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Connectedness in Čech Fuzzy Soft Closure Spaces

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Abstract:
The notion of Čech fuzzy soft closure spaces was defined and its basic properties are introduced very newly by Majeed [1]. In the present paper, we define the notion of fuzzy soft separated sets in Čech fuzzy soft closure spaces and prove some properties concerning to this notion. By using the notion of fuzzy soft separated sets we introduce and study the concept of connected in both Čech fuzzy soft closure spaces and their associative fuzzy soft topological spaces. Then we introduce the concept of feebly connected, and discuss the relationship between the concepts of connected and feebly connected. Finally, we introduce several examples to clarify our results.

Keywords. Fuzzy soft set, Čech fuzzy soft closure operator, Fuzzy soft separated sets, Connected Čech fuzzy soft closure space, Feebly connected Čech fuzzy soft closure space.

Mathematics Subject Classification: 54A40, 54B05, 54C05.

1. Introduction

It is known that Zadeh [2] in 1965 introduced the principal idea of fuzzy sets, which is supply a natural basis for handling mathematically the fuzzy phenomena which exist in our real world, and for constructing new branches of fuzzy mathematics. Later in 1999, Molodtsov [3] initiated the concept of soft set theory, which is a purely new way for modeling uncertainty. Molodtsov [3] established the main results of this new theory and successfully applied the soft set theory into several directions, such as theory of probability, Riemann integration, smoothness of functions, operations research and game theory. The concept of fuzzy soft sets was defined by Maji et al. [4] as fuzzy generalizations of soft sets. Then in 2011, Tanay and Kandemir [5] were gave the concept of topological structure based on fuzzy soft sets. The study of fuzzy soft topological spaces was pursued in recent years by some others [6, 7, 8, 9, 10, 11].

Čech [12] in 1966, introduced the notion of Čech closure spaces $(X,\mathcal{C})$, where $\mathcal{C}: P(X) \to P(X)$ is a mapping satisfying $\mathcal{C}(\emptyset) = \emptyset, A \subseteq \mathcal{C}(A)$ and $\mathcal{C}(A \cup B) = \mathcal{C}(A) \cup \mathcal{C}(B)$, the mapping $\mathcal{C}$ called Čech closure operator on X. After Zadeh introduced the concept of fuzzy sets, in 1985 Mashhour and Ghanim [13] put the concept of Čech fuzzy closure spaces when they exchange sets by fuzzy sets in the definition of Čech closure space. In 2014, Gowri and Jegadeesan [14] using the concept of soft sets to introduced and investigation soft Čech closure spaces, the soft closure operator in that sense was defined from the power set $P(X_{F_A})$ of X_{F_A} to itself (where F_A is a soft set over the universe set X with the set of parameter K, and $A \subseteq K$). Also, in the same year, Krishnaveni and Sekar [15] introduced and study Čech soft closure spaces (where the soft closure operator here defined from the set of all soft sets over X to itself). Very recently Majeed [1] employ the fuzzy set theory to define and study the notion of Čech fuzzy soft closure spaces which is a generalization to Čech soft closure spaces that given by Krishnaveni and Sekar [15]. Also, Majeed and Maibed [16] introduced some structures of Čech fuzzy soft closure spaces. They show that every Čech fuzzy soft closure space gives a parameterized family of Čech fuzzy closure spaces, and defined and studied fuzzy soft exterior (respectively, boundary) in Čech fuzzy soft closure spaces.

On the other hand, the notion of connectedness in closure spaces is introduced and studied. Čech [12] defined the notion of connected spaces in closure spaces. According to Čech a subset A of a closure space X is said to be connected in X if A can not be represent as the union of two nonempty semi-separated subsets of X, that is $A = A_1 \cup A_2$, $(\mathcal{C}(A_1) \cap A_2) \cup (A_1 \cap \mathcal{C}(A_2)) = \emptyset$ implies $A_1 = \emptyset$ or $A_2 = \emptyset$. Plastria [17] studied connectedness and local connectedness of simple extension. Gowri and Jegadeesan [18] introduced the concept of connectedness in soft Čech closure spaces.

In the present paper, we extend the notion of connectedness in Čech fuzzy soft closure spaces. In Section 3, we define the concept of fuzzy soft separated sets in Čech fuzzy soft closure spaces and give some of its basic properties. Then we introduce the notion of disconnected in both Čech fuzzy soft closure spaces and their associative fuzzy soft topological spaces based on fuzzy soft separated sets. In Section 4, we present the concept of feebly disconnected Čech fuzzy soft closure space. We show that the concept of disconnected and feebly disconnected are independent (see Examples 4.11 and 4.12).

2. Preliminaries

In this section we review some basic definitions and results related of fuzzy soft theory and Čech fuzzy soft closure spaces that will be needed in the sequel, and we foresee the reader be familiar with the usual notions and most basic ideas of fuzzy set theory. Throughout our paper, X will refer to the initial universe, $I = [0,1]$, $I_0 = (0,1]$, I^X be the set of all fuzzy sets of X, and K the set of parameters for X.

