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The study of new iterations procedure for expansion mappings

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Abstract:

In this research, we introduce new iteration process for different types of mappings and introduce a concept of expansion mapping, it is independent of non – expansive mapping . Also, we study the convergences for these iterations to common fixed point in real Hilbert spaces.

Keywords: asymptotic fixed point, strong convergence, non-expansive mapping, maximal monotone , weak convergence.

Mathematics Subject Classification: 46S40.

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1. Introduction

Let H be a real Hilbert space $\emptyset \neq C \subseteq H$, and $T: C \longrightarrow C$ is non − expansive. Mapping. That is, if $\|a - b\| \geq \|f(a) - f(b)\|$ for each $a, b \in C$. Also any multivalued operator A is called monotone if the following condition hold:

$< a_1 - a_2, d_1 - d_1 > \geq 0 \ \ \forall a_i \in D(A),\ d_i \in A(z_i)$. And it is called maximal monotone if for all $(a, h) \in H \times H, \langle a - b.,.h - d\rangle \geq 0$ and for all $(b, d) \in gph(A)$ then we get, $h \in A(z)$. The monotone operators has an important role in different branches of mathematics, see.([1]-[5]). On other hand, The convergence of the iteration method studied by many researchers see ([6]-[16]). Define the following mapping as follows:

$J_{r_n} = (I + r_n A^{-1})(a)$ this mapping is called resolvet mapping where $< r_n >$ be a sequence of positive real numbers. Also, the metric projection $P_c(a)$ from H onto C is defined as follows:

For any $a \in H$ there exists a unique element $P_c(a) \in C$ satisfies the following:

$\|a - P_c(a)\| \leq \|a - b\|$, for all $b \in C$.That is, for each $a \in X$, $P_c(a) = b$ iff $b \in C$ and $\|a - b\| = \inf\{\|a - c\|; c \in C\}$.

Now, the following definitions and lemmas are interesting to area of research:

Lemma(1.1) [16]

Let $< \alpha_n >$ and $< \beta_n >$ are sequences of nonnegative real number such that $\alpha_{n+1} \leq \alpha_n + \beta_n$, for each. $n \geq 1$. If $\sum_{n=o}^{\infty} \alpha_n$ converge,then $\lim_{n\to\infty} \alpha_n$ exists.

Definition(1.2) : [17]

Let $\Gamma: C \longrightarrow C$ be a mapping then every $p \in C$ is called asymptotic fixed point of Γ if there exists $\langle\alpha_n\rangle$ is sequence in C such that $\alpha_n \rightharpoonup p$ and $\|\alpha_n - \Gamma(\alpha_n)\| \longrightarrow 0$.

Lemma (1.3) : [18]

Let C be a nonempty convex closed subset of real Hilbert space H and Γ is non-expansive multivalued mapping such that $Fix(\Gamma) \neq \emptyset$.Then Γ is demi-closed, i.e., $\alpha_n \rightharpoonup p$ and $\lim_{n\to\infty} d\big(\alpha_n, \Gamma(\alpha_n)\big) = 0$.Then $p \in \Gamma(p)$.

Lemma(1.4) : [19]

If $\langle\alpha_n\rangle$ be a sequence in H and $\|\alpha_{n+1} - \alpha\| \leq \|\alpha_n - \alpha\|$ for all $\alpha \in C$. Then $\langle P_c(\alpha_n)\rangle$ converges strongly to a point in C.

Now, we introduce the concept of expansion mapping

Main Results

In this section, we define a new iterations for sequence of expansion mapping. Also, we study the convergence for these iterations.

Definition(2.1)

Any mapping f is called expansion mapping if for each sequence,$\langle z_n \rangle$ in (0,1) converges to zero then there exists a nonnegative real number z such that

$$(1 - z_n)||x - w||^2 + z\langle x - f_x, w - f_w \rangle^{k+1}$$
$\geq ||fx - fw||^2$,for all $k > 0$ and $x, w \in C$

The concept of expansion mapping is independent of non − expansive mapping. As shown by the following examples:

Example (2.2)

If $f:(0,\infty) \longrightarrow (0,\infty)$ be a mapping such that $f(x) = x$.Then the mapping f is not non-expansive but it is expansion,mapping. Since, for each sequence $\langle z_n \rangle$ in [0,1] converges to zero then there exists z such that,

$$z = \frac{4}{\langle x - f_x, w - f_w \rangle^{k+1}} ||x - w||^2 \text{ and satisfy}$$
$$(1 - z_n)||x - w||^2 + z\langle x - f_x, w - f_w \rangle^{k+1}$$

Example (2.3)

Let $f: H \longrightarrow H$ be a mapping such that $f(x) = x$.

