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# **Coc-b-connected spaces**

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#### **Abstract :**

We introduce and study the recall the notion of coc-b-connected space. And we prove many of the proposition and remarks which are related to it. And we discuss the definition of coc-b-locally connected, remarks and proposition about this concept . This study presents the definition of hyper connected by coc-b-open set. Also we give some proposition and remarks about this subject and give some important generalizations on this concept and we prove some results on the concept.

## Mathematics subject classification : 54XX

## Introduction

In [5] M.C .Gemignani studied the concept of connected spaces and in [2] R. Engleking studied the characterizations of continuity provided that the continuous image of connected space is connected . Several properties of connected space in [11,10]. We recall that any two subsets *A* and *B* of a space *X* are called  $\tau$  –separated iff  $\overline{A} \cap B = A \cap \overline{B} = \emptyset$  see [8].

# **Definition(1):**

Let *X* be topological space .Then *A* is called cocompact b-open set (notation : coc-b-open set) if for every  $x \in A$ , there exists an b-open set  $U \subseteq X$  and a compact set *K* such that  $x \in U - K \subseteq A$ . The complement of coc-bopen set is called coc-b-closed set .

#### Remark(2):

Everyopen set is coc-b-open set.

But the converse is not true the following example shows: Let  $X = \{a, b, c\}$ 

 $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}.$  The coc-b-open sets are  $\{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ .

Then  $\{a, c\}$  is an coc-b-open but it is not open.

# **Definition(3):**

Let  $f: X \to Y$  be a function of a space X into a space Y then f is called an coc-b-continuous function if  $f^{-1}(A)$ is an coc-b-open set in X for every open set A in Y.

## **Definition(4):**

Let  $f: X \to Y$  be a function of a space X into a space Y, then f is called an coc-b-irresolute (*coc*'-b-continuous for brief) function if  $f^{-1}(A)$  is an coc-b-open set in X for every coc-b-open set A in Y.

# **Definition(5):**

Let  $(X, \tau)$  be topology space .Two subsets *A* and *B* of a space *X* are called coc-b-separated if  $\overline{A}^{b-coc} \cap B = A \cap \overline{B}^{b-coc} = \emptyset$ .

#### Definition(6): [9]

A subset *A* is said to be  $\omega$ -open set if for each  $x \in A$ , there exists an open set  $U_x$  such that  $x \in U_x$  and  $U_x - A$  is countable.

#### Definition(7):

Let *X* be a space and  $A \subseteq X$ . The union of all coc-bopen sets of *X* contained in *A* is called coc-b-Interior of *A* and denoted by  $A^{\circ b-coc}$  or coc-b- $In_{\tau}(A)$ . Coc-b- $In_{\tau}(A)$ =  $\bigcup \{B: B \text{ is } coc - b - open \text{ in } X \text{ and } B \subseteq A\}$ .

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#### **Definition(8):**

Let  $(X, \tau)$  be topology space and  $\emptyset \neq A \subseteq X$ . Then *A* is called coc-b-connected set if is not union of any two coc-b-separated sets .

#### **Definition(9):**

A set is called coc-b-clopen if it is coc-b-open and cocb-closed.

#### **Proposition(10):**

Let  $(X, \tau)$  be topological space ,then the following statements are equivalent :

1-X is coc-b-connected space.

2-The only coc-b-clopen sets in the space are X and  $\emptyset\,$  .

3-There exist no two disjoint coc-b-open sets A and B such that  $X = A \cup B$ .

# **Proof:**

(1)→(2) Let *X* be coc-b-connected space ,suppose that *D* is coc-b-clopen set such that  $D \neq \emptyset$  and  $D \neq X$ .Let E = X - D.Since  $D \neq X$  then  $E \neq \emptyset$ .Since *D* is coc-b-open, then *E* is coc-b-closed.But  $\overline{D}^{b-coc} \cap E = D \cap E = \emptyset$ . (since *D* is coc-b-clopen set and *E* is coc-b-closed set) hence  $D \cap E = D \cap \overline{E}^{b-coc} = \emptyset$ . Then *D* and *E* are two coc-b-separated sets and  $X = D \cup E$ . Hence *X* is not coc-b-connected space which is a contradiction. Therefore the only coc-b-clopen set in the space are *X* and  $\emptyset$ .

