

**The Cyclic decomposition and the Artin characters table of the group  $(Q_{2m} \times C_p)$   
when  $m=2^h$ ,  $h \in Z^+$  and  $p$  is prime number**

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**Abstract :**

The main purpose of this paper, is determination of the cyclic decomposition of the abelian factor group  $AC(G) = \overline{R}(G)/T(G)$  where  $G = Q_{2m} \times C_p$  when  $m=2^h$ ,  $h \in Z^+$  and  $p$  is prime number (the group of all  $Z$ -valued characters of  $G$  over the group of induced unit characters from all cyclic subgroups of  $G$ ).

We have found that the cyclic decomposition  $AC(Q_{2m} \times C_p)$  depends on the elementary divisor of  $m$  as follows.

if  $m = 2^h$ ,  $h$  any positive integer and  $p$  is prime number, then:

$$AC(Q_{2m} \times C_p) = \bigoplus_{i=1}^{2(h+1)} C_2$$

We have also found the general form of Artin's characters table of  $Ar(Q_{2m} \times C_p)$  when  $m=2^h$ ,  $h \in Z^+$  and  $p$  is prime number.

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**Introduction:**

The problem of determining the cyclic decomposition of  $AC(G)$  seem to be untouched. We use the concepts of invariant matrix in linear algebra to find the cyclic decomposition of  $AC(G)$ ,  $G$  is considered to be the group  $Q_2^{h+1} \times C_p$ .

In 1968 T.Y Lam [13] defined  $AC(G)$  and he studied  $AC(G)$ , when  $G$  is a cyclic group.

In 2000 H.R .Yassin [4] studied the cyclic decomposition of  $AC(G)$  when  $G$  is an elementary abelian group . In 2006 A.S. Abid [2] found  $Ar(C_n)$  when  $C_n$  is the cyclic group of order  $n$  .

In this paper, we find the cyclic decomposition of the factor group  $AC(Q_2^{h+1} \times C_p)$  and the Artin characters table where  $Q_{2m}$  is the Quaternion group of order  $4m$  When  $m=2^h$ ,  $h \in Z^+$  and  $C_p$  is the Cyclic group of order  $p$ ,  $p$  is prime number

**1. Some Basic Concepts:**

In this section, we give basic concepts, notations and theorems about the group  $(Q_{2m} \times C_p)$ , a rational valued characters, a rational valued characters table, the Artin characters and the Artin characters table.

**The Group  $(Q_{2m} \times C_p)$  (1.1):**

The direct product group  $(Q_{2m} \times C_p)$  where  $Q_{2m}$  is Quaternion group of order  $4m$  with two generators  $x$  and  $y$  is denoted by

$$Q_{2m} = \{x^k y^j : x^{2m} = y^4 = 1, yx^m y^{-1} = x^{-m}, 0$$

$$0 \leq k \leq 2m-1, j=0,1\}$$

and  $C_p$  is acyclic group of order  $p$  consisting of

elements  $\{1, z, z^2, \dots, z^{p-1}\}$  when  $p$  is prime number. The

generalized the group  $(Q_{2m} \times C_p)$  is denoted by

$$(Q_{2m} \times C_p) = \{(q, c) : q \in Q_{2m}, c \in C_p\} \text{ and}$$

$$|Q_{2m} \times C_p| = |Q_{2m}| \cdot |C_p| = 4m \cdot p = 4mp$$

**Definition (1.2): [8]**

A rational valued character  $\theta$  of  $G$  is a character whose values are in  $Z$ , which is  $\theta(g) \in Z$ , for all  $g \in G$

**Corollary (1.3): [9]**

The rational valued characters  $\theta_j = \sum_{\sigma \in \text{Gal}(Q(\chi_i)/Q)} \sigma(\chi_i)$

form the basis for  $\overline{R}(G)$ , where  $\chi_i$  are the irreducible characters of  $G$  and their numbers are equal to the number of conjugacy classes of cyclic subgroup of  $G$ .

**Proposition (1.4): [7]**

The number of all rational valued characters of a finite group  $G$  is equal to the number of all distinct  $\Gamma$ -classes on  $G$ .

**Definition (1.5): [9]**

The complete information about rational valued characters of a finite group  $G$  is displayed in a table called **rational valued characters table of  $G$** . We refer to it by  $\chi^*(G)$  which is  $n \times n$  matrix whose columns are  $\Gamma$ -classes and rows which are the values of all rational valued characters of  $G$ , where  $n$  is the number of  $\Gamma$ -classes

**Proposition (1.6): [10]**

The general form of the rational valued characters table of the Quaternion group  $Q_{2m}$  when  $m=2^h$ ,  $h$  is any positive integer and it is given by:

$$\chi^*(Q_{2m}) = \chi^*(Q_{2,2^h}) =$$

$\Gamma$ -classes	[1]	$\begin{bmatrix} x \\ 2^h \end{bmatrix}$	$\begin{bmatrix} x \\ 2^{h-1} \end{bmatrix}$	$\begin{bmatrix} x \\ 2^{h-2} \end{bmatrix}$	...	$\begin{bmatrix} x \\ 2 \end{bmatrix}$	[x]	[y]	$\begin{bmatrix} x \\ y \end{bmatrix}$
$\theta_1$	$2^h$	$-2^h$	0	0	...	0	0	0	0
$\theta_2$	$2^{h-1}$	$2^{h-1}$	$-2^{h-1}$	0	...	0	0	0	0
$\theta_3$	$2^{h-2}$	$2^{h-2}$	$2^{h-2}$	$-2^{h-2}$	...	0	0	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\theta_{l-2}$	2	2	2	2	...	-2	0	0	0
$\theta_{l-1}$	1	1	1	1	...	-1	-1	-1	1
$\theta_l$	1	1	1	1	...	-1	1	-1	-1
$\theta_{l+1}$	1	1	1	1	...	-1	-1	1	-1
$\theta_{l+2}$	1	1	1	1	...	-1	1	1	1

Table (1)

Where  $l$  is the number of  $\Gamma$ -classes of  $C_m$ .