Definition 2.1 [9, 10, 19, 20] A fuzzy soft set (fss, for short) λ_A on X is a mapping from K to I^X, i.e., $\lambda_A: K \to I^X$, where $\lambda_A(h) \neq \bar{0}$ if $h \in A \subseteq K$ and $\lambda_A(h) = \bar{0}$ if $h \notin A \subseteq K$, where $\bar{0}$ is the empty fuzzy set on X. The family of all fuzzy soft sets over X denoted by $\mathcal{F}_{ss}(X, K)$.

In the next definition, the basic operations between fuzzy soft sets are given.

Definition 2.2 [9, 10, 20] Let $\lambda_A, \mu_B \in \mathcal{F}_{ss}(X, K)$, then
1. λ_A is said to be a fuzzy soft subset of μ_B, denoted by $\lambda_A \subseteq \mu_B$, if $\lambda_A(h) \leq \mu_B(h)$, for all $h \in K$.
2. λ_A and μ_B are said to be equal, denoted by $\lambda_A = \mu_B$ if $\lambda_A \subseteq \mu_B$ and $\mu_B \subseteq \lambda_A$.
3.The union of λ_A and μ_B, denoted by $\lambda_A \cup \mu_B$ is the fss $\sigma_{(A \cup B)}$ defined by $\sigma_{(A\cup B)}(h) = \lambda_A(h) \vee \mu_B(h)$, for all $h \in K$.
4.The intersection of λ_A and μ_B, denoted by $\lambda_A \cap \mu_B$ is the fss $\sigma_{(A\cap B)}$ defined by $\sigma_{(A\cap B)}(h) = \lambda_A(h) \wedge \mu_B(h)$, for all $h \in K$.

Definition 2.3 [9, 11, 20] The null fss, denoted by $\bar{0}_K$, is a fss defined by $\bar{0}_K(h) = \bar{0}$, for all $h \in K$.

Definition 2.4 [9, 11, 20] The universal fss, denoted by $\bar{1}_K$, is a fss defined by $\bar{1}_K(h) = \bar{1}$, for all $h \in K$, where $\bar{1}$ is the universal fuzzy set of X.

Definition 2.5 [20] The complement of a fss $\lambda_A \in \mathcal{F}_{ss}(X, K)$, denoted $\bar{1}_K - \lambda_A$, is the fss defined by $(\bar{1}_K - \lambda_A)(h) = \bar{1} - \lambda_A(h)$, for each $h \in K$, Its clear that $\bar{1}_K - (\bar{1}_K - \lambda_A) = \lambda_A$.

Definition 2.6 [21] Two fss's $\lambda_A, \mu_B \in \mathcal{F}_{ss}(X, K)$ are said to be disjoint, denoted by $\lambda_A \cap \mu_B = \bar{0}_K$, if $\lambda_A(h) \cap \mu_B(h) = \bar{0}$ for all $h \in K$.

Definition 2.7 [5, 20] A fuzzy soft topological space (fsts, for short) (X, τ, K) where X is a nonempty set with a fixed set of parameters and τ is a family of fuzzy soft sets over X satisfying the following properties:
1.$\bar{0}_K, \bar{1}_K \in \tau$,
2.If $\lambda_A, \mu_B \in \tau$, then $\lambda_A \cap \mu_B \in \tau$,
3.If $(\lambda_A)_i \in \tau$, then $\bigcup_{i\in J\in}(\lambda_A)_i \in \tau$.
τ is called a topology of fuzzy soft sets on X. Every member of τ is called open fuzzy soft set (open-fss, for short). The complement of open-fss is called a closed fuzzy soft set (closed-fss, for short).

Definition 2.8 [1] An operator $\theta: \mathcal{F}_{ss}(X, K) \to \mathcal{F}_{ss}(X, K)$ is called $\check{C}$ech fuzzy soft closure operator ($\check{C}$-fsco, for short) on X, if the following axioms are satisfied.
(C1) $\theta(\bar{0}_K) = \bar{0}_K$,
(C2) $\lambda_A \subseteq \theta(\lambda_A)$, for all $\lambda_A \in \mathcal{F}_{ss}(X, K)$,
(C3) $\theta(\lambda_A \cup \mu_B) = \theta(\lambda_A) \cup \theta(\mu_A)$, for all $\lambda_A, \mu_B \in \mathcal{F}_{ss}(X, K)$.
The triple (X, θ, K) is called a Čech fuzzy soft closure space ($\check{C}\mathcal{F}$-fscs, for short).
A fss λ_A is said to be closed-fss in (X, θ, K) if $\lambda_A = \theta(\lambda_A)$. And a fss λ_A is said to be an open-fss if $\bar{1}_K - \lambda_A$ is a closed-fss.

Proposition 2.9 [1] Let (X, θ, K) be a $\check{C}\mathcal{F}$-scs, and λ_A, $\mu_B \in \mathcal{F}_{ss}(X, K)$ such that $\lambda_A \subseteq \mu_B$, then $\theta(\lambda_A) \subseteq \theta(\mu_B)$.

Definition 2.10 [1] Let (X, θ, K) be a $\check{C}\mathcal{F}$-scs, and let $\lambda_A \in \mathcal{F}_{ss}(X, K)$. The interior of λ_A, denoted by $Int(\lambda_A)$ is defined as $Int(\lambda_A) = \bar{1}_K - (\theta(\bar{1}_K - \lambda_A))$.

