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$L(w\mathcal{C})$-spaces and Some of its Weak Forms

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Abstract:

In this paper, we provide a new generalization of $L\mathcal{C}$-spaces which is $L(w\mathcal{C})$-spaces, $wL(w\mathcal{C})$-spaces also another weak forms of $L(w\mathcal{C})$-spaces which is called wL_i-spaces, $(i = 1,2,3,4)$. In addition, we give the relationships between these new types and studied the heredity property for each type.

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Lindelof space, $L\mathcal{C}$-space, closed set, w-closed set, w-continuous function.

1-Introduction

The concept of Lindelof space was introduced in 1929 by Alexandroof and Urysohn, since there is no relation between Lindelof space and closed sets, so this point stimulated some researchers to introduce a new concept namely $\mathcal{LC}$-space.

The notion of $\mathcal{LC}$-space was first introduced in 1979 by Mukherji and Sarkar [15], that is "Every Lindelof subsets are closed", some authors name it L-closed space [7], [14] and [18]. $\mathcal{LC}$-space different from Lindelof space and there is no relation between Lindelof and $\mathcal{LC}$-space.

Near closed sets has an important role in topological spaces as a generalized of closed sets. Hdeib in 1982 [10] introduce the concept of $\mathcal{w}$-closed set that is " A subset $\mathcal{N}$ is called $\mathcal{w}$-closed , if $\mathcal{N}$ contains all its condensation points.The family of all $\mathcal{W}$-open subset of a space $\mathcal{X}$ denoted by $\mathcal{T}_w$ forms a topology on $\mathcal{X}$ which is finer than $\mathcal{T}$, several characterization and facts of $\mathcal{W}$-closed subset where provided in [3], [1]and [11].

We introduce in this work a new concept which is $\mathrm{L}(\mathcal{wC})$-space that is every Lindelof set in $\mathcal{X}$ is $\mathcal{w}$-closed, and $\mathrm{L}(\mathcal{wC})$-space is a generalized of $\mathcal{LC}$-space also we provided weak forms of $\mathrm{L}(\mathcal{wC})$-space namely $\mathcal{w}\mathrm{L}_i$-space, $i = 1,2,3,4$, and introduce the relationships between themselves also with $\mathrm{L}(\mathcal{wC})$-space and then define $\mathcal{w}\mathrm{L}(\mathcal{wC})$-space which is weaker form of $\mathrm{L}(\mathcal{wC})$-space, several properties and theorem which link between those concepts. Finally, we give several facts and examples to support this concepts.

2-*Preliminaries*

Definition 2.1 [10] Let $\mathcal{N}$ be a subset of a space $\mathcal{X}$, a point $e \in \mathcal{X}$ is called condensation point of $\mathcal{N}$, if for any open set $\mathcal{V}$ and $e \in \mathcal{V}$, the set $\mathcal{V} \cap \mathcal{N}$ is uncountable, if $\mathcal{N}$ contains all its condensation points then it is $\mathcal{w}$-closed.

Definition 2.2 [11] A subset $\mathcal{U}$ of a space $\mathcal{X}$ is said to be $\mathcal{w}$-open set iff for each $x \in \mathcal{U}$ there is $\mathcal{V}$ in $\mathcal{T}$, $x \in \mathcal{V}$ and $\mathcal{V} - \mathcal{U}$ is countable.

Remark 2.3 [16] Every closed (resp., open) set is $\mathcal{w}$-closed (resp.,$\mathcal{w}$- open) set, but the convers is not true.

Example 2.4 Let $(\mathcal{R}, \mathcal{T}_{ind})$ be indiscrete space on a real line $\mathcal{R}$, the set of all irrational numbers $\mathbb{Q}^c$ which is subset of $\mathcal{R}$ $\mathcal{R}$, $\mathbb{Q}^c$ is $\mathcal{W}$-open set but not open. Also the rational numbers $\mathbb{Q}$ is $\mathcal{w}$-closed but not closed.

Definition 2.5 [6] A subset $\mathcal{N}$ of a space $\mathcal{X}$ is said to be clopen set if it is open and closed in $\mathcal{X}$.

Definition 2.6 [10] Let $(\mathcal{X}, \mathcal{T}_w)$ be a topological space and $\mathcal{A}$ be a subset of $\mathcal{X}$, then

1. .
2. .

