Journal of AL-Qadisiyah for computer science and mathematics
Vol.8 No.1 Year 2016

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On Soft Generalized Continuous mappings

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Recived :19\4\2016 Revised : 29\5\2016 Accepted :6\6\2016

Abstract:

In this work ,we study the soft generalized continuous mapping in soft topological spaces and investigate the properties of the restriction , composition and product of this mappings. Also ,we define and investigate the properties of soft strongly generalized continuous and soft generalized irresolute mappings and explain a relationships among these mappings.

Keywords: soft set, soft topological spaces, soft generalized closed set, soft continuous mapping, soft strongly mapping, soft irresolute mapping.

Mathematics subject classification 54 A05 , 54 C08, 54 C10.

I. Introduction

In 1999, D. Molodtsov [1] introduced the soft set theory to solve complicated problems in economics ,engineering, environment, sociology, medical science, etc. He has shown several applications of this theory in solving many practical problems. In 2011, Shabir and Naz[4] introduce the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. They studied some basic concepts of soft topological spaces also some related concepts such as soft interior, soft closure, soft subspace and soft separation axioms . In 2011 Aygunoglu and Aygun [9] introduced soft product topology. Some other studies on soft topological spaces can be listed as [3, 5,4]. More recently ,Kannan [8] defined soft generalized closed and open sets in soft topological spaces. He studied some of their properties. Also, he showed that every soft closed set is soft generalized closed. We use T_{sind} and T_{sd} to denote the indiscrete and discrete soft topology (respectively).

In this work ,we study the concept of soft generalized continuous mapping which introduction in [14] and study the properties of this concept in soft topological spaces. And we introduce the definitions of soft strongly generalized continuous and soft generalized irresolute mappings and study the properties of them. Also, we introduce the relation among these types of soft generalized continuous mapping, soft strongly generalized continuous mapping and soft generalized irresolute mapping .

II. Basic Definitions and Notations.

In this section ,we give some basic definition ,properties and theorems of soft sets.

Definition1.1.[1] Let X be an initial universe set and E be a set of parameters. A pair (F, E) is called a soft set over X if only if F is a mapping from E into the set of all subsets of the set X, i.e. $F:E \rightarrow P(X)$, where P(X) is the power set of X .Note that the set of all soft set over X is denoted by SS(X,E).

Definition 1.2.[1] Let (F,E),(G,E) ∈ SS(X,E) .We say that the pair (F,E) is a soft subset of (G,E) if F(e)⊆ G(e) , for every e ∈ E. Symbolically, we write (F,E) $\tilde{\subseteq}$ (G,E). Also, we say that the pairs (F,E) and (G,E) are soft equal if (F,E) $\tilde{\subseteq}$ (G,E) and (G,E) $\tilde{\subseteq}$ (F,E). Symbolically, we write (F,E) = (G,E).

Definition 1.3.[1] A soft set (F,E) over X is said to be

i. null soft set denoted by $\tilde{\Phi}$ if ∀ e ∈ E, F(e) = ∅.

ii. absolute soft set denoted by $\tilde{X}$, if ∀ e ∈ E, F(e) = X.

Definition 1.4.[2]Let A be a non-empty subset of X, then $\tilde{A}$ denotes the soft set (A,E) over X for which A(e)=A ; for all e ∈ E. In particular (X, E) will be denoted by $\tilde{X}$.

Definition 1.5.[1] Let I be an arbitrary index set and {(F_i ,E) : i ∈ I} ⊆ SS(X,E). The soft union of these soft sets is the soft set(F,E)∈ SS(X,E),where the map F :E→ P(X) defined as follows: F(e) = ∪{ $F_i(e)$: i ∈ I}, for every e ∈ E. Symbolically, we write (F,E) = $\tilde{\cup}$\{(F_i ,E) : i ∈ I}.

Definition 1.6.[1] Let I be an arbitrary index set and {(F_i ,E) : i ∈ I} ⊆ SS(X,E). The soft union of these soft sets is the soft set(F,E) ∈ SS(X,E), where the map F : E→ P(X) defined as follows: F(e) = ∪{ $F_i(e)$: i ∈ I}, for every e ∈ E. Symbolically, we write (F,E) = $\tilde{\cup}$\{(F_i ,E) : i ∈ I}.

Definition1.7.[3] The difference (H, E) of two soft sets (F, E) and (G, E) over X, denoted by (H, E) = (F, E) \ (G, E); is defined by H(e) = F(e) \ G(e) for all e ∈ E.

Definition1.8.[3] The complement of a soft set (F, E), denoted by $(F, E)^c$, is defined by $(F, E)^c = (F^c, E)$, F^c: E → P(X) is a mapping given by $F^c(e) = X - F(e)$,∀e ∈ E . F^c is called the soft complement mapping of F. Clearly,$(F^c)^c$ is the same as F and $((F, E)^c)^c$=(F, E).