**Theorem(1.7):[6]**

The rational valued characters table of the group  $(Q_{2m} \times C_p)$  is equal to the tensor product of the rational valued characters table of  $Q_{2m}$  and the rational valued characters table of  $C_p$  when  $p$  is prime number that is:

$$\equiv(Q_{2m} \times C_p) = \equiv(Q_{2m}) \otimes \equiv(C_p) .$$

**Theorem(1.8): [5]**

Let  $H$  be a cyclic subgroup of  $G$  and  $h_1, h_2, \dots, h_m$  are chosen as representative for  $m$ -conjugate classes of  $H$  contained in  $CL(g)$  in  $G$ , then :

$$1- \varphi'(g) = \frac{|C_G(g)|}{|C_H(g)|} \sum_{i=1}^m \varphi(h_i) \text{ if } h_i \in H \cap CL$$

(g)

$$2- \varphi'(g) = 0 \text{ if } H \cap CL(g) =$$

$\phi$ .

**Definition(1.9):[13]**

Let  $G$  be a finite group, all characters of  $G$  induced from a principal character of cyclic subgroups of  $G$  are called **Artin's characters of  $G$** .

In theorem (1.8) , if  $\phi$  is the principal character , then

$$\varphi(h_i) = \varphi(1) = 1, \text{ where } h_i \in H$$

**Proposition(1.10):[3]**

The number of all distinct Artin's characters on a group  $G$  is equal to the number of  $\Gamma$ -classes on  $G$ .

Furthermore, Artin's characters are constant on each  $\Gamma$ -classes.

**Definition(1.11): [2]**

Artin's characters of finite group  $G$  can be displayed in a table called **Artin's characters table of  $G$**  which is denoted by  $Ar(G)$ .

The first row is the  $\Gamma$ -conjugate classes, the second row is the number of elements in each conjugate classes, the third row is the size of the centralize  $|C_G(CL_\alpha)|$  and the rest rows contain the values of Artin's characters.

**Theorem(1.12):[2]**

The general form of Artin's character table of  $C_{p^s}$  when  $p$  is a prime number and  $s$  is an integer number is given by:

$\Gamma$ -classes	[1]	$[x^{p^{s-1}}]$	$[x^{p^{s-2}}]$	$[x^{p^{s-3}}]$	...	$[x^p]$	$[x]$
$ CL_\alpha $	1	1	1	1	...	1	1
$ C_{p^s}(CL_\alpha) $	$p^s$	$p^s$	$p^s$	$p^s$	...	$p^s$	$p^s$
$\varphi'_1$	$p^s$	0	0	0	...	0	0
$\varphi'_2$	$p^{s-1}$	$p^{s-1}$	0	0	...	0	0
$\varphi'_3$	$p^{s-2}$	$p^{s-2}$	$p^{s-2}$	0	...	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
$\varphi'_s$	$p$	$p$	$p$	$p$	...	$p$	0
$\varphi'_{s+1}$	1	1	1	1	...	1	1

Table (2)

**Proposition (1.13):** [12]

The Artin's characters table of the Quaternion group

$Q_{2m}$  when  $m=2^h$ ,  $h \in \mathbb{Z}^+$  is given as follows

$Ar(Q_{2^{h+1}})=$

$\Gamma$ - classes	$\Gamma$ - classes of $C_{2m}$						[y]	[xy]	
	[1]	$[x^{2^h}]$	...			2			
$ CL_\alpha $	1	1	2	2	...		2	$2^h$	$2^h$
$ C_{Q_{2^{h+1}}}(CL_\alpha) $	$2^{h+2}$	$2^{h+2}$	$2^{h+1}$	$2^{h+1}$	...		$2^{h+1}$	4	4
$\Phi_1$	<b><math>2Ar(C_{2^{h+1}})</math></b>						0	0	
$\Phi_2$							0	0	
⋮							⋮	⋮	
$\Phi_l$							0	0	
$\Phi_{l+1}$							$2^h$	$2^h$	0
$\Phi_{l+2}$	$2^h$	$2^h$	0	0	...		0	0	2

Table (3)

where  $l$  is the number of  $\Gamma$ - classes of  $C_{2m}$  and  $\Phi_j$ ;  $1 \leq j \leq l+2$

$\leq j \leq l+2$  are the Artin characters of the Quaternion

group  $Q_{2m}$  when  $m=2^h$ ,  $h \in \mathbb{Z}^+$

**2.the Factor Group AC(G):**

This section is devoted to the study of the factor group AC(G) of a group G .

**Definition(2.1):[9]**

Let T(G) be the subgroup of  $\overline{R}(G)$  generated by Artin's characters .T (G) is normal subgroup of  $\overline{R}(G)$  and denotes the factor abelian group  $\overline{R}(G)/T(G)$  by AC(G) which is called **Artin cokernel of G**.

**Definition(2.2):[8]**

Let M be a matrix with entries in a principal domain R.

**A k-minor of M** is the determinant of k×k sub matrix preserving row and column order.

**Definition(2.3):[8]**

**A k-th determinant divisor of M** is the greatest common divisor (g.c.d) of all the k-minors of M.

This is denoted by  $D_k(M)$

**Lemma(2.4):[8]**

Let M, P and W be matrices with entries in a principal ideal domain R, let P and W be invertible matrices ,Then  $D_k(P M W)=D_k(M)$  module the group of unites of R.

