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On Minimal and Maximal T_1 –space Via B^*c-open set

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Abstract

In this paper, we introduced study about properties of the spaces minimal T_1 – Space and maximal T_1 – space by using the set open (respace β - open, B^*c– open) sets and we concluded some propositions, remarks and relations between spaces m B^*c – T_1 space and M B^*c – T_1 space and study the relation between m – space and M – space to space T_1 – space. Where we find every m B^*c – T_1 space is M B^*c – T_1 space, but the converse is not true in general. Also we introduced hereditary properties and topological properties.

Key Words: minimal B^*c – T_1 space maximal B^*c – T_1 space minimal β – T_1 space maximal β – T_1 space

Mathematics subject classification: 54XX

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1) Introduction:

The topological idea from study this type of the space came to determine the relation between minimal T_1 – space and maximal T_1 – space In [5] Abd El – Monsef M. E., El – Deeb S. N. Mahmoud R. A., the Category β – open set was defined which considered to study the B^*c – open set. In [1] and [2] F. Nakaoka and N. oda. , has been defined minimal and maximal open set. In [4] M. C. Gemignani, has been defined T_1 – space In[3]S.S Benchalli defined the function category strongly m – open function and m – irresolute function which is study open function continuity respectively. In [6] and [7] proved proposition about T_1–space by the B^*c – open set.

2) Basic Definitions and Remarks

Definition (2.1): [5]

Let X be a topological space, then a sub set A of X is said to be β - open set if A $\subseteq cl(int(cl(A)))$ is β - closed set if A^C is β - open.

Definition (2.2):

Let X be a topological space and $A \subseteq X$.
Then a β - open set A is said a B^*c - open set if ∀ x ∈ A ∃ F_x closed set ∋ x ∈ $F_x \subseteq A$. A is a B^*c – closed set if A^c is a B^*c – open.

Definition (2.3):

The family union of all B^*c - open set of a topological space X contained in A is said B^*c - interior of A is, denoted by A^{oB^*c}. i.e
$A^{oB^*c} = \cup \{ : G \subseteq A \text{ and } G\ B^*c \text{ - open in } X \}$.

Definition (2.4): [1]

Let X be a topological space A proper non empty open set U of X is said to be a minimal open set if any open set which is contained in V is ∅ or U.

Definition (2.5): [2]

Let X be a topological space A proper nonempty open set U of X is called to be a maximal open set if any open set which is contained in U is X or U.

Definition (2.6):

Let X be a topological space A proper non empty β - open set U of X is said to be
i) A minimal β - open set if any β - open set which is contained in U is ∅ or U.
ii) A maximal β - open set if any β - open set which contains in U is X or U.

Definition (2.7):

Let X be a topological space A proper non empty B^*c - open set U of X is said to be
i) A minimal B^*c - open set if any B^*c - open set which contains in U is ∅ or U.
ii) A maximal B^*c - open set if any B^*c - open set which is contained in U is X or U.

Definition (2.8): [4]

A topological space X is called T_1 – space iff for each x ≠ y in X, there exists open sets U and V such that x ∈ U, y ∉ U and y ∈ V. x ∉ V.

Definition (2.9):

A topological space X is called m - T_1 space (respace M - T_1) space iff for each x ≠ y in X, ∃ m – open (respace M – open) sets U and V such that x ∈ U, y ∉ U and y ∈ V. x ∉ V.

Definition (2.10):

A topological space X is called βT_1 – space iff ∀ x ≠ y in X there exists β - open sets U and V such that x ∈ U, y ∉ U and y ∈ V. x ∉ V.

Definition (2.11):

A topological space X is called mβ - T_1 space (respace Mβ T_1 - space) iff for each x ≠ y in X there exists mβ – open (respace Mβ – open) sets U and V such that x ∈ U, y ∉ U and y ∈ V. x ∉ V.

Definition (2.12):

A topological space X is called BcT_1 – space iff for each x ≠ y in X ,∃B*c - open sets U and V such that x ∈ U, y ∉ U and y ∈ V. x ∉ V.

Definition (2.13):

A topological space X is called mB*c - T_1 space (respace MB*c - T_1) space iff for each x ≠ y in X there exists mB*c – open (respace MB*c – open) sets U and V so that x ∈ U, y ∉ U and y ∈ V. x ∉ V.