Definition 2.11 [1] Let V be a non-empty subset of X, then $\bar{V}_K$ denotes the fuzzy soft set V_K over X for which $V(h) = \bar{1}_V$ for all $h \in K$, (where $\bar{1}_V: X \to I$ such that $\bar{1}_V(x) = 1$ if $x \in V$ and $\bar{1}_V(x) = 0$ if $x \notin V$).

Theorem 2.12 [1] Let (X, θ, K) be a $\check{C}\mathcal{F}$-scs, $V \subseteq X$ and let $\theta_V: \mathcal{F}_{ss}(V, K) \to \mathcal{F}_{ss}(V, K)$ defined as $\theta_V(\lambda_A) = \bar{V}_K \cap \theta(\lambda_A)$. Then θ_V is a $\check{C}\mathcal{F}$-sco. The triple (V, θ_V, K) is said to be $\check{C}$ech fuzzy soft closure subspace ($\check{C}\mathcal{F}$-sc subspace, for short) of (X, θ, K).

Theorem 2.13 [1] Let (X, θ, K) be a $\check{C}\mathcal{F}$-scs and let $\tau_\theta \subseteq \mathcal{F}_{ss}(X, K)$, defined as follows

$$\tau_\theta = \{\bar{1}_K - \lambda_A : \theta(\lambda_A) = \lambda_A\}.$$

Then τ_θ is a fuzzy soft topology on X and (X, τ_θ, K) is called an associative fsts of (X, θ, K).

Definition 2.14 [22] Let (X, τ_θ, K) be an associative fsts of (X, θ, K) and let $\lambda_A \in \mathcal{F}_{ss}(X, K)$. The fuzzy soft topological closurer of λ_A with respect to θ, denoted by $\tau_\theta\text{-}cl(\lambda_A)$, is the intersection of all closed fuzzy soft super sets of λ_A. i.e.,

$$\tau_\theta\text{-}cl(\lambda_A) = \cap\{\rho_C : \lambda_A \subseteq \rho_C \ and\ \theta(\rho_C) = \rho_C\}. \quad (2.1)$$

And, The fuzzy soft topological interior of λ_A with respect to θ, denoted by $\tau_\theta\text{-}\ int(\lambda_A)$ is the union of all open fuzzy soft subset of λ_A. i.e.,

$$\tau_\theta\text{-}\ int(\lambda_A) = \cup\{\rho_C : \rho_C \subseteq \lambda_A \ and\ \theta(\bar{1}_K - \rho_C) = \bar{1}_K - \rho_C\}. \quad (2.2)$$

The next theorem give the relation between the $\check{C}$-fsco θ (respectively, interior operator Int) and the fuzzy soft topological closure $\tau_\theta\text{-}cl$ (respectively, interior $\tau_\theta\text{-}int$).

Theorem 2.15 [22] Let (X, θ, K) be $\check{C}\mathcal{F}$-scs and (X, τ_θ, K) be an associative fsts of (X, θ, K). Then for any $\lambda_A \in \mathcal{F}_{ss}(X, K)$

$$\tau_\theta\text{-}int(\lambda_A) \subseteq Int(\lambda_A) \subseteq \lambda_A \subseteq \theta(\lambda_A) \subseteq \tau_\theta\text{-}cl(\lambda_A). \quad (2.3)$$

3.Connected Čech Fuzzy Soft Closure Spaces

In this section we introduce and study fuzzy soft separated sets in $\check{C}\mathcal{F}$−scs, then we use it to introduce the notion of connectedness in $\check{C}\mathcal{F}$−scs's.

Definition 3.1 Let (X,θ,K) be a $\check{C}\mathcal{F}$-scs. If there exist non-empty proper fss's λ_A, $\mu_B \in \mathcal{F}_{ss}(X,K)$, such that $\lambda_A \cap \theta(\mu_B) = \bar{0}_K$ and $\theta(\lambda_A) \cap \mu_B = \bar{0}_K$, then the fss's λ_A and μ_B are called fuzzy soft separated sets.

In other words, two non-empty fuzzy soft set λ_A, μ_B of $\check{C}\mathcal{F}$-scs (X,θ,K) are said to be fuzzy soft separated sets if and only if $(\lambda_A \cap \theta(\mu_B)) \cup (\theta(\lambda_A) \cap \mu_B) = \bar{0}_K$.

Remark 3.2 It is clear that if λ_A and μ_B are fuzzy soft separated sets in (X,θ,K), then λ_A and μ_B are disjoint fuzzy soft sets. The following example shows that the converse is not true.

Example 3.3 Let X={a,b,c}, K={h_1,h_2} and let $\rho_C = \{(h_1,b_{0.5}),(h_2,b_{0.5})\}$. Define $\theta: \mathcal{F}_{ss}(X,K) \to \mathcal{F}_{ss}(X,K)$ as follows:

$$\theta(\lambda_A) = \begin{cases} \bar{0}_K & if\ \lambda_A = \bar{0}_K, \\ \{(h_1, a_{0.5} \vee b_{0.5}),(h_{0.5}, a_{0.5} \vee b_{0.5})\} & if\ \lambda_A \subseteq \rho_C, \\ \bar{1}_K & otherwise. \end{cases}$$

Then θ is $\check{C}$-fsco on X. Here we have $\lambda_A = \{(h_1,b_{0.5})\}$ and $\mu_B = \{(h_1,a_{0.5}),(h_2,c_{0.5})\}$ are non-empty disjoint fuzzy soft sets but λ_A and μ_B are not fuzzy soft separated sets.