Definition 1.9.[2] i. A soft set (F,E) over X is said to be a soft point if there exists e ∈ E such that F(e) is a singleton, say {x}, and F (e`) = ∅, for all $e$`∈ E− {e}. Such a soft point is denoted by x_e.
ii. The soft point $x_e$is said to be in the soft set (G,E) denoted by x_e $\tilde{\in}$(G, E), if x∈G(e).

Definition 1.10.[4] Let (F, E) be a soft set over X and A be a non-empty subset of X. Then the sub soft set of (F, E) over A denoted by (F_A , E), is defined as follows $F_A(e)$ = A ∩ F(e), for all e ∈ E. In other words (F_A, E) = $\tilde{A}$ $\tilde{\cap}$ (F, E).

Definition1.11.[5] Let X and Y be two nonempty sets and f: X → Y be a mapping, and then the following are defined:
i. The image of a soft set (F, E)∈SS(X,E) under the mapping f is defined by f(F,E) = (f (F),E), where [f(F)](e) = f [F(e)] , for all e ∈ E.
ii. The inverse image of a soft set (G,E) ∈ SS(Y,E) under the mapping f is defined by f^{-1}(G,E) = (f^{-1}(G) ,E), where [f^{-1}(G)](e) = f^{-1}[G(e)] ,for all e∈ E.

Proposition 1.12. Let f: X → Y be a mapping, if (F,E) ∈SS(X,E), and (G,E)∈SS(Y,E)then:
i. $f\,(F,E)^c = (f\,(F,E))^c$ if f bijective,
ii. $f^{-1}(G,E)^c = (f^{-1}\,(G,E))^c$.
Proof : Clear.

Definition1.13.[6]Let (F,E) and (G,E) be two soft sets over a common universe X. Then the Cartesian product of (F,E) and (G,E) denoted by (F, E)×(G, E) is a soft set (H, E× E) where H: E× E → P (X× X) and H(e_1, e_2)= F(e_1) × G(e_2) ∀ (e_1, e_2)∈E× E.

Definition 1.14.[6] Let (F_1, E_1) and (F_2, E_2) be two soft sets over X_1 and X_2, respectively and p_i : $X_1 \times X_2 \rightarrow X_i$, q_i : $E_1 \times E_2 \rightarrow E_i$ be projection mappings in classical meaning. Then the soft mappings (p_i , q_i), $i \in \{1, 2\}$, is called soft projection mapping from $X_1 \times X_2$ to Xi and defined by

$(p_i , q_i) ((F_1, E_1) \times (F_2, E_2)) =(p_i , q_i) ((F_1 \times F_2),(E_1 \times E_2)) = p_i (F_1 \times F_2), q_i (E_1 \times E_2) = (F, E)_i$.

Remark1.15.By applying definition1.14 the inverse of soft projection mapping (p_i, q_i) can be defined as following:

$(p_i , q_i)^{-1}$ (F_i ,E_i) = (p_i^{-1} (F_i) , q_i^{-1} (E_i)) , if i = 1,2 we have

$(p_1 , q_1)^{-1}$ (F_1,E_1) = (p_1^{-1} (F_1) , q_1^{-1}(E_1)) ,where ∀ e∈E p_1^{-1}[F_1 (e)]= F_1 (e)×X_2= F_1×X_2 and q_1^{-1} (E_1)=E_1× E_2 thus $(p_1 , q_1)^{-1}$ (F_1, E_1) = (p_1^{-1} (F_1) , q_1^{-1} (E_1)) = ($F_1 \times X_2$), ($E_1 \times E_2$) = (F_1 , E_1)×(X_2 , E_2) = (F_1 , E_1) × $\tilde{X}_2$, more ever (F_1 ,E_1) ×$\tilde{X}_2$ is a soft set over X_1× X_2 .

Definition 1.16.[4] Let T be the collection of soft sets over X ,then T is said to be a soft topology on X, if
i. $\tilde{\Phi}$, $\tilde{X}$ belong to T,
ii. the intersection of any two soft sets in T belongs to T, iii.the union of any number of soft sets in T belongs toT. The triple (X ,T, E) is called a soft topological space, or soft topological space for short over X.

Definition 1.17.[4] The members of T are called soft open sets. The complement of a soft open set is called the soft closed sets.

Proposition 1.18.[4]Let (X, T, E) be a soft topological space over X. Then the collection T^e = {F(e):(F, E)∈T} defines a topology on X for each e ∈ E.

Definition 1.19.[4]Let (X, T, E) be a soft topological space ,and A be a non-empty subset of X. Then T_A ={(F_A, E)|(F, E)∈ T} is said to be the soft relative topology on A and (A, T_A, E) is called a soft subspace of (X, T, E). We can easily verify that T_A is, in fact, a soft topology over A.