 $(2) \rightarrow (3)$  Suppose the only coc-b-clopen set in the space are *X* and  $\emptyset$ . Assume that there exists two disjoint cocb-open *W* and *B* such that  $X = W \cup B$ . Since  $W = B^c$ then *W* is coc-b-clopen set. But  $W \neq \emptyset$  and  $W \neq X$ which is a contradiction. Hence there exists no two disjoint coc-b-open set *W* and *B* such that  $X = W \cup B$ .

(3)→(1) Suppose that *X* is not coc-b-connected space. Then there exist two coc-b-separated sets *A* and *B* such that  $X = A \cup B$ . Since  $\overline{A}^{b-coc} \cap B = \emptyset$  and  $A \cap B \subseteq \overline{A}^{b-coc} \cap B$  thus  $A \cap B = \emptyset$ . Since  $\overline{A}^{b-coc} \subseteq B^c = A$  then *A* is coc-b-closed set and since  $\overline{B}^{b-coc} \cap A = \emptyset$  and  $A \cap B \subseteq \overline{B}^{b-coc} \cap A$  thus  $A \cap B = \emptyset$ , since  $\overline{B}^{b-coc} \subseteq A^c = B$  then *B* is coc-b-closed set, since  $A = B^c$  then *A* and *B* are two disjoint coc-b-open sets such that  $X = A \cup B$  which is a contradiction .Hence *X* is coc-b-connected space.

#### Remark(11):

Every coc-b-connected space is connected space .But the converse is not true in general.

## Example(12):

Let  $X = \{1,2,3\}$  and  $\tau = \{\emptyset, X, \{1\}, \{2\}, \{1,2\}\}$  then X is connected space but X is not coc-b-connected space since  $\{2\}, \{1,3\}$  are coc-b-open set and  $X = \{2\} \cup \{1,3\}$ .

# **Proposition(13):**

Let *A* be coc-b-connected set and *D*, *E* coc-b-separated sets .If  $A \subseteq D \cup E$  then either  $A \subseteq D$  or  $A \subseteq E$ .

## **Proof:**

Suppose *A* be a coc-b-connected set and *D*, *E* coc-bseparated sets and  $A \subseteq D \cup E$ . Let  $A \not\subseteq D$  and  $A \not\subseteq E$ .

Suppose  $A_1 = D \cap A \neq \emptyset$  and  $A_2 = E \cap A \neq \emptyset$  then  $A = A_1 \cup A_2$ . Since  $A_1 \subseteq D$  hence  $\overline{A_1}^{b-coc} \subseteq$   $\overline{D}^{b-coc}$ , since  $\overline{D}^{b-coc} \cap E = \emptyset$  then  $\overline{A_1}^{b-coc} \cap A_2 = \emptyset$ ,since  $A_2 \subseteq E$  hence  $\overline{A_2}^{b-coc} \subseteq \overline{E}^{b-coc}$ , since  $\overline{E}^{b-coc} \cap$   $D = \emptyset$  then  $\overline{A_2}^{b-coc} \cap A_1 = \emptyset$ . But  $A = A_1 \cup A_2$ therefore A is not coc-b-connected space which is a contradiction .Then either  $A \subseteq D$  or  $A \subseteq E$ .

# **Proposition(14):**

Let  $(X, \tau)$  be a topological space such that any two element x and y of X are contained in some coc-bconnected subspace of X. Then X is coc-b-connected.

