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Certain Types of Groupoids

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Abstract:

In this work, we try to construct anew types of groupoids and disucss their properties.

Keywords: Groupoid, Descarts groupoid, Direct sum of two groupoid, Tensor product of two groupoid , Action groupoid.

Classifiacation subject classification: 425F

Introduction:

The conceept of groupoid is one of the means by which the twentieth century reclaims the original domain of appliication of group concept. In 1920's Brandt and Baer gave the algebraic theory of groupoid .In 1950's Ehresmann introduced the groupoid in to the differential geometry. Also the concept of groupoid action is due originally to "Ehreesmann (1959), generalizing the group action in his work on fibre spaces". Dieudonne, J.; introduce to modern analysis to Lie groupoid (1968). Berson, G.E.; introduce to compact groupoid (1972). Al-Taai,A.A. (1988), gave the action of groupoid. Al-Taai, A. A. and Bedaiwi (1999) , gave representation of topological groupoid. Mahdi J.H.; (2006) gave to vector groupoid and isometries. Taghreed H. M. (2010) gave to some results of Lie group and Lie algebra. In this paper, we study the basic construction of groupoid space and we divide into two sections. In section one, we give definition examples and theorems about groupoid and morphism of groupoid . In section two, we introduce subgroupoid , normal subgroupoid and transitive.

1. Category , Fiber product , Groupoid , morphism of groupoid .

(1.1) Definition(2)

"A category C consist of :

(a) A class of objects

(b) For every ordered pair of objects X and Y asset hom(X,Y) of morphism with domain X and range Y , if f∈hom(X,Y) we write f :X→Y.

(c) For every ordered triple of opjects X,Y, and Z a function associating to a pair of morphism f :X→Y and

g :Y→Z their composite gf :X→Z .

These satisfy the following two axioms :

(i) Associativity : if f :X→Y , g:Y→Z and h:Z→W then h(gf) = (hg)f

(ii) Identity :For every object Y there is a morphism I_Y:Y→Y such that if f :X→Y , then I_Yf = f and if h:Y→Z then hI_Y = h"

(1.2) Notation's

(1) The category of sets and maps which we denote by M .

(2) The category of continuous maps and topological spaces which we denote by T .

(1.3) Definition(5)
"A pair of sets (T,Q) is a groupoid on which are given :

(1) Two surjections α,β:T→Q called the source and the target mapping respectively

(2) An injection λ: T→ Q called the mapping of unities satisfying αoλ=I_Q and βoλ=I_Q (whereI_Q: Q→Q is the identity mapping on Q)

(3) A law of partial composition γ in T define as a law of composition

"$T*T=\{(t,t')\in T\times T \mid \alpha(t)=\beta(t')\}$" "fibree product of α and β over Q"

such that
(a)"$\gamma(t,\gamma(t_1,t_2))=\gamma(\gamma(t,t_1,),t_2)$,for all (t,t_1) ,(t_1,t_2) ∈T∗T"
(b) "$\alpha(\gamma(t_1,t_2))= \alpha(t_2)$, $\beta(\gamma(t_1,t_2))=\beta(t_1)$ for each $(t_1,t_2)\in T*T$"
(c) "$\gamma(t,\lambda(\alpha(t)))=t$ and $\gamma(\lambda(\beta(t)),t)=t$,for all $t\in T$"
(4) A biijection σ: T→T called the inveersion of T satisfying
(a) $\alpha(\sigma(t))=\beta(t)$,$\beta(\sigma(t))=\alpha(t)$,for all$t\in T$
(b) $\gamma(\sigma(t),t) = \lambda(\alpha(t))$, $\gamma(t,\alpha(t)) = \lambda(\beta(t))$, for any $t\in T$
We write $\sigma(t)=t^{-1}$ called the inverse element of $t\in T$ and $\lambda(x) = \tilde{x}$ called the unit element in T associated to the element x∈Q .

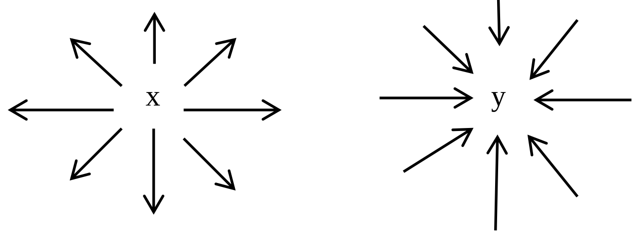
Also, we write $\gamma(t,t')=tt'$. T is called the groupoid and Q is called the base . Also, we say that T is a groupoid on Q" .

(1.4) Remark(1)

"If (T,Q) be any groupoid then :

(i) The sub set of T ; $T_x=\alpha^{-1}(x)$ is called the α-fiber at $x\in Q$,$_yT=\beta^{-1}(y)$ is called the β-fiber at $y\in Q$ and $_yT_x=T_x\cap {_yT}$ the set

of elements in T which have x as a source and y as a target"



α- fiber at x β- fiber at y

(ii) "$_xT_x$ is a group under the restriction γ on $_xT_x\times{_xT_x}$ with unity

λ(x) called the vertex (isotropy) group at $x\in Q$.

(iii) The map $\tau:T\to Q\times Q$; $\tau(t)=(\beta(t),\alpha(t))$ its said to be the transitor of T and $_yT_x=\tau^{-1}(y,x)$, for all $x,y\in Q$".

