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On B^*c– open set and its properties

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Abstract:

In this paper we introduced a new set is said B^*c– open set where we studied and identified its properties and find the relation with other sets and our concluded a new class of the function called B^*c– cont. function, B^*c– open function, B^*c– closed function.

Key words:

B^*c – open set, B^*c – closed set, B^*c – closure, B^*c – interior, B^*c – continuous.

Mathematics subject classification: 54xx.

1- Introduction :

The topological idea from study this set is generalization the properties and using its to prove many of the theorems. In [1]Abd El-Monsef M.E.,El.Deeb S.N. Mahmoud R.A Introduced set of class β- open, β- closed which are considered as in put to study the set of class B^*c – open, B^*c – closed and we introduced the interior and the closure as property of B^*c – open set, B^*c – closed set. In [4] Najasted O (1965) and [5] Andrijecivic D (1986) introduced a study about the set α –open, α –closed, B – open with the set β- open set and through it, we introduced proof many of proposition as the set B^*c – open set with α –closed it can lead to set β- open set. In [6] Ryszard Engelking introduced the function as concept to β- continuous, B^*c – continuous, β– open function, B^*c – open function, β– closed function, B^*c – closed function and find the relation among them.

2. On B^*c – open sets

Definition (2.1) [1]

Let X be a top. sp. Then a sub set A of X is called to be

i) a β- open set if $A \subseteq \overline{\overline{A}^{o}}$.

ii) a β- closed set if $A \supseteq \overline{A^{o}}^{o}$

The all β- open (resp. β- closed) set sub sets of a space X will be as always symbolizes that βo(x) (resp. βc(x)).

Example (2.2):

Let X = {a, b, c, d} with topology t = { ∅ , X, {a} , { a, b}, {a, c, d}. Then the clases of β- open set and βclosed set are:
βo(X) = {∅, X, {a} ,{a, b},{a, c},{a, d}, {a, b, c},{a, b, d},{a, c, d}.
βc(X) = {∅, X, {b} ,{c},{d},{b, c}, {b, d},{c, d},{b, c, d}.

Remark (2.3):

Let X be a top. Sp. If $\overline{A}$ = X, then A is β-open set.

Remark (2.4):

If A β- open set in X, then A^C is β- closed set in X.

Proposition (2.5):

Let X be a top. Sp. Then:
i) Every open set is β- open set in X.
ii) Every closed set is β- closed set in X.

Proof :

i) Let A be open set, then A = A°. Since A ⊆ $\overline{A}$, then $A = A^{\circ} \subseteq \overline{A}^{o}$, there for $A \subseteq \overline{\overline{A}^{o}}$, hence A is β- open set inX.

ii) Let A be closed set, then A^C open set, then A^C β-open set in X by (i), then A β- closed set in X.
The converse of above proposition is not true in general.

Example (2.6):

Let X = {1, 2, 3}, t = {∅, X, {1}, {2,3} }.
βo(X) = {∅, X, {1}, {2}, {3}, {1,2}, {1,3}, {2,3} }.
βC (X) = βo(X).
Note that A = {3} is β- open (resp. β-closed) set, but not open (resp. closed) set.

Theorem (2.7):

Let X be atop. Sp. Then the following statement are holds:
i) The union family of β- open sets is β- open set.
ii) The intersection family of β- closed sets is β-closed set.

Proof:

i) Let $\{A_\alpha : \alpha \in \Lambda\}$ be a family of β-open set in X , then $A_\alpha \subseteq \overline{\overline{A_\alpha}^{o}}$, then

$$\bigcup_{\alpha\in\Lambda} A_\alpha \subseteq \bigcup_{\alpha\in\Lambda} \overline{\overline{A_\alpha}^{o}} = \overline{\bigcup_{\alpha\in\Lambda} \overline{A_\alpha}^{o}} \subseteq \overline{[\bigcup_{\alpha\in\Lambda} \overline{A_\alpha}]^{o}} = \overline{\overline{\bigcup_{\alpha\in\Lambda} A_\alpha}^{o}}$$

, hence $\bigcup_{\alpha\in\Lambda} A_\alpha$ is β-open set .

ii) Let $\{A_\alpha : \alpha \in \Lambda\}$ be a family of β-closed set in X , then $\{A_\alpha^c : \alpha \in \Lambda\}$ be β-open set in X, then $\{ \bigcup_{\alpha\in\Lambda} A_\alpha^c : \alpha \in \Lambda \}$ β - open sets .But $[\bigcap_{\alpha\in\Lambda} A_\alpha]^c = \bigcup_{\alpha\in\Lambda} A_\alpha^c$, then $[\bigcap_{\alpha\in\Lambda} A_\alpha]^c$ β - open sets in X . There for $\bigcap_{\alpha\in\Lambda} A_\alpha$ β-closed set in X.

Remark (2.8):

i) [1] the intersection of any two β- open sets is not β- open set in general.
ii) The union of any two β- closed sets is not β-closed set in general.

Example (2.9):

Let X = {1, 2, 3}, t = {∅, X, {1}, {2}, {1,2} }.
βo(X) = {∅, X, {1}, {2}, {1,2}, {1,3}, {2,3} }.
βc(X) = {∅, X, {1}, {2}, {3}, {1,3}, {2,3} }.
i) Let A = {1,3}, B = {2,3} are β- open sets, but A ∩ B = {3} not β- open in X.
ii) Let A = {1}, B = {2} are β- closed sets, but A ∪B = {1,2} not β- closed in X.

