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Property of oscillation of first order impulsive neutral differential equations with positive and negative coefficients

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ABSTRACT.

In this paper, necessary and sufficient conditions for oscillation are obtained, so that every solution of the linear impulsive neutral differential equation with variable delays and variable coefficients oscillates. Most of authors who study the oscillatory criteria of impulsive neutral differential equations, investigate the case of constant delays and variable coefficients. However the points of impulsive in this paper are more general. An illustrate example is given to demonstrate our claim and explain the results.

Keywords: Oscillation, Impulses effect, Neutral differential equations.

Mathematics Subject Classification: 34K11.

1. INTRODUCTION

The investigation in the theory of impulsive differential equations is now not only wider than the theory of differential equations without impulses effect, but it describes many phenomena and processes more reality so it has a lot of applications in many natural and industrial fields to study different characters and it can used as a tool in mathematical models for instance, in medicine [1], control theory [2], population dynamics [3], in neural networks [4] and etc. In fact, many evolution processes are often developed for immediate perturbations and sudden changes in specific moments of time such as in biological system in heart beats. This period of change is very small compared to the periods of operation, therefore the situation is quite different from what it is in differential equations without impulses in many physical phenomena, and it appears as a sudden change in its state. The consideration of oscillatory solutions for impulsive neutral differential equations is a new and wide object to find the qualitative properties. There is a lot of research and monographs that deal with the conditions to guarantee the oscillation of all solutions the impulsive neutral differential equations with coefficients such as variable coefficients and constant delays see [5, 6]. There are some results of oscillation for this type of equations [7-12] and we noted that the search of oscillation with impulsive neutral differential equations is more difficult than the type without it. Shen etc. al [13-17] obtained sufficient conditions for oscillation of all solutions of first order impulsive neutral differential equation of constant delay with positive and negative coefficients are obtained. Consider neutral differential equation:

$$\left.\begin{aligned}&[y(t)-P(t)y(\tau(t))]'+Q(t)y(\sigma(t))-R(t)y(\alpha(t))=0,\\ &\qquad t\neq t_k \quad k=1,2,\dots\\ &y(t_k^+)+b_k y(t_k)=a_k\, y(t_k),\quad t=t_k \qquad k=1,2,\dots\end{aligned}\right\}\quad(1.1)$$

Where $P\in PC([t_0,\infty);R^+)$ and $Q,R,\in C([t_0,\infty);R^+)$, and $\tau(t),\alpha(t),\sigma(t)\in C([t_0,\infty);R)$, $\lim_{t\to\infty}\tau(t)=\infty, \lim_{t\to\infty}\alpha(t)=\infty, \lim_{t\to\infty}\sigma(t)=\infty$ where τ,α,σ are increasing functions. The functions $\tau^{-1}(t),\sigma^{-1}(t),\alpha^{-1}(t)$ are the inverse of the functions $\tau(t),\alpha(t),\sigma(t)$ respectively.

2. SOME BASIC LEMMAS

The following lemmas will be useful in the proof of our main results: Throughout the paper we assume that $\tau(t),\alpha(t),\sigma(t)<t$, for $t\in(t_k,t_{k+1}], t_1\geq t_0, k=1,2,\dots$

Lemma 2.1. [٨] Suppose that $g,h\in C(R^+,R^+)$, $h(t)<t$ for $t\geq t_0$, $\lim_{t\to\infty}h(t)=\infty$ and

$$\liminf_{t\to\infty}\int_{h(t)}^{t} g(s)ds>\frac{1}{e} \qquad (2.1)$$

then the inequality $y'(t)+g(t)y(h(t))\leq 0$ has no eventually positive solutions.

Lemma 2.2. Let $y(t)$ be an eventually positive solution of equation (1.1) and there exists a continuous function $\delta(t)$ such that:

$$W(t)=y(t)-P(t)y(\tau(t))-\int_{\alpha^{-1}(\delta(t))}^{t}R(u)y(\alpha(u))du-\int_{t}^{\sigma^{-1}(\delta(t))}Q(u)y(\sigma(u))du \qquad (2.2)$$

Where $\delta(t)<t$ and $t\in(t_k,t_{k+1}]$, $0<t_0<t_1<\dots<t_k\to\infty$ as $k\to\infty$.