**Theorem(2.5):[8]**

Let M be an n × n matrix with entries in principal ideal domain R, then there exist two matrices P and W such that:

- 1- P and W are invertible.
- 2-  $P M W = D$ .
- 3- D is diagonal matrix.
- 4- if we denote  $D_{ii}$  by  $d_i$  then there exists a natural number m ;  $0 \leq m \leq n$  such that  $j > m$  implies  $d_j = 0$  and  $j \leq m$  implies  $d_j \neq 0$  and  $1 \leq j \leq m$  implies  $d_j | d_{j+1}$ .

**Definition(2.6):[8]**

Let M be matrix with entries in a principal domain R,

be equivalent to a matrix  $D = \text{diag} \{d_1, d_2, \dots, d_m, 0, 0, \dots, 0\}$  such that  $d_j | d_{j+1}$  for  $1 \leq j < m$

We call D **the invariant factor matrix of M** and  $d_1, d_2, \dots, d_m$  the invariant factors of M

**Theorem(2.7):[8]**

Let K be a finitely generated module over a principal domain R, then K is the direct sum of cyclic sub module with an annihilating ideal

$\langle d_1 \rangle, \langle d_2 \rangle, \dots, \langle d_m \rangle, d_j | d_{j+1}$  for  $j = 1, 2, \dots, K-1$ .

**3.The Matrix M(G) :**

This section is devoted to the study of the matrix M(G),  $M(Q_{2m}), P(Q_{2m})$  and  $W(Q_{2m})$ .

**Proposition(3.1):[9]**

AC(G) is a finitely generated Z- module .Let m be the number of all distinct  $\Gamma$ -classes then Ar(G) and  $\equiv^*(G)$  are

of the rank l. There exists an invertible matrix M(G) with entries in rational number such

That:  $\equiv^*(G)=M^{-1}(G).Ar(G)$  and this implies

$$M(G)=Ar(G).(\equiv^*(G))^{-1}$$

**Theorem(3.2):[4]**

$$AC(G) = \bigoplus_{i=1}^l C_{d_i} \text{ where } d_i = \pm D_i(G) / D_{i-1}(G)$$

where l is the number of all distinct  $\Gamma$ -classes.

**Corollary(3.3):[9]**

$$|AC(G)| = |\det (M(G))|.$$

**Proposition(3.4):[11]**

If p is prime number and s is positive integer, then:

$$M(C_{p^s}) = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 0 & 1 & 1 & \dots & 1 \\ 0 & 0 & 1 & \dots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

which is of the order  $(s+1) \times (s+1)$

**Proposition(3.5):[11]**

The general form of the matrices  $P(C_{p^s})$  and  $W(C_{p^s})$  is:

$$P(C_{p^s}) = \begin{bmatrix} 1 & -1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & -1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & -1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 & -1 \\ 0 & 0 & 0 & 0 & \dots & 0 & 1 \end{bmatrix}$$

which is  $(s+1) \times (s+1)$  square matrix.

$W(C_{p^s}) = I_{s+1}$ , where  $I_{s+1}$  is  $(s+1) \times (s+1)$

identity matrix

and  $D(C_{p^s}) = \text{diag} \underbrace{\{1, 1, \dots, 1\}}_{s+1}$ .

**Remarks(3.6): [1]**

if  $m=2^h$ , h is any positive integer, then we can write

$M(C_m)$  as the following :

$$M(C_m) = \begin{bmatrix} & & & & 1 & 1 \\ & & & & 1 & 1 \\ & & R_1(C_m) & & \vdots & \vdots \\ & & & & 1 & 1 \\ 0 & 0 & \dots & 0 & 1 & 1 \\ 0 & 0 & \dots & 0 & 0 & 1 \end{bmatrix}$$

which is  $(h+1) \times (h+1)$  square matrix,  $R_1(C_m)$  is the matrix obtained by omitting the last two rows

$\{0, 0, \dots, 1, 1\}$  and  $\{0, 0, \dots, 0, 0, 1\}$  and the last two

columns  $\{1, 1, \dots, 1, 0\}$  and  $\{1, 1, \dots, 1, 1\}$  from the matrix

$M(C_{2^h})$  in the Proposition (3.6).

**Proposition(3.7):[12]**

If  $m=2^h$ , h any positive integers, then the matrix  $M(Q_{2m})$  of the quaternion group  $Q_{2m}$  is :

$$M(Q_{2m}) = \begin{bmatrix} & & & & & & & & 1 & 1 & 1 & 1 \\ & & & & & & & & 1 & 1 & 1 & 1 \\ & & & & & & & & 1 & 1 & 1 & 1 \\ & & & & & & & & \vdots & \vdots & \vdots & \vdots \\ & & & & & & & & \vdots & \vdots & \vdots & \vdots \\ & & & & & & & & 1 & 1 & 1 & 1 \\ \hline 0 & 0 & 0 & \dots & \dots & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & \dots & \dots & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & \dots & \dots & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & \dots & \dots & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

which is  $(h+4) \times (h+4)$  square matrix,  $R(C_{2m})$  is similar to the matrix in the remarks (3.8).

**Proposition(3.8):[12]**

If  $m=2^h$ ,  $h$  any positive integer then the matrices  $P(Q_{2m})$

and  $W(Q_{2m})$  are taking the forms:

$$P(Q_{2m}) = \begin{bmatrix} & & & & & 0 & 0 \\ & & & & & 0 & 0 \\ & & & & & \vdots & \vdots \\ & & & & & \vdots & \vdots \\ & & & & & 0 & 0 \\ & & & & & -1 & 1 \\ & & & & & 0 & -1 \\ 0 & 0 & \dots & \dots & 0 & 1 & -1 \\ 0 & 0 & \dots & \dots & 0 & 0 & 1 \end{bmatrix}$$

$$\text{and } W(Q_{2m}) = \begin{bmatrix} & & & & & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 \\ & & & & & \vdots & \vdots & \vdots \\ & & & & & \vdots & \vdots & \vdots \\ & & & & & 0 & 0 & 0 \\ 0 & 1 & 1 & \dots & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & -1 & \dots & -1 & -1 & 0 & 0 & 1 \end{bmatrix}$$

where  $I_{h+1}$  is the identity matrix .They are  $(h+4) \times (h+4)$  square matrix .