Theorem 1.20.[6]Let(A, T_A , E) be a soft subspace of soft topological space (X, T, E) and (F, E) be a soft set over X, then:
i. (F, E) is soft open over A if and only if (F, E) = $\tilde{A}$ $\tilde{\cap}$ (G, E) for some (G, A)∈ T
ii. (F, E) is soft closed over A if and only if (F, E) = $\tilde{A}$ $\tilde{\cap}$ (G, E) for some soft closed set (G, E) over X.

Definition 1.21.**[4]** Let (X, T, E) be a soft topological space and let (G, E) be a soft set over X. Then the soft interior of (G,E) denoted by (G, E)° is the soft set defined as: (G, E)°=$\tilde{\cup}${(F, E):(F, E) is soft open and (F, E) $\tilde{\subseteq}$ (G, E)}. Thus, (G, E)° is the largest soft open set contained in(G, E).

Definition 1.22.[4] Let (X, T, E) be a soft topological space and (G, E) be a soft set over X. Then the soft closure of (G, E) denote by $\overline{(G, E)}$ is the soft set ,defined as: $\overline{(G, E)}$ = $\tilde{\cap}${(F, E):(F, E) is soft closed and (G, E)$\tilde{\subseteq}$ (F,E)}. Note that $\overline{(G, E)}$ is the smallest soft closed set containing (G, E).

Definition1.23.[4] Let (X, T, E) be a soft topological space over X. A soft set (F, E) in (X, T, E) is called a soft neighborhood of the soft point x_e $\tilde{\in}$ (F, E) if there exists a soft open set (G,E) such that x_e $\tilde{\in}$(G,E) $\tilde{\subset}$(F,E).

Definition1.24.[7] Let (X, T, E) and (Y, T′,E) be soft topological spaces.Let f X →Y be a mapping. The mapping f is soft continuous at $x_e \tilde{\in} \tilde{X}$, if for each soft neighbourhood (H, E) of $f(x_e)$ there exists a soft neighborhood (F, E) of x_e such that f((F, E)) ⊂ (H, E). If f is soft continuous mapping for all x_e, then f is called soft continuous mapping.

Theorem1.25.[7]Let (X, T, E) and (Y, T′ , E) be two soft topological spaces, f : (X, T, E) ⟶ (Y, T′, E) be a mapping. Then the following conditions are equivalent: i. f : (X, T, E) → (Y, T′ , E) is a soft continuous mapping,
ii. For each soft open set (G, E) over Y , f^{-1} ((G, E)) is a soft open set over X,
iii. For each soft closed set (H, E) over Y,f^{-1}((H, E)) is a soft closed set over X,
iv. For each soft set (F, E) over X,$f(\overline{(F,E)}) \tilde{\subseteq} \overline{(f(F,E))}$ v. For each soft set (G, E) over Y,$\overline{f-1((G,E))} \tilde{\subseteq} f^{-1}(\overline{(G,E)})$,
vii. For each soft set (G, E) over Y,$f^{-1}((G,E)^{\circ}) \tilde{\subseteq} f^{-1}((G,E))^{\circ}$.

Definition 1.26.A soft constant space (briefly *sc*-space) over X is a soft topology (X,T,E),whose members only $\tilde{A}$ for all A⊆X .

Examples1.26. Let X={x , y}, E={e_1,e_2} . Here ∅, X, {x},{y} all sub sets of X, if

$A_1=\emptyset \Rightarrow \tilde{A}_1 = \tilde{\Phi}, A_2 = X \Rightarrow \tilde{A}_2 = \tilde{X}$

$A_3 = \{x\} \Rightarrow \tilde{A}_3 = \{(e_1,\{x\}),(e_2,\{x\})\}$

$A_4 = \{y\} \Rightarrow \tilde{A}_4 = \{(e_1,\{y\}),(e_2,\{y\})\}$.

Therefore the collection$\{(F,E)_{A_i}, i = 1,2,3,4\}$ is *sc*-space over X.

The following example shows not every soft topological space is *sc*-space.

Let X = { x, y }, E = {e_1, e_2} and T = {$\tilde{X}$, $\tilde{\Phi}$, (F_1, E),(F_2, E),(F_3, E)} where(F_1,E) , (F_2, E),(F_3, E) are soft sets over X defined as follows: $F_1(e_1)$ = X, $F_1(e_2) = \{x\}$, $F_2(e_1) = \{y\}$, $F_2(e_2)$ = X, $F_3(e_1) = \{y\}$, $F_3(e_2) = \{x\}$. Then T defines a soft topology over X.

II. Certain Types of Soft Generalized Continuous Mappings

In this section , we recall that basic definitions and theorems about soft generalized closed set and we introduce the definition of soft generalized continuous mapping , soft strongly generalized continuous mapping and soft generalized irresolute mapping . Also, we give the relation among this types .

Definition 2.1.[8] Let (X, T,E) be a soft topological space. A soft set (F, E) is called a soft generalized closed (briefly soft g-closed) over X if $\overline{(F,E)} \tilde{\subseteq}$ (G,E) whenever (F,E) $\tilde{\subseteq}$ (G,E) and (G,E) is soft open over X .The complement of a soft generalized closed set is called the soft generalized open (briefly soft g-open).

