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Properties of Compact fuzzy Normed Space

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Abstract

In this paper we recall the definition of fuzzy norm then basic properties of fuzzy normed space is recalled after that we introduced the definition of compact fuzzy normed space. Then basic properties of compact fuzzy normed space is proved.

KeyWords: Fuzzy normed space, fuzzy continuous operator, Uniform fuzzy continuous operator, Compact fuzzy normed Space.

Mathematics Subject Classification: **46S40**.

1.Introduction

Through his studying the notion of fuzzy topological vector spaces Katsaras in 1984 [1], was the first researcher who introduced the notion of fuzzy norm on a linear vector space. A fuzzy metric space was introduced by Kaleva and Seikkala in 1984 [2]. The notion of fuzzy norm on a linear space was introduced by Felbin in 1992 [3] in such a way that the corresponding fuzzy metric is of Kaleva and Seikkala type. Another type of fuzzy metric space was introduced by Kramosil and Michalek in [4]. The notion fuzzy norm on a linear space was introduce by Cheng and Mordeson in 1994 [5] so that the corresponding fuzzy metric is of Kramosil and Michalek type.

A finite dimensional fuzzy normed linear spaces was studied by Bag and Samanta [6] in 2003. Some results on fuzzy complete fuzzy normed spaces was studied by Saadati and Vaezpour in 2005 [7]. Fuzzy bounded linear operators on a fuzzy normed space was studied by Bag and Samanta in 2005 [8]. The fixed point theorems on fuzzy normed linear spaces of Cheng and Mordeson type was proved by Bag and Samanta in 2006, 2007 [9], [10]. The fuzzy normed linear space and its fuzzy topological structure of Cheng and Mordeson type was studied by Sadeqi and Kia in 2009 [11]. Properties of fuzzy continuous mapping on a fuzzy normed linear spaces of Cheng and Mordeson type was studied by Nadaban in 2015 [12].

2. Properties of Fuzzy normed space

In this section we recall basic properties of fuzzy normed space

Definition 2.1:[1]

Suppose that U is any set, a fuzzy set $\widetilde{A}$ in U is equipped with a membership function, $\mu_{\widetilde{A}}(u)$: U→ [0,1]. Then $\widetilde{A}$ is represented by $\widetilde{A}=\{(u,\mu_{\widetilde{A}}(u)): u\in U, 0 \leq \mu_{\widetilde{A}}(u) \leq 1\}$.

Definition 2.2: [7]

Let $*: [0,1] \times [0,1] \to [0,1]$ be a binary operation then $*$ is called a continuous **t -norm** (or **triangular norm**) if for all $\alpha, \beta, \gamma, \delta \in [0,1]$ it has the following properties

(1)$\alpha * \beta = \beta * \alpha$, (2)$\alpha * 1 = \alpha$, (3)$(\alpha * \beta) * \gamma = \alpha * (\beta * \gamma)$

(4)If $\alpha \leq \beta$ and $\gamma \leq \delta$ then $\alpha * \gamma \leq \beta * \delta$

Remark 2.3:[8]

(1)If $\alpha > \beta$ then there is γ such that $\alpha * \gamma \geq \beta$

(2)There is δ such that $\delta * \delta \geq \sigma$ where $\alpha, \beta, \gamma, \delta, \sigma \in [0,1]$

Definition 2.4 : [8]

The triple $(V, L, *)$ is said to be a **fuzzy normed space** if V is a vector space over the field $\mathbb{F}$, $*$ is a t-norm and $L: V \times [0,\infty) \to [0,1]$ is a fuzzy set has the following properties for all a, b∈ V and $\alpha, \beta > 0$.

1-$L(a, \alpha) > 0$

2-$L(a, \alpha) = 1 \Leftrightarrow a = 0$

3-$L(ca, \alpha) = L\left(a, \frac{\alpha}{|c|}\right)$ for all $c \neq 0 \in \mathbb{F}$

4-$L(a, \alpha) * L(b, \beta) \leq L(a + b, \alpha + \beta)$

5-$L(a, .): [0, \infty) \to [0,1]$ is continuous function of α.