Also $\alpha^{-1}(\delta(t))<t$ and $\sigma^{-1}(\delta(t))>t$, in addition to the following assumptions:

***H*1**: $Q\left(\sigma^{-1}(\delta(t))\right)(\sigma^{-1}(\delta(t)))'-R\left(\alpha^{-1}(\delta(t))\right)\left(\alpha^{-1}(\delta(t))\right)'\geq 0,$

$t\in(t_k,t_{k+1}], \quad k=1,2,\dots$

***H*2**: There exists two positive real numbers a_k and b_k such that

$0<a_k-b_k\leq 1$, $k=1,2,\dots$ And

$$\begin{cases}P(t_k^+)\geq(a_k-b_k)P(t_k) \text{ for } \tau(t_k)\neq t_i, i<k,\\ P(t_k^+)\geq\dfrac{1}{a_k-b_k}P(t_k) \text{ for } \tau(t_k)=t_i, i<k,\end{cases}$$

Where $a_k=a_i, b_k=b_i$ when $\tau(t_k)=t_i, i<k$

H3: $\limsup_{t\to\infty}\left[P(t)+\int_{\alpha^{-1}(\delta(t))}^{t}R(u)du+\int_{t}^{\sigma^{-1}(\delta(t))}Q(u)du\right]\leq 1,\quad t\in(t_k,t_{k+1}].$

Then $W(t)$ is eventually positive and nonincreasing function.

Proof. Let $y(t)$ be an eventually positive solution of equation(1.1) that is $y(t) > 0, y(\tau(t)) > 0$, $y(\sigma(t)) > 0$ and $y(\alpha(t)) > 0$, $t \ge t_0$,
Differentiate (2.2) for every interval $(t_k, t_{k+1}]$ where $k = 1,2,\dots$
and use (1.1) we get

$$\begin{aligned}
W'(t) &= [y(t) - P(t)y(\tau(t))]' - R(t)y(\alpha(t)) \\
&+ R(\alpha^{-1}(\delta(t)))y(\delta(t))(\alpha^{-1}(\delta(t)))' \\
&- Q(\sigma^{-1}(\delta(t)))y(\delta(t))(\sigma^{-1}(\delta(t)))' \\
&+ Q(t)y(\sigma(t)) \\
&= -Q(t)y(\sigma(t)) + R(t)y(\alpha(t)) \\
&-R(t)y(\alpha(t)) \\
&+ R(\alpha^{-1}(\delta(t)))y(\delta(t))(\alpha^{-1}(\delta(t)))' \\
&- Q(\sigma^{-1}(\delta(t)))y(\delta(t))(\sigma^{-1}(\delta(t)))' \\
&+ Q(t)y(\sigma(t)) \\
&= -\Big[Q\left(\sigma^{-1}(\delta(t))\right)(\sigma^{-1}(\delta(t)))' \\
&- R\left(\alpha^{-1}(\delta(t))\right)\left(\alpha^{-1}(\delta(t))\right)'\Big]y(\delta(t)) \\
&\le 0 \quad (2.3)
\end{aligned}$$

Hence $W(t)$ is nonincreasing function on each $t \in (t_k, t_{k+1}]$ for $k = 1,2,\dots$
To prove that $W(t_k^+) \le W(t_k)$ for $k = 1,2,\dots$ In view of $0 < a_k - b_k \le 1$, $k = 1,2,\dots$, we have $0 < a_k - b_k \le 1$ and from (2.2) with regard to the condition $\boldsymbol{H2}$ when $\tau(t_k) = t_i$, $i < k$, then:

$$\begin{aligned}
W(t_k^+) &= (a_k - b_k)y(t_k) - P(t_k^+)(a_k - b_k)y(\tau(t_k)) \\
&- \int_{\alpha^{-1}(\delta(t_k))}^{t_k} R(u)y(\alpha(u))du \\
&- \int_{t_k}^{\sigma^{-1}(\delta(t_k))} Q(u)y(\sigma(u))du
\end{aligned}$$

$$\le y(t_k) - P(t_k)y(\tau(t_k)) - \int_{\alpha^{-1}(\delta(t_k))}^{t_k} R(u)y(\alpha(u))du - \int_{t_k}^{\sigma^{-1}(\delta(t_k))} Q(u)y(\sigma(u))du = W(t_k)$$