# **Proof:**

Suppose *X* is not coc-b-connected. Then *X* is the union of two coc-b-separated sets *A*, *B*. Since *A*, *B* are nonempty sets. Thus there exists *a*, *b* such that  $a \in A$ ,  $b \in B$ . Let *D* be coc-b-connected subspace of *X* which contains *a*, *b*. Therefore either  $D \subseteq A$  or  $D \subseteq B$  which is a contradiction (since  $A \cap B = \emptyset$ ). Then *X* is coc-b-connected space.

#### **Proposition(15):**

If A is coc-b-connected set then  $\overline{A}^{b-coc}$  is coc-b-connected .

#### **Proof:**

Suppose *A* is coc-b-connected and  $\overline{A}^{b-coc}$  is not coc-bconnected. Then there exist two coc-b-separated set *D*, *E* such that  $\overline{A}^{b-coc} = D \cup E$ . But  $A \subseteq \overline{A}^{b-coc}$ , then  $A \subseteq D \cup E$  and since *A* is coc-b-connected set. Then either  $A \subseteq D$  or  $A \subseteq E$ . If  $A \subseteq D$  then  $\overline{A}^{b-coc} \subseteq$  $\overline{D}^{b-coc}$ . But  $\overline{D}^{b-coc} \cap E = \emptyset$ , hence  $\overline{A}^{b-coc} \cap E = \emptyset$ since  $\overline{A}^{b-coc} = D \cup E$ . Then  $E = \emptyset$  which is a contradiction .If  $A \subseteq E$  then  $\overline{A}^{b-coc} \subseteq \overline{E}^{b-coc}$ . But  $\overline{E}^{b-coc} \cap D = \emptyset$ , hence  $\overline{A}^{b-coc} \cap D = \emptyset$  since  $\overline{A}^{b-coc} = E \cup D$ . Then  $D = \emptyset$  which is a contradiction .Then  $\overline{A}^{b-coc}$  is coc-b-connected.

#### **Proposition(16):**

If *D* is coc-b-connected set and  $D \subseteq E \subseteq \overline{D}^{b-coc}$  then *E* is coc-b-connected.

#### **Proof:**

Let *D* be coc-b-connected set and  $D \subseteq E \subseteq \overline{D}^{b-coc}$ .Suppose *E* is not coc-b-connected ,then there exist two sets *A*, *B* such that  $\overline{A}^{b-coc} \cap B = A \cap \overline{B}^{b-coc} = \emptyset$ ,  $E = A \cup B$ , since  $D \subseteq E$ , thus either  $D \subseteq A$  or  $D \subseteq B$ .Suppose  $D \subseteq A$  then  $\overline{D}^{b-coc} \subseteq \overline{A}^{b-coc}$ , thus  $\overline{D}^{b-coc} \cap$  $B = \overline{A}^{b-coc} \cap B = \emptyset$ . But  $D \subseteq E \subseteq \overline{D}^{b-coc}$ , then  $\overline{D}^{b-coc} \cap B = B$ . Therefore  $B = \emptyset$  which is a contradiction, hence *E* is coc-b-connected set.

#### **Proposition(17):**

If a space X contains a coc-b-connected subspace E such that  $\overline{E}^{bcoc} = X$  then X is coc-b-connected.

#### **Proof:**

Suppose *E* a coc-b-connected subspace of a space *X* such that  $\overline{E}^{b-coc} = X$ , since  $E \subseteq X = \overline{E}^{b-coc}$  then by proposition (2.3.11) then *X* is coc-b-connected.

#### Lemma(18):

If *A* is subset of a space *X* which is both coc-b-open and coc-b-

closed sets, then any coc-b-connected subspace  $C \subseteq X$  which meets A must be contained in A.

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#### **Proof:**

If A is coc-b-open and coc-b-closed sets in X then  $C \cap A$  coc-b-open and coc-b-closed in C, if C is coc-b-connected this implies that  $C \cap A = C$  which says that C is contained in A.

#### Proposition(19):

The coc-b-continuous onto image of coc-b-connected space is connected.