(1.5) Example

Let Q be anon-empty set . The Cartesian product T = Q×é×é×Q is a groupoid of base Q as follows

$\alpha=pr_4$, $\beta=pr_1$, $\lambda(x)=(a,é,é,a)$ for all $a\in Q$ where é×é is the identity element in ή×ή and the partial composition is given by (z,é,é,w′)(w,é,é,x) = (z, γ(é,é,é,é),x) wherever w′=w and γ is a law of composition in ή×ή . The invers element of (a,é,é,y) is (y,é,é,a) , where σ is the inversion law of T . This groupoid is called DeŚcartes groupoid .

(1.6) Definition(8)

"Let T_1 and T_2 be groupoids the Tensor product of $T_1\otimes T_2$ consist of linear combinations of elements of the form $t_1\otimes t_2$ where $t_1\in T_1$ and $t_2\in T_2$ with the following relations :

(i) $c(t_1\otimes t_2)=(ct_1)\otimes t_2=t_1\otimes(ct_2)$,for any scalars c .

(ii) $(t_1\otimes t_2)+(t_1'\otimes t_2')=(t_1+t_1')\otimes t_2$.

(iii) $(t_1\otimes t_2)+(t_1\otimes t_2')=t_1\otimes(t_2+t_2')$ for all $t_1,t_1'\in T_1$ and $t_2,t_2'\in T_2$" .

(1.7) Theorem:

Let (T_1,Q_1) and (T_2,Q_2) are both groupoids then the tensor product $(T_1\otimes T_2,Q_1\otimes Q_2)$ be a groupoid .

Proof:

(1) $\alpha,\beta:T_1\otimes T_2\to Q_1\otimes Q_2$

$\alpha(v_1\otimes v_2)=w_1\otimes w_2$, $\beta(v_1\otimes v_2)=w_1\otimes w_2$,for all $v_1\otimes v_2\in T_1\otimes T_2$ and $w_1\otimes w_2\in Q_1\otimes Q_2$,

(2) $\lambda:Q_1\otimes Q_2\to T_1\otimes T_2$

$\lambda(w_1\otimes w_2)=v_1\otimes v_2$,

$(\alpha o\lambda)(w_1\otimes w_2)=\alpha(\lambda(w_1\otimes w_2))=\alpha(v_1\otimes v_2)=(w_1\otimes w_2)$

$(\beta o\lambda)(w_1\otimes w_2)=\beta(\lambda(w_1\otimes w_2))=\beta(v_1\otimes v_2)=(w_1\otimes w_2)$,

Where

$\alpha o\lambda=I_{Q_1\otimes Q_2}$,$\beta o\lambda=I_{Q_1\otimes Q_2}$ and $I_{Q_1\otimes Q_2}:Q_1\otimes Q_2\to Q_1\otimes Q_2$

(3) $(T_1\otimes T_2)*(T_1\otimes T_2)=\{(v_1\otimes v_2,v_1'\otimes v_2')\in(T_1\otimes T_2)\times(T_1\otimes T_2)$:

$\alpha(v_1\otimes v_2)=\beta(v_1'\otimes v_2')\}$ " fibre product of α and β over $Q_1\otimes Q_2$"

Such that

(a) $\gamma(v\otimes v,\gamma(v_1\otimes v_1,v_2\otimes v_2))$
$=\gamma(v\otimes v,v_2\otimes v_2)=v_2\otimes v_2$
$\gamma(\gamma(v\otimes v,v_1\otimes v_1),v_2\otimes v_2)=\gamma(v_1\otimes v_1,v_2\otimes v_2)$
$=v_2\otimes v_2$

So we have $\gamma(v\otimes v,\gamma(v_1\otimes v_1,v_2\otimes v_2))=\gamma(\gamma(v\otimes v,v_1\otimes v_1),v_2\otimes v_2)$

For all $(v\otimes v,v_1\otimes v_1)$, $(v_1\otimes v_1,v_2\otimes v_2)\in(T_1\otimes T_2)*(T_1\otimes T_2)$

(b) $\alpha(\gamma(v_1\otimes v_1,v_2\otimes v_2))=\alpha(v_2\otimes v_2)=w_2\otimes w_2$

$\alpha(v_2\otimes v_2)=w_2\otimes w_2$

So we get $\alpha(\gamma(v_1\otimes v_1,v_2\otimes v_2))=\alpha(v_2\otimes v_2)$,
$\beta(\gamma(v_1\otimes v_1,v_2\otimes v_2))=\beta(v_1\otimes v_1)=w_1\otimes w_1$
$\beta(v_1\otimes v_1)=w_1\otimes w_1$

So we get , $\beta(\gamma(v_1\otimes v_1,v_2\otimes v_2))=\beta(v_1\otimes v_1)$ for each $(v_1\otimes v_1,v_2\otimes v_2)\in(T_1\otimes T_2)*(T_1\otimes T_2)$

(c) $\gamma(v_1\otimes v_2,\lambda(\alpha(v_1\otimes v_2))) = \gamma(v_1\otimes v_2, \lambda(w_1\otimes w_2)) = \gamma(v_1\otimes v_2, v_1\otimes v_2) = v_1\otimes v_2$
$\gamma(\lambda(\beta(v_1\otimes v_2)), v_1\otimes v_2) = \gamma(\lambda(w_1\otimes w_2), v_1\otimes v_2) =\gamma(v_1\otimes v_2, v_1\otimes v_2) =(v_1\otimes v_2)$
, for all $v_1\otimes v_2 \in T_1\otimes T_2$

(4) $\sigma: T_1\otimes T_2 \to T_1\otimes T_2$

(a) $\alpha(\sigma(v_1\otimes v_2))= \alpha(v_1\otimes v_2) = w_1\otimes w_2$, $\beta(v_1\otimes v_2)= w_1\otimes w_2$