Proposition (2.10)

Let X be atop. Sp. Then:

i) G is an open set in X iff $\overline{G \cap \overline{A}} = \overline{G \cap A}$ for each A ⊆ X. [2]

Proposition (2.11)

Let X be atop. Sp. Then:

i) The intersection a β- open set and open set in X is β- open set.

ii) The union a β- closed set and closed set in X is β- closed set.

Proof:

i) Let A be a β- open set, then $A \subseteq \overline{\overline{A}^{o}}$

Let B open set. Then

$A \cap B \subseteq (\overline{\overline{A}^{o}} \cap B)$.

$\subseteq \overline{(\overline{A}^{o} \cap B)}$.

$= \overline{(\overline{A}^{o} \cap B^{\circ})}$ by proposition (2.10) .

$= \overline{(\overline{A} \cap B)^{o}}$.

$\subseteq \overline{\overline{(\overline{A} \cap B)}^{o}}$ by proposition (2.10)

$= \overline{\overline{A \cap B}^{o}}$ by proposition (2.10)

There fore A ∩ B is β- open set in X.

ii) Let A be a β- closed set in X, then A^C β- open set in X.

Let B be closed set in X, then B^C open set in X, then by (i) we get $A^C \cap B^C$ β- open set in X, but $(A \cup B)^C = (A^C \cap B^C)$, then $(A \cup B)^C$ β- open set in X, then A ∪ B β- closed set in X.

Definition (2.12):

Let X be atop. Sp. and A ⊆ X. Then:

i) A is α – open if $A \subseteq \overline{A^{o}}^{o}$ [4].

ii) A is α – closed if $\overline{A^{o}}^{o} \subseteq A$ [4].

Definition (2.13):

Let X be atop. Sp. X and A ⊆ X. Then a β- open set A is said a B^*c. open set if ∀ x ∈ A ∃ F_x closed set ∋ x ∈ F_x ⊆ A. A is a B^*c– closed set if A^c is a B^*c – open set X.

The all B^*c – open (resp. B^*c – closed) set sub set of a space X will be as always symbolize B^*c O (X) (resp. $B^*cc(X)$).

Example (2.14):

In example (2.9). Note that closed set in X are:

∅, X, {2,3}, {1,3}, {3}. Then

B^*c O (X) = {∅, X, {2,3}, {1,3} }

Remark (2.15):

If A B^*c – open set in X, then A^C is B^*c – closed set in X.

Remark (2.16):

From definition (2.13). Note that:

i) Every B^*c – open set is β- open set.

ii) Every B^*c – closed set is β- closed set.

The converse of above Remark is not true in general.

Example (2.17):

Let X = {a, b, c}, t = {∅, X, {a}, {b, c}}.

β o(X) = {∅, X, {a}, {b}, {c}, {a, b}, {a, c}, {b,c} }.

βc(X) = βo (X)

B^*c O (X) = {∅, X, {a}, {b, c} }.

B^*c C(X) = B^*c O (X). Not that A = {c} is β- open (resp. β- closed) ser. but not B^*c– open (resp. B^*c – closed) set.

Remark (2.18):

i) The B^*c – open set and open set are in-dependent.

ii) The B^*c – closed set and closed set are in-dependent.

Example (2.19):

In example (2.9) not that B^*co (x) = {∅, X, {1,3}, {2,3}, $B^*cc(x)$ = {∅, X, {1}, {2}}. Note that

i) A = {2,3} B^*c – open set, but not open and B = {1} is open, but not B^*c – open.

ii) A = {2} B^*c – closed set, but not closed and B = {3} is closed set, but not B^*c – closed.

Proposition (2.20):

Let X be atop. Sp. and A ⊆ X. If A α – closed. Then A β- open in X iff A B^*c – open.

Proof:

Suppose that A a β-open set in X, then $A \subseteq \overline{\overline{A}^{o}}$. Let x ∈ $A \subseteq \overline{\overline{A}^{o}}$. Since x ∈ $\overline{\overline{A}^{o}}$ and A α – closed set, then $\overline{\overline{A}^{o}} \subseteq A$. Thus x ∈ $\overline{\overline{A}^{o}} \subseteq A$, ∃ $\overline{\overline{A}^{o}}$ closed set ∋ x ∈ $\overline{\overline{A}^{o}} \subseteq A$. Then A B^*c – open set.

Conversely

Suppose that A B^*c – open set, then by definition (2.13), we get A β- open.

Corollary (2.21):

If A open set and α – closed, then A B^*c – open.

Proof:

By proposition (2.5) (i) and proposition (2.20).

Proposition (2.22):

Let X be atop. Sp. and A ⊆ X. If A α – open. Then A β- closed iff A B^*c – closed.

Proof:

Let A be β- closed, then A^C β- open. Since A α – open, then A^C α – closed, then by proposition (2. 20), we get A^C B^*c – open set. There fore A β- closed set.

Corollary (2.23):

If A closed set and α – open, then A B^*c – closed.

Proof:

By proposition (2.5) (ii) and (2.22)

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Proposition (2.24):

Let X be atop. Sp. X. Then
i) The union family of B^*c – open set is B^*c- open set.
ii) The intersection family B^*c – closed set is B^*c – closed set.

