$ and $(n + a)$ are simultaneously primes. We have $2n - p = q$, there exists a positive integer $a = n - p$ such that $n - a = p$ and $n + a = q$.

Conversely, $2n = n + n$, (if n is prime number then $a = 0$). Then by theorem (2.1), $\exists a \in \mathbb{Z}^+$, $s.t.$, $(n - a)$ and $(n + a)$ are simultaneously primes.

Therefore

$$2n = n + n = (n - a) + (n + a). \blacksquare$$

Reference

- [1] L.E.Dickson, "History of the theory of numbers", vol.1, New York, 1934.
- [2] J.Richstein, "Verifying the Goldbach conjecture up to $4 \cdot 10^{14}$.", math.comp,70:236,1745-1749, 2001
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اثبات لمخمة غولدباخ الأقوى (الثنائية)

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المستخلص :

يعبر عن مخمة غولدباخ الأقوى (الثنائية) بأنها " كل عدد صحيح زوجي أكبر من أو يساوي 4 يمكن كتابته كمجموع لعددتين أوليين ". والتي لم يتم اثبات صحتها منذ 1742 ولغاية هذا اليوم حيث تعد من المسائل المفتوحة في الرياضيات . في هذا البحث قدمنا الاثبات على صحة هذه المخمة من خلال إثبات وجود عدد صحيح موجب a لكل عدد صحيح n ، أكبر من أو يساوي 4 بحيث يجعل كل من $n - a$ و $n + a$ اعداد اولية في ذات الوقت.