6-$\lim_{\alpha\to\infty} L(a, \alpha) = 1$

Remark 2.5 : [13]

Assume that $(V, L, *)$ is a fuzzy normed space and let $a \in V, t > 0, 0 < q < 1$. If $L(a,t) > (1-q)$ then there is s with $0 < s < t$ such that $L(a,s) > (1-q)$.

Definition 2.6:[6]

Suppose that $(V, L, *)$ is a fuzzy normed space. Put

$FB(a,p,t) = \{b \in X : L(a-b,t) > (1-p)\}$

$FB[a,p,t] = \{b \in X : L(a-b,t) \geq (1-p)\}$

Then $FB(a,p,t)$ and $FB[a,p,t]$ is called **open and closed fuzzy ball** with the center $a \in V$and radius p, with p>0.

Lemma 2.7 :[7]

Suppose that $(V, L, *)$ is a fuzzy normed space then $L(x-y,t) = L(y-x,t)$ for all $x, y \in V$ and $t > 0$

Definition 2.8: [6]

Assume that $(V, L, *)$ is a fuzzy normed space. W ⊆V is called **fuzzy bounded** if we can find $t > 0$ and $0 < q < 1$ such that $L(w,t) > (1-q)$ for each $w \in W$.

Definition 2.9 :[6]

A sequence (v_n) in a fuzzy normed space $(V, L,*)$ is called **converges to** $v \in V$ if for each $q > 0$ and $t > 0$ we can find N with $L[v_n - v, t] > (1 - q)$ for all $n \geq N$. Or in other word $\lim_{n\to\infty} v_n = v$ or simply represented by $v_n \to v$, v is known the limit of (v_n) or $\lim_{n\to\infty} L[v_n - v, t] = 1$.

Definition 2.10 :[8]

A sequence (v_n) in a fuzzy normed space $(V, L,*)$ is said to be a **Cauchy sequence** if for all $0 < q < 1$, $t > 0$ there is a number N with $L[v_m - v_n, t] > (1 - q)$ for all

$m , n \geq N$.

Definition 2.11:[4]

Suppose that $(V, L,*)$ is a fuzzy normed space and let W be a subset of V. Then the **closure of W** is written by $\overline{W}$ or CL(W) and which is $\overline{W} = \cap\{W \subseteq B$: B is closed in V$\}$.

Lemma 2.12:[13]

Assume that $(V, L,*)$ is a fuzzy normed space and suppose that W is a subset of V. Then $y \in \overline{W}$ if and only if there is a sequence (w_n) in W with (w_n) converges to y.

Definition 2.13:[13]

Suppose that $(V, L,*)$ is a fuzzy normed space and $W \subseteq V$. Then W is called **dense** in V when $\overline{W} = V$.

Theorem 2.14:[13]

Suppose that $(V, L,*)$ is a fuzzy normed space and assume that W is a subset of V. Then W is dense in V if and only if for every $x \in V$ there is $w \in W$ such that

$L[x - w , t] > (1 - \varepsilon)$ for some $0 < \varepsilon < 1$ and $t > 0$.

Definition 2.15:[10]

A fuzzy normed space (V, L, $*$) is said to be **complete** if every Cauchy sequence in V converges to a point in V.

Definition 2.16: [8]

Suppose that $(V, L_V,*)$ and $(W, L_W,*)$ are two fuzzy normed spaces .The operator $S: V \to W$ is said to be **fuzzy continuous at v_0** $\in V$ if for all $t > 0$ and for all $0 < \alpha < 1$ there is s[depends on t , α and v_0] and there is β [depends on t, α and v_0] with, $L_V[v - v_0, s] > (1 - p)$ we have $L_W[S(v) - S(v_0), t] > (1 - \alpha)$ for all $v \in V$.

Theorem 2.17:[13]

Suppose that $(V, L_V,*)$ and $(U, L_U,*)$ are two fuzzy normed spaces.The operator $T: V \to U$ is fuzzy continuous at $a \in X$ if and only if $a_n \to a$ in V implies $T(a_n) \to T(a)$ in U.