Proposition 2.7 [9] If $(\mathcal{X}, \mathcal{T})$ be a topological space. $\mathcal{A}$,$\mathcal{B}$ subsets of $\mathcal{X}$, then

1. is an $\mathcal{w}$-open subset of $\mathcal{X}$.
2. is an $\mathcal{w}$-open set if and only if $\mathcal{A} = Int_w(\mathcal{A})$.
3. $\mathcal{A}$ and $Int(\mathcal{A}) \subseteq Int_w(\mathcal{A})$.
4. is an $\mathcal{w}$-closed set.
5. is an $\mathcal{w}$-closed set if and only if $\mathcal{A} = Cl_w(\mathcal{A})$.
6. $Cl_w(\mathcal{A})$ and $Cl_w(\mathcal{A}) \subseteq Cl(\mathcal{A})$.

Definition 2.8 [16] A subset $\mathcal{A}$ of a space $\mathcal{X}$ is said to be an $\mathcal{W}$-set if $\mathcal{A}=\mathcal{U}\cap\mathcal{V}$, where $Int(\mathcal{V}) = Int_{\mathcal{W}}(\mathcal{V})$ and $\mathcal{U}$ is open set.

Remark 2.9 [16] Let $\mathcal{N}$ be a subset of a space $\mathcal{X}$, then $\mathcal{N}$ is open if and only if $\mathcal{N}$ is w-open and w-set.

Example 2.10 Let $(\mathcal{R}, T_{\mathcal{U}})$ be a usual space on a real line $\mathcal{R}$, the set of all irrational numbers $\mathbb{Q}^c$ is w-open set (since for each $x \in \mathbb{Q}^c$ there is open set $\mathcal{U}$ subset of $\mathcal{R}$ containing x such that $\mathcal{U} - \mathbb{Q}^c$ is countable such as $\sqrt{2} \in \mathbb{Q}^c$ and $(0,2) \subseteq \mathcal{R}$ and $\sqrt{2} \in (0,2)$ and $(0,2) - \mathbb{Q}^c = \mathbb{Q}$ is countable), but $\mathbb{Q}^c$ is not $\mathcal{W}$-set since $\mathbb{Q}^c = \mathcal{R} \cap \mathbb{Q}^c$, where $\mathcal{R}$ is open set but $Int(\mathbb{Q}^c) = \emptyset \neq \mathbb{Q}^c = Int_w(\mathbb{Q}^c)$, since $\mathbb{Q}^c$ is w-open set, hence $\mathbb{Q}^c$ is not open set.

Proposition 2.11 [20] If $\mathcal{N}$ is an w-closed (resp., w-open) subset of $\mathcal{X}$ and $\mathcal{A} \subseteq \mathcal{X}$ then $\mathcal{A} \cap \mathcal{N}$ is an w-closed (resp., w-open) subset of $\mathcal{A}$.

Proposition 2.12 [20] Let $\mathcal{P}$ be an w-closed (resp., w-open) of a space $\mathcal{X}$. If $\mathcal{N}$ is w-closed (resp., w-open) set in $\mathcal{P}$, then $\mathcal{N}$ is w-closed (resp., w-open) set in $\mathcal{X}$. Remark 2.13 [20] If $\mathcal{X}$ is a space and $\mathcal{G}$ is a subspace of $\mathcal{X}$ such that $\mathcal{B} \subseteq \mathcal{G}$ and $\mathcal{B}$ is w-closed (resp., w-open) subset in $\mathcal{X}$. Then $\mathcal{B}$ is w-closed (resp., w-open) set in $\mathcal{G}$.

Definition 2.14 [3] A space $\mathcal{X}$ is called anti-locally countable space if each open set is an uncountable set.

Proposition 2.15[16] If a space $\mathcal{X}$ is anti-locally countable space then:

1. $(\mathcal{S})$, for any w-closed set $\mathcal{S}$ in $\mathcal{X}$.

2. $(\mathcal{N})$, for any w-open set $\mathcal{N}$ in $\mathcal{X}$.

Definition 2.16 [19] A space $\mathcal{X}$ is said to be wT_1-space if for any $, b \in \mathcal{X}, a \neq b$, there exists w-open sets $\mathcal{N}, \mathcal{M}$ with $a \in \mathcal{N}, b \notin \mathcal{N}$ and $b \in \mathcal{M}, a \notin \mathcal{M}$.

Proposition 2.17 [19] A space $\mathcal{X}$ is wT_1-space iff any singleton set is w-closed.