When $\tau(t_k) \ne t_i$, $i < k$ then from (2.2) with regard to the condition $\boldsymbol{H2}$:

$$\begin{aligned}
W(t_k^+) &= (a_k - b_k)y(t_k) - P(t_k^+)y(\tau(t_k)) \\
&- \int_{\alpha^{-1}(\delta(t_k))}^{t_k} R(u)y(\alpha(u))du \\
&- \int_{t_k}^{\sigma^{-1}(\delta(t_k))} Q(u)y(\sigma(u))du \\
&\le (a_k - b_k)y(t_k) - (a_k - b_k)P(t_k)y(\tau(t_k)) \\
&-(a_k - b_k)\int_{\alpha^{-1}(\delta(t_k))}^{t_k} R(u)y(\alpha(u))du \\
&-(a_k - b_k)\int_{t_k}^{\sigma^{-1}(\delta(t_k))} Q(u)y(\sigma(u))du \\
&= (a_k - b_k)W(t_k) \\
&\le W(t_k) \qquad (2.4)
\end{aligned}$$

$W(t)$is non-increasing on $[t_0, \infty)$.Hence $-\infty \le L < \infty$. Where $|L| = sup\{W(t_k^+), lim_{k\to\infty} W(t_k)\}$, $t \in [t_l, \infty)$for some $l \ge t_0$. We claim that $W(t_k) \ge 0$ for k=$l, l+1, \dots$. Otherwise there exists some $m \ge l$ such that:
$W(t_m) = -\mu < 0$, implies that $W(t) \le -\mu$ for $t \ge t_m$, since $W(t)$ is non-increasing on $[t_l, \infty)$,then for each $t \in (t_k, t_{k+1}]$, k $= l, l+1, \dots$ we get from (2.2):

$$y(t) \le -\mu + P(t)y(\tau(t)) + \int_{\alpha^{-1}(\delta(t))}^{t} R(u)y(\alpha(u))du + \int_{t}^{\sigma^{-1}(\delta(t))} Q(u)y(\sigma(u))du. \qquad (2.5)$$

So, we have two cases to consider:
Case 1. If $y(t)$ is unbounded then there exists a sequence of points $\{s_n\}$ such that

$$s_n \ge t_m, \lim_{n\to\infty} y(s_n) = \infty \text{ and}$$
$$y(s_n) = \max\{y(t), t_m \le t \le s_n\}.$$

Then (2.5) reduce to:

$$\begin{aligned}
y(s_n) &\le -\mu + P(s_n)y(\tau(s_n)) \\
&+ \int_{\alpha^{-1}(\delta(s_n))}^{s_n} R(u)y(\alpha(u))du \\
&+ \int_{s_n}^{\sigma^{-1}(\delta(s_n))} Q(u)y(\sigma(u))du. \\
&\le -\mu + \{P(s_n) + \int_{\alpha^{-1}(\delta(s_n))}^{s_n} R(u)du \\
&+ \int_{s_n}^{\sigma^{-1}(\delta(s_n))} Q(u)du\}y(s_n) \\
&\le -\mu + y(s_n)
\end{aligned}$$

Leads to $0 \le -\mu$ which is a contradiction.
Case 2. If $y(t)$ is bounded that is $\limsup_{t\to\infty} y(t) = \mathrm{M} < \infty$.We can choose a sequence of points $\{s_n\}$ such that

$$\lim_{n\to\infty} y(s_n) = \mathrm{M} \text{ and } y(\eta(s_n)) = \max\{\mathrm{y(t)}: \rho_1(s_n) \le t \le \rho_2(s_n)\}.$$

Where $\rho_1(s_n) = min\{\tau(s_n), \sigma(s_n)\}$,
$\rho_2(s_n) = max\{\tau(s_n), \alpha(s_n)\}$ it is obvious that
$\lim_{n\to\infty} \eta(s_n) = \infty$ and $\limsup_{t\to\infty} y(\eta(s_n)) \le M$.