Definition 2.18:[13]

Suppose that $(V, L_V,*)$ and $(W, L_W,*)$ are two fuzzy normed spaces. Let $S: V \to W$ be an operator S is said to be **uniformly fuzzy continuous** if for $t > 0$ and for every $0 < \alpha < 1$ there exists β [depends on t and α] and there exists $s > 0$ [depends on t and α] such that $L_W[S(x) - S(y), t] > (1 - \alpha)$ whenever $L_V[x - y , s] > (1 - \beta)$ for all $x, y \in V$

3. Compact fuzzy normed space

Definition 3.1:

Suppose that $(V, L,*)$ is a fuzzy normed space and W is a subset of V. Assume that $\Psi = \{ A \subseteq V$: A is open sets in V $\}$ where $W \subseteq \cup_{A\in\Psi} A$. Then Ψ is said to be an **open cover** or open covering of W. If $\Psi = \{A_1, A_2, \dots, A_k\}$ with $W = \cup_{i=1}^{k} A_i$ then Ψ is known as a finite **sub covering** of W.

Definition 3.2:

A fuzzy normed space $(V, L,*)$ is called **compact** if $V = \cup_{A\in\Psi} A$ where Ψ is an open covering then we can find $\{A_1, A_2, \dots, A_n\} \subset \Psi$ with $V = \cup_{i=1}^{k} A_i$.

Example 3.3:

The interval (0, 1) in the fuzzy normed space $(\mathbb{R}, L_{|\cdot|},*)$ where $L_{|\cdot|}(x,t) = \frac{t}{t+|x|}$ and $a * b = a \cdot b$ for all $a, b \in [0,1]$ is not compact since the collection $A_n = \{(0,\frac{1}{n}) : n=2, 3. \dots\}$ form an open covering for $(0, 1)$ but has no finite sub covering for (0,1).

Remark 3.4:

When W is a finite subset of the fuzzy normed space $(V, L,*)$ then W is compact

Definition 3.5:

Suppose that $(V, L,*)$ is a fuzzy normed space and $W \subseteq V$ then it is said to be totally fuzzy bounded if for any $\sigma \in (0, 1)$, $t > 0$ we can find $W_\sigma = \{a_1, a_2, \dots, a_n\}$ in W with any $v \in V$ there is some $a_i \in \{a_1, a_2, \dots, a_k\}$ with $L(v - a_i, t) > (1 - \sigma)$. Then W_σ is called σ -fuzzy net.

Proposition 3.6:

Let $(V, L,*)$ be a fuzzy normed space if V is totally fuzzy bounded then V is fuzzy bounded.

Proof:

Suppose that V is a totally fuzzy bounded and let $0 < \varepsilon < 1$ so we can find a finite ε-fuzzy net for V say S. Now put $L\left[s,\frac{t}{2}\right] = \min\left\{L\left(a,\frac{t}{2}\right) : a \in S\right\}$. Let $v \in V$ so we can find $a \in S$ with $L[v - a, t] > (1 - \varepsilon)$. Now we can find $\sigma \in (0, 1)$ with

$(1 - \varepsilon) * L[s, t] > (1 - \sigma)$, it follows that

$$L[v, t] = L[v - a + a, t] \geq L\left[v - a, \frac{t}{2}\right] * L\left[a, \frac{t}{2}\right]$$

$$\geq (1 - \varepsilon) * L\left[s, \frac{t}{2}\right] > (1 - \sigma).$$

Hence V is fuzzy bounded.

Theorem 3.7:

Suppose that $(V, L,*)$ is a fuzzy normed space and assume that $W \subseteq V$. Then W is totally fuzzy bounded if and only if every sequence in W contains a Cauchy subsequence.