#### **Proof:**

Let  $f: (X, \tau) \to (Y, \tau')$  be coc-b-continuous, onto function and X is coc-b-connected .To prove that Y is connected .Suppose Y is a not connected space . So  $Y = A \cup B$  such that  $A \neq \emptyset$ ,  $B \neq \emptyset$  and  $A \cap B = \emptyset$  and  $A, B \in \tau'$  hence  $f^{-1}(Y) = f^{-1}(A \cup B)$ , then X = $f^{-1}(A) \cup f^{-1}(B)$ . Since f is coc-b-continuous hence  $f^{-1}(A)$  and  $f^{-1}(B)$  are coc-b-open in X and sine that  $A \neq \emptyset, B \neq \emptyset$  and f is onto .Then  $f^{-1}(A) \neq$  $\emptyset, f^{-1}(B) \neq \emptyset$  and  $f^{-1}(A) \cap f^{-1}(B) = \emptyset$ , hence X is not coc-b-connected space which is contradiction .Then Y is connected .

#### Corollary(20):

The *coc*<sup>'</sup>-b-continuous image of coc-b-connected space is coc-b-connected.

#### **Proof:**

Let  $f: (X, \tau) \to (Y, \tau')$  be coc'-b-continuous, onto function and X is coc-b-connected. To prove Y is coc-bconnected. Suppose Y is not coc-b-connected space. So,  $Y = A \cup B$  such that  $A \neq \emptyset$ ,  $B \neq \emptyset$  and  $A \cap B = \emptyset$ and A, B are coc-b-open sets, hence  $f^{-1}(Y) =$  $f^{-1}(A \cup B)$  then  $X = f^{-1}(A) \cup f^{-1}(B)$ . Since that  $f \ coc'$ -b-continuous, hence  $f^{-1}(A)$  and  $f^{-1}(B)$  are coc-b-open in X and since that  $A \neq \emptyset$ ,  $B \neq \emptyset$  then  $f^{-1}(A) \neq \emptyset$ ,  $f^{-1}(B) \neq \emptyset$  and  $f^{-1}(A) \cap f^{-1}(B) = \emptyset$ , hence X is not coc-b-connected space which is contradiction .Then Y is coc-b-connected.

#### **Proposition(21):**

Let *X* be topological space and let  $Y = \{0,1\}$  have the discrete space. Then *X* is coc-b-connected iff there is no coc-b-continuous function from *X* onto *Y*.

#### **Proof:**

Suppose  $f: (X, \tau) \to (Y, \tau')$  is coc-b-continuous onto function. So there exists  $x, y \in X$  such that  $x \neq$ y, f(x) = 0, f(y) = 1. Then  $f^{-1}(\{0\}) = A, A \subseteq X$ and  $f^{-1}(\{1\}) = B, B \subseteq X$  therefore A and B are cocb-open set in X. Since f is coc-b-continuous. Hence  $X = A \cup B$  such that  $A \neq \emptyset, B \neq \emptyset$ . A, B are coc-bopen sets which is a contradiction .Since X is coc-bconnected.

Conversely, let *X* be not coc-b-connected. Then  $X = A \cup B$  such that  $A \neq \emptyset$ ,  $B \neq \emptyset$ ,  $A \cap B = \emptyset$  and *A* , *B* are coc-b-open sets. Define  $g: (X, \tau) \rightarrow (Y, \tau')$  such that  $g(x) = 0 \forall x \in A$  and  $g(x) = 1 \forall x \in B$ , hence *g* is coc-b-continuous, which is contradiction. Then *X* is coc-b-connected.

## Definition(22): [4]

Let  $f: X \to Y$  be a function of a space X into a space Y. Then f is called a  $\omega$ -continuous function if  $f^{-1}(A)$  is an  $\omega$ -open set in X for every open set A in Y.

#### Definition(23): [6]

A subset A of a space X is called an  $\omega$ -set if  $A = U \cup V$ when U is open set and  $Int(V) = Int_{\omega}(V)$ .