Proof:

i) Let $\{ A_\alpha : \alpha \in \Lambda \}$ be a family of B^*c – open sets, then $\{A_\alpha : \alpha \in \Lambda\}$ is β- open sets, then $\bigcup_{\alpha\in\Lambda} A_\alpha$ is β- open set by lemma (2.7) (i). Let $x \in \bigcup_{\alpha\in\Lambda} A_\alpha$, then $x \in A_\alpha$ for some $\alpha \in \Lambda$. Since A_α B^*c- open set $\forall\ \alpha \in \Lambda$, then ∃ F closed set in x ∋ x $\in F \subseteq A_\alpha \subseteq \bigcup_{\alpha\in\Lambda}$ A. Then for $\bigcup_{\alpha\in\Lambda} A_\alpha$ is B^*c- open set.
ii) Let $\{ A_\alpha : \alpha \in \Lambda \}$ be a family of B^*c – closed sets, then $\{A^c_\alpha : \alpha \in \Lambda\}$ is a family of B^*c- open sets, then $\bigcup_{\alpha\in\Lambda} A^c_\alpha$ is B^*c- open sets by (i), them $[\bigcup_{\alpha\in\Lambda} A^c_\alpha\ ]$ B^*c – closed. But $\bigcap_{\alpha\in\Lambda} A_\alpha = [\bigcup_{\alpha\in\Lambda} A^c_\alpha\ ]^c$, then $\bigcap_{\alpha\in\Lambda} A_\alpha$ is B^*c – closed in X.

Remark (2.25):

i) Not every intersection of two B^*c – open set is B^*c – open set.
ii) Not every union of two B^*c – closed set is B^*c – closed set.

Example (2.26):

In example (2.9)
B^*c O (X) = {∅, X, {2,3}, {1,3} }.
B^*c C (X) = {∅, X, {1}, {2} }. Not that:
i) Let A = {1,3}, B = {2,3} are B^*c – open set, but A ∩ B = {3} not B^*c – open set in X.
ii) Let A = {1}, B = {2} are B^*c – closed set, but A ∪ B = {1,2} not B^*c – closed set in X.

Definition (2.27):

Let A subset of top. Sp. Then A is called:
i) Clopen set if A closed and open.[6]
ii) β- Clopen set if A β- closed and β- open.
iii) B^*c - Clopen set if A B^*c - closed and B^*c – open.

Proposition (2.28):[4]

Let X be atop. Sp. and A ⊆ X. Then
i) Every closed set is α – closed set.
ii) Every open set is α – open set.

Proposition (2.29):

Let X be atop. Sp. Then:
i) The union B^*c – open set and clopen set is B^*c – open.
ii) The intersection B^*c- closed set and clopen set is B^*c – closed.

Proof:

i) Let A B^*c – open set, then A^C B^*c- closed. Let B clopen, then B^C clopen, then B^C closed and open. Since B^C closed, then B^C β- closed. Since B^C open, then B^C α – open by proposition (2.28) (ii), then B^C B^*c – closed, then $A^C \cap B^C$ B^*c – closed by proposition (2.22) (ii), then $(A^C \cap B^C)^C$ B^*c – open. But $A \cup B = (A^C \cap B^C)^C$, there for A ∪ B B^*c – open set in X.
ii) Let A B^*c – closed, then A^C B^*c – open. Let B clopen, then B^C clopen, then by (i), we get $A^C \cup B^C$ B^*c – open, then $(A^C \cup B^C)^C$ B^*c – closed. But $A \cap B = (A^C \cup B^C)^C$, then A ∩ B B^*c – closed.

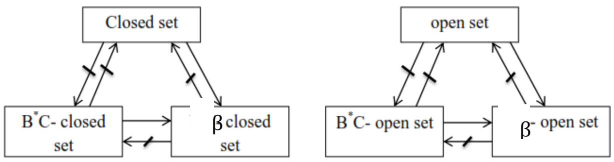
Proposition (2.30):

Let X be atop. Sp. Then:
i) The intersection B^*c – open set and clopen set is B^*c – open set.
ii) The union B^*c- closed set and clopen set is B^*c- closed.

Proof:

i) Let A be B^*c – open set and B clopen, then B open and closed, then A β- open set and B open, then A ∩ B is β- open set by (2.11)(i). Let x ∈ A ∩ B, then x ∈ A and x ∈ B, then ∃ F closed set in x ∋ x ∈ F ⊆ A. Since F ∩ B is closed set in x, then x ∈ F ∩ B ⊆ A ∩ B, hence A ∩ B B^*c – open.
ii) Let A B^*c – closed set, then A^C B^*c- open. Let B clopen in X, then B^C clopen, then by (i) we get $A^C \cap B^C$ B^*c- open in X, then $(A^C \cup B^C)^C$ B^*c – closed. But $A \cup B = (A^C \cap B^C)^C$, then A ∪ B B^*c – closed in X.

The following diagram shows the relation among types of open, closed sets.



(a) (b)
Fig (1)

Definition (2.31):

Let F: X → Y be a function and A ⊆ X. Then:
i) F is called continuous function [6]. If ∀A open subset of Y, then F^{-1} (A) is open subset of X.
ii) F is called β- continuous function. If ∀A open subset of Y , then F^{-1} (A) is β- open subset of X.[1]
iii) F is called B^*c-continuous function. If ∀A open subset of Y , then F^{-1} (A) is B^*c - open subset of X.

