

## Permuting Jordan Left Tri – Derivations On Prime and semiprime rings

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### Abstract :

Let  $\mathfrak{R}$  be a 2 and 3 – torsion free prime ring then if  $\mathfrak{R}$  admits a non-zero Jordan left tri- derivation  $B: \mathfrak{R} \times \mathfrak{R} \times \mathfrak{R} \rightarrow \mathfrak{R}$  , then  $R$  is commutative ,also we give some properties of permuting left tri - derivations.

**Keywords:** prime ring , semiprime ring , left tri-derivation, Jordan left tri-derivations.

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### 1.Introduetion:

Throughout this paper we will use  $\mathfrak{R}$  to represent an associative ring with center  $Z(\mathfrak{R})$  ,  $\mathfrak{R}$  is said to be n-torsion free if  $na = 0$  ,  $a \in \mathfrak{R}$  implies  $a = 0$  [5].

A ring  $\mathfrak{R}$  is called prime(semiprime) if  $a\mathfrak{R}b = 0$  ( $a\mathfrak{R}a = 0$  ) implies that  $a = 0$  or  $b = 0$  ( $a = 0$ ) [3] .A mapping  $D: \mathfrak{R} \times \mathfrak{R} \times \mathfrak{R} \rightarrow \mathfrak{R}$  is said to be permuting if

$$D(x_1, x_2, x_3) = D(x_{\pi_1}, x_{\pi_2}, x_{\pi_3})$$

hold for all  $x_1, x_2, x_3 \in \mathfrak{R}$  and every permute  $\pi_1, \pi_2, \pi_3$  .

A mapping  $d: \mathfrak{R} \rightarrow \mathfrak{R}$  defined by  $d(x) = D(x, x, x)$  is called the trace of  $D(.,.,.)$  where  $D: \mathfrak{R} \times \mathfrak{R} \times \mathfrak{R} \rightarrow \mathfrak{R}$  is permuting tri- additive mapping [3] , a tri-additive mapping  $D: \mathfrak{R} \times \mathfrak{R} \times \mathfrak{R} \rightarrow \mathfrak{R}$  is called tri – derivation if

$$D(x_1x_2, y, z) = x_1D(x_2, y, z) + D(x_1, y, z)x_2 ,$$
$$D(x, y_1y_2, z) = y_1D(x, y_2, z) + D(x, y_1, z)y_2 \text{ and}$$
$$D(x, y, z_1z_2) = z_1D(x, y, z_2) + D(x, y, z_1)z_2$$

are hold for all  $x, y, z, x_i, y_i, z_i \in \mathfrak{R}$  [3] .

The trace  $d$  of  $D$  satisfy the relation

$$d(x + y) = d(x) + d(y) + 3D(x, x, y) + 3D(x, y, y)$$

for all  $x, y \in \mathfrak{R}$  [7].

A. K. Faraj in [1] and R. C. Shaheenin [6] define the permuting left tri- derivation as follows a permuting tri-additive mapping  $D: \mathfrak{R} \times \mathfrak{R} \times \mathfrak{R} \rightarrow \mathfrak{V}$  is called permuting left tri – derivation if

$$D(x_1x_2, y, z) = x_1D(x_2, y, z) + x_2D(x_1, y, z)$$

$$D(x, y_1y_2, z) = y_1D(x, y_2, z) + y_2D(x, y_1, z)$$

and

$$D(x, y, z_1z_2) = z_1D(x, y, z_2) + z_2D(x, y, z_1)$$

are hold for all  $x, y, z, x_i, y_i, z_i \in \mathfrak{R}, i = 1,2$  also  $D$  is called permuting Jordan left tri – derivation if  $D(x^2, y, z) = 2xD(x, y, z), D(x, y^2, z) = 2yD(x, y, z)$  and  $D(x, y, z^2) = 2zD(x, y, z)$  are hold for all  $x, y, z \in \mathfrak{R}$

In this paper , we gave some properties of permeating left tri-derivation, also we prove that if  $\mathfrak{R}$  is a prime ring of characteristic not equal 2 and 3 and  $\mathfrak{R}$  is admit anon-zero Jordan left-tri-derivation on  $\mathfrak{R}$ , Then  $\mathfrak{R}$  is commutative.