Theorem 2.2.[8] Let (X, T,E) be a soft topological space and (F, E) a soft set over X. If (F, E) is soft closed , then (F, E) is soft g-closed.

Theorem2.3.[8]The intersection of soft g-closed set with soft closed set is a soft g-closed set.

Definition2.4.[8] A soft topological space (X, T, E) is a soft $T_{\frac{1}{2}}$ -space if every soft g-closed set is soft closed over X.

Proposition 2.5.[9]Let (X, T_1, E) and (Y, T_2, E) be soft spaces. Let 𝔅 ={(F,E)×(G,E) |(F,E) ∈ T_1, (G, E) ∈T_2} and T be the collection of all arbitrary union of elements of 𝔅. Then T is a soft topology over X × Y .

Definition 2.6.[9] Let (X, T_1, E) and (Y, T_2, E) be soft spaces. Then the soft space (X ×Y, T, E×E) as defined in proposition 2.5 is called soft product topological space over X × Y.

Proposition 2.7.[10] Let (X,T_1, E), (Y,T_2, E) be two soft topological spaces, (F,E) ∈ SS(X,E), and (G,E) ∈ SS(Y,E). Then $\overline{(F,E)\times(G,E)} = \overline{(F,E)} \times \overline{(G,E)}$.

Proposition 2.8. Let$(X,T_1,E)$and (Y,T_2,E) be two soft topological spaces. Then(F, E)and (G, E) are soft g- closed sets over XandY(resp.) if and only if (F,E)×(G,E) is soft g-closed set over X × Y .

Proof:(⟹)Let $(G_1,E)\times(G_2,E)$ be a basic soft open set over X × Y, such that $(F_1,E)\times(F_2,E)\tilde{\subseteq}(G_1,E)\times(G_2,E)$ .To prove $\overline{(F_1,E)\times(F_2,E)}\tilde{\subseteq}(G_1,E)\times(G_2,E)$. Since $(F_1,E)\tilde{\subseteq}(G_1,E)$ and $(F_2,E)\tilde{\subseteq}(G_2,E)$ then $\overline{(F_1,E)}\tilde{\subseteq}(G_1,E)$ and $\overline{(F_2,E)}\tilde{\subseteq}(G_2,E)$. Thus $\overline{(F_1,E)}\times\overline{(F_2,E)}\tilde{\subseteq}$
$(G_1,E)\times(G_2,E)$ and by Proposition 2.7 we have $\overline{(F_1,E)\times(F_2,E)}\tilde{\subseteq}(G_1,E)\times(G_2,E)$. Hence $(F_1,E)\times(F_2,E)$
is soft g-closed set over X × Y .

(⟸) obvious.

Now, we introduce the following definition

Definition 2.9.[14]Let (X, T, E),(Y, T`, E) be soft topological spaces and $f: X \to Y$ be a mapping .Then f is called soft generalized continuous (briefly soft g-continuous) if $f^{-1}(F,E)$ is soft g-closed set over X for every soft closed set (F, E) over Y.

Example 2.10

(i) If (X, T, E) and (Y, T`, E) are sc-spaces, then the constant mapping from X into Y is soft g-continuous.

(ii) If (X, T, E) be a soft topological space ,where T= T_{sind} and $f: X \to Y$ be a mapping of X into a soft topological space (Y, T`, E) , then f is soft is g-continuous.

The following example shows not every soft mapping is soft g-continuous

Example2.11. Let X = {a, b , c}, E={e_1, e_2 }and T={$\tilde{\Phi}$, $\tilde{X}$, (F_1, E),(F_2, E),(F_3,E)}, T′= {$\tilde{\Phi}$, $\tilde{X}$, (G_1, E)} be two soft topologies defined over X, where (F_1, E), (F_2,E) ,(G_1,E) and (G_2, E) are soft sets over X, defined as follows: $F_1(e_1)$ ={ a, b}, $F_1(e_2)$={a, c},$F_2(e_1)$ ={b,c} ,$F_2(e_2)$={a, b}, $F_3(e_1)$ ={ b}, $F_3(e_2)$={a }, and $G_1(e_1)$= {a, c},$G_1(e_2)$ = {b, c}. If we get the mapping f: X →X be identity mapping then f is not soft g-continuous.

Theorem2.12. Let (X, T, E), (Y, T`, E) be soft topological spaces, and $f: X \to Y$ be a mapping. Then f is soft g-continuous if and only if $f^{-1}(F,E)$is soft g-open set over X for every soft open set (F, E)over Y.

Proof:(⟹)Let f be a soft g-continuous ,and (F, E) is soft open set over Y,then $(F,E)^c$ is soft closed set over Y. So by hypothesis $f^{-1}(F,E)^c$ is soft g-closed by Proposition (1.12,ii)
$f^{-1}(F,E)^c = (f^{-1}(F,E))^c$. Thus
$f^{-1}(F,E)$ is soft g − open set over X .

