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Topological study via σ-algebra

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Abstract:

in this research we want to study some of theTopological definitions and theorems by using the concept of σ -algebra. In this paper we define the continuity and separation axioms with respect to this concept .

Key wards :algebra ,σ -algebra , supra topology , supra continuous function , continuous σ -algebra.

Mathematics subject classification : 54XX.

Introduction:

An algebra Ƹ of sets is called a σ – algebra(Borel field) if every union of a countable collection of sets in Ƹ is again in Ƹ[3]

A σ-algebra (σ-field) in mathematical analysis and in Probability theory on a set X is a collection of subsets of X that is closed under countable- set operations (intersection of accountably many sets, union of accountably many sets and complement).

Algebra required only to be closed under finitary set operations. That is, a σ-algebra is an algebra of sets, include accountably infinite operations as complete.

In mathematical analysis this concept is important as the foundation for Lebesgue integration, and in probability theory, where assigned probabilities forthe collection of events.

In1983 A.S.Mashhour introduced the concepts of supra topological spaces , and he defined continuity and separation axioms and discussed some properties about the new space.

Our study In this paper includes anew relation between the concept of topological space and the σ-algebra and for that we will define the interior set and closure set and continuity with respect to σ-algebra.

1-preliminaries:

Definition [1-1][2] :

let X be anon empty set an acollection Ƹ of subset of the set X is said to be algebra(Boolean algebra) in X if the following holds :

a- Ƹ is closed under complementation
b- Ƹ is closed under finite union

Definition 2-1[3]:

let X be a non-empty set an algebra Ƹ in X is called σ –algebra(Borel field) in X if the following holds :

a. Ƹ closed under complementation
b. Ƹ closed under countable union

Remark 1-3

Everyσ -algebrais analgebra[2]

Lemma 1-4

letƸ is σ -algebra in X and $f: X \rightarrow Y$ (y≠ ∅) is onto ,
Then ƸY={ET_Y: $f^{-1}(E)$ ∈Ƹ} is σ -algebra.[2]

Definition 1-5:

A subfamily H of X is said to be a supra topology on Xif:

a. X, Ø ∈ H
b. if A_i∈ H for all i∈ J, then ∪A_i∈ H

(X, H) is called a supra topological space. The elements of H are calledSupra open sets in (X, H) and complement of a supra open set is called a supra closed set. [1,5,4]

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.Definition 1-6 :
let (X,T) and (Y,V) are two topological spaces and Ω is a supra topology such that T⊆Ω then a function f: (X,T) →(Y,V) is supra continuous iff the inverse image of each open set is supra open set .[6]

2-Main result:

In this research we will construct topology , supra topology and б-Algebra bya base ofthe space X as we show in the following example

Example 2-1
By the base {X, {a},{b}} of a space X={a,b,c} we can **construct :**
Topology={Ø,X,{a},{b},{a,b}}
σ -algebra={Ø,X,{a},{b},{b,c},{a,c},{c},{a,b}}
Supra topology={Ø,X,{a},{b},{a,b}}

Remark 2-2 :

1- Clearly that every open set in the topology which constructed is σ-set.
2- In this article every subset of the σ -algebra are called σ -set
3- Every supra open set is σ -set

Definition 2-3:
Let T is atopology constructed by a base of a topological space X then
$int_{\sigma\text{-algebra}}(G)=\cup\{N: N \text{ is } T-\text{open set}, N \sqsubseteq G\}$
$cl_{\sigma\text{-algebra}}(G)=\cap\{F: F \text{ is } T-\text{closed set}, G \sqsubseteq F\}$
Remark 2-4
clearly that $int_{\sigma\text{-algebra}}$ is T-open set and $cl_{\sigma\text{-algebra}}$ is T-closed set.

3-Continuous σ -algebra function

Definition 3-1
let X ,Y are two non-empty sets and Ƹ , Ʊ are two σ -algebra in X ,Y respectively then afunction f:(X,Ƹ) →(Y,Ʊ) is said to be continuousσ -algebra function if and only if the inverse image of each open set in Y is σ -algebra set in X.