It is clear that the mapping f is not expansion mapping but it is non − expansive.

Theorem (2.4):

Let $A_1, A_2, \ldots, A_m$ are maximal monotone multivalued mapping C nonempty convex closed in H, $\langle f_n \rangle$ be a sequence of non-expansive mapping and $\langle T_n \rangle$ is bounded sequence of expansion mapping on C. Let $\langle a_n \rangle, \langle b_n \rangle$ are sequences in (0,1] converges to 0, such that $a_n + b_n = 1$ and $\sum_{i=1}^{m} \gamma_{n,i} = 1$. Define the iteration process$\langle x_n \rangle$ as follows:

$$w_n = b_n v_n + (1 - b_n) \sum_{i=1}^{m} \gamma_{n,i} J_{r_{n,i}}^{i} v_n$$
$$v_{n+1} = a_n T_n v_n + (1 - a_n) f_n w_n$$

If $\cap_{n=1}^{\infty} Fix\left(J_{r_{n,i}}^{i}\right) \cap \left(\cap_{n=1}^{\infty} Fix(T_n)\right) \cap \left(\cap_{n=1}^{\infty} Fix(f_n)\right) \neq \emptyset$.Then $\langle x_n \rangle$ has converges weakly to an asymptotic common fixed point of T_n for each $n \in N$. Moreover $\langle P_C(v_n) \rangle$ converges strongly to a point in C.

Proof :

Let $p \in \cap_{n=1}^{\infty} Fix\left(J_{r_{n,i}}^{i}\right) \cap \left(\cap_{n=1}^{\infty} Fix(T_n)\right) \cap \left(\cap_{n=1}^{\infty} Fix(f_n)\right)$

$$\| w_n - p \|^2 = \left\| \begin{matrix} b_n(v_n - p) + (1 - b_n). \\ \left(\sum_{i=1}^{m} \gamma_{n,i} J_{r_{n,i}}^{i} \; y_n \; (v_n) - p \right) \end{matrix} \right\|^2$$
$$\leq b_n \|v_n - p\|^2$$
$$+ (1 - b_n) \sum_{i=1}^{m} \gamma_{n,i} \; \|v_n - p\|^2$$
$$\leq b_n \|v_n - p\| + (1 - b_n)\|v_n - p\|^2$$
$$= \|v_n - p\|^2$$

Now, for any sequence $\langle z_n \rangle$ in [0,1] converges to zero then there exists a nonnegative real number z such that

$$\|v_{n+1} - p\|^2 = \|T_n v_n + (1 - a_n) f_n w_n - p\|^2$$
$$\leq a_n \|T_n v_n - p\|^2 + (1 - a_n)\|f_n w_n - p\|^2$$
$$\|v_{n+1} - p\|^2 \leq a_n \|T_n v_n - p\|^2 + (1 - a_n)\|w_n - p\|^2$$
$$\|v_{n+1} - p\|^2 \leq a_n (1 - z_n)\|v_n - p\|^2 + b_n z_n \left\|(T_n p - p T_n)(v_n - v_n - (T_n p - p))\right\| + b_n z \langle v_n - f_x, p - f_p \rangle^k + (1 - a_n)\|v_n - p\|^2$$
$$\|v_{n+1} - p\|^2 \leq a_n \{(1 - z_n)\|v_n - p\|^2\} + (1 - a_n)\|v_n - p\|^2$$

$$\leq a_n\|v_n - p\|^2 + (1-a_n)\|v_n - p\|^2$$
$$= \|v_n - p\|^2$$

By lemma (1.1), we get $\lim_{n\to\infty}\|v_n - p\|$ exists and hence $\langle f_n\rangle$ is also bounded. So by lemma (1.4) we get $\langle P_C(v)\rangle$ converges strongly to the point in C.