$$y(s_n) \le -\mu + P(s_n)y(\tau(s_n)) + \int_{\alpha^{-1}(\delta(s_n))}^{s_n} R(u)y(\alpha(u))du + \int_{s_n}^{\sigma^{-1}(\delta(s_n))} Q(u)y(\sigma(u))du$$
$$\le -\mu + \{P(s_n) + \int_{\alpha^{-1}(\delta(s_n))}^{s_n} R(u)du + \int_{s_n}^{\sigma^{-1}(\delta(s_n))} Q(u)du\}y(\eta(s_n))$$
$$\le -\mu + y(\eta(s_n))$$

Taking the superior limit as $n \to \infty$, we get
$M \le -\mu + M$, which is also a contradiction.
Combining the cases 1 and 2, we see that $W(t) \ge 0$ for $t \in (t_k, t_{k+1}]$, k=$l, l+1, \ldots$.
Since $W(t)$ is nonincreasing, so $W(t_k) \ge W(t) \ge 0$ for $t \in (t_k, t_{k+1}]$.
To prove $W(t) > 0$, we first prove that $W(t_k) > 0$ for $k = 1,2, \ldots$. If it is not true, then there exists some $m \ge 0$ such that $W(t_m) = 0$, integrating (2.3) on $(t_m, t_{m+1}]$ yield:

$$W(t_{m+1}) = W(t_m^+) - \int_{t_m}^{t_{m+1}} [Q(\sigma^{-1}(\delta(t)))(\sigma^{-1}(\delta(t)))' - R(\alpha^{-1}(\delta(t)))(\alpha^{-1}(\delta(t)))']y(\delta(t))\,dt < W(t_m^+) \le W(t_m) = 0$$

This contradiction shows that $W(t_k) > 0$ for $k = 1,2 \ldots$, as well as $W(t) \ge W(t_{k+1}) > 0$, for $t \in (t_k, t_{k+1}]$, $k = 1,2, \ldots$. Thus $W(t) > 0$ for $t \ge t_0$. The proof is complete. ■

3. MAIN RESULTS

The next results provide sufficient conditions for the oscillation of all solutions of (1.1):

Theorem 3.1. Let $W(t)$ defined as in (2.2) and the assumptions ***H1 − H3*** hold, and there exist a continuous function $\delta(t) < t$ such that

$$\liminf_{t\to\infty} \int_{\delta(t)}^{t} [Q(\sigma^{-1}(\delta(s)))(\sigma^{-1}(\delta(s)))' - R(\alpha^{-1}(\delta(s)))(\alpha^{-1}(\delta(s)))'][1 + P(\delta(s)) + \int_{\alpha^{-1}(\delta(\delta(s)))}^{\delta(s)} R(u)du + \int_{\delta(s)}^{\sigma^{-1}(\delta(\delta(s)))} Q(u)du]\,ds > \frac{1}{e} \qquad (3.1)$$

Where $\alpha^{-1}(\delta(t)) < t$ and $\sigma^{-1}(\delta(t)) > t$, then every solution of equation (1.1) oscillates.

Proof. Suppose that $y(t)$ is eventually positive solution of (1.1) then by Lemma 2.2 it follows that $W(t)$ is positive nonincreasing function for $t \in (t_k, t_{k+1}]$, k=1,2, …, since $W(t) \le y(t)$, hence it follows from (2.2):

$$y(t) = W(t) + P(t)y(\tau(t)) + \int_{\alpha^{-1}(\delta(t))}^{t} R(u)y(\alpha(u))du + \int_{t}^{\sigma^{-1}(\delta(t))} Q(u)y(\sigma(u))du$$
$$\ge W(t) + P(t)W(\tau(t)) + \int_{\alpha^{-1}(\delta(t))}^{t} R(u)W(\alpha(u))du + \int_{t}^{\sigma^{-1}(\delta(t))} Q(u)W(\sigma(u))du$$
$$\ge W(t) + P(t)W(\tau(t)) + W(\alpha(t))\int_{\alpha^{-1}(\delta(t))}^{t} R(u)du + W(\delta(t))\int_{t}^{\sigma^{-1}(\delta(t))} Q(u)du$$
$$\ge W(t) + P(t)W(t) + W(t)\int_{\alpha^{-1}(\delta(t))}^{t} R(u)du + W(t)\int_{t}^{\sigma^{-1}(\delta(t))} Q(u)du$$
$$= W(t)\,[1 + P(t) + \int_{\alpha^{-1}(\delta(t))}^{t} R(u)du + \int_{t}^{\sigma^{-1}(\delta(t))} Q(u)du]$$