Proposition (2.32):

Let F: X → Y be a function and A ⊆ X. Then:
i) Every cont. function is a β- cont.
ii) Every B^*c -cont. function is a β- cont.

Proof:

Let F: X → Y be a function
i) Let F cont. and Let A be open in Y. Since F is cont. function , then F^{-1} (A) is open in X, then F^{-1} (A) is a β- open in X. Hence F is a β- cont.
ii) Let F B^*c -cont. and Let A be open in Y. Since F B^*c -cont. function then F^{-1} (A) B^*c – open in X, then F^{-1} (A) β- open in X, hence F is a β- cont.

The converse of above proposition is not true in general.

Example (2.33)

Let F: X → Y be a function and let X = {1, 2, 3}

t = {∅, X, {1}, {2,3} }.

βo(X) = {∅, X, {1}, {2}, {3}, {1,2}, {1,3}, {2,3} }.

B^*c o (X) = {∅, X, {1}, {2,3} }.

Y = {a, b, c}, ť = {∅, Y, {a}, {b}, {a, b} }.

βo(Y) = {∅, Y, {a}, {b}, {a, b}, {a, c}, {b ,c}}.

B^*c o (Y) = {∅, Y, {a, c}, {b, c} }.

Define F(1) = a, F(2) = b, F(3) = C.

Note that F is β- cont. But

i) F not cont. Since A ={b} open in Y, but F^{-1}(A) not open in X.

ii) F not B^*c - cont. Since A = {b} open in Y, but F^{-1}(A) not B^*c- open in X.

Remark (2.34):

The continuous function and B^*c-continuous are independent in general.

Example (2.35):

Let F: X → Y be a function

Let X = {1, 2, 3}, t = {∅, X, {1}, {2}, {1,2}, {1,3}}, βo(x) = t.

B^*co (x) = {∅, X, {2}, {1,3} }.

Y = {a, b, c}, ť = {∅, Y, {a}, {b}, {a, b} }.

βo(Y) = {∅, Y, {a}, {b}, {a, b}, {a, c}, {b, c}}.

B^*co (Y) = {∅, Y, {a, c}, {b, c} }.

Define F(1) = a, F(2) = b, F(3) = C.

Note that F is cont. function, but not B^*c - cont. function. Since A={a} open in Y, but F^{-1}(A) not B^*c- open in X.

Example (2.36)

Let F: X → Y be a function and Let X = {1, 2, 3}.

t = {∅, X, {1}, {3}, {1,3}}, βo(X) ={∅, X, {1}, {3}, {1,2}, {1,3}, {2,3}}.

B^*c o (X) = {∅, X, {1,2}, {2,3} }.

Y = {a, b, c}, ť = {∅, Y, {a, b} }.

Define F(1) = a, F(2) = b, F(3) = C.

Note that F is B^*c - cont. Since A = {a, b} is open in Y, but F^{-1}(A) not open in X.

The following diagram shows the relation among type of the continuous function.

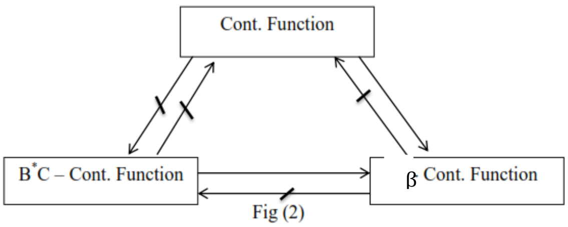


Fig (2)

3- The Closure:

Definition (3.1): [1]

The intersection of all β- closed set of atop. Sp. X which is containing A is called a β- closure of A and denoted by $\overline{A}^{\beta}$.

i. e $\overline{A}^{\beta}$ = ∩{ F: A ⊆ F, F is β- closed in X}.

Definition (3.2):

The intersection of all B^*c - closed set of atop. Sp. X which is containing A is said a B^*c - closure of A and denoted by $\overline{A}^{B^*C}$.

i. e $\overline{A}^{B^*C}$= ∩{ F: A ⊆ F, F is B^*c - closed in X}.

Lemma (3.3):

Let X be atop. Sp. and A ⊆ X. Then

i) X ∈ $\overline{A}^{\beta}$ iff ∀ β- open set G and x ∈ G ∋ G ∩ A ≠ ∅ [1].

ii) X ∈ $\overline{A}^{B^*C}$ iff ∀ B^*c - open set G and x ∈ G ∋ G ∩ A ≠ ∅.

Proof:

ii) Let x ∉ $\overline{A}^{B^*C}$, then x ∉ ∩ F ∋ F is B^*c – closed set and A ⊆ F, then x ∈ $[\cap F]^C$ ∋ $[\cap F]^C$ is B^*c – open containing x. Hence

$[\cap F]^C$ ∩A ⊆ $[\cap F]^C$ ∩ [∩F] = ∅.

Conversely

Suppose that ∃ a B^*c – open set G ∋ x ∈ G and A ∩ G = ∅., then A ⊆ G^c ∋ G^c is B^*c – closed set, hence x ∉ $\overline{A}^{B^*C}$

Remark (3.4):

Let X be a topological space and A ⊆ X. Then

i) $\overline{A}^{\beta}$ is β- closed set and $\overline{A}^{B^*C}$ is B^*c – closed set.

ii) $\overline{A}^{\beta}$ (resp. $\overline{A}^{B^*C}$) is the smallest β- closed (resp. B^*c – closed) set containing A.

iii) A ⊆ $\overline{A}^{\beta}$ also A ⊆ $\overline{A}^{B^*C}$. ∀ A ⊆ X.