## 2.Permuting left tri-derivations:-

In the following theorem we introduce some properties of permuting Jordan left tri –derivation on a ring

### Theorem 2.1:

Let  $\mathfrak{V}$  a 2 - torsion free ring . If  $B: \mathfrak{R} \times \mathfrak{R} \times \mathfrak{R} \rightarrow \mathfrak{R}$  is a Jordan left tri – derivation then for all  $a, b, y, z \in \mathfrak{R}$  we have.

i)  $B(ab + ba, y, z) = 2aB(b, y, z) + 2bB(a, y, z)$

ii)

$$B(aba, y, z) = a^2B(b, y, z) + 3abB(a, y, z) - baB(a, y, z)$$

iii)

$$B(abc + cba, y, z) = (ac + ca)B(b, y, z) + 3abB(c, y, z) + 3cbB(a, y, z) - baB(c, y, z) - bcB(a, y, z)$$

iv)  $[a, b]aB(a, y, z) = a[a, b]B(a, y, z)$

v)  $[a, b]\{B(ab, y, z) - aB(b, y, z) - bB(a, y, z)\} = 0$

### Proof:

i) Since  $B(a^2, y, z) = 2aB(a, y, z)$  replace  $a$  by  $a + b$

$$B((a + b)^2, y, z) = B(a^2 + ba + ab + b^2, y, z) = B(a^2, y, z) + B(b^2, y, z) +$$

$$B(ab + ba, y, z) = 2aB(a, y, z) + 2bB(b, y, z) +$$

$$B(ab + ba, y, z) \dots (1)$$

Now

$$B((a + b)^2, y, z) = 2(a + b)B(a + b, y, z) = 2aB(a + b, y, z) +$$

$$2bB(a + b, y, z) = 2aB(ay, z) + 2aB(b, y, z) +$$

$$2bB(a, y, z) + 2bB(b, y, z) \dots(2)$$

comparing (1) and(2)we get

$$B(ab + ba, y, z) = 2aB(b, y, z) + 2bB(a, y, z)$$

ii) From(i) we have for  $a, b, y, z \in \mathfrak{R}$ .

$$B(a(ab + ba) + (ab + ba)a, y, z) = 2aB(ab + ba, y, z) + 2(ab + ba)B(a, y, z) = 2a\{2aB(b, y, z) + 2bB(a, y, z)\} + 2abB(a, y, z) + 2baB(a, y, z) = 4a^2B(b, y, z) + 6abB(a, y, z) + 2baB(a, y, z) \dots(3)$$

On the other hand , we have

$$B(a(ab + ba) + (ab + ba)a, y, z) = B(a^2b + aba + aba + ba^2, y, z) = B(a^2b + ba^2, y, z) + 2B(aba, y, z) = 2a^2B(b, y, z) + 4baB(a, y, z) + 2B(aba, y, z) \dots(4)$$

Composing (3)and(4) we get

$$2B(aba, y, z) = 2a^2B(b, y, z) + 6abB(a, y, z) - 2baB(a, y, z)$$

So that

$$2B(aba, y, z) = 2(a^2B(b, y, z) + 3abB(a, y, z) - baB(a, y, z))$$

Since  $R$  is 2-torsion free we have

$$B(aba, y, z) = a^2B(b, y, z) + 3abB(a, y, z) - baB(a, y, z).$$

iii) Linearizing (ii) on  $a$  we get

$$B((a + c)b(a + c), y, z) = (a + c)^2B(b, y, z) + 3(a + c)bB(a + c, y, z) - b(a + c)B(a + c, y, z) = a^2B(b, y, z) + acB(b, y, z) + caB(b, y, z) + c^2B(b, y, z) + 3abB(a + c, y, z) + 3cbB(a + c, y, z) - baB(a + c, y, z) - bcB(a + c, y, z) = a^2B(b, y, z) + acB(b, y, z) + caB(b, y, z) + c^2B(b, y, z) + 3abBca, y, z)$$