Example 3-2
let Ƹ is σ -algebra and T is atopological space and let $f:(X,\xi)\to(X,T)$ is the constant function then f is continuous σ -algebra.
Theorem 3-3[1]:
Every continuousfunction issupra continuous function
Remark 3-4

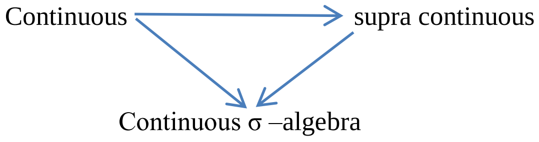
1- if the function$f$:(X,T)→(Y,V) is continuous then$f$:(X,Ƹ)→(Y,Ʊ) is continuousσ -algebra .
Proof:
let U is V-open set in Y , since f is continuous then $f^{-1}(U)$ is open set in X and by remark (2) $f^{-1}(U)$ is б-set , there for f is continuous σ -algebra.
2- let$f$:(X,Ƹ)→(Y,Ʊ) is continuous σ -algebra and g:(X,T)→(Y,V) is continuous then gof is continuous σ -algebra.

Proof: obvious
3- let$\mathcal{M},\mathcal{F}$ are two supra topological space , then if $f:(X,\mathcal{M})\to(Y,\mathcal{F})$ is supra continuous then f is continuous σ -algebra
Proof: obvious

Remark 3-5
The composition of two continuousσ –algebra functionsis not necessary continuousσ -algebra.

Theorem 3-6 :
let $f:(X,\xi)\to(Y,\Psi)$ where T,H are topologies and Ƹ , Ψ are σ -algebra constructed on X, Y respectively is a function then f is continuous σ -algebra if and only if $f^{-1}(V)\subset int_{\sigma-algebra}(f^{-1}(cl_T(V))$ for every open set V in H.
Proof:
now assume that f is continuous σ -algebra , and V is H-open set , then $x\in f^{-1}(V)$, since f is continuous σ -algebra there exist σ -set G in X such that f(G)⊂ $cl_T(V)$ *and then* $x\in G\subset f^{-1}(cl_T(V))$ there for we get that $x\in int_{\sigma-algebra}(f^{-1}(cl_T(V))$ then we get that
$f^{-1}(V)\subset int_{\sigma-algebra}(f^{-1}(cl_T(V))$.
Now we assume that
$f^{-1}(V)\subset int_{\sigma-algebra}(f^{-1}(cl_T(V))$, and V is H-open set in Y , since $int_{\sigma-agebra}(f^{-1}(cl_T(V))\subset f^{-1}(V)$ and by assume we have
$f^{-1}(V)\subset int_{\sigma-algebra}(f^{-1}(cl_T(V))$ we get that
$f^{-1}(V)=int_{\sigma-algebra}(f^{-1}(cl_T(V))$from that we get
$f^{-1}(V)$ is T-open set and then its σ -set , there for f is continuous σ -algebra

The following diagram give us all the above relation :



4- Separation axiom :

Definition 4-1

Let Ƶ is σ -algebra and T is a topological space then the space (X,Ƶ) is called:

1- σ -algebra T_0-space if for every two distinct points ofX , there exist a σ -set of one of them .

2- σ -algebra T_1-space if for every two disjoint points x and y in X there exist two σ -sets G,H such that x$\in G , y \notin G, y \in H, x \notin H$

3- σ -algebra T_2-space if for every two distinct points x and y in X there exist two disjoint σ -sets U,V such that $x \in U, y \in V$.

4- σ -algebra regular space if there exist T-closed set F and appoint x in the space X such that x∉F , there exist two σ -sets U,V such that F$\subset U$ and x∈V , U$\cap V = \phi$.

(X,Ƶ) is called σ -algebra T3-space if (X,Ƶ) is T_1-space and σ -algebra regular space.

5- σ -algebra normal space if for each two T-closed sets F,H such that F$\cap H = \emptyset$ there exist two σ -sets U,V such that F$\subset U, H \subset V,\ U \cap V = \emptyset$.

(X,Ƶ) is called σ -algebra T_4 –space if (X,Ƶ) is T_1-space and σ -algebra normal space.

Theorem 4-2 [1] :

1- Let (X,T) is a topological space thenevery T_2-space is supra T_2-space.
2- Let (X,T) be a topological space then every T1-space is supra T1-space.
3- Let (X,T) be a topological space then every T_0-space is supra T_0-space.

Theorem 4-3:

1-If (X,T) is T_i-topological space then (X,Ƶ) is σ -algebraT_i-space where i=0,1,2

Proof : exist by the definition and remark [3-2]point (1).

2- If(X,T) is T_3-topological space then (X,Ƶ) is σ -algebra T_3-space.