$$\|v_n - T_n v_n\| \leq \|a_{n-1}(b_{n-1}T_{n-1}v_{n-1} + (1-b_{n-1})f_{n-1}w_{n-1} - T_n v_n) + (1-a_{n-1})f_{n-1}v_{n-1} - T_n v_n\|$$
$$\leq a_{n-1}\|b_{n-1}T_{n-1}v_{n-1} + (1-b_{n-1})T_n w_{n-1} - T_n v_n\| + b_{n-1}\|f_{n-1}v_{n-1} - T_n v_n\|$$

Since $\langle f_n\rangle$ and $\langle T_n\rangle$ are also bounded and $\langle a_n\rangle, \langle b_n\rangle$ are sequences in $(0,1]$ converges to zero. As $n \longrightarrow \infty$ we get, $\|v_n - T_n v_n\| \longrightarrow 0$.

Now, since $\langle v_n\rangle$ is bounded then there exists subsequence $\langle v_{nk}\rangle$ of v_n such that $v_{nk} \rightharpoonup z$ and $\|v_n - T_n v_n\| \longrightarrow 0$. Then we get z is an asymptotic common fixed of T_n, for each $n \in N$.Then the iteration, $\langle v_n\rangle$ has converges weakly to an asymptotic common fixed point of T_n , for each $n \in N$. ■

Now, we consider property $\mathcal{P}$ for any sequence as follows:

Let $\langle T_n\rangle$ be a sequence, of mapping we say that $\langle T_n\rangle$ has property $\mathcal{F}$ if $\langle T_n\rangle$ satisfies the condition:

$\|T_n - z\|^2 \leq \|T_n\|^2$,for each $z \in (\cap_{n=1}^{\infty} Fix(T_n))$.

In the following theorem we study the convergence for the iteration process

$$w_n = \acute{b}_n\left[\acute{a}_n v_n + (1-\acute{a}_n)\sum_{i=1}^{m}\gamma_{n,i}\, J_{r_{n,i}}^{i}\ v_n\right] + (1-\acute{b}_n)g_n v_n$$

$$v_{n+1} = \acute{a}_n[a_n T_n v_n + b_n f_n v_n + c_n f_n g_n v_n] + \acute{b}_n g_n w_n \qquad (2.1)$$

where $\langle \acute{a}_n\rangle$ and $\langle \acute{b}_n\rangle$ are sequences in $[0,1]$ such that $\langle \acute{a}_n\rangle, \langle \acute{b}_n\rangle$ converges to zero, $\acute{a}_n \geq \acute{b}_n$. Such that $\acute{a}_n + \acute{b}_n = 1$, $a_n + b_n + c_n = 1$, $\sum_{i=1}^{m}\gamma_{n,i}$.

Theorem (2.5) :

Let A_1, A_2 , ….. , A_m are maximal monotone multivalued mapping and $\emptyset \neq C$ convex closed in X, $\langle T_n\rangle$ is bounded,sequences of expansion mappings on C and $\langle f_n\rangle$, $\langle g_n\rangle$ are sequences of non-expansive mapping on C. If the iteration process defined as (2.1) and $\left(Fix\left(J_{r_{n,i}}^{i}\right)\right) \cap (\cap_{n=1}^{\infty} Fix(T_n)) \cap (\cap_{n=1}^{\infty} Fix(f_n)) \cap (\cap_{n=1}^{\infty} Fix(g_n)) \neq \emptyset$.Then $\langle x_n\rangle$ has converges weakly to an asymptotic common fixed point of T_n , for each $n \in N$. Moreover $\langle P_C(x_n)\rangle$ converges strongly to a point in C.