$$+ 3abB(c, y, z) + 3cbB(a, y, z) + 3cbB(c, y, z) - baB(a, y, z)$$

$$- baB(c, y, z) - bcB(a, y, z) - bcB(c, y, z) \dots(5)$$

In other hand

$$B((a + c)b(a + c), y, z) = B(aba + abc + cba + cbc, y, z) = B(aba, y, z) + B(abc + cba, y, z) + B(cbc, y, z) = a^2B(b, y, z) + 3abB(a, y, z) - baB(a, y, z) + B(abc + cba, y, z) + c^2B(b, y, z) + 3cbB(c, y, z) - bcB(c, y, z) \dots(6)$$

Comparing (5) and( 6) we have.

$$B(abc + cba, y, z) = (ac + ca)B(b, y, z) + 3abB(c, y, z) + 3cbB(a, y, z)$$

$$- baB(c, y, z) - bcB(a, y, z)$$

(iv) Assume that  $w = B(ab(ab) + (ab)ba, y, z)$   
 Then by (iii) we obtain  
 $w = (a(ab) + (ab)a)B(b, y, z) + 3abB(ab, y, z)$   
 $\quad + 3(ab)bB(a, y, z)$   
 $\quad - baB(ab, y, z)$   
 $\quad - b(ab)B(a, y, z)$   
 $w = (a^2b + aba)B(b, y, z) + 3abB(ab, y, z)$   
 $\quad + 3ab^2B(a, y, z)$   
 $\quad - baB(ab, y, z)$   
 $\quad - babB(a, y, z) \quad \dots(7)$

On the other hand  
 $w = B((ab)(ab) + (ab)ba, y, z)$   
 $\quad = B((ab)^2, y, z) + B(ab^2a, y, z)$   
 $\quad = 2abB(ab, y, z) + a^2B(b^2, y, z) +$   
 $3ab^2B(a, y, z) - b^2aB(a, y, z)$   
 So by definition of B  
 $w = 2abB(ab, y, z) + 2a^2bB(b, y, z) +$   
 $3ab^2B(a, y, z) - b^2aB(a, y, z) \quad \dots(8)$   
 By comparing (7) and (8)  
 $[a, b]B(ab, y, z) + abaB(b, y, z) - b(ab)B(a, y, z)$   
 $\quad - a^2bB(b, y, z)$   
 $\quad + b(ba)B(a, y, z) = 0$

Then  
 $[a, b]B(ab, y, z) - a[a, b]B(b, y, z)$   
 $\quad - b[a, b]B(a, y, z) = 0$

so  
 $[a, b]B(ab, y, z) = a[a, b]B(b, y, z) +$   
 $b[a, b]B(a, y, z) \quad \dots(9)$   
 Replace b by  $a + b$  in (9)  
 $[a, a + b]B(a(a + b), y, z) = [a, b]B(a^2, y, z) +$   
 $[a, b]B(ab, y, z)$   
 $\quad = 2[a, b]aB(a, y, z) + a[a, b]B(b, y, z) +$   
 $b[a, b]B(a, y, z)$   
 $\quad = a[a, a + b]B(a + b, y, z) + (a + b)[a, a +$   
 $b]B(a, y, z)$   
 $\quad = a[a, b]B(a, y, z) + a[a, b]B(b, y, z) +$   
 $a[a, b]B(a, y, z) + b[a, b]B(a, y, z)$

Hence  
 $2[a, b]aB(a, y, z) = 2a[a, b]B(a, y, z)$   
 Since  $\mathfrak{R}$  is 2-torsion free, then  
 $[a, b]aB(a, y, z) =$   
 $a[a, b]B(a, y, z) \quad \dots(10)$