Theorem 3.4 Let (X,θ,K) be a $\check{C}\mathcal{F}$-scs. Then every fuzzy soft subset of fuzzy soft separated sets are also fuzzy soft separated sets.

Proof. Let λ_A and μ_B are fuzzy soft separated sets in (X,θ,K), and let $\rho_C \subseteq \lambda_A$ and $\eta_D \subseteq \mu_B$. Since $\rho_C \subseteq \lambda_A$ and $\eta_D \subseteq \mu_B$, then by Proposition 2.9, we have $\theta(\rho_C) \subseteq \theta(\lambda_A)$ and $\theta(\eta_D) \subseteq \theta(\mu_B)$. This implies $\theta(\rho_C) \cap \eta_D \subseteq \theta(\lambda_A) \cap \mu_B$ and $\theta(\eta_D) \cap \rho_C \subseteq \theta(\mu_B) \cap \lambda_A$. But λ_A and μ_B are fuzzy soft separated sets, it follows $\theta(\rho_C) \cap \eta_D \subseteq \theta(\lambda_A) \cap \mu_B = \bar{0}_K$ and $\theta(\eta_D) \cap \rho_C \subseteq \theta(\mu_B) \cap \lambda_A = \bar{0}_K$. Hence $\theta(\rho_C) \cap \eta_D = \bar{0}_K$ and $\theta(\eta_D) \cap \rho_C = \bar{0}_K$. Thus ρ_C and η_D are fuzzy soft separated sets. ■

Theorem 3.5 Let (V,θ_V,K) be a $\check{C}\mathcal{F}$-sc subspace of (X,θ,K) and let λ_A, $\mu_B \in \mathcal{F}_{ss}(V,K)$, then λ_A and μ_B are fuzzy soft separated sets in (X,θ,K) if and only if λ_A and μ_B are fuzzy soft separated sets in (V,θ_V,K).

Proof. Let (X,θ,K) be a $\check{C}\mathcal{F}$-scs and (V,θ_V,K) be a $\check{C}\mathcal{F}$-sc subspace of (X,θ,K). Assume that that λ_A and μ_B are fuzzy soft separated sets in (X,θ,K), this implies that $\lambda_A \cap \theta(\mu_B) = \bar{0}_K$ and $\theta(\lambda_A) \cap \mu_B = \bar{0}_K$. Which means $(\lambda_A \cap \theta(\mu_B)) \cup (\theta(\lambda_A) \cap \mu_B) = \bar{0}_K$.
Now,

$$(\lambda_A \cap \theta_V(\mu_B)) \cup (\theta_V(\lambda_A) \cap \mu_B) = (\lambda_A \cap (\bar{V}_K \cap \theta(\mu_B))) \cup ((\bar{V}_K \cap \theta(\lambda_A)) \cap \mu_B)$$

$$= ((\lambda_A \cap \bar{V}_K) \cap \theta(\mu_B)) \cup ((\bar{V}_K \cap \mu_B) \cap \theta(\lambda_A))$$

$$= (\lambda_A \cap \theta(\mu_B)) \cup (\mu_B \cap \theta(\lambda_A))$$

$$= \bar{0}_K.$$

Therefore, λ_A and μ_B are fuzzy soft separated sets in (X,θ,K) if and only if λ_A and μ_B are fuzzy soft separated sets in (V,θ_V,K). ■

Definition 3.6 A $\check{C}\mathcal{F}$-scs (X,θ,K) is said to be disconnected Čech fuzzy soft closure space (disconnected-$\check{C}\mathcal{F}$-scs, for short) if there exist fuzzy soft separated sets λ_A and μ_B such that $\theta(\lambda_A) \cap \theta(\mu_B) = \bar{0}_K$ and $\theta(\lambda_A) \cup \theta(\mu_B) = \bar{1}_K$.

Definition 3.7 A $\check{C}\mathcal{F}$-scs (X,θ,K) is said to be connected Čech fuzzy soft closure space (connected-$\check{C}\mathcal{F}$-scs, for short) if it is not disconnected-$\check{C}\mathcal{F}$-scs.

Now we give two examples one is disconnected-$\check{C}\mathcal{F}$-scs and the other is connected-$\check{C}\mathcal{F}$-scs.

Example 3.8 Let X={a,b}, K={h_1,h_2}. Define $\theta: \mathcal{F}_{ss}(X,K) \to \mathcal{F}_{ss}(X,K)$ as follows:

$$\theta(\lambda_A) = \begin{cases} \bar{0}_K & if\ \lambda_A = \bar{0}_K, \\ \{(h_1, a_1 \vee b_1)\} & if\ \lambda_A \subseteq \{(h_1,a_1)\}, \\ \{(h_2, a_1 \vee b_1)\} & if\ \lambda_A \subseteq \{(h_2,a_1)\}, \\ \bar{1}_K & other\ wise. \end{cases}$$

Then (X,θ,K) is disconnected-$\check{C}\mathcal{F}$-scs. To explain that taking $\lambda_A = \{(h_1,a_{0.5})\}$ and $\mu_B = \{(h_2,a_{0.2})\}$. It is clear that λ_A and μ_B are fuzzy soft separated sets such that $\theta(\lambda_A) \cap \theta(\mu_B) = \bar{0}_K$ and $\theta(\lambda_A) \cup \theta(\mu_B) = \bar{1}_K$.