Definition (2.14):

Let X,Y be a topological spaces and let F: X → Y be a function Then:
i) F is called strongly m – open [3], if ∀ m – open set U in X, then F(U) is m - open set in Y.
ii) F is called strongly M – open ,if ∀ M – open set U in X, then F(U) M - open set in Y.
iii) F is called strongly mβ – open (respace strongly Mβ – open),if for all mβ – open (respace Mβ – open) set U in X, then F(U) is mβ – open (respace Mβ – open) set in Y.[8].

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iv) F is called strongly $mB^{*}c$–open (respace strongly M $B^{*}c$ – open), if ∀m $B^{*}c$ – open (respace MB*c – open) set U in X, then F(U) is mB*c – open (respace MB*c – open) set in Y.

Definition (2.15):

Let X,Y be a topological spaces and let F: X → Y be a function Then:

i) F is called m – irresolute function [3], if ∀ m – open U in Y, then $F^{-1}(U)$ is m – open in X.

ii) F is called M –irresolute function, if ∀M –open U in Y, then $F^{-1}(U)$ is M – open in X.

iii) F is called mβ – irresolute (respace Mβ – irresolute) function. If ∀ mβ –open (respace Mβ – open) U in Y, then $F^{-1}(U)$ mβ – open (respace Mβ – open) set in X.[8].

iv) F is called $mB^{*}c$ – irresolute (respace $MB^{*}c$ – irresolute) function, if ∀ $mB^{*}c$ –open (respace $MB^{*}c$ – open) U in Y, then $F^{-1}(U)$ $mB^{*}c$ – open (respace $MB^{*}c$ – open) set in X.

Notion: We will use the symbol m to minimal sets and the symbol M to maximal sets.

The family a fall β – open set is denoted by βo(x) and the family of all $B^{*}c$ – open is denoted by $B^{*}co(x)$.

Theorem (2.1):

Let X be a topological space and A ⊆ X. Then:

i) Every open set is β - open.

ii) Every $B^{*}c$ – open set is β - open.

Proof:

i) Let A be open set, then A =int(A). Since A ⊆ cl(A), then A = int(A) ⊆ int(cl(A)), there for A ⊆cl(int(cl(A))), hence A β - open set in X.

ii) By definition (2.2)

The converse of above Theorem is not true in general.

Example (2.1):

Let X = {a, b, c}, t = {∅, X, {a}, {b}, {a, b} }.

βo (X) = {∅, X, {a}, {b}, {a, b}, {a, c},{b, c} }.

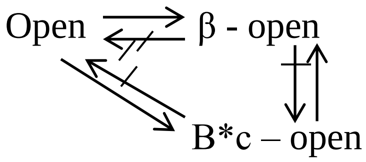
B*co (X) = {∅, X, {b,c}, {a, c} }.

Then let A = {b, c}, B = {b}. Note that

i) A is β - open, but not open.

ii) B is β - open, but not B*c – open.

The following diagram shows the relation among types of open sets



Remark (2.1):

Let X be a topological space . Then:

i) Every m – open (respace M – open) is open.

ii) Every mβ – open (respace Mβ – open) is β - open.

iii) Every mB*c – open (respace MB*c – open) is B*c - open.

The converse of above Theorem is not true in general.

Example (2.2):

In example (2.1) ,we notice:

i) A = {a,b} open, but not m – open also B = {a} open, but not M – open.

ii) A = {a,b}β - open, but not m β – open also B = {a} β - open, but not M – open.

iii) A = ∅ B*c - open, but not m B*c – open also B = X B*c - open, but not M – open.

Corollary (2.1):

i) Every m –open (respace M–open) is β - open.

ii) Every mB*c – open (respace MB*c – open) is β - open.

Remark (2.2):

Let X be a topological space Then

i) Every T_1 – space is β - T_1 space

ii) Every B*c – T_1 space is β - T_1 space

The converse of above Theorem is not true in general.

Example (2.3):

In example (2.1), we see that X is β - T_1 space But

i) X is not T_1 – space, since ∀ a, c ∈ X ∋ a ≠ c, but ∄ U,V open in X∋ a ∈ U, c ∉ U and c ∈ V, a ∉ V.

ii) X is not B*c – T_1 space ,since ∀ a, c ∈ X ∋ a ≠ c, but ∄ U, V B*c – open ∋ a ∈ U, c ∉ U and c ∈ V, a ∉ V.