Proof :

Let

$$p \in \left(Fix\left(J_{r_{n,i}}^{i}\right)\right) \cap \left(\bigcap_{n=1}^{\infty} Fix(T_n)\right) \cap \left(\bigcap_{n=1}^{\infty} Fix(f_n)\right) \cap \left(\bigcap_{n=1}^{\infty} Fix(g_n)\right)$$

$$\|w_n - p\|^2 \leq \left\|\acute{b}_n\left[\acute{a}_n(v_n - p) + (1-\acute{a}_n)\left(\sum_{i=1}^{m}\gamma_{n,i}\, J_{r_{n,i}}^{i}\ v_n - p\right)\right] + (1-\acute{b}_n)g_n v_n - p\right\|^2$$

$$\leq \acute{b}_n\left\|\acute{a}_n(v_n - p) + (1-\acute{a}_n)\left(\sum_{i=1}^{m}\gamma_{n,i}\, J_{r_{n,i}}^{i}\ v_n - p\right)\right\|^2 + (1-\acute{b}_n)\|P_c g_n v_n - p\|^2$$

$$\|w_n - p\|^2 \le \acute{b}_n \begin{bmatrix} \acute{a}_n\|v_n - p\|^2 + (1-\acute{a}_n) \\ \sum_{i=1}^{m} \gamma_{n,i} \left\|J^i_{r_{n,i}}\ v_n - p\right\|^2 \end{bmatrix}$$
$$+ (1-\acute{b}_n)\|v_n - p\|^2$$
$$\le \acute{b}_n[\acute{a}_n\|v_n - p\|^2$$
$$+ (1-\acute{a}_n)\|v_n - p\|^2]$$
$$+ (1-\acute{b}_n)\|v_n - p\|^2$$
$$\|w_n - p\|^2 = \acute{b}_n\|v_n - p\|^2 + (1-\acute{b}_n)\|v_n - p\|^2$$
$$= \|v_n - p\|^2$$

Hence, $\|w_n - p\|^2 \le \|v_n - p\|^2$

Now, by (2.1) then we have

$$\|v_{n+1} - p\|^2 \le \acute{a}_n\|a_nT_nv_n + b_nf_nv_n + c_nf_ng_nv_n - p\|^2 + \acute{b}_n\|g_nw_n - p\|^2$$
$$\|v_{n+1} - p\|^2 \le \acute{a}_na_n\|T_nv_n - p\|^2$$
$$+ \acute{a}_nb_n\|f_nv_n - p\|^2$$
$$+ \acute{a}_nc_n\|f_ng_nv_n - p\|^2$$
$$- \acute{a}_na_nb_n\|T_nv_n - f_nv_n\|^2$$
$$- \acute{a}_nb_nc_n\|f_nv_n - f_nT_nv_n\|^2$$
$$- \acute{a}_nc_na_n\|f_nT_nv_n - T_nv_n\|^2$$
$$+ \acute{b}_n\|g_nw_n - p\|^2$$
$$\le \acute{a}_na_n\|T_nv_n - p\|^2 + \acute{a}_nb_n\|f_nv_n - p\|^2$$
$$+ \acute{a}_nc_n\|f_ng_nv_n - p\|^2$$
$$+ \acute{a}_na_nb_n\|T_nv_n - f_nv_n\|$$
$$+ \acute{a}_nb_nc_n\|f_nv_n - f_nT_nv_n\|^2$$
$$-\acute{a}_nc_na_n\|f_nT_nv_n - T_nv_n\|^2$$
$$- \acute{a}_na_nb_n\|T_nv_n - f_nv_n\|$$
$$- \acute{a}_nb_nc_n\|f_nv_n - f_nT_nv_n\|^2$$
$$- \acute{a}_nc_na_n\|f_nT_nv_n - T_nv_n\|^2$$
$$+ \acute{b}_n\|g_nw_n - p\|^2$$

For any sequence $\langle z_n\rangle$ in [0,1] converges to zero there exists a nonnegative real number z such that

$$\|v_{n+1} - p\|^2$$
$$\le \acute{a}_na_n[(1-z_n)\|v_n - p\|^2$$
$$+ z_n\|p - T_np\|.\|(p - T_np)({}_cv_n$$
$$- T_nv_n - (p - T_np))\|$$
$$+ z(\langle p - T_np, v_n - T_nv_n\rangle)^{k+1}]$$
$$+ \acute{a}_nb_n\|v_n - p\|^2$$
$$+ \acute{a}_nc_n\|v_n - p\|^2$$
$$+\acute{b}_n\|w_n - p\|^2$$