$$y(\delta(t)) \geq W(\delta(t))[1 + P(\delta(t)) + \int_{\alpha^{-1}(\delta(\delta(t)))}^{\delta(t)} R(u)du + \int_{\delta(t)}^{\sigma^{-1}(\delta(\delta(t)))} Q(u)du] \quad (3.2)$$

Substituting the last inequality (3.2) in (2.3) we get

$$W'(t) \leq -[Q\left(\sigma^{-1}(\delta(t))\right)(\sigma^{-1}(\delta(t)))' - R\left(\alpha^{-1}(\delta(t))\right)\left(\alpha^{-1}(\delta(t))\right)'][1 + P(\delta(t)) + \int_{\alpha^{-1}(\delta(\delta(t)))}^{\delta(t)} R(u)du + \int_{\delta(t)}^{\sigma^{-1}(\delta(\delta(t)))} Q(u)du]\ W(\delta(t))$$

$$W'(t) + [Q\left(\sigma^{-1}(\delta(t))\right)(\sigma^{-1}(\delta(t)))' - R\left(\alpha^{-1}(\delta(t))\right)\left(\alpha^{-1}(\delta(t))\right)'][1 + P(\delta(t)) + \int_{\alpha^{-1}(\delta(\delta(t)))}^{\delta(t)} R(u)du + \int_{\delta(t)}^{\sigma^{-1}(\delta(\delta(t)))} Q(u)du]\ W(\delta(t)) \leq 0, \quad (3.3)$$

By Lemma 2.1, and the condition (3.1) the last inequality cannot has eventually positive solution, which is a contradiction. The proof is complete. ∎

Corollary 3.2: Let $W(t)$ defined as in (2.2) and the assumptions H1-H3 hold, and there exist a continuous function $\delta(t) < t$ such that

$$[Q(\sigma^{-1}(\delta(t)))(\sigma^{-1}(\delta(t)))' - R(\alpha^{-1}(\delta(t)))(\alpha^{-1}(\delta(t)))'] \geq \frac{1}{e\ \min_{t\geq t_0}\{t-\delta(t)\}}. \quad (3.4)$$

Where $\alpha^{-1}(\delta(t)) < t$ and $\sigma^{-1}(\delta(t)) > t$, then every solution of equation (1.1) oscillates.

Proof. It is obvious that condition (3.4) implies that

$$\int_{\delta(t)}^{t} [Q(\sigma^{-1}(\delta(s)))(\sigma^{-1}(\delta(s)))' - R(\alpha^{-1}(\delta(s)))(\alpha^{-1}(\delta(s)))']ds \geq \frac{1}{e\ \min_{t\geq t_0}\{t-\delta(t)\}}(t - \delta(t)) \geq \frac{1}{e}.$$

Which leads to condition (3.1) holds. Hence according to Theorem 3.1 every solution of (1.1) oscillates.

4. EXAMPLE

In this section we give an example to illustrate the obtained results.

Example 4.1. Consider the impulsive neutral differential equation:

$$\left[y(t) - \frac{1}{9}(1 + e^{-t})y(t - 2\pi)\right]' + \frac{1}{9}(8 - e^{-t})y\left(t - \frac{5\pi}{2}\right) - \frac{1}{9}e^{-t}y(t - 2\pi) = 0,$$

$t \neq t_k$ and $k = 1,2,\ldots$

$$y(t_k^+) + b_k y(t_k) = a_k\, y(t_k), \quad t = t_k \text{ and } k = 1,2,\ldots \quad (4.1)$$