Theorem (2.2):

Let X be a topological space and A ⊆ X. Then x ∈ $A^{oB^{*}c}$ iff ∃ B*c – open in X ∋ x ∈ G ⊆ A.

Proof:

Let $x \in A^{oB^*c}$

Since $A^{oB^*C} = \bigcup \{ G : G \subseteq A$, G is B*c - open set in X }.

Then $x \in \bigcup \{ G : G \subseteq A$, G is B*c - open set in X }, and hence ∃ G B*c – open in X ∋ x ∈ G ⊆ A.

Conversely

Let x ∈ G ⊆ A and G is B*c – open ∋ x ∈ G ⊆ A. Then $x \in \bigcup \{ G : G \subseteq A$, G is B*c - open set in X },therefore $x \in A^{oB^*C}$.

Definition (2.16):

Let X be a topological space and A ⊆ X, x ∈ X. Then

i) The point x is called limit point of A [7] iff ∀ U open set ∋ x ∈ U, then (U ∩ A) - {x} ≠ ∅.

ii) The point x is called β-limit point of A iff ∀ U β -open set ∋x ∈U,then (U∩ A) - {x} ≠ ∅.

iii) The point x is called B*c - limit point of A iff ∀ U B*c - open set ∋ x ∈ U, then (U ∩ A) - {x} ≠ ∅.

Remark (2.2):

i) The set of all limit point of A is denoted by Á.

ii) The set of all β - limit point of A is denoted that $Á^{\beta}$.

iii) The set of all B*c - limit point of A is denoted that $Á^{B^*c}$.

Lemma (2.1):

Let X be a topological space and A ⊆ X, x ∈ X. Then

i) A is closed set iff Á ⊆ A. [7].

ii) A is β - closed set iff $Á^{\beta} \subseteq A$. [5].

Theorem (2.3):

Let X be a topological space and A ⊆ X. Then A is B*c – closed set iff $Á^{B^*c} \subseteq A$.

Proof:

Let A be B*c – closed and b ∉ A, then $b \in A^C$ is B*c – open set, hence ∃ B*c – open set A^C ∋ $A^C \cap A = \emptyset$. Hence $x \notin Á^{B^*c}$, therefore $Á^{B^*c} \subseteq A$.

Conversely

Let $Á^{B^*c} \subseteq A$ and b ∉ A then $b \notin Á^{B^*c}$, hence ∃ B*c – open set G ∋ b ∈ G ∩ A = ∅, hence $b \in G \subseteq A^C$. Therefore A^C is B*c – open set in X by Theorem(2.2), hence A is B*c– closed .

3) m – T_1 (M – T_1) space by using the set (open, β - open, B*c – open).

Lemma (3.1) [6]

Let X be a topological space Then X is T_1 – space iff {x} is closed set in X ∀ x ∈ X.

Theorem (3.1):

Let X be a topological space Then X is β–T_1 space iff {x} is β– closed set in X ∀ x ∈ X.

Proof:

Let X be β– T_1 space and let y ∉ {x}, the x ≠ y. Since X β– T_1 space, then ∃ U, V β - open in X ∋ x ∈ U, y ∉ U and y ∈ V, x ∉ V. Then V β - open in X and y ∈ V, then (V ∩ {x}) – {Y} = ∅, then y not β - limit point of {x}, then $y \notin [\{x\}]^{\beta}$, then $[\{x\}]^{\beta} \subseteq \{x\}$. Hence {x} is β - closed by lemma (2.1) (ii).

Conversely

Let x, y ∈ X ∋ x ≠ y. Let {x} β - closed in X, then $\{x\}^C$ β - open in X. Let $U = \{x\}^C$, $V = \{y\}^C$ are β - open in X ∋ x ∈ V, y ∉ V and y ∈ X, x ∉ U, hence X β–T_1 space

Theorem (3.2):

Let X be a topological space Then X B*c –T_1 space iff {x} is B*c – closed set in X ∀ x ∈ X.

Proof:

Similarly of Theorem(3.2).