(⟸)Let (F,E) be a soft closed set over Y.Then $(F,E)^c$ is soft open set , by hypothesis $f^{-1}(F,E)^c$ is soft g-open set over X ,so f^{-1}(F, E) is soft g-closed set over X . Hence f is soft g-continuous.

Remark 2.13.[14]Every soft continuous mapping is soft g-continuous, but the converse is not true in general as the following example shows:

Let X = Y = {a, b, c},E={ e_1,e_2}and T = T_{sind} ,T′ = T_{sd} be soft topologies spaces over X and Y(respectively). Let $f: X \to Y$ be an identity mapping, then $f$is soft g-continuous but it's not soft continuous.

Now ,we put a condition on soft topological spaces to satisfy the converse .

Theorem 2.14. Let f be soft g-continuous mapping from a soft $T_{\frac{1}{2}}$-space(X, T, E) onto a soft topological space $(Y, T`, E)$, then f is soft continuous.

Proof : Let (F,E) be a soft open set over Y, then $(F,E)^c$ is soft closed set . Since f is soft g-continuous ,then $f^{-1}(F,E)^c$ soft g-closed set over X, and by Proposition (1.12,ii) $f^{-1}(F,E)^c = (f^{-1}(F,E))^c$ is soft closed set over X ,so $f^{-1}(F,E)$ is soft open set. Hence f is soft continuous.

Remark 2.15. The restriction of soft g-continuous mapping is not necessary being a soft g-continuous mapping.

Let X = $\{x_1, x_2, x_3\}$, Y= $\{y_1, y_2\}$, E= $\{e_1, e_2\}$ and T={ $\tilde{\Phi}$,$\tilde{X}$,(F,E)}, T ′={$\tilde{\Phi}$, $\tilde{Y}$,(G,E)} be two soft topologies defined over X and Y, respectively. Here (F,E)and (G, E) are soft sets over X and Y(respectively), defined as follows: F(e_1)={ x_1}, F(e_2) = $\{x_1, x_2\}$ and G(e_1)={ y_1}, G (e_2) =$\{y_2\}$. Now we define the mapping f: X → Y as $f(x_1) = f(x_2) = y_1$, $f(x_3) = y_2$. It is clear that f is soft g-continuous. If A = $\{x_1, x_2\}$ then $T_A = \{\tilde{\Phi}, \tilde{A}, (F_A, E)\}$,where $F_A(e_1)$=$\{x_1\}$,$F_A(e_1)$= $\{x_1, x_2\}$ then the restriction map $f|_A$: A ⟶ Y is not soft g-continuous mapping. Since (G ,E)^c is soft closed set over Y but $(f|_A)^{-1}(G, E)^c$is not soft g-closed over A.

Now , we give a condition on set $\tilde{A}$ to satisfy the restriction mapping is soft g-continuous.

Theorem 2.16. Let f be a soft g-continuous mapping from soft topological space (X, T, E)into soft topological space $(Y, T`, E)$,A ⊆ X and $\tilde{A}$ is soft closed set over X. Then the restriction map$f|_A$ is soft g-continuous.

Proof: Let (F,E) be a soft closed set over Y . Since f is soft g-continuous ,then $f^{-1}(F,E)$ is soft g-closed set over X ,so by theorem 2.3 $f^{-1}(F,E)\tilde{\sqcap}\tilde{A}$ is soft g-closed set. Since $(f|_A)^{-1}(F,E) = f^{-1}(F,E)\tilde{\sqcap}\tilde{A}$, thus $(f|_A)^{-1}(F,E)$ is soft g-closed set over X .So $f|_A$ is soft g-continuous.

Recall that if $f: X \longrightarrow Y$ be a mapping and K⊆Y, then $f_K: f^{-1}(K) \to K$ which defined by $f_K(x) = f(x)$ for all $x \in f^{-1}(K)$.[11]

Theorem2.17. Let f be a soft g-continuous mapping from a soft $T_{\frac{1}{2}}$ –space (X,T,E) into soft topological space $(Y, T`, E)$,and $\tilde{K}$ is soft closed set over Y, then $f_K: f^{-1}(K) \longrightarrow K$, is soft g-continuous mapping.

Proof: Let (F,E) be a soft closed set over K, to prove ${f_K}^{-1}$(F,E) is soft g-closed set over f^{-1}(K) . Since $\tilde{K}$ is soft closed set over Y, then (F,E) is soft closed set over Y.But f is soft g-continuous and (X, T, E) is a soft $T_{\frac{1}{2}}$ -space ,then f^{-1}(F,E) is soft closed set over X, thus $f^{-1}(\tilde{K})\,\tilde{\sqcap} f^{-1}(F,E)$ is soft closed set over f^{-1}(K) .So by theorem2.2 ${f_K}^{-1}$(F,E)= $f^{-1}(\tilde{K})\,\tilde{\sqcap} f^{-1}(F,E)$ is soft g-closed set. Hence f_K is soft g-continuous mapping .