Proof:

Clear.

Proposition (3.5):

Let X be a top. Sp. X and A ⊆ X. Then:

i) A β- closed set iff A = $\overline{A}^{\beta}$ [1].

ii) A B^*c – closed set iff A = $\overline{A}^{B^*C}$.

Proof:

ii) Let A be B^*c – closed set

Let X ∉ A, then X ∈ A^C, then ∃ B^*c – open set A^C ∋ A^C ∩ A = ∅, then X ∉ $\overline{A}^{B^*C}$, then $\overline{A}^{B^*C}$ ⊆ A. Since A ⊆ $\overline{A}^{B^*C}$ by Remark (3.4) (iii). Hence A = $\overline{A}^{B^*C}$.

Conversely

Let A = $\overline{A}^{B^*C}$. Since $\overline{A}^{B^*C}$ B^*c – closed set in X and A = $\overline{A}^{B^*C}$, then A B^*c – closed set.

Proposition (3.6):

Let X be a top. Sp. X and A ⊆ X. Then:

i) $\overline{\overline{A}^{\beta}}^{\beta}$ = $\overline{A}^{\beta}$ [1].

ii) If A ⊆ B, then $\overline{A}^{\beta}$ ⊆ $\overline{B}^{\beta}$.

Proof:

ii) Let A ⊆ B. Since B ⊆ $\overline{B}^{\beta}$ by Remark (3.4) (iii), then A ⊆ $\overline{B}^{\beta}$.

Since $\overline{B}^{\beta}$ is β- closed in X and $\overline{A}^{\beta}$ is the smallest β- closed set containing A. There for $\overline{A}^{\beta}$ ⊆ $\overline{B}^{\beta}$.

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Proposition (3.7):

Let X be a top. Sp. X and A ⊆ X. Then

i) $\overline{\overline{A}^{B^*c}}^{B^*c} = \overline{A}^{B^*C}$.

ii) If A ⊆ B, then $\overline{A}^{B^*C} \subseteq \overline{B}^{B^*C}$.

Proof:

i) Since $\overline{A}^{B^*C}$ is B^*c – closed, then by proposition (3.5) (ii), we get the result.

ii) Let A ⊆ B. Since B ⊆ $\overline{B}^{\beta}$by Remark (3.4) (iii), then A ⊆ $\overline{B}^{B^*C}$.

Since $\overline{B}^{B^*C}$ is B^*c – closed and $\overline{A}^{B^*C}$ is the smallest B^*c – closed containing A. Then fore $\overline{A}^{B^*C} \subseteq \overline{B}^{B^*C}$.

Proposition (3.8)

Let F: X → Y be function. Then the following statements are equivalent.

i) F is β- continuous.

ii) F^{-1} (B) is β- closed in X ∀ B is closed set in Y.

iii) F ($\overline{A}^{\beta}$) ⊆ $\overline{F(A)}$ ∀ A ⊆ X.

iv) $\overline{F^{-1}(B)}^{\beta} \subseteq F^{-1}(\overline{B})$ ∀ B ⊆ Y.

Proof:

(i) ——— (ii)

Let B be closed set in Y, then Y–B is open set in Y, then F^{-1} (Y–B) is a β- open in X by (i), then X – F^{-1} (B) is a β- open in X.

Then F^{-1} (B) is a β- closed in X.

(ii) ——— (iii)

Let A ⊆ X, then F(A) ⊆ Y, then $\overline{F(A)}$ is closed set in Y, then F^{-1} ($\overline{F(A)}$) is a β- closed set in X by (ii). Since F(A) ⊆ $\overline{F(A)}$, Then A ⊆ F^{-1} ($\overline{F(A)}$), then $\overline{A}^{\beta} \subseteq F^{-1}$ ($\overline{F(A)}$), there fore F ($\overline{A}^{\beta}$) ⊆ $\overline{F(A)}$.

(iii) ——— (iv)

Let B ⊆ Y, then F^{-1}(B) ⊆ X, then $F[\overline{F^{-1}(B)}^{\beta}] \subseteq \overline{F[F^{-1}(B)]}$ by (iii), then $F[\overline{F^{-1}(B)}^{\beta}] \subseteq (\overline{B})$, then $\overline{F^{-1}(B)}^{\beta} \subseteq F^{-1}(\overline{B})$.

(iv) ——— (i)

Let B be open set in Y, then Y – B is closed set in Y. Then

$\overline{F^{-1}(Y-B)}^{\beta} \subseteq F^{-1}(\overline{Y-B}) = F^{-1}(Y-B)$. Since F^{-1} (Y – B) = X – F^{-1} (B) is a β- closed set in X, then F^{-1} (B) is a β- open set in X.

There fore F is a β- continuous.

Proposition (3.9)

Let F: X → Y be a function. Then the following statements are equivalent.

i) F is BC- continuous.

ii) F^{-1} (B) is B^*c - closed in X ∀ B is closed set in Y.

iii) F ($\overline{A}^{B^*C}$) ⊆ $\overline{F(A)}$ ∀ A ⊆ X.

iv) $\overline{F^{-1}(B)}^{B^*c} \subseteq F^{-1}(\overline{B})$ ∀ B ⊆ Y.