We conclude $\alpha(\sigma(v_1\otimes v_2))= \beta(v_1\otimes v_2)$, $\beta(\sigma(v_1\otimes v_2))= \beta(v_1\otimes v_2) =w_1\otimes w_2$, $\alpha(v_1\otimes v_2) =w_1\otimes w_2$

For any $v_1\otimes v_2 \in T_1\otimes T_2$

(b) $\gamma(\sigma(v_1\otimes v_2), v_1\otimes v_2) =\gamma(v_1\otimes v_2, v_1\otimes v_2) =v_1\otimes v_2$, $\lambda(\alpha(v_1\otimes v_2)) =\lambda(w_1\otimes w_2) = v_1\otimes v_2$

So we get $\gamma(\sigma(v_1\otimes v_2), v_1\otimes v_2)= \lambda(\alpha(v_1\otimes v_2))$,

And $\gamma(v_1\otimes v_2,\sigma(v_1\otimes v_2)) =\gamma(v_1\otimes v_2, v_1\otimes v_2) =v_1\otimes v_2$,$\lambda(\beta(v_1\otimes v_2)) =\lambda(w_1\otimes w_2) =v_1\otimes v_2$

Hence we have $\gamma(v_1\otimes v_2,\sigma(v_1\otimes v_2)) =\lambda(\beta(v_1\otimes v_2))$ for all $v_1\otimes v_2 \in T_1\otimes T_2$

We write $\sigma(v_1\otimes v_2) =(v_1\otimes v_2)^{-1}$ it refers to the invers element of $v_1\otimes v_2 \in T_1\otimes T_2$ and $\lambda(x)= \tilde{x}$ is said to be the unite an item in $T_1\otimes T_2$ associaated to the element $x \in Q_1\otimes Q_2$.Also we write

$\gamma(v_1\otimes v_2,v_1'\otimes v_2')= (v_1\otimes v_2)(v_1'\otimes v_2')$, $T_1\otimes T_2$ it refers to the groupoid and $Q_1\otimes Q_2$ is said to be the base

One more time , we say that $T_1\otimes T_2$ is a groupoid on $Q_1\otimes Q_2$.

(1.8) Corollary:

Let $(T_1,Q_1),\ldots,(T_n,Q_n)$ are groupoids then the Tensor product $(\otimes_{i=1}^{n} T_i, \otimes_{i=1}^{n} Q_i)$ is a groupoid .

Proof:

Same the prove of theorem (1.7) .

(1.9) Definition(8)

Let A and B are both a subgroupoids of T (where T be a groupoid) then the direct sum $A\oplus B$ satisfy the following : (1) A+B = T , (2) $A\cap B=\{0\}$.

(1.7) Theorem:

Let (T_1,Q_1) and (T_2,Q_2) are both groupoids then the direct sum $(T_1\oplus T_2,Q_1\oplus Q_2)$ be a groupoid .

Proof:

(1) $\alpha,\beta:T_1\oplus T_2 \to Q_1\oplus Q_2$

$\alpha(v_1\oplus v_2)=w_1\oplus w_2$, $\beta(v_1\oplus v_2)=w_1\oplus w_2$,for all $v_1\oplus v_2 \in T_1\oplus T_2$ and $w_1\oplus w_2 \in Q_1\oplus Q_2$,

(2) $\lambda:Q_1\oplus Q_2\to T_1\oplus T_2$

$\lambda(w_1\oplus w_2) = v_1\oplus v_2$,
$(\alpha o\lambda)(w_1\oplus w_2)=\alpha(\lambda(w_1\oplus w_2))=\alpha(v_1\oplus v_2)=(w_1\oplus w_2)$

$(\beta o\lambda)(w_1\oplus w_2) = \beta(\lambda(w_1\oplus w_2)) = \beta(v_1\oplus v_2) = (w_1\oplus w_2)$,

Where $\alpha o\lambda= I_{Q_1\oplus Q_2}$,$\beta o\lambda= I_{Q_1\oplus Q_2}$ and $I_{Q_1\oplus Q_2}:Q_1\oplus Q_2\to Q_1\oplus Q_2$

(3) $(T_1\oplus T_2)*(T_1\oplus T_2) =\{(v_1\oplus v_2,v_1' \oplus v_2') \in (T_1\oplus T_2)\times (T_1\oplus T_2)$:

$\alpha(v_1\oplus v_2)=\beta(v_1' \oplus v_2')\}$ " fibre product of α and β over $Q_1\oplus Q_2$"

Such that

(a) $\gamma(v\oplus v, \gamma(v_1\oplus v_1,v_2\oplus v_2)) =\gamma(v\oplus v, v_2\oplus v_2)= v_2\oplus v_2$
$\gamma(\gamma(v\oplus v, v_1\oplus v_1), v_2\oplus v_2)=\gamma(v_1\oplus v_1,v_2\oplus v_2) = v_2\oplus v_2$

So we have $\gamma(v\oplus v, \gamma(v_1\oplus v_1,v_2\oplus v_2)) = \gamma(\gamma(v\oplus v, v_1\oplus v_1), v_2\oplus v_2)$

For all $(v\oplus v, v_1\oplus v_1)$, $(v_1\oplus v_1,v_2\oplus v_2)\in (T_1\oplus T_2)*(T_1\oplus T_2)$

(b) $\alpha(\gamma(v_1\oplus v_1,v_2\oplus v_2)) =\alpha(v_2\oplus v_2)= w_2\oplus w_2$
$\alpha(v_2\oplus v_2) = w_2\oplus w_2$

So we get $\alpha(\gamma(v_1\oplus v_1,v_2\oplus v_2))= \alpha(v_2\oplus v_2)$, $\beta(\gamma(v_1\oplus v_1,v_2\oplus v_2)) = \beta(v_1\oplus v_1) = w_1\oplus w_1$
$\beta(v_1\oplus v_1) = w_1\oplus w_1$