Example 3.9 Let $X=\{a,b\}$, $K=\{h_1,h_2\}$. Define $\theta: \mathcal{F}_{ss}(X,K) \to \mathcal{F}_{ss}(X,K)$ as follows:

$$\theta(\lambda_A) = \begin{cases} \bar{0}_K & if\ \lambda_A = \bar{0}_K, \\ \{(h_1, a_1 \vee b_1)\} & if\ \lambda_A \subseteq \{(h_1, a_1 \vee b_1)\}, \\ \bar{1}_K & otherwise. \end{cases}$$

Then (X,θ,K) is connected-$\check{C}\mathcal{F}$-scs.

Remark 3.10 Connectedness in $\check{C}\mathcal{F}$-scs is not hereditary property. The following example explain that.

Example 3.11 Let $X=\{a,b,c\}$, $K=\{h_1,h_2\}$ and let $(\lambda_A)_1, (\lambda_A)_2 \in \mathcal{F}_{ss}(X,K)$ such that
$(\lambda_A)_1 = \{(h_1, a_1 \vee b_1 \vee c_{0.4})\}$ and $(\lambda_A)_2 = \{(h_2, a_1 \vee b_1 \vee c_{0.7})\}$.
Define $\theta: \mathcal{F}_{ss}(X,K) \to \mathcal{F}_{ss}(X,K)$ as follows:

$$\theta(\lambda_A) = \begin{cases} \bar{0}_K & if\ \lambda_A = \bar{0}_K, \\ \{(h_1, a_1 \vee b_1 \vee c_{0.4})\} & if\ \lambda_A \subseteq (\lambda_A)_1, \\ \{(h_2, a_1 \vee b_1 \vee c_{0.7})\} & if\ \lambda_A \subseteq (\lambda_A)_2, \\ \theta((\lambda_A)_1) \cup \theta((\lambda_A)_2) & if\ \lambda_A \subseteq (\lambda_A)_1 \cup (\lambda_A)_2, \\ \bar{1}_K & otherwise. \end{cases}$$

Then (X,θ,K) is connected-$\check{C}\mathcal{F}$-scs. Let $V=\{a,b\}$, then $\theta_V: \mathcal{F}_{ss}(V,K) \to \mathcal{F}_{ss}(V,K)$ defined as

$$\theta_V(\lambda_A) = \begin{cases} \bar{0}_K & if\ \lambda_A = \bar{0}_K, \\ \{(h_1, a_1 \vee b_1)\} & if\ \lambda_A \subseteq \{(h_1, a_1 \vee b_1)\}, \\ \{(h_2, a_1 \vee b_1)\} & if\ \lambda_A \subseteq \{(h_2, a_1 \vee b_1)\}, \\ \bar{V}_K & otherwise. \end{cases}$$

Then (V,θ_V,K) is disconnected-$\check{C}\mathcal{F}$-sc subspace of (X,θ,K). Since there exist $\lambda_A = \{(h_1, a_1 \vee b_1)\}$ and $\mu_B = \{(h_2, a_1 \vee b_1)\}$ are fuzzy soft separated sets such that $\theta_V(\lambda_A) \cap \theta_V(\mu_B) = \bar{0}_K$ and $\theta_V(\lambda_A) \cup \theta_V(\mu_B) = \bar{V}_K$.

Now, we introduce the concept of fuzzy soft separated sets in the associative fsts's of $\check{C}\mathcal{F}$-scs's.

Definition 3.12 Two non-empty fss's λ_A and μ_B are said to be fuzzy soft separated sets in the associative fsts (X,τ_θ,K), if $\lambda_A \cap \tau_\theta\text{-}cl(\mu_B) = \bar{0}_K$ and $\tau_\theta\text{-}cl(\lambda_A) \cap \mu_B = \bar{0}_K$.

Theorem 3.13 If λ_A and μ_B are fuzzy soft separated sets in the associative fsts (X,τ_θ,K), then λ_A and μ_B are also fuzzy soft separated sets in (X,θ,K).

Proof. Let λ_A and μ_B are fuzzy soft separated sets in (X,τ_θ,K). Then $\lambda_A \cap \tau_\theta\text{-}cl(\mu_B) = \bar{0}_K$ and $\tau_\theta\text{-}cl(\lambda_A) \cap \mu_B = \bar{0}_K$. By Theorem 2.15, we get, $\lambda_A \cap \theta(\mu_B) = \bar{0}_K$ and $\theta(\lambda_A) \cap \mu_B = \bar{0}_K$. This implies λ_A and μ_B are fuzzy soft separated sets in (X,θ,K). ∎

Definition 3.14 An associative fsts (X,τ_θ,K) of $\check{C}\mathcal{F}$-scs (X,θ,K) is said to be disconnected fsts, if there exist two fuzzy soft separated sets λ_A and μ_B in (X,τ_θ,K) such that $\tau_\theta\text{-}cl(\lambda_A) \cap \tau_\theta\text{-}cl(\mu_B) = \bar{0}_K$ and $\tau_\theta\text{-}cl(\lambda_A) \cup \tau_\theta\text{-}cl(\mu_B) = \bar{1}_K$.