Proof:

Let W be totally fuzzy bounded. Suppose that $(w_n) \in W$. Choose a finite 0.5- fuzzy net in W then we can find a fuzzy open ball of radius 0.5 its center in the 0.5- fuzzy net contains infinite members of (w_n). Let $\left(w_n^{(1)}\right)$ denote this subsequence. Choose finite 0.25- fuzzy net in W. So we can find a fuzzy balls of radius 0.25- where its center in the finite 0.25 fuzzy net contains infinite members of $\left(w_n^{(1)}\right)$. Let $\left(w_n^{(2)}\right)$ denote this subsequence. Continue in this process we get a sequence of sequences each is a subsequence of proceeding one, so that $\left(w_n^{(j)}\right)$ lies in the fuzzy ball of radius $\frac{1}{2^j}$ with center in the $\frac{1}{2^j}$ fuzzy net. Now $\left(w_n^{(n)}\right) \subseteq (w_n)$. Now when $0 < \varepsilon < 1$ be given and $t > 0$, let $\left(1 - \frac{1}{2^j}\right) * \left(1 - \frac{1}{2^{j+1}}\right) * \dots * \left(1 - \frac{1}{2^{k-1}}\right) > (1 - \varepsilon)$. then for all $k > j \geq N$ where N is positive number, we have $L\left[w_j^{(j)} - w_k^{(k)}, t\right] \geq L\left[w_j^{(j)} - w_{j+1}^{(j+1)}, \frac{t}{k-j}\right] * L\left[w_{j+1}^{(j+1)} - w_{j+2}^{(j+2)}, \frac{t}{k-j}\right] * \dots * L\left[w_{k-1}^{(k-1)} - w_k^{(k)}, \frac{t}{k-j}\right] \geq \left(1 - \frac{1}{2^j}\right) * \left(1 - \frac{1}{2^{j+1}}\right) * \dots * \left(1 - \frac{1}{2^{k-1}}\right) > (1 - \varepsilon)$.

Hence $\left(w_j^{(j)}\right)$ is a Cauchy.

Conversely, suppose that every sequence in W has a Cauchy subsequence. Let $0 < \varepsilon < 1$ be given and $t > 0$. Let $w_1 \in W$. if $W - FB(w_1, \varepsilon, t) = \emptyset$, we find an ε- fuzzy net, namely $\{w_1\}$ otherwise choose $w_2 \in W - FB(w_1, \varepsilon, t)$. if $W - [FB(w_1, \varepsilon, t) \cup FB(w_2, \varepsilon, t)] = \emptyset$. we found an ε -fuzzy net namely $\{w_1, w_2\}$. After finite steps this process will stops.

If it does not stop, we will get $(w_n) \in W$ with $L[w_n - w_m, t] \le (1-\varepsilon), n \ne m$. That is (w_n) has no Cauchy subsequence, which is contradiction.

Proposition 3.8:

If the fuzzy normed space $(V, L, *)$ is compact then it is totally fuzzy bounded.

Proof:

For any given $0 < \varepsilon < 1$ and $t > 0$ the collection of all fuzzy balls $FB(v, \varepsilon, t)$ is an open cover for V. But V is compact hence this cover contains a finite sub cover say $\{FB(v_1, \varepsilon, t), FB(v_2, \varepsilon, t), \dots, FB(v_k, \varepsilon, t)\}$ thus the finite set $\{v_1, v_2, \dots, v_k\}$ is ε-fuzzy net for V. Hence V is totally fuzzy bounded.

Proposition 3.9:

If $(V, L, *)$ is compact fuzzy normed space then it is complete.

Proof:

Suppose that $(V, L, *)$ is not complete then there exists a Cauchy sequence (v_n) in V has no limit in V. Let $v \in V$ and since (v_n) is not converge to v so we can find $\sigma \in (0, 1)$, $t > 0$ with $L[v_n - v, t] \le (1-\sigma)$ for infinite members. But (v_n) is Cauchy so we can find $N \in \mathbb{N}$ with $L[v_n - v_m, t] > (1-\sigma)$ for all $n, m \ge N$. Choose $m \ge N$ for which $L[v_m - v, t] > (1-\sigma)$ so the open fuzzy ball $FB(v, \sigma, t)$ contains finite members of v_n. In this manner, so for any $v \in V$ we can find a fuzzy open ball $FB(v, \sigma(v), t)$, with $\sigma(v) \in (0, 1)$ depends on v and an open fuzzy ball $FB(v, \sigma(v), t)$ which contains finite of v_n. Now $V = \cup_{v \in V} FB(v, \sigma(v), t)$ that is $\{FB(v, \sigma(v), t) : v \in V\}$ is an open covering for V using V is compact we have

$V = \cup_{j=1}^{k} FB(v^{(j)}, \sigma(v^{(j)}), t)$ but any $FB(v^{(j)}, \sigma(v^{(j)}), t)$ contains finite of v_n. This means that V must contain finite of v_n. But this is impossible hence V must be complete.