So we get , $\beta(\gamma(v_1\oplus v_1,v_2\oplus v_2)) = \beta(v_1\oplus v_1)$ for each $(v_1\oplus v_1,v_2\oplus v_2)\in (T_1\oplus T_2)*(T_1\oplus T_2)$

(c) $\gamma(v_1\oplus v_2,\lambda(\alpha(v_1\oplus v_2))) = \gamma(v_1\oplus v_2, \lambda(w_1\oplus w_2)) = \gamma(v_1\oplus v_2, v_1\oplus v_2) = v_1\oplus v_2$
$\gamma(\lambda(\beta(v_1\oplus v_2)), v_1\oplus v_2) = \gamma(\lambda(w_1\oplus w_2), v_1\oplus v_2) =\gamma(v_1\oplus v_2, v_1\oplus v_2) =(v_1\oplus v_2)$
, for all $v_1\oplus v_2 \in T_1\oplus T_2$

(4) $\sigma: T_1\oplus T_2 \to T_1\oplus T_2$

(a) $\alpha(\sigma(v_1\oplus v_2))= \alpha(v_1\oplus v_2) = w_1\oplus w_2$, $\beta(v_1\oplus v_2)= w_1\oplus w_2$

We conclude $\alpha(\sigma(v_1\oplus v_2))= \beta(v_1\oplus v_2)$, $\beta(\sigma(v_1\oplus v_2))= \beta(v_1\oplus v_2) =w_1\oplus w_2$, $\alpha(v_1\oplus v_2) =w_1\oplus w_2$

For any $v_1\oplus v_2 \in T_1\oplus T_2$

(b) $\gamma(\sigma(v_1\oplus v_2), v_1\oplus v_2) =\gamma(v_1\oplus v_2, v_1\oplus v_2) =v_1\oplus v_2$, $\lambda(\alpha(v_1\oplus v_2)) =\lambda(w_1\oplus w_2) = v_1\oplus v_2$

So we get $\gamma(\sigma(v_1 \oplus v_2), v_1 \oplus v_2) = \lambda(\alpha(v_1 \oplus v_2))$,
And $\gamma(v_1 \oplus v_2, \sigma(v_1 \oplus v_2)) = \gamma(v_1 \oplus v_2, v_1 \oplus v_2) = v_1 \oplus v_2$, $\lambda(\beta(v_1 \oplus v_2)) = \lambda(w_1 \oplus w_2) = v_1 \oplus v_2$
Hence we have $\gamma(v_1 \oplus v_2, \sigma(v_1 \oplus v_2)) = \lambda(\beta(v_1 \oplus v_2))$ for all $v_1 \oplus v_2 \in T_1 \oplus T_2$
We write $\sigma(v_1 \oplus v_2) = (v_1 \oplus v_2)^{-1}$ it refers to the invers element of $v_1 \oplus v_2 \in T_1 \oplus T_2$ and $\lambda(x) = \tilde{x}$ is said to be the unite an item in $T_1 \oplus T_2$ associaated to the element $x \in Q_1 \oplus Q_2$.Also we write

$\gamma(v_1 \oplus v_2, v_1' \oplus v_2') = (v_1 \oplus v_2)(v_1' \oplus v_2')$, $T_1 \oplus T_2$ it refers to the groupoid and $Q_1 \oplus Q_2$ is said to be the base

One more time , we say that $T_1 \oplus T_2$ is a groupoid on $Q_1 \oplus Q_2$.

(1.11) Corollary:

Let $(T_1,Q_1),\ldots,(T_n,Q_n)$ are groupoids then the Tensor product $(\oplus_{i=1}^{n} T_i, \oplus_{i=1}^{n} Q_i)$ is a groupoid .

Proof :

Same way the proof of theorem (1.10) .

(1.12)Definition(9)

Let T_1 and T_2 are both a groupoids then the Cartesian Products of groupoids define by $T_1 \times T_2 = \{(a,b): a \in T_1 , b \in T_2\}$.

(1.13) Theorem:

Let (T_1,Q_1) and (T_2,Q_2) are both groupoids then the Cartesian Products $(T_1 \times T_2, Q_1 \times Q_2)$ be a groupoid .

Proof:

(1) $\alpha,\beta: T_1 \times T_2 \to Q_1 \times Q_2$
$\alpha(v_1, v_2) = (w_1, w_2)$, $\beta(v_1, v_2) = (w_1, w_2)$,for all $(v_1, v_2) \in T_1 \times T_2$ and $(w_1, w_2) \in Q_1 \times Q_2$,
(2) $\lambda: Q_1 \times Q_2 \to T_1 \times T_2$
$\lambda(w_1, w_2) = (v_1, v_2)$,
$(\alpha o \lambda)(w_1, w_2) = \alpha(\lambda(w_1, w_2)) = \alpha(v_1, v_2) = (w_1, w_2)$
$(\beta o \lambda)(w_1, w_2) = \beta(\lambda(w_1, w_2)) = \beta(v_1, v_2) = (w_1, w_2)$,
Where $\alpha o \lambda = I_{Q_1 \times Q_2}$,$\beta o \lambda = I_{Q_1 \times Q_2}$ and $I_{Q_1 \times Q_2}$: $Q_1 \times Q_2 \to Q_1 \times Q_2$
(3) $(T_1 \times T_2) * (T_1 \times T_2) = \{((v,v),(v',v')) \in (T_1 \times T_2) \times (T_1 \times T_2)$:

$\alpha(v,v) = \beta(v',v')\}$ “ fiber product of α and β over $Q_1 \times Q_2$”