Definition 3.15 An associative fsts (X,τ_θ,K) of $\check{C}\mathcal{F}$-scs(X,θ,K) is said to be connected fsts, if it is not disconnected fsts.

Theorem 3.16 If (X,τ_θ,K) is a disconnected fsts, then (X,θ,K) is disconnected-$\check{C}\mathcal{F}$-scs.

Proof. Let (X,τ_θ,K) be disconnected fsts, then there exist two fuzzy soft separated sets λ_A and μ_B in (X,τ_θ,K) such that $\tau_\theta\text{-}cl(\lambda_A) \cap \tau_\theta\text{-}cl(\mu_B) = \bar{0}_K$ and $\tau_\theta\text{-}cl(\lambda_A) \cup \tau_\theta\text{-}cl(\mu_B) = \bar{1}_K$. Since $\tau_\theta\text{-}cl(\lambda_A)$ and $\tau_\theta\text{-}cl(\mu_B)$ are closed-fss's, then $\theta(\tau_\theta - cl(\lambda_A)) = \tau_\theta\text{-}cl(\lambda_A)$ and $\theta(\tau_\theta\text{-}cl(\mu_B)) = \tau_\theta\text{-}cl(\mu_B)$. Let $\rho_C = \tau_\theta\text{-}cl(\lambda_A)$ and $\eta_D = \tau_\theta\text{-}cl(\mu_B)$. Then we have ρ_C and η_D are fuzzy soft separated sets in (X,θ,K) such that $\theta(\rho_C) \cap \theta(\eta_D) = \rho_C \cap \eta_D = \bar{0}_K$ and $\theta(\rho_C) \cup \theta(\eta_D) = \rho_C \cup \eta_D = \bar{1}_K$. Hence, (X,θ,K) is disconnected-$\check{C}\mathcal{F}$-scs. ∎

Corollary 3.17 If (X,θ,K) is connected-$\check{C}\mathcal{F}$-scs, then (X,τ_θ,K) is a connected fsts.

Proof. The proof follows by suppose (X,τ_θ,K) is disconnected fsts. From Theorem 3.16, we get (X,θ,K) is disconnected-$\check{C}\mathcal{F}$-scs which is a contradiction with hypothesis. Hence, the result. ∎

Remark 3.18 The converse of Theorem 3.16 and its corollary is not true in general. That is, if (X,θ,K) is disconnected-$\check{C}\mathcal{F}$-scs, then (X,τ_θ,K) need not to disconnected fsts. The following example shows that.

Example 3.19 In Example 3.8, (X,θ,K) is disconnected-$\check{C}\mathcal{F}$-scs. But its associative fsts (X,τ_θ,K) is connected fsts, because $\tau_\theta = \{\bar{0}_K, \bar{1}_K\}$.

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4. Feebly Connected Čech Fuzzy Soft Closure Spaces

Definition 4.1 A $\check{C}\mathcal{F}$-scs (X,θ,K) is said to be feebly disconnected-$\check{C}\mathcal{F}$-scs, if there two non-empty disjoint fuzzy soft sets λ_A and μ_B such that $\lambda_A \cup \theta(\mu_B) = \bar{1}_K$ and $\theta(\lambda_A) \cup \mu_B = \bar{1}_K$.

Definition 4.2 A $\check{C}\mathcal{F}$-scs (X,θ,K) is said to be feebly connected-$\check{C}\mathcal{F}$-scs if it is not feebly disconnected-$\check{C}\mathcal{F}$-scs.

Remark 4.3 Feebly disconnectedness in $\check{C}\mathcal{F}$-scs is not hereditary property. The following example explains that.

Example 4.4 Let $X=\{a,b,c\}$, $K=\{h_1,h_2\}$ and let $(\lambda_A)_1, (\lambda_A)_2 \in \mathcal{F}_{ss}(X,K)$ such that

$(\lambda_A)_1 = \{(h_1, a_1 \vee c_1)\}$ and

$(\lambda_A)_2 = \{(h_1,b_1),(h_2, a_1 \vee b_1 \vee c_1)\}$.

Define $\theta: \mathcal{F}_{ss}(X,K) \to \mathcal{F}_{ss}(X,K)$ as follows:

$$\theta(\lambda_A) = \begin{cases} \bar{0}_K & if\ \lambda_A = \bar{0}_K, \\ \{(h_1, a_1 \vee c_1)\} & if\ \lambda_A \subseteq (\lambda_A)_1, \\ \{(h_1,b_1),(h_2,a_1 \vee b_1 \vee c_1)\} & if\ \lambda_A \subseteq (\lambda_A)_2, \\ \bar{1}_K & otherwise. \end{cases}$$

Then (X,θ,K) is feebly disconnected-$\check{C}\mathcal{F}$-scs. Since there exist $\lambda_A = \{(h_1, a_1 \vee c_1)\}$ and $\mu_B = \{(h_1,b_1),(h_2,a_1 \vee b_1 \vee c_1)\}$ are disjoint fuzzy soft sets such that $\theta(\mu_B) \cup \lambda_A = \bar{1}_K$ and $\mu_B \cup \theta(\lambda_A) = \bar{1}_K$.
Let $V = \{b\}$, then $\theta_V: \mathcal{F}_{ss}(V,K) \to \mathcal{F}_{ss}(V,K)$ defined as:

$$\theta_V(\lambda_A) = \begin{cases} \bar{0}_K & if\ \lambda_A = \bar{0}_K, \\ \bar{V}_K & otherwise. \end{cases}$$

Then (V,θ_V,K) is feebly connected-$\check{C}\mathcal{F}$-sc subspace of (X,θ,K).

