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On the NBC-Bases of product hypersolvable arrangements

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Abstract

This paper aims centered around the product of hypersolvable arrangements by using the hypersolvable partition analogue by proving that each of A and B are hypersolvable if $A\times B$ is a hypersolvable, also each of A and B are supersolvable if $A\times B$ is supersolvable. Moreover, this paper show how to prove that the dimension of the first non-vanishing higher homotopy groups of the complement $M(A\times B)$ is $p(A\times B)=\min\{p(A),p(B)\}$.

Keyword: Hypersolvable**,** Supersolvable**,** Hypersolvable arrangements

List of symbols: *rk(A)* = rank of *A*, *T(A)* = maximal element of central *A*, *M(A)*= complement of A.

Mathematics subject classification: 55Q20

1. Introduction:

Let $A=\{H_1,\dots,H_n\}$ be a complex hyperplane r-arrangement, with complement $M(A)=C^r\setminus\bigcup_{i=1}^{n}H_i$. The cohomology ring for the complement $M(A)$, with arbitrary constant coefficients was given by Arnold [1] and Brieskorn [2].

For a given total order $\trianglelefteq$ on A, if $C\subseteq A$ is a minimal (with respect to inclusion) dependent set, we call C a circuit of A and $\overline{C}=C\setminus\{H\}$ a broken circuit of C, where H is the smallest hyperplane in C via $\trianglelefteq$ and by NBC base $B\subseteq A$ we mean that B contains no broken circuit.

The hypersolvable class of hyperplane arrangements were originally introduced by M. Jambu and S. Papadima [3,4], as a combinatorial generalization of the supersolvable class of hyperplane arrangements and they showed that all the major results on the topology of the complements together with their algebraic and combinatorial aspects, may be extended and refined in this new framework. The hypersolvable class of hyperplane arrangements contains the supersolvable ones, the generic ones and many others.

We used the hypersolvable partition, the hypersolvable ordering which are defined by Ali and Al-Ta'ai [5], and their study of the NBC bases of a hypersolvable arrangement to complete the study of the product of hypersolvable arrangements which is studied by Mahdi in [7] he suggested a conjecture , namely , if $(A\times B, V\oplus W)$ is a hypersolvable arrangement , then A and B are hypersolvable arrangements . This conjecture is proved under some condition, namely, all the exponents of $A\times B$ are equal to 1. In section three we prove this conjecture without any condition, also we prove that the dimension of the first non-vanishing higher homotopy groups for complement $M(A\times B)$ is $p(A\times B)=\min\{p(A),p(B)\}$.

2. A hypersolvable partition of an arrangement

A hypersolvable class of arrangements was originally introduced by Jambu and Papadima ([3], [4]) Ali and Al-Ta'ai redefine this concept by using a partition which is called a hypersolvable partition as follows:

(2.1)Definition: [5]

Let A be an essential central complex r-arrangement(i.e. $\bigcap_{i=1}^{n}H_i=T(A)=T\neq\phi$ and $\text{rk}(A)=\text{rk}(T(A))=\text{co}\dim(\bigcap_{H\in A}H=r=\dim(C^r))$. A partition $\Pi=(\Pi_1,\dots,\Pi_\ell)$ of A is said to be a *hypersolvable partition* of A with *length* $\ell(A)=\ell$ denoted by Hp, if $|\Pi_1|=1$, (i.e. Π_1 is a singleton), and for fixed $2\le j\le\ell$, the block Π_j satisfies the following properties:

(***j-closed property of*** Π_j) For each $H_1,H_2\in\Pi_1\cup\cdots\cup\Pi_j$, there is no hyperplane $H\in\Pi_{j+1}\cup\cdots\cup\Pi_\ell$ such that $\text{rk}(H_1,H_2,H)=2$.

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(*j-complete property of* Π_j) For each $H_1, H_2 \in \Pi_j$, there is a hyperplane $H \in \Pi_1 \cup \cdots \cup \Pi_{j-1}$ such that $\text{rk}(H_1, H_2, H) = 2$. Note that, from the closed properties of the blocks $\Pi_2, \ldots, \Pi_{j-1}$, the hyperplane H is unique and it is denoted in this case by $H_{1,2}$.