Lemma 3.10:

Suppose that $(V, L, *)$ is fuzzy normed space and $W \subset V$. If V is totally fuzzy bounded then so is W.

Proof:

Let $S = \{v_1, v_2, \dots, v_k\}$ be ε -fuzzy net for V then for any $v \in V$

$L[v - v_j, t] > (1-\varepsilon)$ for $t > 0$ and some $v_j \in S$. now let $S_1 = \{e_1, e_2, \dots, e_m\} \subset W$. Then $L[e_j - v_n, t] > (1-\varepsilon)$ for each $1 < j < m$

and for some $v_n \in S$. Now

$$L[v - e_j, t] = L[v - v_n + v_n - e_j, t]$$
$$\ge L\left[v - v_n, \frac{t}{2}\right] * L\left[v_n - e_j, \frac{t}{2}\right]$$
$$\ge (1-\varepsilon) * (1-\varepsilon) > (1-r)$$

For some $0 < r < 1$ hence W is totally fuzzy bounded

Theorem 3.11:

If $(V, L, *)$ is totally fuzzy bounded and complete fuzzy normed space then V is compact.

Proof:

Suppose V is not compact then $V = \cup_{\lambda \in \Lambda} G_\lambda$ and $V \ne \cup_{i=1}^{n} G_i$. But V is totally fuzzy bounded it is fuzzy bounded by proposition (3.6), hence for some $\sigma \in (0, 1)$ and some $v \in V, t > 0$, we have $V \subseteq FB(v, \sigma, t)$ which implies that $V = FB(v, \sigma, t)$. let $\varepsilon_n = \frac{\sigma}{2^n}$ since V is totally fuzzy bounded so it can be covered by finite many fuzzy balls of radius ε_1 but by our assumption there is $FB(v_1, \varepsilon_1, t) \ne \cup_{i=1}^{n} G_i$. But $FB(v_1, \varepsilon_1, t)$ is it self totally fuzzy bounded by Lemma (3.10), so we can find $v_2 \in FB(v_1, \varepsilon_1, t)$ such that $FB(v_2, \varepsilon_2, t) \ne \cup_{i=1}^{n} G_i$. Thus there is a sequence $(v_n) \in V$ with $FB(v_n, \varepsilon_n, t) \ne \cup_{i=1}^{n} G_i$ and $v_{n+1} \in FB(v_n, \varepsilon_n, t)$. Since $v_{n+1} \in FB(v_n, \varepsilon_n, t)$ it follows that

$L[v_n - v_{n+1}, t] > (1 - \varepsilon_n)$, let $0 < \varepsilon < 1$ be given with

$(1-\varepsilon_n)*(1-\varepsilon_{n+1})*\ldots*(1-\varepsilon_m)>(1-\varepsilon)$.

Hence for $m>n$ $L[v_n-v_m,t] \geq L\left[v_n-v_{n+1},\frac{t}{m-n}\right]*$

$$L\left[v_{n+1}-v_{n+2},\frac{t}{m-n}\right]*\ldots*L\left[v_{m-1}-v_m,\frac{t}{m-n}\right]$$
$$\geq(1-\varepsilon_n)*(1-\varepsilon_{n+1})*\ldots*(1-\varepsilon_m)>(1-\varepsilon)$$