Theorem 2.18. Let (X, T, E) be a soft topological space and A be a non-empty subset of X then the inclusion soft mapping $i: (A, T_A, E) \longrightarrow (X, T, E)$ is soft g-continuous if and only if $\tilde{A}$ is a soft closed set over X .

Proof:(⟹) Let (F,E) be a soft closed set over X ,then by theorem 2.2 (F,E) soft g-closed set and by theorem 2.3 $\tilde{A}\,\tilde{\sqcap}(F,E)$ is soft g-closed sets over$X$ .But i^{-1}(F,E) = $\tilde{A}\,\tilde{\sqcap}(F,E)$.Thus i^{-1}(F,E) is soft g-closed set over X. Therefore i_A is soft g-continuous.

(⟸) clear .

Remark2.19. The composition of two soft g-continuous mapping not necessary be soft g-continuous mapping as the following example shows:

Let X = Y = Z = $\{a, b, c\}$ .Where T = $\{\tilde{\Phi}, \tilde{X}, (F_1, E), (F_2, E)\}$, T ′= {$\tilde{\Phi}$, $\tilde{Y}$} and T″= {$\tilde{\Phi}$, $\tilde{Z}$,(G,E)} are a soft topological spaces over X,Y and Z (respectively). The soft open sets (F_1,E),(F_2,E) and

(G,E) are defined as follows: $F_1(e_1)$ ={a}, $F_1(e_2)$= $\{b\}$ $F_2(e_1)$ =$\{a,c\}$,$F_2(e_2)$ =$\{b,c\}$,and $G(e_1)$ =$\{a,b\}$,G $(e_2)=\{a,b\}$, Define $f: X \to Y$ by $f(a)=c$, $f(b)=a$,
$f(c)=b$ and $g: Y \to Z$ by $g(a)=a$, $g(b)=c$,
$g(c)=b$ .Then f and g are soft g-continuous mappings but gof is not soft g-continuous mapping , since $(G,E)^c$ is a soft closed set over Z , but $(gof)^{-1}(G,E)^c$ is not soft g-closed over X.

Now, we put the condition either on soft topological spaces or on mappings to satisfy the composition of two soft g-continuous mappings is soft g-continuous.

Theorem 2.20. Let (X,T,E) ,$(Y,T`,E)$and$(Z,T`,E)$ be soft topological spaces,$f: X \to Y, h: Y \to Z$ be mappings then:

i. If f is soft g-continuous and h is soft continuous ,then hof is a soft g-continuous.

ii. If f and h are soft g-continuous,(Y,T`, E)is soft $T_{\frac{1}{2}}$ -space , then hof is soft g-continuous.

(iii) If f is soft continuous and h is soft g-continuous,$(Y,T`,E)$is soft $T_{\frac{1}{2}}$ –space then hof is soft g-continuous.

Proof: (i) Let (F,E) be a soft closed set over Z,and since h is soft continuous ,then h^{-1}(F,E) is soft closed set over Y ,since f is soft g-continuous then $f^{-1}(h^{-1}$(F,E))= $(hof)^{-1}$(F,E)is soft g-closed set over X ,thus hof is a soft g-continuous.

(ii) Let (F,E) be a soft closed set over Z , since h is soft g-continuous then h^{-1}(F,E) is soft g-closed set over Y ,and by definition 2.4, h^{-1}(F,E) is soft closed set , so $f^{-1}(h^{-1}$(F,E)) = $(hof)^{-1}$(F,E)is soft g-closed set over X ,thus hof is a soft g-continuous.

(iii) clear.

Now, we study the product of soft g-continuous mappings.

Proposition 2.21. Let f_i: $(X_i, T_i, E) \to (Y_i, T_i^`, E)$,i=1,2 be a soft mapping .If $f_1 \times f_2$ is soft g-continuous then f_1 and $f_2$are soft g-continuous .

Proof:(⟹)To prove f_1: $(X_1,T_1, E) \longrightarrow (Y_1, T_1^`, E)$ is soft g-continuous. Let (F, E) be a soft closed set over Y_1, then (F, E) × $\tilde{Y}_2$ is an soft closed set over $Y_1 \times Y_2$. Since $f_1 \times f_2$ is soft g-continuous, then $(f_1 \times f_2)^{-1}$((F, E)× $\tilde{Y}_2$) = f_1^{-1}(F, E) × $f_2^{-1}(\tilde{Y}_2)$ is an soft g-closed set over $X_1 \times X_2$,thus by Proposition 2.8, we have f_1^{-1}(F, E) is an soft g-closed set over X_1. Hence f_1 is soft g-continuous mapping.
In similar way we can prove $f_2$is soft g-continuous mapping .