(*j-solvable property of* Π_j) If $H_1, H_2, H_3 \in \Pi_j$, the hyperplanes $H_{1,2}, H_{1,3}, H_{2,3} \in \Pi_1 \cup \cdots \cup \Pi_{j-1}$ are equal or $\text{rk}(H_{1,2}, H_{1,3}, H_{2,3}) = 2$. Observe that, if $\text{rk}(H_1, H_2, H_3) = 2$, then from the closed properties of the blocks $\Pi_2, \ldots, \Pi_{j-1}$, we have $H_{1,2} = H_{1,3} = H_{2,3}$.

The vector of integers $d = (d_1, \ldots, d_\ell)$, is called the *exponent vector* of Π, where $d_i = |\Pi_i|$, $i = 1, \ldots, \ell$. The *rank* of Π_i is defined to be

$$\text{rk}(\Pi_i) = \text{rk}(\Pi_1 \cup \cdots \cup \Pi_i) = \text{rk}\Big(\bigcap_{H \in \Pi_1 \cup \cdots \cup \Pi_i} H\Big)$$

, for $1 \le i \le \ell$. We call the block Π_i, a *singular block* of Π if $\text{rk}(\Pi_i) = \text{rk}(\Pi_{i-1})$ and we call it *non-singular block* otherwise. Notice that, in general $\text{rk}(\Pi_i) \le \text{rk}(\Pi_{i-1}) + 1$.

(2.2) Proposition: [6]

Let A be an essential central complex $r-$arrangement. A is hypersolvable if, and only if, A has a Hp $\Pi = (\Pi_1, \ldots, \Pi_\ell)$.

(2.3) Definition:[5]

Let A be a hypersolvable $r-$arrangement with Hp $\Pi = (\Pi_1, \ldots, \Pi_\ell)$. For a fixed $1 \le j \le \ell$, the properties of the hypersolvable partition give rise to a natural partition Π_j as follows:

1- Let $\Pi_{j*1} = \{H_{i_1}, \ldots, H_{i_k}\}$ such that $rk(H_{i_1}, \ldots, H_{i_k}) = 2$ and

2- Let $\Pi_{j*2} = \Pi_j \setminus \Pi_{j*1}$.

Define the *hypersolvable ordering* of A that is denoted by $\trianglelefteq$ as follows:

1- $H \in \Pi_i$ and $H' \in \Pi_j$ such that $1 \le i < j \le \ell$, put $H \trianglelefteq H'$.

2- For a fixed $1 < j \le \ell$, give the hyperplanes of the block Π_{j*1} *of* Π_j an arbitrary total order with preserving the order of Π_i in Π for each $1 \le i \le j-1$ and preserving the order of Π_{j*2} as if $H_1, H_2, H_3 \in \Pi_j$ with $rk(H_1, H_2, H_3) = 3$, put $H_{i_1} \trianglelefteq H_{i_2} \trianglelefteq H_{i_3}$ if, and only if, $H_{i_1,i_2} \trianglelefteq H_{i_2,i_3} \trianglelefteq H_{i_1,i_3}$ such that $\{H_{i_1}, H_{i_2}, H_{i_3}\} = \{H_1, H_2, H_3\}$. Observe that, since $rk(H_1, H_2, H_3) = 3$ then there is at least one of $H_1, H_2, H_3 \in \Pi_{j*2}$.

(2.4) proposition:[3]

Let A be a hypersolvable arrangement. Then A is said to be *supersolvable* if, and only if, $\ell(A) = rk(A)$.

3. The Product of Hypersolvable Arrangement

(3.1) Definition:

Let (A,V) and (B,W) be two hyperplane arrangements. Define the product $(A\times B,V\oplus W)$ by $A\times B=\{H\oplus W:H\in A\}\cup\{V\oplus K:K\in B\}$.

Note that, $|A\times B|=|A|+|B|$. If we denote the sets $\{H\oplus W:H\in A\}$ and $\{V\oplus K:K\in B\}$ by $A\oplus W$ and $V\oplus B$ respectively, then one can easily denote the hyperplane arrangement $A\times B$ by $A\times B=(A\oplus W)\cup(V\oplus B)$.