**Example (3.9):**

To find  $P(Q_{32})$  and  $W(Q_{32})$ , by the proposition (3.8).

$$P(Q_{32}) = P(Q_{2^5}) = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

which is  $8 \times 8$  square matrix

and

$$W(Q_{32}) = W(Q_{2^5}) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & -1 & -1 & -1 & 0 & 0 & 1 \end{bmatrix}$$

which is  $8 \times 8$  square matrix

**4.The Main Results**

In this section we give the general form of the Artin's characters table of the group  $(Q_2^{h+1} \times C_p)$  and the cyclic decomposition of the factor group  $AC(Q_{2m} \times C_p)$  when  $m=2^h$ ,  $h \in \mathbb{Z}^+$  and  $p$  is prime number.

**Proposition(4.1):**

The general form of the Artin's characters table of the group  $(Q_2^{h+1} \times C_p)$  when  $m=2^h, h \in \mathbb{Z}^+$  and  $p$  is prime number is give as follows:

$Ar(Q_2^{h+1} \times C_p) =$

$\Gamma$ - classes of $(Q_2^{h+1}) \times \{I\}$						$\Gamma$ - classes of $(Q_2^{h+1}) \times \{z\}$						
$\Gamma$ - classes	$[1, I]$	$[x^{2^h}, I]$	...	$[x, I]$	$[y, I]$	$[xy, I]$	$[1, z]$	$[x^{2^h}, z]$	...	$[x, z]$	$[y, z]$	$[xy, z]$
$ CL_\alpha $	1	1	..	2	$2^h$	$2^h$	1	1	...	2	$2^h$	$2^h$
$ C_{Q_2^{h+1} \times C_p}(C_2)$	$p2^{h+2}$	$p2^{h+2}$	..	$p2^{h+1}$	$4p$	$4p$	$p2^{h+2}$	$p2^{h+2}$	...	$p2^{h+1}$	$4p$	$4p$
$\Phi_{(1,1)}$	<b><math>pAr(Q_2^{h+1})</math></b>						<b>0</b>					
$\Phi_{(2,1)}$												
$\vdots$												
$\Phi_{(l,1)}$												
$\Phi_{(l+1,1)}$												
$\Phi_{(l+2,1)}$												
$\Phi_{(1,2)}$	<b><math>Ar(Q_2^{h+1})</math></b>						<b><math>Ar(Q_2^{h+1})</math></b>					
$\Phi_{(2,2)}$												
$\vdots$												
$\Phi_{(l,2)}$												
$\Phi_{(l+1,2)}$												
$\Phi_{(l+2,2)}$												

Table (4)



**Proof :**

Let  $g \in (Q_{2^m} \times C_p)$  ;  $g=(q,I)$  or  $g=(q,z)$  or  $g=(q,z^2) \dots$

$g=(q,z^{p-1}), q \in Q_{2^m}$  when  $m=2^h, h \in \mathbb{Z}^+, I, z, z^2, \dots, z^{p-1} \in C_p$

**Case (I):**

If  $H$  is a cyclic subgroup of  $Q_{2^m} \times \{I\}$ , then:

$$2. H = \langle (y, I) \rangle$$

$$3. H = \langle (xy, I) \rangle \quad 1. H = \langle (x, I) \rangle$$

And  $\varphi$  the principal character of  $H$ ,  $\Phi_j$  Artin characters

of  $Q_{2^m}$  where  $1 \leq j \leq l+2$  then by using Theorem

(1.8)

$$\Phi_j(g) = \begin{cases} \frac{|C_G(g)|}{|C_H(g)|} \sum_{i=1}^m \varphi(h_i) & \text{if } h_i \in H \cap CL(g) \\ 0 & \text{if } H \cap CL(g) = \phi \end{cases}$$

1. IF  $H = \langle (x, I) \rangle$

(i) if  $g=(1,I), g \in H$

$$\begin{aligned} \Phi_{(j,1)}((1, I)) &= \frac{|C_{Q_{2^{h+1}} \times C_p}(g)|}{|C_H(g)|} \cdot \varphi(g) \\ &= \frac{p \cdot 2^{h+2}}{|C_H(1, I)|} \cdot 1 = \frac{p \cdot |C_{Q_{2^{h+1}}}(1)|}{|C_{\langle x \rangle}(1)|} \cdot \varphi(1) = p \cdot \Phi_j(1), \end{aligned}$$

since  $H \cap CL(1, I) = \{(1, I)\}$

(ii) if  $g=(x^{2^h}, I), g \in H$

$$\begin{aligned} \Phi_{(j,1)}(g) &= \frac{|C_{Q_{2^{h+1}} \times C_p}(g)|}{|C_H(g)|} \cdot \varphi(g) \\ &= \frac{p \cdot 2^{h+2}}{|C_H(g)|} \cdot 1 = \frac{p \cdot |C_{Q_{2^{h+1}}}(x^{2^h})|}{|C_{\langle x \rangle}(x^{2^h})|} \cdot \varphi(g) = p \cdot \Phi_j(x^{2^h}) \end{aligned}$$