Such that
(a) $\gamma((v,v), \gamma((v_1,v_1),(v_2,v_2))) = \gamma((v,v),(v_2,v_2)) = (v_2,v_2)$
$\gamma(\gamma((v,v),(v_1,v_1)),(v_2,v_2)) = \gamma((v_1,v_1),(v_2,v_2)) = v_2,v_2)$
So we have $\gamma((v,v), \gamma((v_1,v_1),(v_2,v_2))) = \gamma(\gamma((v,v),(v_1,v_1)),(v_2,v_2))$
For all $((v,v),(v_1,v_1))$, $((v_1,v_1),(v_2,v_2)) \in (T_1 \times T_2) * (T_1 \times T_2)$
(b) $\alpha(\gamma((v_1,v_1),(v_2,v_2))) = \alpha(v_2,v_2) = (w_2,w_2)$
$\alpha(v_2,v_2) = (w_2,w_2)$
So we get $\alpha(\gamma((v_1,v_1),(v_2,v_2))) = \alpha(v_2,v_2)$,
$\beta(\gamma((v_1,v_1),(v_2,v_2))) = \beta(v_1,v_1) = (w_1,w_1)$
$\beta(v_1,v_1) = (w_1,w_1)$
So we get , $\beta(\gamma((v_1,v_1),(v_2,v_2))) = \beta(v_1,v_1)$ for each $((v_1,v_1),(v_2,v_2)) \in (T_1 \times T_2) * (T_1 \times T_2)$
(c) $\gamma((v_1,v_2), \lambda(\alpha(v_1,v_2))) = \gamma((v_1,v_2), \lambda(w_1,w_2)) = \gamma((v_1,v_2),(v_1,v_2)) = (v_1,v_2)$
$\gamma(\lambda(\beta(v_1,v_2)),(v_1,v_2)) = \gamma(\lambda(w_1,w_2),(v_1,v_2)) = \gamma((v_1,v_2),(v_1,v_2)) = (v_1,v_2)$
, for all $(v_1,v_2) \in T_1 \times T_2$
(4) $\sigma: T_1 \times T_2 \to T_1 \times T_2$
(a) $\alpha(\sigma(v_1,v_2)) = \alpha(v_1,v_2) = (w_1,w_2)$, $\beta(v_1,v_2) = (w_1,w_2)$
We conclude $\alpha(\sigma(v_1,v_2)) = \beta(v_1,v_2)$,
$\beta(\sigma(v_1,v_2)) = \beta(v_1,v_2) = (w_1,w_2)$, $\alpha(v_1,v_2) = (w_1,w_2)$
For any $(v_1,v_2) \in T_1 \times T_2$
(b) $\gamma(\sigma(v_1,v_2),(v_1,v_2)) = \gamma((v_1,v_2),(v_1,v_2)) = (v_1,v_2)$, $\lambda(\alpha(v_1,v_2)) = \lambda(w_1,w_2) = (v_1,v_2)$
So we get $\gamma(\sigma(v_1,v_2),(v_1,v_2)) = \lambda(\alpha(v_1,v_2))$,
And $\gamma((v_1,v_2),\sigma(v_1,v_2)) = \gamma((v_1,v_2),(v_1,v_2)) = (v_1,v_2)$, $\lambda(\beta(v_1,v_2)) = \lambda(w_1,w_2) = (v_1,v_2)$
Hence we have $\gamma((v_1,v_2),\sigma(v_1,v_2)) = \lambda(\beta(v_1,v_2))$ for all $(v_1,v_2) \in T_1 \times T_2$
We write $\sigma(v_1,v_2) = (v_1, v_2)^{-1}$ it refers to the invers element of $(v_1,v_2) \in T_1 \times T_2$ and $\lambda(x) = \tilde{x}$ is said to be the unite an item in $T_1 \times T_2$ associaated to the element $x \in Q_1 \times Q_2$.Also we write

$\gamma((v_1,v_2),(v_1',v_2')) = (v_1,v_2)(v_1',v_2')$, $T_1 \times T_2$ it refers to the groupoid and $Q_1 \times Q_2$ is said to be the base

One more time , we say that $T_1 \times T_2$ is a groupoid on $Q_1 \times Q_2$.

(1.14) Corollary:

Let $(T_1,Q_1),\ldots,(T_n,Q_n)$ are groupoids then the Cartisean product $(\times_{i=1}^{n} T_i, \times_{i=1}^{n} Q_i)$ is a groupoid .
Proof:

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The prove same the way of proposition (1.13) .

(1.15) Theorem

Let (T_1,Q_1) and (T_2,Q_2) are both a groupoids then

(1) $((T_1\otimes T_2)\oplus(T_1\otimes T_2), (Q_1\otimes Q_2)\oplus(Q_1\otimes Q_2))$

(2) $((T_1\oplus T_2)\otimes(T_1\oplus T_2),(Q_1\oplus Q_2)\otimes(Q_1\oplus Q_2)$ are both groupoid

Proof : of (1)

(1) $\alpha,\beta: (T_1\otimes T_2)\oplus(T_1\otimes T_2) \to (Q_1\otimes Q_2)\oplus(Q_1\otimes Q_2)$, $\alpha((v_1\otimes v_2)\oplus(v_1\otimes v_2)) = ((w_1\otimes w_2)\oplus(w_1\otimes w_2))$

$\beta((v_1\otimes v_2)\oplus(v_1\otimes v_2)) = ((w_1\otimes w_2)\oplus(w_1\otimes w_2))$