(3.2) Proposition: [7]

Let $(A\times B,V\oplus W)$ be the product of (A,V) and (B,W) such that, $rk(A)=r$ and $rk(B)=k$. Then we have the following:

1. If each one of A and B is a hypersolvable arrangement, then $(A\oplus W,V\oplus W)$, $(V\oplus B,V\oplus W)$ and $(A\times B,V\oplus W)$ are hypersolvable arrangements.
2. If each one of A and B is a supersolvable arrangement, then $(A\oplus W,V\oplus W)$, $(V\oplus B,V\oplus W)$ and $(A\times B,V\oplus W)$ are supersolvable arrangements .

(3.3) Remark:

Suppose (A,V) and (B,W) be hypersolvable arrangements with hypersolvable partitions say; $\prod^A=(\prod_1^A,\ldots,\prod_{\ell_1}^A)$ and $\prod^B=(\prod_1^B,\ldots,\prod_{\ell_2}^B)$ respectively. From [7], then $(A\times B,V\oplus W)$ is a hypersolvable arrangement with a hypersolvable composition series ;

$\prod_1^A\oplus W\subseteq(\prod_1^A\cup\prod_2^A)\oplus W\subseteq\ldots$
$\subseteq(\prod_1^A\cup\ldots\cup\prod_{\ell_1}^A)\oplus W=(A\oplus W)$
$\subseteq(A\oplus W)\cup V\oplus\prod_1^B\subseteq$ From
$\ldots\subseteq(A\oplus W)\cup V\oplus(\prod_1^B\cup\ldots\cup\prod_{\ell_2}^B)$
$=(A\oplus W)\cup(V\oplus B)$……(3.1)

[5], $A\times B$ has a hypersolvable partition $\prod^{A\times B}=(\prod_1^{A\times B},\ldots,\prod_{\ell_1+\ell_2}^{A\times B})$ induced from the composition series (3.1), as follows:

- For $1\le k\le\ell_1$; $\prod_k^{A\times B}=\prod_k^A\oplus W$ and;
- For $\ell_1+1\le k\le\ell_1+\ell_2$; $\prod_k^{A\times B}=V\oplus\prod_{k-\ell_1}^B$.

Ali in [5] showed that such partition forms a hypersolvable partition.

(3.4)Remark:[7]

There are no collinear relations among the hyperplanes of $A\oplus W$ and $V\oplus B$. Thus, for each $H_1,H_2\in A$, there is no hyperplane $K\in B$ such that $rk\{H_1\oplus W,H_2\oplus W,V\oplus K\}=2$ and for each $K_1,K_2\in B$, there is no $H\in A$ such that $rk\{H\oplus W,V\oplus K_1,V\oplus K_2\}=2$.

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(3.5) Lemma:

Every broken circuit C in $A\oplus W$ has the following property; there is no hyperplane K in B such that $C\bigcup\{V\oplus K\}$ forms a circuit in $A\times B$. As well as, for any broken circuit C' in $V\oplus B$, there is no hyperplane H in A such that $C'\bigcup\{H\oplus W\}$ forms a circuit in $A\times B$. Thus,

$$NBC(A\oplus W)\bigcap NBC(V\oplus B)=\phi.$$

Proof: directly result of proposition (2.4) and remark (3.3).

(3.6) Proposition :

Let $A\times B$ be a hypersolvable $r+k-$ arrangement. Then;

$$NBC(A\oplus W)\subseteq NBC(A\times B) \text{ and}$$

$$NBC(V\oplus B)\subseteq NBC(A\times B).$$

Proof: By contrary, for $1\leq k\leq r$, let $S_k=\{H_{i_1}\oplus W,\ldots,H_{i_k}\oplus W\}$ be a k-section of $\Pi^{A\times B}$, such that $S_k\in NBC(A\oplus W)$ and $S_k\notin NBC(A\times B)$. Then S_k be a broken circuit in $A\times B$. That is, there exists a hyperplane $H'\in A\times B$ such that $H'\trianglerighteq H_{i_j}\oplus W$, $1\leq j\leq k$ and $\{H'\}\bigcup S_k$ form a circuit, i.e. $rk\{H'\bigcup S_k\}=k$. It is clear that, $H'\notin A\oplus W$, since $S_k\in NBC(A\oplus W)$. On the other hand, $H'\notin V\oplus B$ as shown in lemma (3.5) above. Therefore, S_k must be an NBC base of $A\times B$. Similarly, it is easy to show that $NBC(V\oplus B)\subseteq NBC(A\times B)$.