Corollary 2.22.[5] Let (F, E) and (G, E) be soft closed set in soft topological spaces (X, T_1, E) and (Y, T_2, E), respectively. Then (F, E) × (G, E) is soft closed set in soft product space (X × Y, T, E × E).

Theorem 2.23. Let $\{(X_i, T_i, E)\}_{i\in I}$, i =1,2 be soft topological spaces, then the soft projection mappings (p_i, q_i): $(X_1,T_1,E) \times (X_2,T_2,E) \to (X_i,T_i,E)$ is soft g-continuous , for each i=1,2.

Proof: To prove that (p_1, q_1) is soft g-continuous . Let $(F,E)_1$ be a soft closed set over X_1 , then $(p_1, q_1)^{-1}(F,E)_1$= $(F,E)_1 \times \tilde{X}_2$,is soft closed set over X×Y by Corollary2.22 .So $(p_1, q_1)^{-1}(F,E)_1$ is soft g-closed over X×Y . Hence (p_1, q_1) is soft g-continuous.

In similar way we can prove that (p_2, q_2) is soft g-continuous.

Recall that, A function $f: X \to Y$ is g-continuous if and only if $f^{-1}(V)$is g-open set in $X$for every open set Vin Y.[13]

Theorem 2.24. If f: (X, T, E) ⟶(Y, T′ , E) is a soft g-continuous mapping, then for each $e \in E$, f_e:(X, T_e) → (Y, T_e') is g-continuous mapping.

Proof: Let A∈ T'_e.Then there exists a soft open set (G,E) over Y such that A = G(e). Since f: (X, T, E) ⟶(Y, T′ , E) is a soft g-continuous mapping ,then f^{-1}(G, E) is a soft g-open set over X and f^{-1}(G, E)(e) = f^{-1}(G(e)) = f^{-1}(A) is an g-open set. This implies that f_e is a g-continuous mapping.

Now, we introduce the following definitions.

Definition 2.25. Let (X, T, E) and (Y,T`,E) be soft topological spaces, $f: X \to Y$ be a mapping, then:

i. f is called soft strongly generalized continuous mapping (briefly soft sg-continuous) if f^{-1}(F, E)is soft closed set over X for every soft g-closed set(F,E)over Y.

ii. $f$is called soft generalized irresolute mapping (briefly soft g-irresolute) if f^{-1}(F, E) is soft g-closed set over X for every soft g-closed set (F, E) over Y.

Example 2.26.

i. Let$X = Y = \{a, b, c\}$, $E = \{e_1, e_2\}$and T = T_{sd} , T′ = T_{sind} be soft topologies on $X$and Y(resp.) .The identity mapping $f: X \to Y$ soft sg-continuous.

ii. Let X = $\{x_1, x_2, x_3\}$,Y = $\{y_1, y_2, y_3\}$, E = $\{e_1, e_2\}$ and T = T_{sd}, T′= {$\widetilde{\Phi}$, $\widetilde{Y}$,(G,E)} be soft topologies over X Y(respectively) and (G,E) be a soft set over Y where G(e_1)= $\{y_1\}$ G(e_2)= $\{y_1\}$. A mapping $f: X \to Y$ which defined by $f(x_1) = y_1$; $f(x_2) = y_3$; $f(x_3) = y_2$is soft g- irresolute.

The following example shows that not every soft mapping is soft sg-continuous (soft g- irresolute) .

Example2.27.

i. Let X = $\{x_1, x_2, x_3\}$, Y = $\{y_1, y_2, y_3\}$ and E = $\{e_1, e_2\}$ where $T = \{\widetilde{\Phi}, \widetilde{X}, (F, E)\}$,$T' = \{\widetilde{\Phi}, \widetilde{Y}, (G, E)\}$ are soft topologies over X and Y(respectively).The soft open sets (F,E) and(G,E) are defined as follows: F (e_1)= $\{x_2, x_3\}$ F(e_2)=$\{x_1, x_3\}$and G(e_1)={ y_2},G(e_2)={ y_1}. A mapping $f: X \to Y$ which defined by $f(x_1) = y_1$; $f(x_2) = y_3$; $f(x_3) = y_2$ is not soft sg-continuous.

ii. Let $X = \{x_1, x_2, x_3\}$,Y = $\{y_1, y_2, y_3\}$, E = $\{e_1, e_2\}$ and T={ $\widetilde{\Phi}$, $\widetilde{X}$,(F_1,E), (F_2,E), (F_3,E) } ,= T′{ $\widetilde{\Phi}$, $\widetilde{Y}$, (G,E)} are soft topological spaces over X, and Y respectively . The soft open sets (F_1, E), (F_2, E),(F_3,E), (F_4,E) and (G,E) are defined as follows: F₁(e_1)=$\{x_1\}$,F₁(e_2)= $\{x_2\}$, F₂(e_1)= $\{x_3\}$,F₂(e_2)= $\{x_3\}$, F₃(e_1)= $\{x_1, x_3\}$,F₃(e_2) = $\{x_2, x_3\}$,and G(e_1)=$\{y_1\}$, G (e_2)= $\{y_1\}$. A mapping $f: X \to Y$ which defined by $f(x_1) = y_2$; $f(x_2) = y_3$; $f(x_3) = y_1$is not soft g- irresolute.