(v) In (10) replace  $a$  by  $a + b$ ,  
 the left hand give  
 $w = [a + b, b](a + b)B(a + b, y, z)$   
 $\quad = [a, b]aB(a, y, z) + [a, b]aB(b, y, z) +$   
 $[a, b]bB(a, y, z) + [a, b]bB(b, y, z) \dots(11)$   
 The right hand give  
 $w = (a + b)[a + b, b]B(a + b, y, z)$   
 $\quad = a[a, b]B(a, y, z) + a[a, b]B(b, y, z) +$   
 $b[a, b]B(a, y, z) + b[a, b]B(b, y, z) \dots(12)$   
 from(9)we have  
 $[a, b]B(ab, y, z) = a[a, b]B(b, y, z)$   
 $\quad + b[a, b]B(a, y, z)$

So that by using (10)  
 $[a, b]B(ab, y, z) = [a, b](aB(b, y, z)$   
 $\quad + bB(a, y, z))$   
 $[a, b]\{B(ab, y, z) - aB(b, y, z) - bB(a, y, z)\} = 0$

### 3.The Main Results:

#### Theorem 3.1:-

let  $\mathfrak{R}$  be prime ring of char  $\mathfrak{R} \neq 2, 3$ , then if  $R$  admits a non-zero Jordan left-tri derivation  $B: \mathfrak{R} \times \mathfrak{R} \times \mathfrak{R} \rightarrow \mathfrak{R}$ , then  $\mathfrak{R}$  is commutative

#### Proof:

we divide proof to some steps.

**Step1:** If  $B(a, y, z) \neq 0$  for some  $a, y, z \in \mathfrak{R}$   
 then  $(a[a, x] - [a, x]a)^2 = 0$  for all  $x \in \mathfrak{R}$ .  
 Let  $a$  be a fixed element in  $\mathfrak{R}$  and  $\emptyset: \mathfrak{R} \rightarrow \mathfrak{R}$  be a mapping defined by

$$\emptyset(x) = [a, x]$$

for all  $x \in \mathfrak{R}$   
 Now [Theorem 2.1 ,iv] can be written in the form  
 $\emptyset^2(x)B(a, y, z) = 0$   
 for all  $x \in \mathfrak{R} \quad \dots\dots(13)$

Since the mapping  $\emptyset(x)$  is a derivation, we have  
 $\emptyset^2(x_1x_2) = \emptyset^2(x_1)x_2 + 2\emptyset(x_1)\emptyset(x_2) + x_1\emptyset^2(x_2)$   
 And from (13) it follows that  $\emptyset^2(x_1x_2)B(a, y, z) = 0$

hence  
 $(\emptyset^2(x_1)x_2 + 2\emptyset(x_1)\emptyset(x_2) + x_1\emptyset^2(x_2))B(a, y, z)$   
 $\quad = 0$

So that  
 $(\emptyset^2(x_1)x_2 + 2\emptyset(x_1)\emptyset(x_2))B(a, y, z) = 0$   
 Hold for all  $x_1, x_2 \in \mathfrak{R}$ .

In the above relation replace  $x_2$  by  $\emptyset(x_2x_3)$  and the relation (13) we get  
 $(\emptyset^2(x_1)\emptyset(x_2)x_3 + \emptyset^2(x)x_2\emptyset(x_2))B(a, y, z)$   
 $\quad = 0 \dots\dots(14)$

For all  $x_1, x_2, x_3 \in \mathfrak{R}$ .  
 In (14) substitute  $\emptyset(x_3)$  for  $x_3$ , we get  
 $\emptyset^2(x_1)\emptyset(x_2)\emptyset(x_3)B(a, y, z) =$   
 $0 \quad \dots(15)$