So (v_n) is a Cauchy sequence in V but V is complete so $v_n\to y$ since $y\in V$ there is $\lambda_0\in\Lambda$ such that $y\in G_{\lambda_0}$. since G_{λ_0} is open it contains $FB(y,\delta,t)$ for some $0<\delta<1$. Choose a positive number N such that $L[v_n-y,t]>(1-\delta)$ for all $n\geq N$ and $(1-\varepsilon_n)>(1-\delta)$ then for any $v\in V$ with $L\left[v-v_n,\frac{t}{2}\right]>(1-\varepsilon_n)$. So

$$L[v-y,t]\geq L\left[v-v_n,\frac{t}{2}\right]*L\left[v_n-y,\frac{t}{2}\right]$$
$$\geq(1-\delta)*(1-\delta)>(1-r)$$

for some $0<r<1$. So that $FB(v_n,\varepsilon_n,t)\subseteq FB(y,r,t)$. Therefore $FB(v_n,\varepsilon_n,t)$ has a finite sub covering namely the set $G_{\lambda_\circ}$. This contradicts that $V\neq\cup_{i=1}^{n}G_i$.

Proposition 3.12:

Suppose that $(V,L,*)$ is a fuzzy normed space. Then for any set S=$\{v_n : 1\leq n<\infty\}$ in V has at least one limit point v in V if and only if every (v_n) in V contains (v_{n_k}) with $v_{n_k}\to v$.

Proof:

Let $(v_n)\in V$ when S=$\{v_1,v_2,\ldots v_k\}$ then choose $v_j\in S$. Thus $(v_j,v_j,\ldots)\in(v_n)$ and converges to v_j. Suppose that the set S is infinite. Then by our assumption it has at least one limit point $v\in V$. Let $n_1\in\mathbb{N}$ with

$L[v_{n_1}-v,t]>0$. let $n_{k+1}\in\mathbb{N}$ with $n_{k+1}>n_k$ and

$L[v_{n_{k+1}}-v,t]>\left(1-\frac{1}{(k+1)}\right)$. Then $v_{n_k}\to v$.

Conversely let S=$\{v_n : 1\leq n<\infty\}\subset V$. Then we can find $(v_n)\in V$ with $v_i\neq v_j$ so by our assumption (v_n) has a subsequence (v_{n_k}) of distinct with $v_{n_k}\to v\in V$. Thus any FB(v,σ,t) contains an infinite members of (v_{n_k}). Hence any FB(v,σ,t) contains infinite members of S. This means that $v\in V$ is a limit point of S.

Theorem 3.13:

The fuzzy normed space $(V,L,*)$ is compact if and only if for any (v_n) in V contains (v_{n_k}) with $v_{n_k}\to v$.

Proof:

Let V be compact then V is totally fuzzy bounded and complete by proposition (3.8) and proposition (3.9). Suppose that $(v_n)\in V$ since V is totally fuzzy bounded using theorem (3.7) we have (v_n) contains a Cauchy (v_{n_k}). So $v_{n_k}\to v\in V$ since V is complete. Hence every (v_n) in V contains (v_{n_k}) with $v_{n_k}\to v$.

To prove the converse let every (v_n) in V contains (v_{n_k}) with $v_{n_k}\to v$.

Now by using theorem (3.7) we have V is totally fuzzy bounded. To prove that V is complete. Let (v_n) be a Cauchy sequence in V so (v_n) contains (v_{n_k}) with $v_{n_k}\to v\in V$. We now prove that $v_n\to v$. Let $0<\varepsilon<1$ be given and $t>0$ by remark (2.3) there is $0<r<1$ such that $(1-r)*(1-r)>(1-\varepsilon)$. Now $v_{n_k}\to v$ there is N_1 such that $L\left[v_{n_k}-v,\frac{t}{2}\right]>(1-r)$ for all $n_k>N_1$. But (v_n) is Cauchy there is N_2 with $L\left[v_n-v_m,\frac{t}{2}\right]>(1-r)$ for any $m,n>N_2$. Now let $N=N_1\wedge N_2$ then for all $n\geq N$, $L[v_n-v,t]\geq L\left[v_n-v_{n_k},\frac{t}{2}\right]*L\left[v_{n_k}-v,\frac{t}{2}\right]>(1-r)*(1-r)>(1-\varepsilon)$

hence (v_n) converges to $v\in V$.