Proof :

Let F is closed set and y$\in X$ *such that* $y \notin F$, since X is regular space then there exist two T-open sets G,H such that $y \in G$, $F \subset H$, $G \cap H = \emptyset$. by remark [3-2] ,(1) we het that X is isσ -algebra T_3-space.

3- if (X,T) is T_4-topological space then (X,Ƶ) is σ -algebra T_4-space.

Proof :

Let F , M are closed sets in X such that $F \cap M = \emptyset$ and , since X is normal there exist two T-open sets G,H such that $F \subset G$, $M \subset H$, $G \cap H = \emptyset$. now by remark [3-2] (1) we get that (X,Ƶ) is σ-algebra T_4-space.

Theorem 4-4:

let (X,T) be a topological space then

1- everyσ -algebra T_3-space is σ -algebra T_2-space.

Proof:

Let x≠y in X , by assume X is T_1-space then {x} is closed set and $y \notin \{x\}$

And since X is regular there exist T-open sets G,H such that $G \cap H = \emptyset$ *and* $\{x\} \subset H$, $y \in G$ *and then* $x \in H$, $y \in G$, $G \cap H = \emptyset$. by remark [3-2] , (1) we get that (X,Ƶ) is σ -algebra T_2-space.

2-Everyσ -algebra T_4-space is σ -algebra T_3-space

Proof:

Let F is closed set and $y \in X$, *such that* $y \notin F$, now since X is T_1 –space then {y} is closed set and $\{y\} \cap F = \emptyset$, by assume X is б-Algebra normal space then there exist two σ -sets G,H such that $\{y\} \subset G, F \subset H$, $G \cap H = \emptyset$ *and then* $y \in G$, $F \subset H$, $G \cap H = \emptyset$ there for X is σ-algebra regular space and then X is σ -Algebra T_3-space.

3-Everyσ -algebra T_3-space is σ -algebra T_2-space

Proof:

Let $x \neq y$ are two points in X , since X is T_1-space then {x} is closed set and $y \notin \{x\}$

Since X is σ -regular space then there exist two σ -sets V, W such that $y \in V$, $\{x\} \subset W,\ V \cap W = \emptyset$ *and then* $x \in W, y \in V$, $V \cap W = \emptyset$, and then X is σ -algebra T_2-space .

4-Everyσ -algebra T_2-space is σ -algebra T_1-space

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Proof:exist bydefinition.

5-Everyσ -algebra T_1-space is σ -algebra T_0-space
Proof:exist by definition

6- Every T_3-space is σ -algebra T_2-space

Proof:

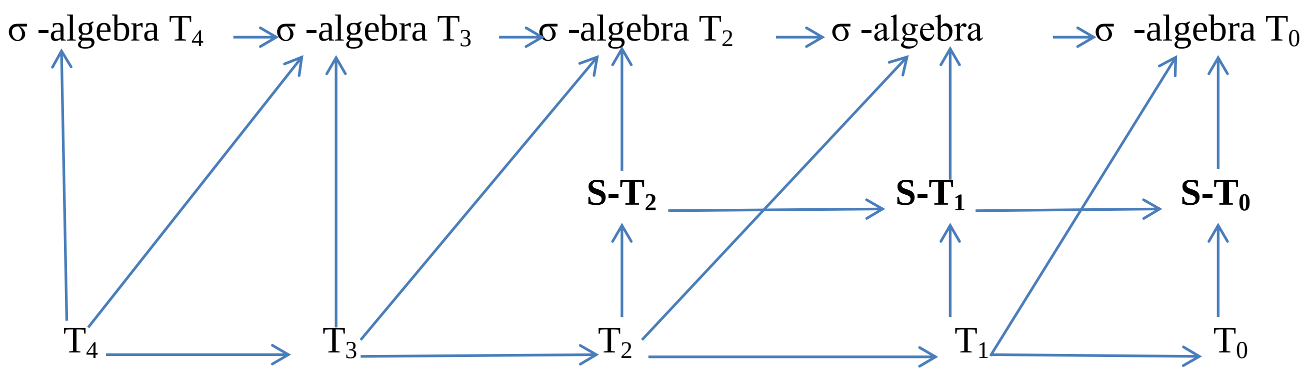
Let $x \neq y$ are two points in X , since X is T_1-space then {x} is closed set and $y \notin \{x\}$ .Since X is regular space then there exist two open set V, W such that $y \in V$,$\{x\} \subset W$, $V \cap W = \emptyset$ *and then* $x \in W, y \in V , V \cap W = \emptyset$, and by remark [3-2] (1) we get that X is σ -algebra T_2-space .