Theorem (3.3):

Let X be a topological space Then

i) Every m - T_1 space is T_1 space

ii) Every m β - T_1 space is β–T_1 space

ii) Every mB*c - T_1 space is B*c –T_1 space

Proof:

i) Let X be m - T_1 space and let x, y ∈ X ∋ x ≠ y. Since X m - T_1 space, then ∃ U, V m - open in X ∋ x ∈ U, y ∉ U and y ∈ V, x ∉ V, then by Remark (2.1) (i) and definition (2.8), we get the resulte.

ii) Let X be mβ- T_1 space and let x, y ∈ X ∋ x ≠ y. Since X is mβ- T_1 space, then ∃ U, V mβ- open sets in X ∋ x ∈ U, y ∉ U and y ∈ V, x ∉ V. then by Remark (2.1) (ii) and definition (2.10), we get the resulte.

iii) Let X be mB*c - T_1 space and let x, y ∈ X ∋ x ≠ y. Since X is mB*c - T_1 space, then ∃ U, V mB*c - open sets in X ∋ x ∈ U, y ∉ U and y ∈ V, x ∉ V. then by Remark (2.1) (iii) and definition (2.12), we get the resulte .The converse of above Theorem is not true in general.

Example (3.1)

Let X = IR with a usual top. Then X is T_1 – space, but not m – T_1 space

Proof:

Let x ∈ IR and let (x, ∞), (-∞, x) ∈ T. Since (x, ∞) ∪ (-∞, x) ∈ T, then IR – {(x, ∞) ∪ (-∞, x)} closed set in IR.

But IR – {(x, ∞) ∪ (-∞, x)} = {x}, then ∃ {x} closed set in IR ∀ x ∈ IR. Hence X is T_1 – space by lemma (3.1).

T. P X not m – T_1 space

Let x, y ∈ X ∋ x ≠ y, but ∄ U, V m – open in X ∋ x ∈ U, y ∉ U and y ∈ V, x ∉ V.

Example (3.2):

In example (3.1). Note that X is βT_1 – space by remark (2.2) (i), but not mβ - T_1 space Since ∀ x, y ∈ X , x ≠ y, but ∄ U, V mβ – open in X ∋ x ∈ U, y ∉ U and y ∈ V, x ∉ V.

Example (3.3)

Let X = IR with a usual topology. Then X is B*cT_1 – space, but not mB*c – T_1 space

Proof:

Let x, y ∈ Y, x ≠ y and let |x – y| = ε and let U = $(x - \frac{\varepsilon}{4}, x + \frac{\varepsilon}{4})$, V = $(y - \frac{\varepsilon}{4}, y + \frac{\varepsilon}{4})$, then U, V ∈ T ∋ x ∈ U, Y ∉ U and Y ∈ V, X ∉ V.

Choose U = (x, Y], V = [x, Y), then U, V are β - open set, then

∀ a ∈ U ∃ {a} closed set ∋ a ∈ {a} ⊆ U.

∀ b ∈ V ∃ {b} closed set ∋ a ∈ {b} ⊆ V.

Then U, V B*c – open sets in X ∋ y ∈ U, x ∉ U and x ∈ V, Y ∉ V.

Then X is B*c – T_1 space

T. P. X not mB*c – T_1 space Let x, y ∈ X ∋ x ≠ Y, but ∄ mB*c – open set U, V in X ∋ y ∈ U, x ∉ U and x ∈ V, Y ∉ V.

Corollary (3.1):

Let X be a topological space .Then:

i) If X m–T_1 space , then {x} closed set in X∀ x∈ X.

ii) If X mβ – T_1 space, then {x} β - closed set in X ∀ x ∈ X.

iii) If X mB*c – T_1 space, then {x} B*c - closed set in X ∀ x ∈ X.

Proof:

i) Follows from theorem (3.3) (i) and lemma (3.1).

ii) Follows from theorem (3.3) (ii) and Theorem(3.2).

iii) Follows from theorem (3.3) (iii) and Theorem(3.2).

Theorem (3.4):

Let X be a topological space Then:

i) If X is m – T_1 space, then X is β - T_1 space

ii) If X is mB*c – T_1 space, then X is β - T_1 space

Proof:

i) Follows from Theorem(3.3) (i) and remark (2.2) (i).

ii) Follows from Theorem(3.3) (iii) and remark (2.2) (ii).