Now,

$$\|v_{n+1} - p\|^2 \le \acute{a}_na_n$$
$$\|v_n - p\|^2 + \acute{a}_nb_n\|v_n - p\|^2 + \acute{a}_nc_n\|v_n - p\|^2$$
$$+ \acute{b}_n\|v_n - p\|^2$$
$$\|v_{n+1} - p\|^2 = \acute{a}_n\|v_n - p\|^2 + (1-\acute{a}_n)\|v_n - p\|^2$$
$$= \|v_n - p\|^2$$

By lemma (1.1),

we get $\lim_{n\to\infty}\|v_n - p\|$ exists. Hence, $\langle v_n\rangle$ is bounded sequence, so that $\langle g_n\rangle$ and $\langle f_n\rangle$ are also bounded sequences.

So, by lemma (1.4) we deduce $\langle P_C(x_n)\rangle$ converges strongly to the point in C.

$$\|v_n - T_nv_n\| = \|\acute{a}_{n-1}[a_{n-1}T_{n-1}v_{n-1}$$
$$+ b_{n-1}f_{n-1}v_{n-1}$$
$$+ c_{n-1}f_{n-1}g_{n-1}v_{n-1}]$$
$$+ \acute{b}_{n-1}[a_{n-1}b_{n-1}(T_{n-1}v_{n-1}$$
$$- f_{n-1}v_{n-1})$$
$$+ b_{n-1}c_{n-1}(f_{n-1}v_{n-1}$$
$$- f_{n-1}T_{n-1}v_{n-1})$$
$$+ c_{n-1}a_{n-1}(f_{n-1}T_{n-1}v_{n-1}$$
$$- T_{n-1}v_{n-1}) + d_{n-1}\ g_{n-1}w_{n-1}]$$
$$- T_nw_n\|$$

$$\|v_n - T_n v_n\| \leq a'_{n-1}\|a_{n-1}T_{n-1}v_{n-1} + b_{n-1}f_{n-1}v_{n-1} + c_{n-1}f_{n-1}T_{n-1}v_{n-1} - g_n w_n\| + b'_{n-1}\|a_{n-1}b_{n-1}(T_{n-1}v_{n-1} - f_{n-1}v_{n-1}) + b_{n-1}c_{n-1}(f_{n-1}v_{n-1} - f_{n-1}T_{n-1}v_{n-1}) + c_{n-1}\alpha_{n-1}(f_{n-1}T_{n-1}v_{n-1} - T_{n-1}v_{n-1}) + d_{n-1}\, g_{n-1}w_{n-1} - T_n w_n\|$$

Since $\acute{a}_n, \acute{b}_n \longrightarrow 0$ and $\langle T_n\rangle, \langle f_n\rangle$and $\langle g_n\rangle$ are bounded then we get

$$\|v_n - T_n v_n\| \longrightarrow 0 \; as \; n \longrightarrow \infty$$

Now, since $\langle v_n\rangle$ is bounded sequence then there exists subsequence $\langle v_{nk}\rangle$ of $\langle v_n\rangle$ such that $v_{nk} \rightharpoonup z$ and since $\|v_n - T_n v_n\| \longrightarrow 0$,then we get,

z is asymptotic common fixed point of T_n, for all $n \in N$.

Then the iterations$\langle v_n\rangle$ has converges weakly to an asymptotic common fixed point of T_n, for all $n \in N$. ■

In the following theorem we give a new iteration process and we study the convergence for this iteration to an asymptotic common fixed point.

Theorem (2.6) :