since  $H \cap CL(g) = \{g\}, \varphi(g) = 1$

(iii) if  $g \notin (x^{2^h}, I), g \in H$

$$\begin{aligned} \Phi_{(j,1)}(g) &= \frac{|C_{Q_{2^{h+1}} \times C_p}(g)|}{|C_H(g)|} \cdot (\varphi(g) + \varphi(g^{-1})) = \\ &= \frac{p \cdot 2^{h+1}}{|C_H(g)|} \cdot (1 + 1) = \end{aligned}$$

$$\frac{p \cdot 2^{h+1}}{|C_H(g)|} \cdot (1 + 1) = \frac{p \cdot |C_{Q_{2^{h+1}}}(q)|}{|C_{\langle x \rangle}(q)|} \cdot (\varphi(g) + \varphi(g^{-1})) = p \cdot \Phi_j(q)$$

since  $H \cap CL(g) = \{g, g^{-1}\}$  and  $\varphi(g) = \varphi(g^{-1}) = 1, g=(q, I), q \in$

$Q_{2^{h+1}}$  and  $q \neq x^{2^h}$

(iv) if  $g \notin H$

$$\Phi_{(j,1)}(g) = p \cdot 0 = p \cdot \Phi_j(q), \quad \text{Since } H \cap CL(g) = \phi$$

2. IF  $H = \langle (y, I) \rangle = \{(1, I), (y, I), (y^2, I), (y^3, I)\}$

(i) If  $g=(1, I)$   $H \cap CL(1, I) = \{(1, I)\}$

$$\begin{aligned} \Phi_{(1+1,1)}(g) &= \frac{|C_{Q_{2^{h+1}} \times C_p}(g)|}{|C_H(g)|} \cdot \varphi(g) \\ &= \frac{p \cdot 4 \cdot 2^h}{4} \cdot 1 = p \cdot 2^h = p \cdot \Phi_{1+1}(1) \end{aligned}$$

(ii) If  $g = (x^{2^h}, I) = (y^2, I)$  and  $g \in H$

$$\begin{aligned} \Phi_{(l+1,1)}(g) &= \frac{|C_{Q_{2^{h+1}} \times C_p}(g)|}{|C_H(g)|} \cdot \varphi(g) \\ &= \frac{p \cdot 4 \cdot 2^h}{4} \cdot 1 = p \cdot 2^h = p \cdot \Phi_{l+1}(x^{2^h}) \end{aligned}$$

Since  $H \cap CL(g) = \{g\}$ ,  $\varphi(g) = 1$

(iii) If  $g \neq (x^{2^h}, I)$  and  $g \in H$ , i.e.  $\{g = (y, I) \text{ or } g = (y^3, I)\}$

Otherwise

$$\begin{aligned} \Phi_{(l+1,1)}(g) &= \frac{|C_{Q_{2^{h+1}} \times C_p}(g)|}{|C_H(g)|} \cdot (\varphi(g) + \varphi(g^{-1})) \\ &= \frac{p \cdot 4}{4} \cdot (1 + 1) = p \cdot 2 = p \cdot \Phi_{l+1}(y) \end{aligned}$$

since  $H \cap CL(g) = \{g, g^{-1}\}$  and  $\varphi(g) = \varphi(g^{-1}) = 1$

Otherwise

$$\Phi_{(l+1,1)}(g) = 0 \quad \text{since } H \cap CL(g) = \emptyset$$

3-IF  $H = \langle (xy, I) \rangle$

$$= \{(1, I), (xy, I), ((xy)^2, I) = (y^2, I), ((xy)^3, I) = (xy^3, I)\}$$

(i) If  $g = (1, I)$   $H \cap CL(1, I) = \{(1, I)\}$

$$\begin{aligned} \Phi_{(l+2,1)}(g) &= \frac{|C_{Q_{2^{h+1}} \times C_p}(g)|}{|C_H(g)|} \cdot \varphi(g) \\ &= \frac{p \cdot 4 \cdot 2^h}{4} \cdot 1 = p \cdot 2^h = p \cdot \Phi_{l+2}(1) \end{aligned}$$

(ii) If  $g = (x^{2^h}, I) = ((xy)^2, I) = (y^2, I)$  and  $g \in H$

$$\begin{aligned} \Phi_{(l+2,1)}(g) &= \frac{|C_{Q_{2^{h+1}} \times C_p}(g)|}{|C_H(g)|} \cdot \varphi(g) \\ &= \frac{p \cdot 4 \cdot 2^h}{4} \cdot 1 = p \cdot 2^h = p \cdot \Phi_{l+2}(x^{2^h}) \end{aligned}$$

Since  $H \cap CL(g) = \{g\}$ ,  $\varphi(g) = 1$

(iii) If  $g \neq (x^{2^h}, I)$  and  $g \in H$ , i.e.  $\{g = (xy, I) \text{ or } g = ((xy)^3, I)\}$

$$\begin{aligned} \Phi_{(l+2,1)}(g) &= \frac{|C_{Q_{2^{h+1}} \times C_p}(g)|}{|C_H(g)|} \cdot (\varphi(g) + \varphi(g^{-1})) \\ &= \frac{p \cdot 4}{4} \cdot (1 + 1) = p \cdot 2 = p \cdot \Phi_{l+2}(xy) \end{aligned}$$

since  $H \cap CL(g) = \{g, g^{-1}\}$  and  $\varphi(g) = \varphi(g^{-1}) = 1$

$$\Phi_{(l+2,1)}(g) = 0 \quad \text{since } H \cap CL(g) = \emptyset$$

Case (II):