7-every T_4-space isσ -algebra T_3-space

Proof:

Let F is closed set and $y \in X$ *such that* $y \notin F$, now since X is T_1-space then {y} is closed set and $F \cap \{y\} = \emptyset$, since X is normal space then there exist two open sets G,H such that $F \subset G$,$\{y\} \subset H$,$G \cap H = \emptyset$ *and then* $y \in H$, by remark [3-2] (1) we that X is σ -algebra T_3-space

From the above we get the following diagram



Theorem 4-5:

(X,Ƹ) is σ -algebra normal space if and only if for any T-closed set F and T-open set G containing F there exist σ -set V such that F⊂V , cl_T(V)⊂G

Proof:

assume that (X,Ƹ) is σ -algebra normal space , since G is T-open set containing F then $F \cap G^c = \emptyset$ *and* G^c *is closed set*. Now since X is σ -algebra normal space there exist two σ-sets U,V such that $F \subset V , G^c \subset U , U \cap V = \emptyset$, from that we get $U^c \subset G$ *and* $V \subset U^c$.

Since U^c *is closed set then* $cl_T(V) \subset U^c$ from that we get the result $F \subset V , cl_T(V) \subset G$.

Now we assume that there exist T-closed set F and T-open set G containing F , then $F \cap G^c = \emptyset$, $G^c$is T-closed set , by assume there exist σ -set V *such that* $F \subset V$ *and* $G^c \subset (cl_T(V))^c$ and $(cl_T(V))^c$ is T-open set also $(cl_T(V))^c \cap V = \emptyset$ and then X is σ -algebra normal space .

Theorem 4-6

let $f: (X, \xi) \to (Y, V) is$ one-one onto continuous σ -algebra function such that Y is T_0-space then X isσ -algebra T_0-space.

Proof:

let x , m are two distinct points in X , then there exist two points y , r in Y such that $y = f(x)$, $r = f(m)$, since y$\neq r$ *and* Y is T_0-space then there exist open sets U *such that* $y \in U$,$r \notin U$ *or there exist* $V -$ *open set* H *such that* $y \notin H$,r ∈H . from that we get $f(x) \in U$ *then* $x \in f^{-1}(U)$, $m \notin f^{-1}(U)$*or* $f(m) \in H$ *then* $m \in f^{-1}(H)$,$x \notin f^{-1}(H)$, since f is

Continuousσ -algebra then $f^{-1}(U)$ and $f^{-1}(H)$ are σ -sets and then X is σ -algebra T_0-space .

Theorem 4-7:

let $f:(X,\xi)\rightarrow(Y,V) is$ one-one onto continuous б-Algebra function such that Y is T_1-space (T_2-space) then X is σ-algebra T_1-space(T_2-space) .

Proof: exist by definition

Theorem4-8:

let $f:(X,\xi)\rightarrow(Y,V) is$ one-one onto continuous σ -algebra function and closed mapping such that Y is T_3-space then X is σ -algebra T_3-space .

Proof:

Letx is any point in X and G is any T-closed set in X such that $x\notin G$.

Then there exist appoint y in Y such that y=f(x), now since f is closed mapping then f(G) is closed set in Y and f(x)∉f(G), and since Y is regular space there exist two disjoint points U,H such that $f(x)\in U,\ f(G)\subset H$, from that we get

$x\in f^{-1}(U)$, $G\subset f^{-1}(H)$, $f^{-1}(U)\ \cap f^{-1}(H)\ =\emptyset$, now since f is continuous σ -algebra function then $f^{-1}(U)$, $f^{-1}(H)$ are σ -sets and then X is σ -algebra regular space .

Theorem 4-9:

let $f:(X,\xi)\rightarrow(Y,V) is$ one-one onto continuous σ -algebra function and closed mapping such that Y is T_4-space then X is σ -algebra T_4-space.

Proof: the proof is similar to theorem (5-8)

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دراسة تبولوجية من خلال الجبر- σ

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المستخلص :

يتناول هذا البحث تعريف بعض المفاهيم التبولوجية مثل الاستمرارية وبديهيا ت الفصل باستخدام مجموعة الجبر- σ ايضا تم دراسة بعض النظريات والخواص التي يمكن الربط فيما بينها