# **Definition(24):** [7]

A space  $(X, \tau)$  is said to be satisfy  $\omega$ -condition if every  $\omega$ -open is  $\omega$ -set.

#### Lemma(25): [6]

A subset A of a space X is open iff A  $\omega$ -open set and  $\omega$ -set.

#### Definition(26): [1]

A space X is said to be  $\omega$ -connected provided that X is not the union of two nonempty disjoint  $\omega$ -open sets.

# **Proposition(27):**

Let  $(X, \tau)$  and  $(Y, \tau')$  be two topological spaces. If satisfy  $\omega$ - condition ,then the coc-b-continuous, onto image of coc-b-connected space is  $\omega$ -connected .

#### Proof:

Let  $f: (X, \tau) \to (Y, \tau')$  be coc-b-continuous, onto function and X be coc-b-connected. To prove Y is  $\omega$ connected. Suppose Y is not  $\omega$ -connected space. So,  $Y = \{A \cup B\}$  such that  $A \neq \emptyset$ ,  $B \neq \emptyset$  and  $A \cap B = \emptyset$ and A, B  $\omega$ -open sets since Y satisfy  $\omega$ -condition, then A, B are open sets. Hence  $f^{-1}(Y) = f^{-1}(A \cup B)$ .

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Then  $X = f^{-1}(A) \cup f^{-1}(B)$ . Since that f coc-bcontinuous hence  $f^{-1}(A)$  and  $f^{-1}(B)$  are coc-b-open in

*X* .Since that  $A \neq \emptyset$ ,  $B \neq \emptyset$  and *f* is onto then  $f^{-1}(A) \neq \emptyset$ ,  $f^{-1}(B) \neq \emptyset$  and  $f^{-1}(A) \cap f^{-1}(B) = \emptyset$ , hence *X* is not coc-b-connected space which is contradiction .Then *Y* is  $\omega$ -connected.

### **Definition(28):** [3]

A space  $(X, \tau)$  is said to be locally connected if for each point  $x \in X$  and each open set U such that  $x \in U$ . There is a connected open set  $V, x \in V \subseteq U$ .

#### **Definition(29):**

A space  $(X, \tau)$  is said to be coc-b-locally connected if for each point  $x \in X$  and each coc-b-open set U such that  $x \in U$ . There is a coc-b-connected open set V $x \in V \subseteq U$ .

#### **Proposition(30):**

Every coc-b-locally connected space is locally connected space.

Proof: Clear

#### Remark(31):

The convers of the proposition (24) is not true in general.

#### Example(32):

Let  $X = \{1,2,3\}$ ,  $\tau = \{X, \emptyset, \{2,3\}\}$ . The coc-b-open sets are  $X, \emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}$  then  $(X, \tau)$  is locally connected but  $(X, \tau)$  is not coc-b-locally. Since  $1 \in \{1,2\}$ . There is no coc-b-connected open set V such that  $1 \in V \subseteq \{1,2\}$ .

#### Remark(33):

If  $(X, \tau)$  is a coc-b-locally connected space. Then it need not be coc-b-connected.

# Example(34):

Let  $X = \{1,2,3\}$ ,  $\tau = \{X, \emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}\}$ . The coc-b-open sets is discrete topology then  $(X, \tau)$  is a coc-b-locally but  $(X, \tau)$  is not coc-b-connected ,since  $\{1\}, \{2,3\}$  are coc-b-open sets in *X* such that  $X = \{1\} \cup \{2,3\}$  and  $\{1\} \cap \{2,3\} = \emptyset$ .

# **Definition(35):**

Let  $(X, \tau)$  be any space, a maximal coc-b-connected of *X* is said to be coc-b-component of *X*.

#### Theorem(36):

For a space  $(X, \tau)$ . The following condition are equivalent:

1-X is a coc-b-locally connected.

2-Every coc-b-component of every coc-b-open set is open.