Proof:

(i) ——— (ii)

Let B be closed set in Y, then Y–B is open set in Y, then F^{-1} (Y–B) is a B^*c - open in X by (i), then X – F^{-1} (B) is a B^*c - open in X.

Hence F^{-1} (B) is a B^*c - closed in X.

(ii) ——— (iii)

Let A ⊆ X, then F(A) ⊆ Y, then $\overline{F(A)}$ is closed set in Y, then F^{-1} ($\overline{F(A)}$) is a BC- closed set in X by (ii). Since F(A) ⊆ $\overline{F(A)}$, Then A ⊆ F^{-1} ($\overline{F(A)}$), then $\overline{A}^{B^*C} \subseteq F^{-1}$ ($\overline{F(A)}$), hence F ($\overline{A}^{B^*C}$) ⊆ $\overline{F(A)}$.

(iii) ——— (iv)

Let B ⊆ Y, then F^{-1}(Y) ⊆ X, then $\overline{F[F^{-1}(B)]}^{B^*c} \subseteq F[\overline{F^{-1}(B)}]$, then $\overline{F[F^{-1}(B)]}^{B^*c} \subseteq \overline{B}$, then $\overline{F^{-1}(B)}^{B^*c} \subseteq F^{-1}(\overline{B})$.

(iv) ——— (i)

Let B be open set in Y, then Y – B is closed set in Y. Then

$\overline{F^{-1}(Y-B)}^{B^*c} \subseteq F^{-1}(\overline{Y-B}) = F^{-1}(Y-B)$, then F^{-1} (Y – B) = X – F^{-1} (B) is a BC- closed set in X. F^{-1} (B) is a BC- open set in X.

There fore F is a B^*c - continuous.

Definition (3.10):

Let F: X → Y be function and A ⊆ X. Then:

i) F is called open (resp. closed) [6] . If ∀ A open (resp. closed), subset of X, then F(A) is open (resp. closed) subset of Y.

ii) F is called β- open (resp. β- closed). If ∀ A open (resp. closed), subset of X, then F(A) is β- open (resp. β- closed) subset of Y.

iii) F is called B^*c - open (resp. B^*c - closed). If ∀ A open (resp. closed), subset of X, then F(A) is B^*c - open (resp. B^*c - closed) subset of Y.

Proposition (3.11):

Let F: X → Y be a function and A ⊆ X. Then:

i) Every open function is β- open.

ii) Every closed function is β- closed.

iii) Every B^*c – open function is β- open.

iv) Every B^*c – closed function is β- closed.

Proof:

i) Let F: X → Y be a function.

Suppose that F open function and let A open in X. Since F open, then F(A) open in Y, then F(A) β- open in Y. Thus F is β- open.

ii) Similarly part (i).

iii) Suppose F is B^*c - open function and let A open in X. Since F B^*c - open, then F(A) B^*c - open in Y, then F(A) β- open in Y. Thus F is β- open.

iv) Similarly part (iii).

The Converse above proposition is not true in general.

Example (3.12):

In example (2.34)

Closed set in X are: ∅, X, {2,3},{1}.

Closed set in Y are: ∅, Y: ∅, X, {b, c}, {a,c},{c}.

βc(Y) = {∅, Y, {b, c}, {a, c}, {c},{b}, {a} }.

B^*c (Y) = {∅, Y, {b}, {a} }.

Not that:
i) F β- open, but not open since A = {2, 3} open in X, but F(A) not open in Y.
ii) F β- closed, but not closed. Since A = {1} closed set in X, but F(A) not closed set in Y.
iii) F β- open, but not B^*c – open. Since A = {1} open in X, but F(A) not B^*c – open set in Y.
iv) F β- closed, but not B^*c – closed. Since A = {2, 3} is closed in X, but F(A) not B^*c – closed in Y.

Remark (3.13):

i) The open function and B^*c – open function are independent.
ii) The closed function and B^*c – open function are independent.

We can showing that with two the following examples.

Example (3.14):

i) Let F: X → Y be function and let X = { a, b, c}
t = {∅, X, {b}, {b, c}}, βo (x) = {∅, X, {b}, {a, b}, {b, c}}.
B^*co (X) = {∅, X}.
Y = {1, 2, 3}, ť = {∅, Y, {2}, {3}, {2, 3}, {1, 2}}.
βo(Y) = {∅, Y, {2}, {3}, {1, 2}, {2, 3} }.
B^*co (Y) = {∅, Y, {3}, {1, 2}}.

Define F(a) = 1, F(b) = 2, F(c) = 3.

ii) Let F: X → Y be a function and let X = { a, b, c}
t = {∅, X, {b, c}}, βo(x) = {∅, X, {b}, {c}, {a, b}, {a, c}, {b, c}}.
B^*co (X) = {∅, X}.
Y = {1, 2, 3}, ť = {∅, Y, {1}, {3}, {1, 3}}.
βo(Y) = {∅, Y, {1}, {3}, {1, 3}, {2, 3} }, B^*co (Y) = {∅, Y, {2, 3}}.

Define F(a) = 1, F(b) = 2, F(c) = 3.