(3.7) Theorem*:*

Let $A\times B$ be a hypersolvable $r+k-$ arrangement then;

$$NBC(A\times B)=\{C\in A\times B\,|\,C=C_1\bigcup C_2$$
$$:C_1\in NBC(A\oplus W) \text{ and } C_2\in NBC(V\oplus B)\}$$

Proof: By contrary, suppose that $C\in NBC(A\times B)$, such that C cannot be written as a union of an NBC base of $A\oplus W$ and NBC base of $V\oplus B$, i.e. either;

$$C\bigcap(A\oplus W)\notin NBC(A\oplus W) \text{ or}$$

$$C\bigcap(V\oplus B)\notin NBC(V\oplus B).$$

If $C\bigcap(A\oplus W)\notin NBC(A\oplus W)$, then there exists a hyperplane $H'\in A\times B$ such that $H'\bigcup\{C\bigcap(A\oplus W)\}$ forms a circuit in $A\times B$. But this contradicts our assumption that $C\in NBC(A\times B)$. By the same way, we deduce that $C\bigcap(V\oplus B)\notin NBC(V\oplus B)$.

(3.8) Corollary :

Let $A\times B$ be a hypersolvable $r+k-$ arrangement then $p(A)=p(A\oplus W)$ and $p(B)=p(V\oplus B)$.

(3.9) Theorem :

Let $A\times B$ be a hypersolvable $r+k-$ arrangement then

$$p(A\times B)=\min\{p(A),p(B)\}.$$

Proof: In general, deduce that $p(A\times B)\leq p(A)$ and $p(A\times B)\leq p(B)$. So by contrary suppose that, $p(A\times B)<\min\{p(A),p(B)\}$. So suppose that, there exists a section $S\in S_{p(A\times B)+1}$ such that S is a $(p(A\times B)+1)$-broken circuit and from our construction of $\prod^{A\times B}$ then $S=S^{A\oplus W}\bigcup S^{V\oplus B}$ where $S^{A\oplus W}=S\bigcap A\oplus W$ and $S^{V\oplus B}=S\bigcap V\oplus B$. It is clear that $S^{A\oplus W}\in NBC(A\oplus W)$ and $S^{V\oplus B}\in NBC(V\oplus B)$ since $p(A\times B)+1<\min\{p(A)+1,p(B)+1\}$.

Now, let H be the minimal hyperplane of $A\times B$ such that $\{H\}\bigcup S$ forms a $(p(A\times B)+1)$-circuit. If $S^{A\oplus W}\neq\phi$, then H minimal than H' via the hypersolvable ordering $\trianglerighteq$ on the hyperplanes of $A\times B$, for each $H'\in S^{A\oplus W}$. Thus, $\{H\}\bigcup S^{A\oplus W}$ is a circuit and this contradicts the fact that $S^{A\oplus W}$ is an NBC base of $A\oplus W$. On the other hand, if $S^{A\oplus W}=\phi$ then $S=S^{V\oplus B}$. That is, the hyperplane H minimal than K via hypesovable ordering $\trianglerighteq$ for each $K\in S^{V\oplus B}$, thus $\{H\}\bigcup S^{V\oplus B}$ is a circuit which contradicts that $S^{V\oplus B}\in NBC(V\oplus B)$. This ends the proof.

(3.10)Theorem:

If $A\times B$ be a hypersolvable $r+k-$arrangement, then each of $A\oplus W$ and $V\oplus B$ are hypersolvable.