(2) $\lambda: (Q_1\otimes Q_2)\oplus(Q_1\otimes Q_2) \to (T_1\otimes T_2)\oplus(T_1\otimes T_2)$,$\lambda((w_1\otimes w_2)\oplus(w_1\otimes w_2)) = ((v_1\otimes v_2)\oplus(v_1\otimes v_2))$

$(\alpha o\lambda)((w_1\otimes w_2)\oplus(w_1\otimes w_2))=\alpha(\lambda((w_1\otimes w_2)\oplus(w_1\otimes w_2))= \alpha((v_1\otimes v_2)\oplus(v_1\otimes v_2)) = ((w_1\otimes w_2)\oplus(w_1\otimes w_2))$

$(\beta o\lambda)((w_1\otimes w_2)\oplus(w_1\otimes w_2))=\beta(\lambda((w_1\otimes w_2)\oplus(w_1\otimes w_2)))= \beta((v_1\otimes v_2)\oplus(v_1\otimes v_2)) =((w_1\otimes w_2)\oplus(w_1\otimes w_2))$

So $\alpha o\lambda= I_{(Q_1\otimes Q_2)\oplus(Q_1\otimes Q_2)}$ and $\beta o\lambda= I_{(Q_1\otimes Q_2)\oplus(Q_1\otimes Q_2)}$

And $I_{(Q_1\otimes Q_2)\oplus(Q_1\otimes Q_2)}$: $(Q_1\otimes Q_2)\oplus(Q_1\otimes Q_2)\to (Q_1\otimes Q_2)\oplus(Q_1\otimes Q_2)$

(3) $(T_1\otimes T_2)\oplus(T_1\otimes T_2) *(T_1\otimes T_2)\oplus(T_1\otimes T_2) = \{(((v_1\otimes v_2)\oplus(v_1\otimes v_2)), ((v_1' \otimes v_2')\oplus(v_1' \otimes v_2'))) \in$
$((T_1\otimes T_2)\oplus(T_1\otimes T_2))\times((T_1\otimes T_2)\oplus(T_1\otimes T_2))$: $\alpha((v_1\otimes v_2)\oplus(v_1\otimes v_2)) =\beta((v_1' \otimes v_2')\oplus(v_1' \otimes v_2'))\}$ " fibre product of α and β over $((Q_1\otimes Q_2)\oplus(Q_1\otimes Q_2))$ "such that

(a) $\gamma(((v\otimes v)\oplus(v\otimes v)),\gamma(((v_1\otimes v_2)\oplus(v_1\otimes v_2)), ((v_1' \otimes v_2')\oplus(v_1' \otimes v_2')))) =$

$\gamma(((v\otimes v)\oplus(v\otimes v)),((v_1' \otimes v_2')\oplus(v_1' \otimes v_2'))) = ((v_1' \otimes v_2')\oplus(v_1' \otimes v_2'))$

And

$\gamma(\gamma(((v\otimes v)\oplus(v\otimes v)),((v_1\otimes v_2)\oplus(v_1\otimes v_2))),(((v_1' \otimes v_2')\oplus(v_1' \otimes v_2'))) =$

$\gamma(((v_1\otimes v_2)\oplus(v_1\otimes v_2)),((v_1' \otimes v_2')\oplus(v_1' \otimes v_2'))) = ((v_1' \otimes v_2')\oplus(v_1' \otimes v_2'))$

So we get

$\gamma(((v\otimes v)\oplus(v\otimes v)),\gamma(((v_1\otimes v_2)\oplus(v_1\otimes v_2)), ((v_1' \otimes v_2')\oplus(v_1' \otimes v_2')))) = \gamma(\gamma(((v\otimes v)\oplus(v\otimes v)),((v_1\otimes v_2)\oplus(v_1\otimes v_2))),(((v_1' \otimes v_2')\oplus(v_1' \otimes v_2')))$

For all $((v\otimes v)\oplus(v\otimes v)), ((v_1\otimes v_2)\oplus(v_1\otimes v_2)), (((v_1\otimes v_2)\oplus(v_1\otimes v_2)),((v_1' \otimes v_2')\oplus(v_1' \otimes v_2')))$

$\in(T_1\otimes T_2\oplus T_1\otimes T_2)*(T_1\otimes T_2\oplus T_1\otimes T_2)$

(b)
$\alpha(\gamma(((v_1\otimes v_2)\oplus(v_1\otimes v_2)),((v_1' \otimes v_2')\oplus (v_1' \otimes v_2')))) = \alpha((v_1' \otimes v_2')\oplus(v_1' \otimes v_2'))$
$= ((w_1'\otimes w_2')\oplus(w_1'\otimes w_2'))$
And $\alpha((v_1' \otimes v_2')\oplus(v_1' \otimes v_2'))$
$= ((w_1'\otimes w_2')\oplus(w_1'\otimes w_2'))$
So we get
$\alpha(\gamma(((v_1\otimes v_2)\oplus(v_1\otimes v_2)),((v_1' \otimes v_2')\oplus(v_1' \otimes v_2')))) = \alpha((v_1' \otimes v_2')\oplus(v_1' \otimes v_2'))$
$\beta(\gamma(((v_1\otimes v_2)\oplus(v_1\otimes v_2)),((v_1' \otimes v_2')\oplus(v_1' \otimes v_2')))) = \beta((v_1\otimes v_2)\oplus(v_1\otimes v_2)) = ((w_1\otimes w_2)\oplus(w_1\otimes w_2))$
$\beta((v_1\otimes v_2)\oplus(v_1\otimes v_2)) =((w_1\otimes w_2)\oplus(w_1\otimes w_2))$
So we get
$\beta(\gamma(((v_1\otimes v_2)\oplus(v_1\otimes v_2)),((v_1' \otimes v_2')\oplus(v_1' \otimes v_2'))))= \beta((v_1\otimes v_2)\oplus(v_1\otimes v_2))$
For all
$(((v_1\otimes v_2)\oplus(v_1\otimes v_2)),((v_1' \otimes v_2')\oplus(v_1' \otimes v_2'))) \in (T_1\otimes T_2\oplus T_1\otimes T_2)*(T_1\otimes T_2\oplus T_1\otimes T_2)$