If  $H$  is a cyclic subgroup of  $(Q_2^{h+1} \times \{z\})$ , then

$$1.H = \langle (x, z) \rangle = \langle (x, z^2) \rangle =$$

$$\langle (x, z^3) \rangle = \dots = \langle (x, z^{p-1}) \rangle$$

$$2.H = \langle (y, z) \rangle = \langle (y, z^2) \rangle =$$

$$\langle (y, z^3) \rangle = \dots = \langle (y, z^{p-1}) \rangle$$

$$3.H = \langle (xy, z) \rangle = \langle (xy, z^2) \rangle =$$

$$\langle (xy, z^3) \rangle = \dots = \langle (xy, z^{p-1}) \rangle$$

And  $\varphi$  the principal character of  $H$ ,  $\Phi_j$  Artin characters

of  $Q_2^{h+1}$   $1 \leq j \leq l+2$ , then by using theorem (1.8)

$$\Phi_j(g) = \begin{cases} \frac{|C_G(g)|}{|C_H(g)|} \sum_{i=1}^m \varphi(h_i) & \text{if } h_i \in H \cap CL(g) \\ 0 & \text{if } H \cap CL(g) = \emptyset \end{cases}$$

$$1.IFH = \langle (x, z) \rangle = \langle (x, z^2) \rangle =$$

$$\langle (x, z^3) \rangle = \dots = \langle (x, z^{p-1}) \rangle$$

(i) If  $g=(1,I)$  or  $g=(1,z)$  or  $g=(1,z^2), \dots$  or  $g=(1,z^{p-1})$  and  $g \in H$

$$\begin{aligned} \Phi_{(j,2)}(g) &= \frac{|C_{Q_2^{h+1} \times C_p}(g)|}{|C_H(1,I)|} \cdot \varphi(g) \\ &= \frac{p \cdot 2^{h+2}}{|C_H(1,I)|} = \frac{p \cdot |C_{Q_2^{h+1}}(1)|}{p \cdot |C_{\langle x \rangle}(1)|} \cdot \varphi(1) = \Phi_j(1) \end{aligned}$$

since  $H \cap CL(g) = \{(1,I), (1,z), (1,z^2), \dots, (1,z^{p-1})\}$

(ii) if  $g =$

$(1,I)$  or  $g=(1,z), \dots$  or  $g=(1,z^{p-1})$  or  $g=(x^{2^h}, I)$  or  $g=(x^{2^h}, z)$  or  $g=(x^{2^h}, z^{p-1})$ ,  $g \in H$

(iii) if  $g=(1,I)$  or  $g=(1,z)$  or  $g=(1,z^2), \dots$  or  $g=(1,z^{p-1})$

$$\Phi_{(j,2)}(g) = \frac{|C_{Q_2^{h+1} \times C_p}(g)|}{|C_H(g)|} \cdot \varphi(g) =$$

$$\frac{p \cdot 2^{h+2}}{|C_H(g)|} \cdot 1 = \frac{p \cdot 2^{h+2}}{|C_{\langle x,z \rangle}(g)|} \cdot 1 = \frac{p \cdot |C_{Q_2^{h+1}}(1)|}{p \cdot |C_{\langle x \rangle}(1)|} \cdot \varphi(1) = \Phi_j(1)$$

since  $H \cap CL(g) = \{g\}$ ,  $\varphi(g) = 1$

if  $g=(x^{2^h}, I)$  or  $g=(x^{2^h}, z), \dots$  or  $g=(x^{2^h}, z^{p-1})$ ,  $g \in H$

$$\begin{aligned} \Phi_{(j,2)}(g) &= \frac{|C_{Q_2^{h+1} \times C_p}(g)|}{|C_H(g)|} \cdot \varphi(g) \\ &= \frac{p \cdot 2^{h+2}}{|C_H(g)|} \cdot 1 = \frac{p \cdot |C_{Q_2^{h+1}}(x^{2^h})|}{p \cdot |C_{\langle x \rangle}(x^{2^h})|} \cdot \varphi(x^{2^h}) = \Phi_j(x^{2^h}) \end{aligned}$$

since  $H \cap CL(g) = \{g\}$ ,  $\varphi(g) = 1$

(iii) if  $\{g=(x^{2^h}, I)$  or  $g \neq (x^{2^h}, z)\} \dots$  or  $g \neq (x^{2^h}, z^{p-1})$ ,  $g \in H$

$$\begin{aligned} \Phi_{(j,2)}(g) &= \frac{|C_{Q_2^{h+1} \times C_p}(g)|}{|C_H(g)|} \cdot (\varphi(g) + \varphi(g^{-1})) = \frac{p \cdot 2^{h+2}}{|C_H(g)|} (1+1) = \\ &= \frac{p \cdot |C_{Q_2^{h+1}}(q)|}{p \cdot |C_{\langle x \rangle}(q)|} \cdot (\varphi(g) + \varphi(g^{-1})) = \Phi_j(q) \end{aligned}$$

since  $H \cap CL(g) = \{g, g^{-1}\}$  and  $\varphi(g) = \varphi(g^{-1}) = 1$

$g=(q,z), q \in Q_2^{h+1}$  and  $q \neq x^{2^h}$

(iv) if  $g \notin H$

$$\Phi_{(j,2)}(g) = 0 \quad \text{Since } H \cap CL(g) = \emptyset$$

2- IF  $H = \langle (y, I) \rangle$

$= \{(1,I), (y,I), (y^2,I), (y^3,I), (1,z), (y,z), (y^2,z), (y^3,z), \dots$   
 $\dots (1,z^{p-1}), (y,z^{p-1}), (y^2,z^{p-1}), (y^3,z^{p-1})\}$