Proof: Since $A\times B$ be a hypersolvable $r+k-$arrangement, hence $A\times B$ has an Hp, $\prod^{A\times B}=(\prod_1,\ldots,\prod_\ell)$. From lemma (3.4), the partition $\prod^{A\times B}$ splits into two partitions as follows:

- Let $\prod_i^A=\prod_{j_i}^{A\times B}\subseteq A\oplus W$, for $1\le i\le\ell_1, 1\le j_1<j_2<\ldots<j_{\ell_1}\le\ell$ and;
- $\prod_i^B=\prod_{j_i}^{A\times B}\subseteq V\oplus B$, for $1\le i\le\ell_2, 1\le j_1<j_2<\ldots<j_{\ell_2}\le\ell$; where $\ell_1+\ell_2=\ell$.

Deduce that $\prod^A=(\prod_1^A,\ldots,\prod_{\ell_1}^A)$ form a partition of $A\oplus W$. We need to show that $\prod^A$ is a hypersolvable partition as follows:

1. If $\prod_1^A$ contains two hyperplanes say $H_1\oplus W$ and $H_2\oplus W$, then there exists a hyperplane $H\in\prod_1^{A\times B}\bigcup\ldots\bigcup\prod_{j_{n-1}}^{A\times B}$ such that $rk\{H_1\oplus W,H_2\oplus W,H\}=2$, from the complete property of block $\prod_{j_1}^{A\times B}$. Therefore, $H\in A\oplus W$, see lemma(3.4). But this contradicts our assumption that $\prod_{j_1}^{A\times B}$ is the first block of $\prod^{A\times B}$ such that $\prod_{j_1}^{A\times B}\subseteq A\oplus W$. Thus, $\left|\prod_1^A\right|=1$.
2. For $2\le k\le\ell_1$; it is clear that the block $\prod_k^A$ satisfies the closed , complete and solvable properties since it is a block from an Hp. Thus $A\oplus W$ is hypersolvable since it is has an Hp. In the same way $V\oplus B$ is a hypersolvable.

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(3.11) Corollary :

The product $r+k$-arrangement $A\times B$ is hypersolvable if, and only if, each of A and B are hypersolvable.

Proof: It is known that, if A and B are hypersolvable, then $A\times B$ is hypersolvable (see [7]). Conversely, If $A\times B$ is a hypersolvable arrangement, the canonical projections $q_A: V\times W\to V$ defined as $q_A(H\oplus W)=H$ and $q_B: W\times V\to V$ defined by $q_B(V\oplus K)=K$ preserve the dependent and independent relations. Therefore, each one of A and B are hypersolvable arrangements.

(3.12) Corollary:

$A\times B$ is supersolvable if, and only if, each of A and B is supersolvable.

Proof: It is known that, if A and B are supersolvale, then $A\times B$ is supersolvable (see [7]). Conversely if $A\times B$ is supersolvable then $\ell(A\times B)=\ell=r+k$ where $r=rk(A)$ and $k=rk(B)$, since $\ell_1\geq rk(A)=r$ and $\ell_2\geq rk(B)=k$, then $\ell=\ell_1+\ell_2\geq r+k$, but $\ell=r+k$ which means ℓ_1 and ℓ_2 cannot be greater than r and k respectively. Hence, each of A and B is supersolvable.

(3.13) Example:

Let A be central complex 6-arrangements, define as follows:

$$Q(A) = x_2x_3(x_1-x_3)(x_1+x_3)(x_2-x_1)(x_2+x_1)(x_5+3x_6)(x_5+2x_6)x_5(x_5-x_6)x_6(x_4+x_5+x_6)(x_5-x_4+x_6)$$

A is a hypersolvable arrangement in C^6 since we can find a hypersolvable Hp as follows:

$$\Pi^A=(\Pi_1,\Pi_2,\Pi_3,\Pi_4,\Pi_5,\Pi_6,\Pi_7,\Pi_8) =$$

$$(\{H_1\},\{H_2,H_3\},\{H_4\},\{H_5H_6\},\{H_7\},\{H_8\},\{H_9\},\{H_{10}\},\{H_{11},H_{12},H_{13}\})$$ where