In example (i). Note that:
1) F β- open, but not B^*c – open. Since A = {b} open in X, but F(A) not B^*c – open set in Y.
2) F b^*-closed, but not B^*c–closed. Since A= {a} closed set in X, but F(A) not B^*c–closed in Y.
In example (ii). Note that:
1) F B^*c – open, but not open. Since A = {b, c} open in X, but F(A) not open set in Y.
2) F B^*c – closed, but not closed. Since A = {a} closed in X, but F(A) not closed in Y.

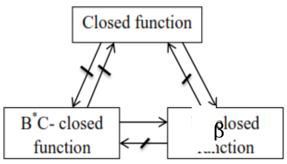

Fig (5)

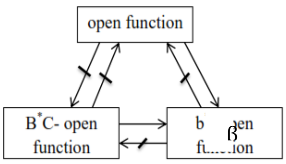

Fig (4)

4- The interior:

Definition (4.1):[1]

The union of all β - open set of atop. Sp. X contained in A is called β- interior of A and denoted $A^{o\beta}$.
i.e
$A^{o\beta} = \cup$ { U : U ⊆ A and U β- open set in X}.

Definition (4.2):

The union of all B^*c- open set of atop. Sp. X contained in A is called B^*c - interior of A and denoted A^{oB^*c}.
i.e
$A^{oB^*c} = \cup$ { U : U ⊆ A and U BC- open set in X }.

Proposition (4.3):

Let X be atop. Sp. and A ⊆ X. Then:
i) X ∈ $A^{o\beta}$ iff ∃ G β- open in X∋ x ∈G ⊆ A. [1]
ii) X ∈ A^{oB^*C} iff ∃ G B^*c-open in X∋ x ∈ G ⊆ A.

Proof:

ii) Let X ∈ A^{oB^*C}
Since A^{oB^*C} = ∪ { G : G ⊆ A, G is B^*c - open set in X }.
Then z ∈ ∪ {G: G ⊆ A, G is B^*c - open set in X}.
Then ∃ G B^*c – open in X ∋ X ∈ G ⊆ A.
Conversely

Let X ∈ G ⊆ A and G is B^*c – open in x ∈ G ⊆ A. Then
X ∈ U { G : G ⊆ A, G is B^*c - open set in X }.
There fore X ∈ A^{oB^*C}.

Remark (4.4):

Let x be atop. Sp. and A ⊆ X. Then:
i) $A^{o\beta}$ is β- open set and A^{oB^*C} is B^*c - open set.
ii) $A^{o\beta}$ (resp. A^{oB^*C}) is the largest β- open (resp. B^*c – open) set contained A.
iii) $A^{o\beta}$ ⊆ A also A ⊆ A^{oB^*C}.

Proof:

Clear.

Lemma (4.5): [1]

Let X be atop. Sp. and A ⊆ X. Then
i) $[A^{o\beta}]^C = \overline{A^C}^{\beta}$.
ii) $[\overline{A}^{\beta}]^C = A^{C^{o\beta}}$

Remark (4.6):

i) $[A^{oB^*C}]^C = \overline{A^C}^{B^*C}$
ii) $[\overline{A}^{B^*C}]^C = A^{C^{oB^*C}}$.

Proposition (4.7):

Let X be atop. Sp. and A ⊆ X. Then:
i) A β- open set iff A = $A^{o\beta}$.[1]
ii) i) A B^*c - open set iff A = A^{oB^*C}.

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Proof:

ii) Let A B^*c – open set, then A^C B^*c – closed set, then by Remark (3.4) (iii), we have $A^C = \overline{A^C}^{B^*C}$. Since $\overline{A^C}^{B^*C} = [A^{oB^*C}]^C$ by Remark (4.6) (ii), then $A^C = [A^{oB^*C}]^C$, hence $A = A^{oB^*C}$.

Conversely

Supposedly that $A = A^{oB^*C}$.

Since A^{oB^*C} is B^*c – open set and $A = A^{oB^*C}$, then B^*c – open set.

Proposition (4.8):

Let X be atop. Sp. and A, B ⊆ X. Then

i) $[A^{o\beta}]^{o\beta} = A^{o\beta}$ [1].

ii) If A ⊆ B, then $A^{o\beta} \subseteq B^{o\beta}$.

Proof:

ii) Let A ⊆ B. Since $A^{o\beta} \subseteq A \subseteq B$, then $A^{o\beta} \subseteq B$. Since $B^{o\beta}$is the largest β- open set contained B, them $A^{o\beta} \subseteq B^{o\beta}$.

Proposition (4.9):

Let X be atop. Sp. and A, B ⊆ X. Then

i) $[A^{oB^*C}]^{oB^*C} = A^{oB^*C}$.

ii) If A ⊆ B, then $A^{oB^*C} \subseteq B^{oB^*C}$.

Proof:

i) Since A^{oB^*C} is BC– open set, then $A^{oB^*C} \subseteq$ B. Since B^{oB^*C} is the largest B^*c – open set contained B, then $A^{oB^*C} \subseteq B^{oB^*C}$.

Proposition (4.10):

Let F: X → Y be function. Then the following statement are equivalent.

i) F β- open function.

ii) $F(A^\circ) \subseteq [F(A)]^{o\beta}$ ∀ A ⊆ X.

iii) $[F^{-1}(A)]^\circ \subseteq F^{-1}(A^{o\beta})$ ∀ A ⊆ Y.