Now in (14) replace  $x_2$  by  $\emptyset(x_2)$  and using(15) we have  
 $\emptyset^2(x_1)\emptyset^2(x_2)x_3B(a, y, z) = 0 \quad \dots(16)$

holds for all  $x_1, x_2, x_3 \in \mathfrak{R}$   
 since the relation (16) hold for all  $x_3 \in \mathfrak{R}$ , we are forced to conclude that  $B(a, y, z) = 0$  to which implies that

$\emptyset^2(x_1)\emptyset^2(x_2) = 0$  for all  $x_1, x_2 \in \mathfrak{R}$   
 in particular  $(\emptyset^2(x_1))^2 = 0$  as required

**Step 2:** If  $a^2 = 0$  then  $B(a, y, z) = 0$  for all  $y, z \in \mathfrak{R}$

Let  $w = B(a(xay + yax)a, y, z)$   
 Then by using (ii) in Theorem 2.1 We get  
 $w = a^2B(xay + yax, y, z)$   
 $\quad + 3a(xay + yax)B(a, y, z)$   
 $\quad - (xay + yax)aB(a, y, z) \quad \dots(17)$

Since  $a^2 = 0$  we have

$$B(a^2, y, z) = 0 = 2aB(a, y, z)$$

But  $\text{char } \mathfrak{R} \neq 2$ , then  $aB(a, y, z) = 0$

hence (17) becomes

$$w = 3a(xay + yax)B(a, y, z)$$

$$= 3axayB(a, y, z) +$$

$$3ayaxB(a, y, z) \dots(18)$$

From (ii) Theorem 2.1 we have

$$B(axa, y, z) = 3axB(a, y, z)$$

According to (iii) in Theorem 2.1, we find

$$B(ax(axa) + (axa)xa, y, z)$$

$$= (a^2xa + axa^2)B(x, y, z)$$

$$+ 3axB(axa, y, z)$$

$$+ 3(axa)xB(a, y, z)$$

$$- x(axa)B(a, y, z)$$

Since  $a^2 = 0$ , we have

$$w = 9axaxB(a, y, z) + 3axaxB(a, y, z) \dots(19)$$

Comparing (18) and (19) we get

$$6axaxB(a, y, z) = 0$$

But  $\mathfrak{R}$  is of characteristic not equal 2 and 3 so that

$$axaxB(a, y, z) = 0 \text{ for all } x, y \text{ and } z \text{ are}$$

arbitrary elements of  $\mathfrak{R}$ .

so that

$$B(a, y, z) = 0 \text{ or } a = 0$$

If  $a = 0$  we have  $B(a, y, z) = 0$

So that in any case we have

$$B(a, y, z) = 0 \text{ for all } y, z \in \mathfrak{R}$$

**Step3 :**  $\mathfrak{R}$  is commutative.

Take:  $a, y, z \in \mathfrak{R}$  such that  $B(a, y, z) \neq 0$ .

From step1 and step2, it follows in this case that

$$B(a[a, x] - [a, x]a, y, z) = 0 \text{ for all } x \in \mathfrak{R}$$

$$\dots(20)$$

By using (i) and (ii) in Theorem 2.1, and since

$$a[a, x] - [a, x]a = a^2x - 2axa + xa^2$$

So we obtain from (20)