#### **Proof:**

 $(1) \rightarrow (2)$  Let *X* be coc-b-locally connected and let *C* be coc-b-component of *X* such that  $x \in C$ . Let  $x \in X$  and *A* is coc-b-open set in *X* such that  $x \in C \subseteq A$ . Then  $x \in A$  and *A* is coc-b-open set in *X*. Since *X* is a coc-b-locally connected, then there exist coc-b-connected open set *V* in *X* such that  $x \in V \subseteq A$ , since that *C* is coc-b-component, then  $V \subseteq C$  and  $\bigcup_{x \in C} V_x \subseteq C$ , hence  $C = \bigcup_{x \in X} \{V_x : x \in C\}$  therefore *C* is open set.

 $(2) \rightarrow (1)$  Let  $x \in X$  and U be coc-b-open set in X such that  $x \in U$  and let C coc-b-component of U such that  $x \in C \subseteq U$ . Then C is open set in X by (2). Since that C is coc-b-component, hence C is coc-b-connected. Therefore X is a coc-b-locally connected.

#### **Proposition(37):**

The coc-b-continuous, open, image of coc-b-locally connected space is locally connected.

#### **Proof:**

Let  $f: (X, \tau) \to (Y, \tau')$  be coc-b-continuous open and onto function and  $(X, \tau)$  is coc-b-locally connected space. To prove  $(Y, \tau')$  is locally connected. Let  $y \in Y$ and U be open set in  $Y \ni y \in U$ . Since f is onto there exist  $x \in X$  such that f(x) = y, since f is coc-bcontinuous then  $f^{-1}(U)$  is coc-b-open set in X such that  $x \in f^{-1}(U)$ , since X is coc-b-locally connected then there exist V is coc-b-connected open set in X such that  $x \in V \subseteq f^{-1}(U)$  since f open function, then  $f(x) \in$  $f(V) \subseteq U$  such that f(V) is open and f(V) is connected by corollary(14). Therefore Y is a locally connected .

#### Remark(38):

The coc-b-continuous image of coc-b-locally connected need not be coc-b-locally connected.

## Example(39):

Let  $X = \{1,2,3\}$ ,  $Y = \{a, b, c\}$ ,  $\tau = \{\emptyset, X, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}\}$  and

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 $\tau' = \{\emptyset, Y, \{a\}\}$ . The coc-b-open set in *X* and *Y* are discrete topology. Define  $f: (X, \tau) \to (Y, \tau')$  such that f(1) = a, f(2) = b, f(3) = c is coc-b-continuous, onto function .Then  $(X, \tau)$  is coc-b-locally connected but  $(Y, \tau')$  is not coc-b-locally connected since  $b \in \{a, b\}$  and exists no coc-b-connected open set *V* in *X* such that  $b \in V \subseteq \{a, b\}$ .

## **Proposition(40):**

The *coc*<sup>'</sup>-b-continuous, open, image of coc-b-locally connected space is coc-b-locally connected.

#### **Proof:**

Let  $f: (X, \tau) \to (Y, \tau')$  be coc'-b-continuous, open and onto function and  $(X, \tau)$  is coc-b-locally connected space. To prove  $(Y, \tau')$  is coc-b-locally connected, let  $y \in Y$  and U is coc-b-open set in Y, such that  $y \in U$ . Since f onto there exist  $x \in X$  such that f(x) = y for each  $y \in Y$ , since f is coc'-b-continuous, hence  $f^{-1}(U)$ is coc-b-open set in X such that  $x \in f^{-1}(U)$ . Since X is coc-b-locally connected then  $\exists V$  coc-b-connected open set in X such that  $x \in V \subseteq f^{-1}(U)$ , since f is open then f(V) is open set in Y and f(V) is coc-b-connected by proposition (15). Hence  $f(V) \subseteq U$ . Therefore Y is a coc-blocally connected space.