(i) If  $g=(1,I)$  or  $g=(1,z), \dots$  or  $g=(1,z^{p-1})$

$H \cap CL(g) = \{(1,I), (1,z), \dots, (1,z^{p-1})\}$

$$\Phi_{(l+1,2)}(g) = \frac{|C_{Q_2^{h+1} \times C_p}(g)|}{|C_H(g)|} \cdot \varphi(g)$$

$$= \frac{4p \cdot 2^h}{4p} \cdot 1 = 2^h = \Phi_{l+1}(1)$$

(ii) If  $g=(x^{2^h}, I) = (y^2, I)$ , or  $g=(y^2, z)$  or  $g=(y^2, z^{p-1})$

and  $g \in H$

$$\Phi_{(l+1,2)}(g) = \frac{|C_{Q_2^{h+1} \times C_p}(g)|}{|C_H(g)|} \cdot \varphi(g)$$

$$= \frac{4p \cdot 2^h}{4p} \cdot 1 = 2^h = \Phi_{l+1}(x^{2^h})$$

Since  $H \cap CL(g) = \{g\}$ ,  $\varphi(g) = 1$

(iii) If  $g \neq (x^{2^h}, I)$  and  $g \in$

$H$ , i.e.  $\{g=(y,I), (y,z), \dots, (y,z^{p-1})$  or  $g=(y^3,I), (y^3,z), \dots, (y^3,z^{p-1})\}$

$$\Phi_{(l+1,2)}(g) = \frac{|C_{Q_2^{h+1} \times C_p}(g)|}{|C_H(g)|} \cdot (\varphi(g) + \varphi(g^{-1}))$$

$$= \frac{4p}{4p} \cdot (1+1) = 2 = \Phi_{l+1}(y)$$

since  $H \cap CL(g) = \{g, g^{-1}\}$  and  $\varphi(g) = \varphi(g^{-1}) = 1$

Otherwise

$$\Phi_{(l+1,2)}(g) = 0 \quad \text{since } H \cap \text{CL}(g) = \emptyset$$

3. IF  $H = \langle (xy, I) \rangle =$

$$\{(1, I), (xy, I), ((xy)^2, I) = (y^2, I), ((xy)^3, I) = (xy^3, I), (1, z), (xy, z), ((xy)^2, z), ((xy)^3, z), \dots, (1, z^{p-1}), (xy, z^{p-1}), ((xy)^2, z^{p-1}), ((xy)^3, z^{p-1})\}$$

(i) If  $g = (1, I)$  or  $g = (1, z)$  ..... or  $g = (1, z^{p-1})$   $H \cap \text{CL}(g) = \{g\}$

$$\Phi_{(l+2,2)}(g) = \frac{|C_{Q_{2^{h+1}} \times C_p}(g)|}{|C_H(g)|} \cdot \varphi(g)$$

$$= \frac{4p \cdot 2^h}{4p} \cdot 1 = 2^h = \Phi_{l+2}(1)$$

(ii) If  $g = (x^{2^h}, I) = ((xy)^2, I) = (y^2, I), ((xy)^2, z), \dots, ((xy)^2, z^{p-1})$

$^1)$  and  $g \in H$

$$\Phi_{(l+2,2)}(g) = \frac{|C_{Q_{2^{h+1}} \times C_p}(g)|}{|C_H(g)|} \cdot \varphi(g)$$

$$= \frac{4p \cdot 2^h}{4p} \cdot 1 = 2^h = \Phi_{l+2}(x^{2^h})$$

Since  $H \cap \text{CL}(g) = \{g\}$ ,  $\varphi(g) = 1$

(ii) If  $g \neq (x^{2^h}, I)$  and  $g \in H$

i.e.  $g = \{(xy, I), ((xy)^3, I), (xy, z), ((xy)^3, z), (xy, z^{p-1}),$

(iii)  $\{(xy)^3, z^{p-1}\}$

$$\Phi_{(l+2,2)}(g) = \frac{|C_{Q_{2^{h+1}} \times C_p}(g)|}{|C_H(g)|} \cdot (\varphi(g) + \varphi(g^{-1}))$$

$$= \frac{4p}{4p} \cdot (1 + 1) = 2 = \Phi_{l+2}(xy)$$

since  $H \cap \text{CL}(g) = \{g, g^{-1}\}$  and  $\varphi(g) = \varphi(g^{-1}) = 1$

Otherwise

$$\Phi_{(l+2,2)}(g) = 0 \quad \text{since } H \cap \text{CL}(g) = \emptyset$$

**Example(4.2):**

To construct  $\text{Ar}(Q_{32} \times C_7)$  by using the theorem (4.1) we get the following table:

$\Gamma$ - classes	[1,I]	[x <sup>16</sup> ,I]	[x <sup>8</sup> ,I]	[x <sup>4</sup> ,I]	[x <sup>2</sup> ,I]	[x,I]	[y,I]	[xy,I]	[1,z]	[x <sup>16</sup> ,z]	[x <sup>8</sup> ,z]	[x <sup>4</sup> ,z]	[x <sup>2</sup> ,z]	[x,z]	[y,z]	[xy,z]
$ CL_\alpha $	1	1	2	2	2	2	16	16	1	1	2	2	2	2	16	16
$ C_{Q_2, C_7}(CL_\alpha) $	448	448	224	224	224	224	28	28	448	448	224	224	224	224	28	28
$\Phi_{(1,1)}$	448	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(2,1)}$	224	224	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(3,1)}$	112	112	112	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(4,1)}$	56	56	56	56	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(5,1)}$	28	28	28	28	28	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(6,1)}$	14	14	14	14	14	14	0	0	0	0	0	0	0	0	0	0
$\Phi_{(7,1)}$	112	112	0	0	0	0	14	0	0	0	0	0	0	0	0	0
$\Phi_{(8,1)}$	112	112	0	0	0	0	0	14	0	0	0	0	0	0	0	0
$\Phi_{(1,2)}$	64	0	0	0	0	0	0	0	64	0	0	0	0	0	0	0
$\Phi_{(2,2)}$	32	32	0	0	0	0	0	0	32	32	0	0	0	0	0	0
$\Phi_{(3,2)}$	16	16	16	0	0	0	0	0	16	16	16	0	0	0	0	0
$\Phi_{(4,2)}$	8	8	8	8	0	0	0	0	8	8	8	8	0	0	0	0
$\Phi_{(5,2)}$	4	4	4	4	4	0	0	0	4	4	4	4	4	0	0	0
$\Phi_{(6,2)}$	2	2	2	2	2	2	0	0	2	2	2	2	2	2	0	0
$\Phi_{(7,2)}$	16	16	0	0	0	0	2	0	16	16	0	0	0	0	2	0
$\Phi_{(8,2)}$	16	16	0	0	0	0	0	2	16	16	0	0	0	0	0	2