Lemma (3.3)

Let X be a topological space and a ∈ X. Then:

i) [1]. If {a} open (respace closed), then {a} m – open (respace m – closed) set. So $[\{a\}]^C$ M – closed (respace M – open).

ii) If {a} β - open (respace β - closed), then {a} mβ – open (respace mβ– closed). So $[\{a\}]^C$ Mβ – closed (respace Mβ – open).

iii) If {a} B*c - open (respace B*c - closed), then {a} mB*c – open (respace mB*c – closed). So $[\{a\}]^C$ MB*c – closed (respace MB*c – open).

Theorem (3.5):

Let X be a topological space .Then X is M – T_1 space iff {x} closed set in X ∀ x ∈ X.

Proof:

Let X be M– T_1 space and let Y ∉ {x}, the x ≠ y. Since X M– T_1 space, then ∃ U, V M - open in X ∋ x ∈ U, y ∉ U and y ∈ V, x ∉ V, then by Lemma (3.1) we get {x} closed set in X.

Conversely

Let x, y ∈ X and x ≠ y. Let {x} be closed set in X, then by Lemma (3.3) (i), we get {x} m – closed, then $[\{x\}]^C$ M – open in X. Let U = $\{x\}^C$, V = $\{Y\}^C$ are M - open in X, then ∃ U, V M - open in X ∋ x ∈ V, y ∉ V and y ∈ U, x ∉ U, hence X is M – T_1 space

Theorem (3.6):

Let X be a topological space Then

i) X Mβ–T_1 space iff{x}β–closed set in X∀x∈ X.

ii) X MB*c – T_1 space iff {x} B*c – closed set in X ∀ x ∈ X.

Proof:

Similarly Theorem(3.5).

Corollary (3.2):

Let X be a topological space Then

i) X M – T_1 space iff X is T_1 – space

ii) X Mβ – T_1 space iff X is β–T_1 space

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iii) X MB*c – T_1 space iff X is B*c –T_1 space

Proof:

i) Follows from Theorem(3.5) and Lemma (3.1).
ii) Follows from Theorem(3.6) (i) and Theorem(3.1).
iii) Follows from Theorem(3.6) (ii) and Theorem(3.2).

Theorem (3.7):

Let X be a topological space .Then
i) Every m – T_1 space is M –T_1 space
ii) Every mβ – T_1 space is Mβ–T_1 space
iii) Every mB*c – T_1 space is MB*c –T_1 space

Proof:

i) Follows from Theorem (3.3)(i)and Corollary. (3.13) (i).
ii)Follows from Theorem (3.3)(ii)and Corollary.(3.2) (ii).
iii) Follows from Theorem(3.3) (iii) and Corollary . (3.2) (iii).
The converse of above Theorem is not true in general.

Example (3.4):

i) In example (3.1), we note that X is T_1 – space, then X is M - T_1 space, by Coro. (3.13) (i), but not m - T_1 space Since ∀ x, Y ∈ X , x ≠ Y, but ∄ U, V m – open in X ∋ x ∈ U, Y ∉ U and Y ∈ V, X ∉ V.
ii) In example (3.2), note that X is βT_1 – space, then X is Mβ - T_1 space by (3.13) (ii), but not mβ - T_1 space Since ∀ x, Y ∈ X , x ≠ Y, but ∄ U, V m – open in X ∋ x ∈ U, Y ∉ U and Y ∈ V, X ∉ V.
iii) In example (3.3), note that X is B*c – T_1 space, then X is MB*c - T_1 space by corollary (3.2) (iii), but not mB*c - T_1 space Since ∀ x, Y ∈ X , x ≠ Y, but ∄ U, V mB*c – open in X ∋ x ∈ U, Y ∉ U and Y ∈ V, X ∉ V.

Theorem (3.8):

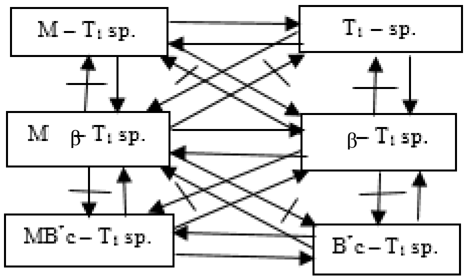
Let X be a topological space . Then
i) If X M – T_1 space, then X is β– T_1 space
ii) If X M – T_1 space, then X is Mβ– T_1 space
iii) If X MB*c – T_1 space, then X is β– T_1 space
iv) If X MB*c – T_1 space, then X is Mβ– T_1 space

Proof:

It is clear.
The converse of above theorem is not true in general.