$$H_1=\{(x_1,x_2,x_3,x_4,x_5,x_6): x_1+x_3=0\}$$

$$H_2=\{(x_1,x_2,x_3,x_4,x_5,x_6): x_1-x_3=0\}$$

$$H_3=\{(x_1,x_2,x_3,x_4,x_5,x_6): x_1+x_2=0\}$$

$$H_4=\{(x_1,x_2,x_3,x_4,x_5,x_6): x_2=0\}$$

$$H_5=\{(x_1,x_2,x_3,x_4,x_5,x_6): x_2-x_1=0\}$$

$$H_6=\{(x_1,x_2,x_3,x_4,x_5,x_6): x_3=0\}$$

$$H_7=\{(x_1,x_2,x_3,x_4,x_5,x_6): x_5+3x_6=0\}$$

$$H_8=\{(x_1,x_2,x_3,x_4,x_5,x_6): x_5+2x_6=0\}$$

$$H_9=\{(x_1,x_2,x_3,x_4,x_5,x_6): x_5=0\}$$

$$H_{10}=\{(x_1,x_2,x_3,x_4,x_5,x_6): x_5-x_6=0\}$$

$$H_{11}=\{(x_1,x_2,x_3,x_4,x_5,x_6): x_6=0\}$$

$$H_{12}=\{(x_1,x_2,x_3,x_4,x_5,x_6): x_5-x_4+x_6=0\}$$

$$H_{13}=\{(x_1,x_2,x_3,x_4,x_5,x_6): x_4+x_5+x_6=0\}$$

From new hypersolvable ordering we rewrite a defining polynomial as

$$Q(A) = (x_1+x_3)(x_1-x_3)(x_1+x_2)x_2(x_2-x_1)x_3(x_5+3x_6)(x_5+2x_6)x_5(x_5-x_6)x_6(x_5-x_4+x_6)(x_4+x_5+x_6).$$

Journal of AL-Qadisiyah for computer science and mathematics Vol.10 No.3 Year 2018
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Note that by applying our construction we can split A into two arrangements A_1 and A_2 where:

$$Q(A_1) = (x_1 + x_3)(x_1 - x_3)(x_1 + x_2)x_2 (x_2 - x_1)x_3$$

and

$$Q(A_2) = (x_5 + 3x_6)(x_5 + 2x_6)x_5 (x_5 - x_6)x_6(x_5 - x_4 + x_6)(x_4 + x_5 + x_6).$$

Observe that both of A_1 and A_2 are hypersolvable 3-arrangements since they have Hp as follows:

$$\Pi^{A_1} = (\Pi_1, \Pi_2, \Pi_3, \Pi_4) = (\{H_1\}, \{H_2, H_3\}, \{H_4\}, \{H_5, H_6\})$$

$$\Pi^{A_2} = (\Pi_1, \Pi_2, \Pi_3, \Pi_4) = (\{K_1\}, \{K_2\}, \{K_3\}, \{K_4\}, \{K_5, K_6, K_7\})$$

Where;

$$H_1 = \{(x_1, x_2, x_3) : x_1 + x_3 = 0\}$$
$$H_2 = \{((x_1, x_2, x_3)) : x_1 - x_3 = 0\}$$
$$H_3 = \{(x_1, x_2, x_3) : x_1 + x_2 = 0\}$$
$$H_4 = \{(x_1, x_2, x_3) : x_2 = 0\}$$
$$H_5 = \{(x_1, x_2, x_3) : x_2 - x_1 = 0\}$$
$$H_6 = \{(x_1, x_2, x_3) : x_3 = 0\}$$
$$K_1 = \{(x_1, x_2, x_3) : x_2 + 3x_3 = 0\}$$
$$K_2 = \{(x_1, x_2, x_3) : x_2 + 2x_3 = 0\}$$
$$K_3 = \{(x_1, x_2, x_3) : x_2 = 0\}$$
$$K_4 = \{(x_1, x_2, x_3) : x_2 - x_3 = 0\}$$
$$K_5 = \{(x_1, x_2, x_3) : x_3 = 0\}$$
$$K_6 = \{(x_1, x_2, x_3) : x_2 - x_1 + x_3 = 0\}$$
$$K_7 = \{(x_1, x_2, x_3) : x_1 + x_2 + x_3 = 0\}$$

(3.14) Example :

Let A be central complex 6-arrangements, define as follows:

$$Q(A) = x_2x_1(x_1 + x_2)x_3(x_2 - x_3)x_4x_5x_6 (x_4 - x_5)(x_4 + x_5)(x_6 - x_5)(x_6 + x_5)$$