$$0 = B(a^2x + xa^2, y, z) - 2B(axa, y, z)$$

$$= 2a^2B(x, y, z) + 2xB(a^2, y, z) - 2a^2B(x, y, z)$$

$$- 6axB(a, y, z) + 2xaB(a, y, z)$$

$$= 4xaB(a, y, z) - 6axB(a, y, z) + 2xab(a, y, z)$$

$$= 6xaB(a, y, z) - 6axB(a, y, z)$$

hence

$$6[x, a]B(a, y, z) = 0$$

since  $\text{char } (\mathfrak{R}) \neq 2, 3$ , we get

$$[x, a]B(a, y, z) = 0 \text{ for all } x, y \in \mathfrak{R}$$

For all  $x, y \in \mathfrak{R}$  we have

$$0 = [yx, a]B(a, y, z) = y[x, a]B(a, y, z) +$$

$$[y, a]xB(a, y, z) = [y, a]xB(a, y, z)$$

Since we have assumed that  $B(a, y, z) \neq 0$

So that  $[y, a] = 0$  for all  $y \in \mathfrak{R}$

consequently  $a \in Z(\mathfrak{R})$

Thus we have proved that  $\mathfrak{R}$  is the union of its

proper subsets  $Z(\mathfrak{R})$  and

$$\ker B = \{a \in R \mid B(a, y, z) = 0 \text{ for all } y, z \in \mathfrak{R}\}$$

It is clear that both subsets  $Z(\mathfrak{R})$  and  $\ker B$  are

additive subgroups of  $\mathfrak{R}$ , but a group cannot be the

union of it is two proper subgroups, so that either

$\ker B = \mathfrak{R}$  or  $Z(\mathfrak{R}) = \mathfrak{R}$  and since  $B \neq 0$  by hypothesis we have  $\ker B \neq \mathfrak{R}$

Consequently  $\mathfrak{R} = Z(\mathfrak{R})$  and hence  $\mathfrak{R}$  is commutative

**Theorem 3.2 :** Let  $\mathfrak{R}$  be a prime ring and  $B: \mathfrak{R} \times \mathfrak{R} \times \mathfrak{R} \rightarrow \mathfrak{R}$  a left tri-derivation. If  $B \neq 0$  then  $\mathfrak{R}$  is commutative.

**Proof:**

Consider  $B(aba, y, z)$  for all  $a, b, y, z \in \mathfrak{R}$

Then

$$B(a(ba), y, z) = aB(ba, y, z) + baB(a, y, z) \\ = a^2B(b, y, z) + abB(a, y, z)$$

$$+ baB(a, y, z) \dots(21)$$

On the other hand

$$B((ab)a, y, z) = abB(a, y, z) + aB(ab, y, z) \\ = abB(a, y, z) + a^2B(b, y, z) +$$

$$abB(a, y, z) \\ = 2abB(a, y, z) +$$

$$a^2B(b, y, z) \dots(22)$$

Comparing (21) and (22) we have

$$[a, b]B(a, y, z) = 0 \text{ for all } a, b, y, z \in \mathfrak{R} \dots(23)$$

Replace  $b$  by  $cb$  in (23) we get

$$0 = [a, cb]B(a, y, z) \\ = (a(cb) - (cb)a)B(a, y, z) \\ = (ac - ca)bB(a, y, z) + c(ab - ba)B(a, y, z) \\ = [a, c]bB(a, y, z) \text{ for all } a, b, c, y, z \in \mathfrak{R}$$

So that

$$[a, c]bB(a, y, z) = 0 \text{ for all } a, b, c, z, y \in \mathfrak{R} \dots(24)$$

It follows that for each  $a \in \mathfrak{R}$  we have either

$$a \in Z(\mathfrak{R}) \text{ or } B(a, y, z) = 0$$

But, since  $Z(\mathfrak{R})$  and

$$\ker B =$$

$\{a \in \mathfrak{R} \mid B(a, y, z) = 0 \text{ for all } y, z \in \mathfrak{R}\}$  are additive subgroups of  $\mathfrak{R}$ .

We have either  $\mathfrak{R} = Z(\mathfrak{R})$  or  $\mathfrak{R} = \ker B$  but  $B \neq 0$ , so that  $\mathfrak{R} = Z(\mathfrak{R})$  and hence  $\mathfrak{R}$  is commutative.

**Theorem 3.3\_:** Let  $\mathfrak{R}$  be a semi-Prime ring and  $B: \mathfrak{R} \times \mathfrak{R} \times \mathfrak{R} \rightarrow \mathfrak{R}$  be a left tri-derivations then  $B$  is tri-derivation that maps  $\mathfrak{R}$  into its center.