Example (3.5):

In example (3.1), note that X is βT_1 – space, and Mβ - T_1 space But
i) and (ii) Not M - T_1 space Since a, c ∈ X ∋ a ≠ c, but ∄ U, V M – open in X ∋ a ∈ U, c ∉ U and c ∈ V, a ∉ V.
iii) and (iv) Not MB*c - T_1 space Since a, c ∈ X ∋ a ≠ c, but ∄ U, V MB*c – open in X ∋ a ∈ U, c ∉ U and c ∈ V, a ∉ V.
The following diagram shows the relation among types of M - T_1 space



4) Hereditary properties:

Lemma (4.1): [7]

Let X be a topological space then
i) If V open in Y and Y open in X, then V open in X.
ii) If V closed in Y and Y closed in X, then V closed in X.

Lemma (4.2): [6]

Let X be a topological space Then G open set in X if and only if cl(G∩cl(A))=cl(G∩A) ∀ A ⊆ X.

Theorem (4.1):

Let X be a topological space and Y open in X. If A β - open in Y, then A β- open in X.

Proof:

Let A be β - open in Y
Let x ∈ A, then there exists U β - open in Y such that x ∈ U ⊆ A. Then

$$U \subseteq \overline{\left(\overline{U}^{Y}\right)^{oY}}^{Y} = [\overline{\overline{U}\cap Y}^{Y}]^{oY}.$$

$$\subseteq [\overline{(\overline{U}\cap Y)^{oY}}\cap Y].$$

$$\subseteq [\overline{\overline{(\overline{U}\cap Y)^{oY}}}\cap Y].$$

$$= [\overline{(\overline{U}\cap Y)^{oY}}\cap Y^{o}]. \text{ by lemma (4.2).}$$

$= \overline{\overline{(\overline{U} \cap Y)}^{o}}$

$= \overline{(U \cap Y)}^{o}$ by lemma (4.2).

$\subseteq \overline{\overline{U}^{o}}$.

Then U is β - open. Since x ∈ U ⊆ X and U is β - open in X. Therefore A β - open in X.

Theorem (4.2):

Let X be a topological space and Y clopen in X. If A B*c - open in Y, then A B*c - open in X.

Proof:

Let A be B*c - open in Y, then A β - open in Y. Since Y clopen in X, then Y open and closed in X, then A β - open in X by Theorem(4.1). Let x ∈ A, then ∃ F closed set in Y such that x ∈ F_y ⊆ A, then F closed set in X by lemma (4.1) (ii). Then x ∈ F_x ⊆ A, hence A B*c – open in X.

Theorem (4.3):

Let X be a topological space and Y open in X. Then:

i) If U m–open in X, then U ∩ Y is m– open in Y.

ii) If U M–open in X, then U∩Y is M–open in Y.

Proof:

i) Let V open in Y ∋ V ⊆ U ∩ Y. T. P V = ∅ or V = U ∩ Y. Then V open in X by lemma (4.1). Since V ⊆ U and U m – open in X, then V = ∅ or V = U. Since V = V ∩ Y = U ∩ Y, then U ∩ Y m – open in Y.

ii) Let V open in Y ∋ U ∩ Y ⊆ V. T. P V = X or V = U ∩ Y. Then V open in X by lemma (4.1). Since U ⊆ V and U M – open in X, then V = X or V = U. Since V = V ∩ Y = U ∩ Y, then U ∩ Y M – open in Y.

Theorem (4.4):

Let X be a topological space and Y clopen in X. Then:

i) If U mB^*c–open in X, then U∩Y is mB*c– open in Y.

ii) If U MB*c– open in X, then U∩Y is MB*c – open in Y.

Proof : Similarly Theorem(4.3).