A is a hypersolvable arrangement in C^6 since we can find a hypersolvable Hp as follows:

$$\Pi^A = (\Pi_1, \Pi_2, \Pi_3, \Pi_4, \Pi_5, \Pi_6) = (\{H_1\}, \{H_2, H_3\}, \{H_4, H_5\}, \{H_6\}, \{H_7, H_8, H_9\}, \{H_{10}, H_{11}, H_{12}\})$$ where

$$H_1 = \{(x_1, x_2, x_3, x_4, x_5, x_6) : x_2 = 0\}$$
$$H_2 = \{(x_1, x_2, x_3, x_4, x_5, x_6) : x_1 = 0\}$$
$$H_3 = \{(x_1, x_2, x_3, x_4, x_5, x_6) : x_1 + x_2 = 0\}$$
$$H_4 = \{(x_1, x_2, x_3, x_4, x_5, x_6) : x_3 = 0\}$$
$$H_5 = \{(x_1, x_2, x_3, x_4, x_5, x_6) : x_2 - x_3 = 0\}$$
$$H_6 = \{(x_1, x_2, x_3, x_4, x_5, x_6) : x_5 = 0\}$$
$$H_7 = \{(x_1, x_2, x_3, x_4, x_5, x_6) : x_4 = 0\}$$
$$H_8 = \{(x_1, x_2, x_3, x_4, x_5, x_6) : x_6 = 0\}$$
$$H_9 = \{(x_1, x_2, x_3, x_4, x_5, x_6) : x_4 - x_5 = 0\}$$
$$H_{10} = \{(x_1, x_2, x_3, x_4, x_5, x_6) : x_4 + x_5 = 0\}$$
$$H_{11} = \{(x_1, x_2, x_3, x_4, x_5, x_6) : x_6 - x_5 = 0\}$$
$$H_{12} = \{(x_1, x_2, x_3, x_4, x_5, x_6) : x_6 + x_5 = 0\}$$

Note that A is supersolvable arrangement since $\ell(A) = rk(A) = 6$

From new hypersolavble ordering we rewrite the defining polynomial of A as follow:

$$Q(A) = x_2x_1(x_1 + x_2)x_3(x_2 - x_3)x_4 x_5(x_4 - x_5)(x_4 + x_5)x_6(x_6 - x_5)(x_6 + x_5)$$

Journal of AL-Qadisiyah for computer science and mathematics Vol.10 No.3 Year 2018
ISSN (Print): 2074 – 0204 ISSN (Online): 2521 – 3504

Note that by applying our construction we can split A into two 3-arrangements A_1 and A_2 where:

$Q(A_1) = x_2x_1(x_1+x_2)x_3(x_2-x_3)$ and

$Q(A_2) = x_2x_1(x_1-x_2)(x_1+x_2)x_3(x_3-x_2)(x_3+x_2)$

Observe that both of A_1 and A_2 are supersolvable arrangements since they have Hp as follows:

$\prod^{A_1} = (\prod_1, \prod_2, \prod_3) = (\{H_1\}, \{H_2, H_3\}, \{H_4, H_5\})$

$\prod^{A_2} = (\prod_1, \prod_2, \prod_3, \prod_4) = (\{K_1\}, \{K_2, K_3, K_4\}, \{K_5, K_6, K_7\})$.

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(حول قواعد-NBC لضرب الترتيبة القابلة للحل الفوقية)

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المستخلص :

الهدف من هذا البحث يتمركز حول دراسه ضرب الترتيبه القابله للحل فوقيا التي تم دراستها باستخدام مفهوم تجزئه الترتيبه القابله للحل فوقيا ففي هذا البحث تمكنا من برهان انه اذا كان A×B ترتيبه قابله للحل فوقيا فأن كل من AوBترتيبه قابله للحل فوقيا. كذلك في هذا البحث تطرقنا الى برهان اذا كان A×Bترتيبه السوبر القابله للحل فأن كل من AوB تكون ترتيبه سوبر قابله للحل وايضا كيفية برهان ان بعد اول زمرة غير متلاشية الاعلى هوموتوبي لمتممة M(A×B) تكون $p(A\times B)=\min\{p(A),p(B)\}$.