Corollary 3.14:

Suppose that $(V, L, *)$ a compact fuzzy normed space and $W \subset V$. If W is closed then W is compact

Proof:

Assume that $(w_n) \in W$ then $(w_n) \in V$ so (w_n) has a subsequence (w_{n_k}) converges to $w \in W$. Then $w \in W$ since W is closed. Hence W is compact by theorem (3.13)

Proposition 3.15:

Suppose that $(V, L, *)$ a fuzzy normed space and $W \subset V$. If W is compact then W is closed

Proof:

Assume that $v \in V$ be a limit point of W then there is $(w_n) \in W$ with $w_n \to v$ so (w_n) is Cauchy sequence in W. Since W is complete by proposition (3.9) so (w_n) converges to $w \in W$. Therefore $v = w \in W$ this implies that W has all it limit points. Hence W is closed.

Theorem 3.16:

Suppose that $(V, L_V, *)$ and $(U, L_U, *)$ are two fuzzy normed spaces and $T: V \to U$ be fuzzy continuous operator. If V is compact then T(V) is compact

Proof:

Assume that $(T(v_n)) \in T(V)$ then $(v_n) \in V$. So $v_{n_k} \to v$ since V is compact. Hence by Theorem 2.17 $Tv_{n_k} \to T(v) \in T(V)$ since T is continuous. Thus by Theorem 3.13 T(V) is compact.

Theorem 3.17:

Suppose that $(V, L_V, *)$ is a compact fuzzy normed space and assume that $(U, L_U, *)$ is a fuzzy normed space. Suppose that $T: V \to U$ is a fuzzy continuous operator. Then T is uniformly fuzzy continuous that is for each $0 < \varepsilon < 1$ and $t > 0$ there exists $\delta, 0 < \delta < 1$ and $s > 0$ [δ depending on ε only] such that $T(FB(v, \delta, s)) \subset FB(T(v), \varepsilon, t)$ for all $v \in V$.

Proof:

Let $\sigma \in (0, 1)$ with $(1 - \sigma) * (1 - \sigma) > (1 - \varepsilon)$ for some $0 < \varepsilon < 1$ and $t > 0$. then the collection of fuzzy balls $\{FB(u, \sigma, t): u \in U\}$ from an open cover for U. since T is fuzzy continuous then the set $\{T^{-1}[FB(u, \sigma, t): u \in U]\}$ from an open cover for V but V is compact so the set $\{T^{-1}[FB(u_1, \sigma_1, t)], T^{-1}[FB(u_2, \sigma_2, t)], \dots, T^{-1}[FB(u_k, \sigma_k, t)]\}$ cover V that is $V = \bigcup_{j=1}^{k} T^{-1}[FB(u_j, \sigma_j, t)]$. Now let $0 < \delta < 1$ be $\delta > \sigma_j$ for some $1 < j < k$. Thus for each $v \in V$ the fuzzy ball $FB(v, \delta, s)$ lies in $T^{-1}[FB(u_j, \sigma_j, t)]$ so $T[FB(v, \delta, s)] \subseteq FB(u, \sigma, t)$ for some $u \in U$. Since $T(v) \in FB(u, r, t)$ we can find $z \in B(v, \delta, s)$ with

$$L_U[T(z) - T(v), t] \geq L_U\left[T(z) - u, \frac{t}{2}\right] * L_U\left[u - T(v), \frac{t}{2}\right] \geq (1 - \sigma) * (1 - \sigma) > (1 - \varepsilon)$$

Thus $T[FB(v, \delta, s)] \subseteq FB(T(v), \varepsilon, t)$

Conclusion

The principle goal of this research is to continue the study of fuzzy normed space and introduce more notions or results. In this paper the notion compact fuzzy normed is introduced and basic results properties of this space is proved.

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خواص فضاء القياس الضبابي المتراص

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المستخلص
في هذا البحث تم اعادة استخدام تعريف القياس الضبابي ثم تم استعراض الخواص الاساسية لفضاء القياس الضبابي بعد ذلك عرفنا الفضاء القياسي الضبابي المتراص. وتم برهان الخواص الاساسية لفضاء القياس الضبابي المتراص.