Theorem (4.5):

i) Every open sub space of m – T_1 space is m – T_1 space

ii) Every open sub space of M – T_1 space is M – T_1 space

Proof:

i) Let X be m – T_1 space and A is open sub space of X T. P A m – T_1 space

Let a_1, a_2 ∈ A ∋ $a_1 \neq a_2$. Since A ⊆ X and a_1, a_2 ∈ A, then a_1, a_2 ∈ X ∋ $a_1 \neq a_2$. Since m – T_1 space, then ∃ U, V m - open in X ∋ a_1 ∈ U, a_2 ∉ U and a_2 ∈ V, a_1 ∉ V. Since U, V m - open in X and A open sub space of X, then. Let U^* = U ∪ A, V^* = V ∪ A, then U^*, V^* are m – open in A by Theorem(4.3) (i). Since a_1 ∈ U, a_2 ∉ U and a_1, a_2 ∈ A, then a_1 ∈ A ∩ U = U^* and a_2 ∉ A ∩ U = U^*, then a_1 ∈ U^*, a_2 ∉ U^*. Since a_2 ∈ V, a_1 ∉ V and a_1, a_2 ∈ A, then a_2 ∈ A ∩ V = V^* and a_1 ∉ A ∩ V = V^*, then a_2 ∈ V^*, a_1 ∉ V^*. Hence A is m – T_1 space

ii) Similarly part (i)

Remark (4.1):

i) If A m – T_1 space sub space of X, the X not necessary m – T_1 space

ii) If A M – T_1 space sub space of X, the X not necessary M – T_1 space

Example (4.1)

Let X = {1,2,3}, t = {∅, X, {1}, {2}, {1,2} }.

Let A = {1,2}, t_A = {∅, X, {1}, {2}}. Note that

i) A m – T_1 space, but X not m – T_1 space Since 2, 3 ∈ X ∋ 2 ≠ 3, but ∄ U, V m – open in X ∋ 2 ∈ U, 3 ∉ U and 3 ∈ V, 2 ∉ V.

ii) A M – T_1 space, but X not M – T_1 space Since 2, 3 ∈ X ∋ 2 ≠ 3, but ∄ U, V M – open in X ∋ 2 ∈ U,3 ∉ U and 3∈ V,2 ∉ V.

Theorem (4.6):

i) Every open sub space of mβ–T_1 space is mβ–T_1 space

ii) Every open sub space of Mβ–T_1 space is mB*c–T_1 space

Proof:

Similarly Theorem(4.5).

Theorem (4.7):

i) Every clopen sub space of mB*c – T_1 space is mB*c – T_1 space

ii) Every clopen sub space of MB*c – T_1 space is MB*c – T_1 space

Proof:

i) Let X be a mB*c – T_1 space and A is clopen sub space of X T. P A mB*c – T_1 space

Let a_1, a_2 ∈ A ∋ $a_1 \neq a_2$. Since A ⊆ X and a_1, a_2 ∈ A, then a_1, a_2 ∈ X ∋ $a_1 \neq a_2$. Since X mB*c – T_1 space, then ∃ U, V mB*c - open in X ∋ a_1 ∈ U, a_2 ∉ U and a_2 ∈ V, a_1 ∉ V. Since U, V mB*c - open in X and A clopen sub space of X, then. Let $U^* = U \cup A$, $V^* = V \cup A$, then U^*, V^* are mB*c – open in A by Theorem(4.3) (i).

Since $a_1 \in U, a_2 \notin U$ and $a_1, a_2 \in A$, then $a_1 \in A \cap U = U^*$ and $a_2 \notin A \cap U = U^*$, then $a_1 \in U^*$, $a_2 \notin U^*$.

Since $a_2 \in V$, $a_1 \notin V$ and $a_1, a_2 \in A$, then $a_2 \in A \cap V = V^*$ and $a_1 \notin A \cap V = V^*$, then $a_2 \in V^*$, $a_1 \notin V^*$, hence A is mB*c – T_1 space

Remark (4.2):

i) If A mB*c – T_1 space sub space of X, the X not necessary mB*c – T_1 space

ii) If A MB*c – T_1 space sub space of X, the X not necessary MB*c – T_1 space

Example (4.2):

Let X = {a, b, c, d, e}

t = { ∅ , X, {a} , {b}, {e}, {a, b}, {a, e}, {b, e} {a, b, e}, {a, c, d}, {a,b,c,d}, {a, c, d, e} }.