**Proof :**

Linearize (24)

$$[a + d, c]bB(a + d, y, z) = 0$$

Which gives that

$$[a, c]bB(d, y, z) + [d, c]bB(a, y, z) = 0$$

Which implies that

$$[a, c]bB(d, y, z) = -[d, c]bB(a, y, z)$$

Since  $\mathfrak{R}$  is a ring, for all  $a, b, c, d, y, z \in \mathfrak{R}$  we have

$$[a, c]bB(d, y, z)x[a, c]bB(a, y, z) \\ = -[d, c]bB(d, y, z)x[a, c]bB(a, y, z) = 0$$

Since  $\mathfrak{R}$  is semiprime, this relation yields

$$[a, c]bB(d, y, z) = 0$$

In particular

$$\{aB(d, y, z) - B(d, y, z)a\}b\{aB(d, y, z) - B(d, y, z)a\} = 0$$

semiprimeness of  $\mathfrak{R}$  implies that

$$aB(d, y, z) - B(d, y, z)a = 0$$

Consequently  $B$  is a tri-derivation and  $B(d, y, z) \in Z(\mathfrak{R})$

**Theorem 3.4:** Let  $\mathfrak{R}$  be a prime ring of  $\text{char } \mathfrak{R} \neq 2, 3$  and  $B: \mathfrak{R} \times \mathfrak{R} \times \mathfrak{R} \rightarrow \mathfrak{R}$  a Jordan left tri-derivation, suppose that  $ax = 0, a, x \in \mathfrak{R}$  implies that  $a = 0$  or  $x = 0$  if  $B \neq 0$  then  $\mathfrak{R}$  is commutative .

**Proof:**

From (v) of Theorem 2.1 , we have for each pair  $a, b \in \mathfrak{R}$

$$[a, b] = 0 \text{ or } B(ab, y, z) = aB(b, y, z) + bB(a, y, z)$$

Given  $a \in \mathfrak{R}$  and let

$$G_a = \{b \in \mathfrak{R} | a[a, b] = 0\} \text{ and}$$

$$H_a = \{b \in \mathfrak{R} | B(ab, y, z) = aB(b, y, z) + bB(a, y, z)\}$$

We see that  $\mathfrak{R}$  is the union of it's additive subgroups  $G_a$  and  $H_a$

Hence  $\mathfrak{R} = G_a$  or  $\mathfrak{R} = H_a$  , on the other words ,  $\mathfrak{R}$  is the union of its subgroup's

$$G = \{a \in \mathfrak{R} | G_a = \mathfrak{R}\} = Z(\mathfrak{R}) \text{ and}$$

$$H = \{a \in \mathfrak{R} | H_a = \mathfrak{R}\}$$

$$= \{a \in \mathfrak{R} | B(ab, y, z) = aB(b, y, z) + bB(a, y, z), \text{ for all } a, b, y, z \in \mathfrak{R}\}$$

Clearly  $G$  and  $H$  are additive subgroups of  $\mathfrak{R}$

Hence  $G = \mathfrak{R}$  or  $H = \mathfrak{R}$

If  $G = \mathfrak{R}$  then  $\mathfrak{R}$  is commutative.

If  $H = \mathfrak{R}$  then  $B$  is a left tri-derivation and hence  $\mathfrak{R}$  is commutative by Theorem 3.3

Thus in any way  $\mathfrak{R}$  is commutative.

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## اشتقاقات جوردان اليسارية الثلاثية التبادلية على الحلقات الاولية والحلقات شبه الاولية

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### الملخص :

لتكن  $R$  حلقة اولية تطبيقية الالتواء من النمط 2 و 3 . اذا كانت  $R$  تسمح بوجود اشتقاق جوردان اليساري الثلاثي  $B: R \times R \times R \rightarrow R$  فانها تكون ابدالية .  
كذلك قدمنا بعض الخواص لمشتقات جوردان اليسارية الثلاثية التبادلية