(c) $\gamma(((v_1\otimes v_2)\oplus(v_1\otimes v_2)), \lambda(\alpha((v_1\otimes v_2)\oplus(v_1\otimes v_2)))) =$

$\gamma(((v_1\otimes v_2)\oplus(v_1\otimes v_2)), \lambda((w_1\otimes w_2)\oplus(w_1\otimes w_2))) = \gamma(((v_1\otimes v_2)\oplus(v_1\otimes v_2)),((v_1\otimes v_2)\oplus(v_1\otimes v_2)))$

$=((v_1\otimes v_2)\oplus(v_1\otimes v_2))$

$\gamma(\lambda(\beta((v_1 \otimes v_2) \oplus (v_1 \otimes v_2))),$
$((v_1 \otimes v_2) \oplus (v_1 \otimes v_2))) =$
$\gamma(\lambda((w_1 \otimes w_2) \oplus (w_1 \otimes w_2)),((v_1 \otimes v_2) \oplus (v_1 \otimes v_2)))$

$= \gamma(((v_1 \otimes v_2) \oplus (v_1 \otimes v_2)),((v_1 \otimes v_2) \oplus (v_1 \otimes v_2))) =$
$((v_1 \otimes v_2) \oplus (v_1 \otimes v_2))$

For all $((v_1 \otimes v_2) \oplus (v_1 \otimes v_2)) \in (T_1 \otimes T_2 \oplus T_1 \otimes T_2)$

(4) $\sigma : (T_1 \otimes T_2) \oplus (T_1 \otimes T_2) \to (T_1 \otimes T_2) \oplus (T_1 \otimes T_2)$

(a) $\alpha(\sigma((v_1 \otimes v_2) \oplus (v_1 \otimes v_2)))$
$= \alpha(v_1 \otimes v_2) \oplus (v_1 \otimes v_2) = ((w_1 \otimes w_2) \oplus (w_1 \otimes w_2))$

$\beta((v_1 \otimes v_2) \oplus (v_1 \otimes v_2)) = ((w_1 \otimes w_2) \oplus (w_1 \otimes w_2))$

So we have $\alpha(\sigma((v_1 \otimes v_2) \oplus (v_1 \otimes v_2))) =$
$\beta((v_1 \otimes v_2) \oplus (v_1 \otimes v_2))$ for any
$((v_1 \otimes v_2) \oplus (v_1 \otimes v_2)) \in (T_1 \otimes T_2 \oplus T_1 \otimes T_2)$

(b) $\gamma(\sigma((v_1 \otimes v_2) \oplus (v_1 \otimes v_2))),$
$((v_1 \otimes v_2) \oplus (v_1 \otimes v_2))) =$
$\gamma((v_1 \otimes v_2) \oplus (v_1 \otimes v_2)),((v_1 \otimes v_2) \oplus (v_1 \otimes v_2)))$

$= ((v_1 \otimes v_2) \oplus (v_1 \otimes v_2))$

$\lambda(\alpha((v_1 \otimes v_2) \oplus (v_1 \otimes v_2))) =$
$\lambda((w_1 \otimes w_2) \oplus (w_1 \otimes w_2)) = ((v_1 \otimes v_2) \oplus (v_1 \otimes v_2))$

So we get $\gamma(\sigma(((v_1 \otimes v_2) \oplus (v_1 \otimes v_2))$
$),((v_1 \otimes v_2) \oplus (v_1 \otimes v_2))) =$
$\lambda(\alpha((v_1 \otimes v_2) \oplus (v_1 \otimes v_2)))$

And
$\gamma(\sigma((v_1 \otimes v_2) \oplus (v_1 \otimes v_2))), ((v_1 \otimes v_2) \oplus (v_1 \otimes v_2)))$
$= \gamma(((v_1 \otimes v_2) \oplus (v_1 \otimes v_2)),((v_1 \otimes v_2) \oplus (v_1 \otimes v_2)))$

$= ((v_1 \otimes v_2) \oplus (v_1 \otimes v_2))$

$\lambda(\beta((v_1 \otimes v_2) \oplus (v_1 \otimes v_2))) =$
$\lambda(w_1 \otimes w_2) \oplus (w_1 \otimes w_2) = ((v_1 \otimes v_2) \oplus (v_1 \otimes v_2))$

So we get
$\gamma(\sigma((v_1 \otimes v_2) \oplus (v_1 \otimes v_2))), ((v_1 \otimes v_2) \oplus (v_1 \otimes v_2)))=$
$\lambda(\beta((v_1 \otimes v_2) \oplus (v_1 \otimes v_2)))$

For all $((v_1 \otimes v_2) \oplus (v_1 \otimes v_2)) \in (T_1 \otimes T_2 \oplus T_1 \otimes T_2)$

$\sigma((v_1 \otimes v_2) \oplus (v_1 \otimes v_2)) = ((v_1 \otimes v_2) \oplus (v_1 \otimes v_2))^{-1}$
called the inverse element of $((v_1 \otimes v_2) \oplus (v_1 \otimes v_2))$

$\in (T_1 \otimes T_2 \oplus T_1 \otimes T_2)$ and $\lambda(x) = \tilde{x}$ refers to the unite element in $(T_1 \otimes T_2) \oplus (T_1 \otimes T_2)$

associated to the element
$x \in ((Q_1 \otimes Q_2) \oplus (Q_1 \otimes Q_2))$.