Proof:

i) ------- ii)

Let A ⊆ X. Since A° open in X, then F(A°) β- open in Y by (i). Then $F(A^\circ) = [F(A^\circ)]^{o\beta} \subseteq [F(A)]^{o\beta}$. Hence $F(A^\circ) \subseteq [F(A)]^{o\beta}$.

ii) ------- (iii)

Let A ⊆ Y, then $F^{-1}(A) \subseteq X$, then $F[(F^{-1}(A^\circ))^o] \subseteq [F(F^{-1}(A))]^{o\beta}$by (ii). Then $F[(F^{-1}(A))^\circ] \subseteq A^{o\beta}$. Then $[F^{-1}(A)]^\circ \subseteq F^{-1}(A^{o\beta})$.

iii) ------ (i)

Let A open in X, then A = A°. Let F (A) ⊆ Y, then

$[F^{-1}(F(A))]^\circ \subseteq F^{-1}[(F(A))^{o\beta}]$, by (iii). Then $A = A^\circ \subseteq F^{-1}[(F(A))^{o\beta}]$, then $F(A) \subseteq [F(A)]^{o\beta}$. But $[F(A)]^{o\beta} \subseteq F(A)$, then $F(A) = [F(A)]^{o\beta}$. Hence F (A) β- open in Y, there fore F β- open function.

Proposition (4.11):

Let F: X → Y be function. Then the following statement are equivalent.

i) F B^*c – open function.

ii) $F(A^\circ) \subseteq [F(A)]^{oB^*C}$ ∀ A ⊆ X.

iii) $[F^{-1}(A)]^\circ \subseteq F^{-1}(A^{oB^*C})$ ∀ A ⊆ Y.

Proof:

i) ------- ii)

Let A ⊆ X. Since A° open in X, then F(A°) B^*c - open in Y by (i). Then $F(A^\circ) = [F(A^\circ)]^{oB^*C} \subseteq [F(A)]^{oB^*C}$. Hence $F(A^\circ) \subseteq [F(A)]^{oB^*C}$.

ii) ------- (iii)

Let A ⊆ Y, then $F^{-1}(A) \subseteq X$, then $F[(F^{-1}(A^\circ))^o] \subseteq [F(F^{-1}(A))]^{oB^*C}$ by (ii). Then $F[(F^{-1}(A))^\circ] \subseteq A^{oB^*C}$, hence $[F^{-1}(A)]^\circ \subseteq F^{-1}(A^{oB^*C})$.

iii) ------ (i)

Let A open in X, then A = A°. Let F (A) ⊆ Y, then

$[F^{-1}(F(A))]^\circ \subseteq F^{-1}[(F(A))^{oB^*C}]$, by (iii). Then $A = A^\circ \subseteq F^{-1}[(F(A))^{oB^*C}]$, then $F(A) \subseteq [F(A)]^{oB^*C}$. But $[F(A)]^{oB^*C} \subseteq F(A)$, then $F(A) = [F(A)]^{oB^*C}$. Hence F (A) BC- open in Y, there fore F Bc- open function.

Proposition (4.10):

A function F: X → Y is a β- closed iff $\overline{F(A)}^{\beta} \subseteq F(\bar{A})$ ∀ A ⊆ X.

Proof:

Suppose F is a β- closed. Let A ⊆ X, then $\bar{A}$ closed in X, then $F(\bar{A})$ is a β- closed in Y.

Then $\overline{F(A)}^{\beta} \subseteq \overline{F(\bar{A})}^{\beta} = F(\bar{A})$.

Conversely

Let A be closed in X, then $A = \bar{A}$ Since $\overline{F(A)}^{\beta} \subseteq F(\bar{A}) = F(A)$, then $\overline{F(A)}^{\beta} \subseteq F(A)$. But $F(A) \subseteq \overline{F(A)}^{\beta}$, then $F(A) = \overline{F(A)}^{\beta}$. There fore F(A) is a β- closed set in Y. Hence F is a β- closed.

Proposition (4.11):

A function F: X → Y is a B^*c- closed iff $\overline{F(A)}^{B^*c} \subseteq F(\bar{A})$ ∀ A ⊆ X.

Proof:

Suppose that F is a B^*c - closed.

Let A ⊆ X, then $\bar{A}$ closed set in X, then $F(\bar{A})$ is a B^*c - closed in Y.

Then $\overline{F(A)}^{B^*C} \subseteq \overline{F(\bar{A})}^{B^*C} = F(\bar{A})$.

Conversely

Let A be closed in X, then $A = \bar{A}$ Since $\overline{F(A)}^{B^*C} \subseteq F(\bar{A}) = F(A)$, then $\overline{F(A)}^{B^*C} \subseteq F(A)$. But $F(A) \subseteq \overline{F(A)}^{B^*C}$, then $F(A) = \overline{F(A)}^{B^*C}$. There fore F(A) is a B^*c - closed set in Y. Hence F is a B^*c - closed.

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حول خصائص المجموعة B^*c– open set

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المستخلص:

في هذا البحث قدمنا صنف جديد من المجموعات يسمى B^*c– open set تم دراسته والتعرف على خواص وايجاد العلاقات مع المجموعات الاخرى ودراسة صنف جديد من الدوال يسمى B^*c– continuous ، B^*c– open function، B^*c– closed function. حيث حصلنا على بعض النتائج التي تظهر العلاقة بين المجموعات من خلال النظريات التي تم الحصول عليها باستخدام المجموعة من النمط B^*c– open.