Where $a_k = \frac{2k}{k+1}$ and $b_k = \frac{k}{k+1}$ we can see that

$$a_k - b_k = \frac{2k}{k+1} - \frac{k}{k+1} = \frac{k}{k+1} < 1$$

Let $P(t) = \begin{cases} \left(\frac{1}{9} + \frac{1}{9}e^{-t}\right), & t \neq t_k \\ \frac{1}{20k}, & t = t_k \end{cases}$

Let $\delta(t) = t - \frac{9\pi}{4}$, $\sigma^{-1}(\delta(t)) = t + \frac{\pi}{4}$ and $\alpha^{-1}(\delta(t)) = t - \frac{\pi}{4}$ to see condition ***H1***

$$Q\left(\sigma^{-1}(\delta(t))\right)(\sigma^{-1}(\delta(t)))' - R\left(\alpha^{-1}(\delta(t))\right)\left(\alpha^{-1}(\delta(t))\right)' = \frac{1}{9}\left(8 - e^{-t-\frac{\pi}{4}}\right) - \frac{1}{9}e^{-t+\frac{\pi}{4}} \geq 0.5945.$$

Let $t_k = k$, $P(t_k^+) = P(k^+) = \lim_{t\to k^+} P(t) = \lim_{t\to k^+}\left(\frac{1}{9} + \frac{1}{9}e^{-t}\right) = \left(\frac{1}{9} + \frac{1}{9}e^{-k}\right) \geq \frac{1}{9}$.

$$(a_k - b_k)P(t_k) = \frac{k}{k+1}P(k) = \frac{k}{k+1}\frac{1}{20k} = \frac{1}{20(k+1)} \leq 0.025, \text{ and}$$

$$\frac{1}{(a_k - b_k)}P(t_k) = \frac{k+1}{k}P(k) = \frac{k+1}{k}\frac{1}{20k} = \frac{k+1}{20k^2} \leq 0.1,$$

so ***H2*** holds.

And the condition ***H3***leads to

$$\liminf_{t\to\infty} \int_{\delta(t)}^{t} [Q\left(\sigma^{-1}(\delta(\xi))\right)(\sigma^{-1}(\delta(\xi)))' - R\left(\alpha^{-1}(\delta(\xi))\right)\left(\alpha^{-1}(\delta(\xi))\right)'][1 + P(\delta(\xi)) + \int_{\alpha^{-1}(\delta(\delta(\xi)))}^{t} R(u)du + \int_{\delta(\xi)}^{\sigma^{-1}(\delta(\delta(\xi)))} Q(u)du]d\xi$$

$$= \frac{1}{9}\lim_{t\to\infty} \int_{t-\frac{9\pi}{4}}^{t} \left[8 - e^{-\xi-\frac{9\pi}{4}}\right][1 + \frac{1}{9} + \frac{1}{9}e^{-\xi+\frac{9\pi}{4}} + \frac{1}{9}\int_{\xi-\frac{10\pi}{4}}^{\xi-\frac{9\pi}{4}} e^{-u}du + \frac{1}{9}\int_{\xi-\frac{9\pi}{4}}^{\xi-\frac{8\pi}{4}} (8 - e^{-u})du]\, d\xi$$

$$= 11.3662 > \frac{1}{e}$$

Hence all conditions of theorem 3.1 hold, so all solutions of equation (3.1) are oscillatory. For instance the solution $y(t) = \begin{cases} sint, & t \neq t_k \\ 2 + \frac{1}{k}, & t = t_k \end{cases}$ is such a solution.

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خاصية التذبذب لمعادلات تفاضلية محايدة متسارعة من الرتبة الاولى ذات المعاملات الموجبة والسالبة

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قسم الرياضيات

المستخلص:

في هذا البحث حصلنا على الشروط الضرورية والكافية للتذبذب، ذلك أنه كل حل لمعادلات تفاضلية محايدة متسارعة خطية ذات تباطؤات متغيرة ومعاملات متغيرة يتذبذب.إن أغلب المؤلفين والباحثين الذين درسوا ظاهرة التذبذب لمعادلات تفاضلية محايدة متسارعه كانوا قد اجروا دراسة لمعادلات ذات تباطؤات ثابتة ومعاملات ثابتة. على اية حال، إن نقاط التسارع هنا أكثر عمومية. لقد قدمنا مثال توضيحي ليبرهن صحة النتائج.