$Ar(Q_2^5 \times C_7) =$

Table (5)





**Example(4.6):**

To find the matrices  $P(Q_{32} \times C_7)$  and  $W(Q_{32} \times C_7)$  by the proposition (4.5) from Example (3.9) to find  $P(Q_{32})$  and  $W(Q_{32})$  :

$$P(Q_{32} \times C_7) = \left[ \begin{array}{c|c} P(Q_{32}) & -P(Q_{32}) \\ \hline 0 & P(Q_{32}) \end{array} \right] =$$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

And

$$W(Q_{32} \times C_7) = \left[ \begin{array}{c|c} W(Q_{32}) & 0 \\ \hline 0 & W(Q_{32}) \end{array} \right] =$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & -1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

**Example(4.7)**

To find  $D(Q_{32} \times C_7)$  and the cyclic decomposition of the factor group

We find the matrices  $P(Q_{32} \times C_7)$  and  $W(Q_{32} \times C_7)$  as in example (4.6) and  $M(Q_{32} \times C_7)$  as in example (4.4), then :

$$P(Q_{32} \times C_7) \cdot M(Q_{32} \times C_7) \cdot W(Q_{32} \times C_7) =$$

$$\text{diag}\{2,2,2,2,2,2,2,2,2,2,2,2,1,1,1,1,1,1\} = D(Q_{32} \times C_7)$$

Then by Theorem (3.2) we have

$$AC(D(Q_{32} \times C_7)) = \bigoplus_{i=1}^{10} C_2$$

The following theorem gives the cyclic decomposition of the factor group  $AC(D(Q_{2m} \times C_7))$  when  $m=2^h$ ,  $h \in \mathbb{Z}^+$  and  $p$  is prime number .

**Theorem(4.8):**

If  $m=2^h$ ,  $h$  any positive integer and  $p$  is prime number then the cyclic decomposition of  $AC(Q_{2m} \times C_p)$  is :

$$AC(D(Q_{2m} \times C_p)) = \bigoplus_{i=1}^{2(h+1)} C_2$$

**Proof :**

By using the proposition (4.3), we can find matrix

$M(Q_{2m} \times C_p)$  and by the proposition (4.5), we find

$P(Q_{2m} \times C_p)$  and  $W(Q_{2m} \times C_p)$  when  $m=2^h$ ,  $h \in \mathbb{Z}^+$  and  $p$  is prime number :

$$P(Q_{2m} \times C_p) \cdot M(Q_{2m} \times C_p) \cdot W(Q_{2m} \times C_p) =$$

$$\text{diag}\{2,2,2,2,2,2, \dots, 2,2,2,1,1,1,1,1,1\}$$

Then ,by the theorem (3.2) we have :

$$AC(D(Q_{2m} \times C_p)) = \bigoplus_{i=1}^{2(h+1)} C_2$$



**Example(4.9) :**

Consider the groups  $(Q_{16384} \times C_{11})$ ,  $(Q_{134217728} \times C_5)$ , then :

$$1. AC(Q_{16384} \times C_{11}) = AC(Q_{2^{14}} \times C_{11}) = \bigoplus_{i=1}^{30} C_2$$

$$2. AC(Q_{134217728} \times C_5) = AC(Q_{2^{27}} \times C_5) = \bigoplus_{i=1}^{56} C_2$$

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التجزئة الدائرية وجدول شواخص ارتن للزمرة  $h \in \mathbb{Z}^+, m=2^h$  عندما  $(\mathbb{Q}_{2m} \times \mathbb{C}_p)$  و  $p$  عدد اولي

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جامعة الكوفة / كلية التربية للبنات / قسم الرياضيات

**المستخلص :**

الهدف الأساسي من هذا البحث هو تحديد التجزئة الدائرية للزمرة الايبليه الكسرية المنتهية  $AC(G) = \overline{R}(G)/T(G)$  حيث ان  $G = \mathbb{Q}_{2m} \times \mathbb{C}_p$  عندما  $h \in \mathbb{Z}^+, m=2^h$  و  $p$  عدد اولي ( زمرة كل الشواخص العمومية ذات القيم الصحيحة للزمرة  $G$  على زمرة الشواخص المحتته من الشواخص الأحادية للزمرة الجزئية الدائرية). وجدنا ان التجزئة الدائرية للزمرة  $AC(\mathbb{Q}_{2m} \times \mathbb{C}_p)$  تعتمد على القواسم الأولية للعدد  $m$  حيث انه اذا كان  $h, m=2^h$  عدد صحيح موجب و  $p$  عدد اولي فان:

$$AC(\mathbb{Q}_{2m} \times \mathbb{C}_p) = \bigoplus_{i=1}^{2(h+1)} \mathbb{C}_2$$

كذلك وجدنا الصيغة العامة لجدول شواخص ارتن  $Ar(\mathbb{Q}_{2m} \times \mathbb{C}_p)$  عندما  $h \in \mathbb{Z}^+, m=2^h$  و  $p$  عدد اولي