#### **Definition(41):**

Let X be a space  $A \subseteq X$ , A is called coc-b-dence set in X if  $\overline{A}^{b-coc} = X$ .

We recall that a space X is said to be hyper connected if for every nonempty open subset of X is dence see [5].

#### **Definition(42):**

A space *X* is said to be coc-b-hyper connected if for every nonempty coc-b-open subset of *X* is coc-b-dence.

Now, we explain the relation between an coc-b-hyper connected space and hyper connected space .

#### **Proposition(43):**

Every coc-b-hyper connected space is hyper connected.

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## **Proof:**

Let X be coc-b-hyper connected space. Then every nonempty coc-b-open subset of X is coc-b-dence in X, hence every nonempty open subset of X is dence.

Therefore X is hyper connected (since every coc-bdence set is dence).

#### Remark(44):

The convers of the proposition (43) is not true in general.

## Example(45):

Let  $X = \{1,2,3\}$ ,  $\tau = \{X, \emptyset\}$ . The coc-b-open sets  $\{X, \emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}\}$ . Then  $(X, \tau)$  is hyper connected but  $(X, \tau)$  is not coc-b-hyper connected since  $\{1\}$  is coc-b-open set and  $\overline{\{1\}}^{b-coc} = \{1\} \neq X$ .

## **Proposition(46):**

Every coc-b-hyper connected space is coc-b-connected.

## **Proof:**

Let *X* be coc-b-hyper connected space and suppose *X* is not coc-b-connected. Then there exists *A* is coc-b-clopen subset in *X* such that  $A \neq \emptyset$  and  $A \neq X$ , hence  $A = \overline{A}^{b-coc}$  which is a contradiction, since *X* is coc-b-hyper connected. Therefore *X* is coc-b-connected.

# **Definition(47):** [2]

A space  $(X, \tau)$  is said to be extremally disconnected if the closure of every open subset of X is open in X.

#### **Definition(48):**

A space  $(X, \tau)$  is said to be coc-b-extremally disconnected if the closure of every open is coc-b-open.

#### Remark(49):

Every extremally disconnected space is coc-bextremally disconnected space and the convers is not true in general.

## Example(50):

Let  $X = \{a, b, c\}$ ,  $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$ . The coc-bopen set  $\{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ . Then  $(X, \tau)$  is coc-b-extremally disconnected, but  $(X, \tau)$  is not extremally disconnected since  $\overline{\{a\}} = \{a, c\} \notin \tau$ .

#### Remark(51):

Every coc-b-hyper connected is a coc-b-extremally disconnected space but the convers is not true in general.

# Example(52):

Let  $X = \{1, 2, 3\}, \tau =$ 

{*X*,  $\emptyset$ , {1}, {2}, {3}, {1,2}, {1,3}, {2,3}}. The coc-b-open set *X*,  $\emptyset$ , {1}, {2}, {3}, {1,2}, {1,3}, {2,3}. Then (*X*,  $\tau$ ) is coc-b-extremally disconnected since the closure of every open subset of X is coc-b-open. But (*X*,  $\tau$ ) is not coc-b-hyper connected since  $\overline{A}^{b-coc} = A \neq X \forall A$  coc-b-open.

The following diagram explain the relationship among these types of connected spaces



Locally connected

Hyper connected

# Raad, A/Ghadeer, k

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# الفضاءات المتصلة من النمط coc-b

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#### المستخلص :

نحن نقدم و ندرس الفضاء المتصل من النمط coc-b ،حيث نقدم عدد من المبر هنات والملاحظات حول هذا المفهوم والنتائج التي تخص ذلك و كذلك نقوم بمناقشة تعريف المتصل محليا من النمط coc-b أيضا نقدم بعض الملاحظات والمبرهنات حول المفهوم الجديد. في هذه الدراسة أيضا نقدم تعريف hyper connected باستخدام المجموعة المفتوحة من النمط co-b ونعطى المبر هنات والملاحظات التي تخص ذلك المفهوم.