Also we rewrite
$\gamma(((v_1 \otimes v_2) \oplus (v_1 \otimes v_2)),((v_1' \otimes v_2') \oplus (v_1' \otimes v_2')))$
$= (v_1 \otimes v_2) \oplus (v_1 \otimes v_2))((v_1' \otimes v_2') \oplus (v_1' \otimes v_2'))$.

$(T_1 \otimes T_2 \oplus T_1 \otimes T_2)$ it refers to the groupoid and $(Q_1 \otimes Q_2) \oplus (Q_1 \otimes Q_2)$ is called base .

Also we say that $(T_1 \otimes T_2 \oplus T_1 \otimes T_2)$ is a groupoid on $((Q_1 \otimes Q_2) \oplus (Q_1 \otimes Q_2))$.

The prove of **(2)** same the prove of **(1)** .

2.

Subgroupoid , normal subgroupoid , morphism of groupoid , Krenal .

(2.1) Definition(5)

"Let (T,Q) be any groupoid .A subgroupoid of (T,Q) is a pair (H,A) of sub sets, $H \subseteq T$ and $A \subseteq Q$ with

$\alpha(H) \subseteq A$,$\beta(H) \subseteq A$, $\lambda(A) \subseteq H$ and H is closed under partial composition γ and inversion σ of T . In other

word H is a groupoid of base A such that"

(i) $\alpha_H = \alpha|_H$, $\beta_H = \beta|_H$ and $\lambda_A = \lambda|_A$.

(ii) $\gamma_H = \gamma|_{H*H}$; $\sigma_H = \sigma|_H$.

and a subgroupoid (H,A) of a groupoid (T,Q) is called wide if A= Q .

(2.2) Example(5)

Let (T,Q) be any groupoid then a set $\tilde{Q} = \{\tilde{x} : x \in Q\} = \lambda(Q)$ is a wide subgroupoid of T that is called the

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base subgroupoid of (T,Q) .

(2.3) Definition(3)

Let (T,Q) be any groupoid . A normal subgroupoid of (T,Q) is a wide subgroupoid H such that for

all h∈H and t∈T with α(t)=α(h)=β(h) we have tht^{-1}∈H .

(2.4) Definition(7)

"A morphism of groupoid is a pair of maps (f,f_0):(T,Q)→(T′,Q′) such that α′of=f_0oα ,β′of=f_0oβ and f(γ(t,t′))=γ′(f(t),f(t′)) for all (t,t′)∈T∗T" . An isomorphism of groupoids is a morphism of groupoids such that

f :T → T′ is bijeective map

(2.5) Proposition

Let f':(f,f_0): (T_1,Q_1) →(T_2,Q_2) be a morrphism and Let g' :(g,g_0) : (T_2,Q_2)⟶(T_3,Q_3) be a morphim

then $g'of'$: (T_1,Q_1) →(T_3,Q_3) is a morphiism .

Proof:

Since f′ and g' are both a morphism then

α′of =f_0 oα and β′of=f_0 oβ , and

f_0 oα =β′of , then α′of= f_0 oβ

so $g'$of′ it's a morphism .

(2.6)corollary

Let g:(f_1,f_1'):(T_1,Q_1)→(T_2,Q_2) and g_1: (T_2,Q_2)→(T_3,Q_3) …. g_n:(T_{n-1},Q_{n-1})→(T_n,Q_n) are a morphism

then the composition $g_n o g_{n-1} o \ldots o g$:S→$S_n$ be a morphism .

Proof

The prove same the way of proposition **(2.5)** .

(2.7)Definition(6)

"Let (f,f_0): (T,Q)→(T′,Q′) be a morphism of groupoids then

(i) The kernal of f is the set kerf={t∈T ∣ f(t)∈λ′(Q′)}.

(ii) If Q=Q′ and $f_0=I_Q$ then f is called a morphism over Q".

(2.8) Remark(6)

If (f,f_0): (T,Q)→(T′,Q′) be a morphism of groupoids then the kerf is normal subgroupoid of T but f(T)

is not necessarily subgroupoid of T′ since whenever γ′(f(t),f(t′)) is defined in T′ ,γ(t ,t′) is not necessarily

the base map f_0 :Q→Q′ is injective .Also (f, f_0) induces a homomorphism of group say $_xf_x$: $_xT_x$ →$_{f0(x)}T'_{f0(x)}$,

for all x∈T.

(2.9) Definition(4)

Let (T,Q) be any groupoid ,we say that T is transitive groupoid if transitor τ:T→Q×Q ,τ(t)=(β(t),α(t)) is

surjective map i.e. agroupoid is transitive if any two points of its base can be joined by an element of

the groupoid .

(2.10) Example(4)

Any trivial groupoid its transitive groupoid

Solution:

Let ή be a group and T = Q×ή×Q is a groupoid

α,β: Q×ή×Q → Q

α(q,x,q)=q , β(q,x,q)=q

τ(q,x,q)=(q,q)

we have τ(q,x,q)=(β(q),α(q))

So any trivial agroupoid its transitive groupoid .

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أنواع معينه من الفئات الزمروية

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المستخلص:

نحاول الاهتمام في هذا البحث و النظر لأهميته في الحصول على بعض النتائج و النظريات و نسعى لإعطاء بعض الأمثلة و توضيحها للفئه الزمرويه .

الكلمات المفتاحية:

الفئات الزمروية ، فئة دسكاريتس الزمروية ، الجمع المباشر لفئتين زمروية ، الضرب التنسوري لفئتين زمروية ، فعل الفئة الزمروية .