B*co (X) = {∅, X, {b},{e},{b, e}, {a, c, d},{a, b, c, d},{a, c, d, e} }.

Let A = {b, e}, t_A = {∅, A, {b}, {e}}. B*co (A) = t_A Note that:

i) A mB*c – T_1 space, but X not mB*c – T_1 space Since a, d ∈ X ∋ a ≠ d, but ∄ U, V mB*c – open in X ∋ a ∈ U, d ∉ U and d ∈ V, a ∉ V.

ii) A MB*c – T_1 space, but X not MB*c – T_1 space Since a, d ∈ X ∋ a ≠ d, but ∄ U, V MB*c – open in X ∋ a ∈ U, d ∉ U and d ∈ V, a ∉ V.

5) Topological properties

Lemma (5.1): [3]

Let F: X → Y be abjection, strongly m – open, m – irresolute function. Then X is m – T_1 space iff Y is m – T_1 space

Theorem (5.1):

Let F: X → Y be abjection, strongly mβ – open. If X mβ – T_1 space, then Y is mβ – T_1 space

Proof:

Let X be mβ – T_1 space T. P Y mβ – T_1 space Let $y_1, y_2 \in Y \ni y_1 \neq y_2$. Since F on to, then $\exists\, x_1, x_2 \in X \ni F(x_1) = y_1$, $F(x_2) = y_2$. If $x_1 = x_2$, then $F(x_1) = F(x_2)$, then $y_1 = y_2$ which is contradiction, then $x_1 \neq x_2$. Since X mβ – T_1 space, then ∃ U, V mβ- open set in X ∋ $x_1 \in U$, $x_2 \notin U$ and $x_2 \in V$, $x_1 \notin V$. Since F strongly mβ - open, then F (U) mβ-open in Y, then $y_1 = F(x_1) \in F(U)$, $y_2 = F(x_2) \notin F(U)$ and $y_2 = F(x_2) \in F(V)$, $y_1 = F(x_1) \notin F(V)$. Hence Y mβ–T_1 space

Theorem (5.2):

Let F:X→Y be abjection,mB*c–irresolute function. If Y is mB*c–T_1 space ,then X is mB*c–T_1 space

Proof:

Let X be mB*c – T_1 space T. P X mB*c – T_1 space Let $x_1, x_2 \in X \ni x_1 \neq x_2$ and let $F(x_1) = y_1$, $F(x_2) = y_2$, then $y_1, y_2 \in Y$.

If $y_1 = y_2$, then $F(x_1) = F(x_2)$, then $x_1 = x_2$ which is contradiction, then $y_1 \neq y_2$. Since F bijection, then $x_1 = F^{-1}(y_1)$, $x_2 = F^{-1}(y_2)$. Since Y mB*c – T_1 space, then ∃ U, V mB*c - open set in Y ∋ $y_1 \in U$, $y_2 \notin U$ and $y_2 \in V$, $y_1 \notin V$. Since F mB*c – irresolute function, then $F^{-1}(U)$ mB*c - open in X, then $x_1 = F^{-1}(y_1) \in F^{-1}(U)$, $x_2 = F^{-1}(y_2) \notin F^{-1}(U)$ and $x_2 = F^{-1}(y_2) \in F^{-1}(V)$, $x_1 = F^{-1}(y_1) \notin F^{-1}(V)$. Hence Y mB*c – T_1 space

Rephrases :

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حول فضاءات T_1 العضمى والصغرى بأستخدام مجموعات B^*c المفتوحه

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المستخلص :

في هذا البحث قدمنا دراسه حول خصائص الفضاء التبولوجي Maximal T_1-space and Minimal T_1-Space بأستخدام المجموعه B^*c-Open set وأستنتجنا بعض المبرهنات والملاحظات والعلاقات بالنسبه للفضاء MB^*c-T_1-Space و mB^*c-T_1-Space ودرسنا العلاقه بين الفضاء الأعظم MB^*c-T_1-Space والفضاء الأصغر mB^*c-T_1-Space حيث وجدنا أن كل فضاء mB^*c-T_1-Space يؤدي الى MB^*c-T_1-Space لكن العكس لايتحقق بصوره عامه . كذالك قدمنا دراسه حول الصفات الوراثيه والصفات التبولوجيه لهذا الفضاء .