If $\langle f_n\rangle$ be a sequence of non-expansive mapping on C and $\langle T_n\rangle$ be a bounded sequence of expansion mappings on C. Define the iteration $\langle v_n\rangle$ as follows:

$$w_n = \acute{a}_n f_n v_n + (1 - \acute{a}_n)(T_n v_n)$$

$$v_{n+1} = a_n \sum_{i=1}^{m} \gamma_{n,i}\, J^i_{r_{n,i}}\, v_n + (1 - a_n) f_n w_n \qquad (2.2)$$

where $\langle \acute{a}_n\rangle, \langle \acute{b}_n\rangle, \langle a_n\rangle, \langle b_n\rangle$ are sequences in [0,1] such that$\langle a_n\rangle, \langle b_n\rangle$ converges to 0 such that . If $\left(\bigcap_{n=1}^{\infty} Fix\left(J^i_{r_{n,i}}\right)\right) \cap \left(\bigcap_{n=1}^{\infty} Fix(T_n)\right) \cap \left(\bigcap_{n=1}^{\infty} Fix(f_n)\right) \neq \emptyset$. Then the iteration process $\langle v_n\rangle$ has converges weakly to an asymptotic common fixed point of T_n , for all $n \in N$. Moreover $\langle P_C(v_n)\rangle$ converges, strongly to a point in C.

Proof :

Let $p \in \left(Fix(P_c)\right) \cap \left(\bigcap_{n=1}^{\infty} Fix(T_n)\right) \cap \left(\bigcap_{n=1}^{\infty} Fix(f_n)\right)$

Since $w_n = \acute{a}_n f_n v_n + (1 - \acute{a}_n)\left(\acute{b}_n P_c T_n v_n + (1 - \acute{b}_n) f_n P_c T_n v_n\right)$ then we have,

$$\|w_n - p\|^2 \leq \acute{a}\|f_n v_n - p\|^2 + (1 - \acute{a}_n)[\|T_n v_n - p\|^2]$$

$$\|w_n - p\|^2 \leq \acute{a}_n\|v_n - p\|^2 + (1 - \acute{a}_n)[\|T_n v_n - p\|^2] = \acute{a}_n\|v_n - p\|^2 + (1 - \acute{a}_n)\|T_n v_n - p\|^2$$

For any sequence $\langle z_n\rangle$ in [0,1] converges to zero there exists a nonnegative real number z such that

$$\|w_n - p\|^2 \leq \acute{a}_n\|v_n - p\|^2 + (1 - \acute{a}_n)[(1 - z_n)\|v_n - p\|^2 + z_n\|(p - T_n p)(v_n - T_n v_n - (p - T_n p))\| + z(\langle v_n - T_n v_n, p - T_n p\rangle)^k]$$

$$\|w_n - p\|^2 \leq \acute{a}\|v_n - p\|^2 + (1 - \acute{a}_n)\|v_n - p\|^2 = \|v_n - p\|^2$$

Hence, $\|w_n - p\|^2 \leq \|v_n - p\|^2$

$$\|v_{n+1} - p\|^2 \leq a_n \sum_{i=1}^{m} \gamma_{n,i} \left\|J^i_{r_{n,i}}\, v_n - p\right\|^2 + (1 - a_n)\|f_n w_n - p\|^2$$

$$\|v_{n+1} - p\|^2 \leq a_n \sum_{i=1}^{m} \gamma_{n,i} \ \| \ v_n - p\|^2 + (1 - a_n)\|w_n - p\|^2$$

$$\|v_{n+1} - p\|^2 \leq a_n\| \ v_n - p\|^2 + (1 - a_n)\|v_n - p\|^2 = \|v_n - p\|^2$$

By lemma (1.1), we get $\lim_{n\to\infty}\|v_n - p\|$ exists

Hence, the iteration $\langle x_n \rangle$ is bounded sequence. So $\langle f_n \rangle$ and $\langle g_n \rangle$ also bounded sequences. And hence, by lemma (1.4) we deduce $\langle P_C(v_n) \rangle$ converges strongly to a point in C. ■

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دراسة إجراءات التكرارات الجديدة للتطبيقات التوسعية

زينة حسين معيبد

قسم الرياضيات
كلية التربية للعلوم الصرفة ابن الهيثم
جامعة بغداد

رياض دلفي علي

قسم الرياضيات
كلية التربية للعلوم الصرفة
جامعة كربلاء

المستخلص :

في هذا البحث سنقدم عمليات تكرارية جديدة لانواع مختلفة من التطبيقات وسنقدم مفهوم التطبيقات التوسعية والتي تكون مستقلة عن التطبيقات الغير توسيعية .ايضا سندرس التقارب لهذا النوع من التكرارات الى نقطة صامدة مشتركة في فضاء هلبرت