Remark2.28. The restriction of soft sg-continuous(soft g- irresolute) mapping is not necessary being a soft sg-continuous(soft g- irresolute) mapping.

Theorem 2.29. Let f be a soft sg-continuous (soft g-irresolute) mapping from a soft topological space (X, T, E)into soft topological space $(Y, T`, E)$,A ⊆ X and $\widetilde{A}$ is soft closed set over X. Then the restriction map$f|_A$ is soft sg-continuous(soft g- irresolute).

Proof: clear.

Theorem 2.30. Let (X, T, E), (Y, T`, E) be soft spaces, and $f: X \to Y$ be a mapping. Then

i. f is soft g-irresolute if and only if $f^{-1}(G, E)$is soft g-open over X for every soft g-open set (G, E)over Y.

ii. f is soft sg-continuous if and only if $f^{-1}(G, E)$is soft open over X for every soft g-open set (G, E)over Y.

Proof: i. (⟹)Let f be soft g- irresolute mapping, and (G, E) is soft g-open set over Y, then $(G, E)^c$ is soft g-closed set over Y.So by hypothesis $f^{-1}(G, E)^c$ is soft g-closed ,and by Proposition(1.12,ii),$f^{-1}(F, E)^c = (f^{-1}(F, E))^c$. Thus $f^{-1}(G, E)$ is soft g-open over X.

(⟸) Let (G,E) be a soft g-closed set over Y. Then $(G, E)^c$ is soft g-open set , by hypothesis $f^{-1}(G, E)^c$ is soft g-open set over X ,so f^{-1}(G, E) is soft g-closed set over X . Hence f is soft g- irresolute.

ii. Clear.

Theorem2.31 Let (X, T, E)and (Y, T′ , E) be a soft topological spaces and f:X ⟶ Y is soft g-continuous mapping. Then: i. If (Y,T′,E) is soft $T_{\frac{1}{2}}$-space ,then f is soft g-irresolute**.** ii. If (X,T,E) is soft $T_{\frac{1}{2}}$-space,then f is soft sg-continuous **.**

Proof: i. Let (F,E)be a soft g-closed set over Y ,then by hypothesis (F,E) is soft closed over Y. Since f is soft g-continuous then $f^{-1}(F,E)$ is soft g-closed set X, thus f is soft g- irresolute.

ii. Clear.

Theorem 2. 32. Let $\{(X_i, T_i, E)\}_{i\in I}$, i =1,2 be a soft topological spaces, and$(p_i\ ,q_i)$: $(X_1, T_1, E) \times (X_2, T_2, E) \rightarrow (X_i, T_i, E)$ are soft projection mappings then :

i. $(p_i\ ,q_i)$ are soft g- irresolute for each i=1,2.

ii. If $\{(X_i, T_i, E)\}_{i\in I}$ are soft $T_{\frac{1}{2}}$-space ,then $(p_i\ ,q_i)$ are soft sg-continuous ,for each i=1,2**.**

Proof: i. Similar to the proof of theorem 2.23. **ii.** To prove that $(p_1\ ,q_1)$ is soft sg-continuous . Let $(F,E)_1$ be a soft g-closed set over X_1 ,then $(F,E)_1$is soft closed set over X_1.Thus $(p_1\ ,q_1)^{-1}(F,E)_1$= $(F,E)_1 \times \widetilde{X}_2$,is soft closed set over X×Y by Corollary 2.22 . Hence $(p_1\ ,q_1)$ is soft sg-continuous.

In similar way we can prove that $(p_2\ ,q_2)$ is soft g-continuous.

The following diagram shows that the relations among the different types of soft continuous mappings.

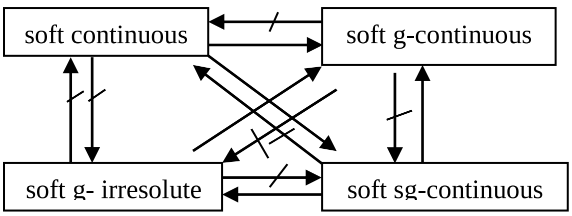

Fig. (1)

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حول الدوال الواهنة المستمرة المعممة

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المستخلص :

في هذا البحث درسنا الدالة الواهنة المستمرة المعممة في الفضاءات التبولوجية الواهنة ،ودرسنا خواص القصر و التركيب والضرب لهذه الدالة، وقدمنا تعريف وخواص الدوال الواهنة المستمرة المعممة بقوه و الواهنة المنحلة المعممة ودرسنا العلاقة بين هذه الدوال.