Definition 4.5 An associative fsts (X,τ_θ,K) of $\check{C}\mathcal{F}$-scs (X,θ,K) is said to be feebly disconnected fsts, if there exist two non-empty disjoint fuzzy soft sets λ_A and μ_B such that $\lambda_A \cup \tau_\theta\text{-}cl(\mu_B) = \bar{1}_K$ and $\tau_\theta\text{-}cl(\lambda_A) \cup \mu_B = \bar{1}_K$.

Theorem 4.6 If (X,θ,K) is feebly disconnected -$\check{C}\mathcal{F}$-scs, then (X,τ_θ,K) is feebly disconnected fsts.

Proof. The proof follows from the definition 4.1 and Theorem 2.19. ∎

Corollary 4.7 If (X,τ_θ,K) is feebly connected fsts, then (X,θ,K) is feebly connected-$\check{C}\mathcal{F}$-scs.

Proof. The proof follows by suppose (X,θ,K) is feebly disconnected-$\check{C}\mathcal{F}$-scs. From Theorem 4.6, we get (X,τ_θ,K) is feebly disconnected fsts which is a contradiction with hypothesis. Hence, the result. ∎

Next we discuss the relationship between disconnectedness and feebly disconnectedness in $\check{C}\mathcal{F}$-scs's.

Remark 4.10 The concept of disconnected-$\check{C}\mathcal{F}$-scs and feebly disconnected-$\check{C}\mathcal{F}$-scs are independent. The next two examples explain our clime.

The following example shows that if (X,θ,K) is disconnected-$\check{C}\mathcal{F}$-scs, then (X,θ,K) need not to be feebly disconnected-$\check{C}\mathcal{F}$-scs.

Example 4.11 Let $X=\{a,b\}$, $K=\{h\}$. Define $\theta: \mathcal{F}_{ss}(X,K) \to \mathcal{F}_{ss}(X,K)$ as follows:

$$\theta(\lambda_A) = \begin{cases} \bar{0}_K & if\ \lambda_A = \bar{0}_K, \\ \{(h,a_1)\} & if\ \lambda_A = \{(h,a_t);\ 0 < t < 1\}, \\ \{(h,b_1)\} & if\ \lambda_A = \{(h,b_s);\ 0 < s < 1\}, \\ \bar{1}_K & other\ wise. \end{cases}$$

Then (X,θ,K) is disconnected-$\check{C}\mathcal{F}$-scs, since there exist $\lambda_A = \{(h,a_{0.5})\}$ and $\mu_B = \{(h,b_{0.3})\}$ are fuzzy soft separated sets such that $\theta(\lambda_A) \cap \theta(\mu_B) = \bar{0}_K$ and $\theta(\lambda_A) \cup \theta(\mu_B) = \bar{1}_K$. However, (X,θ,K) is not feebly disconnected-$\check{C}\mathcal{F}$-scs since for any non-empty disjoint fss's λ_A and μ_B, we have $\lambda_A \cup \theta(\mu_B) \neq \bar{1}_K$.

The next example shows that if (X,θ,K) is feebly disconnected-$\check{C}\mathcal{F}$-scs, then (X,θ,K) need not to be disconnected-$\check{C}\mathcal{F}$-scs.

Example 4.12 Let $X=\{a,b\}$, $K=\{h_1,h_2\}$. Define $\theta: \mathcal{F}_{ss}(X,K) \to \mathcal{F}_{ss}(X,K)$ as follows:

$$\theta(\lambda_A) = \begin{cases} \bar{0}_K & if\ \lambda_A = \bar{0}_K, \\ \{(h_1, a_1 \vee b_1),(h_2,b_1)\} & if\ \lambda_A \subseteq \{(h_1,b_1)\}, \\ \{(h_1,a_1),(h_2,a_1 \vee b_1)\} & if\ \lambda_A \subseteq \{(h_2,a_1)\}, \\ \bar{1}_K & other\ wise. \end{cases}$$

Then (X,θ,K) is feebly disconnected-$\check{C}\mathcal{F}$-scs. Since there are non-empty disjoint fuzzy soft sets $\lambda_A = \{(h_1,b_1)\}$ and $\mu_B = \{(h_2,a_1)\}$ such that $\theta(\lambda_A) \cup \mu_B = \bar{1}_K$ and $\lambda_A \cup \theta(\mu_B) = \bar{1}_K$.
And (X,θ,K) is connected-$\check{C}\mathcal{F}$-scs. Since for any fuzzy soft separated sets λ_A and μ_B, we have $\theta(\lambda_A) \cup \theta(\mu_B) = \bar{1}_K$ but $\theta(\lambda_A) \cap \theta(\mu_B) \neq \bar{0}_K$.

Remark 4.13 It is worth noting that the definitions of disconnected-$\check{C}\mathcal{F}$-scs and feebly disconnected-$\check{C}\mathcal{F}$-scs (see Definitions 3.6 and 4.1, respectively) turn to be every disconnected-$\check{C}\mathcal{F}$-scs is feebly disconnected-$\check{C}\mathcal{F}$-scs, if the fuzzy soft separated sets which are satisfying the conditions of disconnected-$\check{C}\mathcal{F}$-scs are closed-fss's.

References

[1] Majeed RN, Čech fuzzy soft closure spaces, International Journal of Fuzzy System Applications 2018; 7(2): 62-74.
[2] Zadeh LA, Fuzzy sets, Information and Control 1965; 8: 338-353.
[3] Molodtsov DA, Soft set theory first results, Computers and Mathematics with Applications 1999; 37: 19-31.
[4] Maji PK, Biswas R, Roy AR, Fuzzy soft sets, J. Fuzzy Math. 2001; 9(3): 589-602.
[5] Tanay B, Kandemir MB, Topological structures of fuzzy soft sets, Computers and Mathematics with Applications 2011; 61: 412-418.
[6] Roy S, Samanta TK, An introduction to open and closed sets on fuzzy topological spaces, Ann. Fuzzy Math. Inform. 2013; 2(6):425-431.
[7] Borgohain K, Extended approach to fuzzy soft topological spaces, International Journal of Mathematics Trends and Technology 2014; 7(1): 12-18.
[8] Mishra S, Srivastava R, Hausdorff fuzzy soft topological spaces, Ann. Fuzzy Math. Inform. 2015; 9(2): 247-260.
[9] Mukherjee P, Chakraborty RP, Park C, Fuzzy soft θ-closure operator in fuzzy soft topological spaces, Electronic Journal of Mathematical Analysis and Applications, 2015; 3(2): 227-236.
[10] Kandil A, El-tantawy OA, El-Sheikh SA, El-Sayed SS, Fuzzy soft connected sets in fuzzy soft topological spaces I, J. Adv. Math. 2016; 8(12):6473-6488.
[11] Kandil A, El-tantawy OA, El-Sheikh SA, El-Sayed SS, Fuzzy soft connected sets in fuzzy soft topological spaces II, Journal of the Egyptian Mathematical Society 2017; 25: 171-177.
[12] $\check{C}$ech E, Topological spaces, Inter Science Publishers, John Wiley and Sons, New York; 1966.
[13] Mashhour AS, Ghanim MH, Fuzzy closure spaces, J. Math. Anal. Appl. 1985; 106: 154-170.
[14] Gowri R, Jegadeesan G, On soft $\check{C}$ech closure spaces, International Journal of Mathematics Trends and Technology 2014; 9(2): 122-127.
[15] Krishnaveni J, Sekar C, $\check{C}$ech Soft Closure Spaces, International Journal of Mathematical Trends and Technology 2014; 6: 123-135.
[16] Majeed RN, Maibed LH, Some structures of Čech fuzzy soft closure spaces, Journal of engineering and applied sciences, 2018 (Accepted).
[17] Plastria F, Connectedness and local connectedness of simple extensions, Bull. Belg. Math. Soc. Simon Stevin 1976; 28: 43-51.
[18] Gowri R, Jegadeesan G, Connectedness in soft Čech closure spaces, Annals of Pure and Applied Mathematics 2016; 11(1): 115-122.
[19] Roy S, Samanta TK, A note on fuzzy soft topological spaces, Ann. Fuzzy Math. Inform. 2012; 3(2): 305-311.
[20] Varol BP, Aygün H, Fuzzy soft Topology, Hacet. J. Math. Stat. 2012; 41(2): 407-419.
[21] Mahanta J, Das PK, Results on fuzzy soft topological spaces, arXiv:1203.0634v, 2012.
[22] Majeed RN, Maibed LH, Lower separation axioms in Čech fuzzy soft closure spaces,Submitted.

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الاتصال في فضاءات الاغلاق الضبابية الناعمة من النوع – تشيك

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المستخلص :

يعتبر مفهوم فضاءات الاغلاق الضبابية الناعمة من النوع –تشيك من المفاهيم الحديثة حيث تم تعريفه ودراسة خواصه من قبل مجيد [1] . في هذا البحث قمنا بتعريف ودراسة مفهوم المجموعات الضبابية الناعمة القابلة للفصل في فضاءات الاغلاق الضبابية الناعمة من النوع – تشيك. بأستخدام المجموعات الضبابية القابلة للفصل تم تعريف ودراسة مفهوم الاتصال في كلا من فضاءات الاغلاق الضبابية الناعمة من النوع –تشيك والفضاء الضبابي الناعم المشتق منه. كذلك عرفنا مفهوم الاتصال الضعيف ودرسنا العلاقة بين مفهوم الاتصال ومفهوم الاتصال الضعيف . واخيرا، اعطينا العديد من الامثلة لتوضيح النتائج التي تم التوصل اليها في البحث.

الكلمات المفتاحية: مجموعة ضبابية ناعمة، مؤثر فضاء الاغلاق الضبابي الناعم، مجموعات ضبابية ناعمة قابلة للفصل، فضاء الاغلاق الضبابي الناعم من النوع –تشيك المتصل، فضاء الاغلاق الضبابي الناعم من النوع –تشيك المتصل الضعيف.