Definition 2.18 [19] A space $\mathcal{X}$ is said to be w-T_2-space if for any two points x, y of $\mathcal{X}$, with $x \neq y$ there exist w-open sets $\mathcal{U}, \mathcal{V}$ and $x \in \mathcal{U}, y \in \mathcal{V}$, such that $\mathcal{U} \cap \mathcal{V} = \emptyset$.

Remark 2.19 [19] Every T_2-space is w-T_2-space.

The next example refers to the invers direction of Remark 2.17 not hold:

Example 2.20 If $\mathcal{X}$ is a finite set contain more than one point and T be a discrete topology defined on $\mathcal{X}$, then $\mathcal{X}$ is w-T_2-space but not T_2-space.

Proposition 2.21 [13] Let $\mathcal{X}$ be an anti-locally countable space then $\mathcal{X}$ is an w-T_2-space iff $\mathcal{X}$ is T_2-space.

Definition 2.22 [6] Let $\mathcal{U}$ be a collection of a subset of a space $\mathcal{X}$, $\mathcal{U}$ is called open cover of $\mathcal{X}$, if $\mathcal{U}$ is cover $\mathcal{X}$ and $\mathcal{U}$ is a subfamily of a topology T.

Definition 2.23 [6] A space $\mathcal{X}$ is said to be Lindelof space if for any open cover of $\mathcal{X}$, there is a countable sub cover.

Example 2.24 A countable topological space is a Lindelof space.

Proposition 2.25 [10] In a Lindelof space every closed subset is Lindelof subset.

Proposition 2.26 [17] If $\mathcal{A}$ is Lindelof in $\mathcal{X}$ and $\mathcal{B}$ is w-closed in $\mathcal{X}$ then $\mathcal{A} \cap \mathcal{B}$ is Lindelof in $\mathcal{X}$.

Definition 2.27 [4] A space $\mathcal{X}$ is w-Lindelof space, if for each w-open cover of $\mathcal{X}$ has a countable sub cover.

$Int(\mathcal{S}) = Int_w$

$Cl(\mathcal{N}) = Cl_w$

Remark 2.28 Any w-Lindelof space is Lindelof space.

Proposition 2.29 [10] In Lindelof space, every w-closed set is Lindelof set.

Definition 2.30 [7, 15] A space $\mathcal{X}$ is said to be $L\mathcal{C}$-space if any Lindelof set in $\mathcal{X}$ is closed.

Example 2.31 A discrete space on a non-empty set $\mathcal{X}$, $(\mathcal{X}, T_D)$ is $L\mathcal{C}$-space.

Example 2.32 Let $(\mathcal{R}, T_{cof})$ be a co-finite topology on a real line $\mathcal{R}$, the set of rational numbers $\mathbb{Q}$ is Lindelof, not closed, so $(\mathcal{R}, T_{cof})$ is not $L\mathcal{C}$-space.

Proposition 2.33 [2] Let $f:(\mathcal{X},T) \rightarrow (\mathcal{Y},T')$ be a bijective open function, if $\mathcal{X}$ is $L\mathcal{C}$-space then $\mathcal{Y}$ is $L\mathcal{C}$-space.

Definition 2.34 [8] A space $\mathcal{X}$ is said to be Locally $L\mathcal{C}$-space if any point has a neighborhood which is an $L\mathcal{C}$-subspace.

Remark 2.35 [8] Any $L\mathcal{C}$-space is Locally $L\mathcal{C}$-space

3- $\boldsymbol{L(\omega\mathcal{C})}$-space and $\boldsymbol{\omega L_i}$-spaces , i=1, 2, 3, 4.

In this section, we define $L(\omega\mathcal{C})$-space and state four weaker forms of $L(\omega\mathcal{C})$-space also we study their relationship with $L\mathcal{C}$-space.

Definition 3.1 A space $\mathcal{X}$ is called $L(\omega\mathcal{C})$-space if for each Lindelof set in $\mathcal{X}$ is ω-closed.

Example 3.2 The integer numbers $\mathcal{Z}$ defined on a topology T as follows: $T_{Exc}=\{\mathcal{U}\subseteq\mathcal{Z}, x_0 \notin \mathcal{U}$, for some $x_0 \in \mathcal{Z}\} \cup \{\mathcal{Z}\}$ be excluded point topology on $\mathcal{Z}$, let $x_0 = 6$, so $\mathcal{Z} - \{6\} \subseteq \mathcal{Z}$ is countable, so it is Lindelof and ω-closed hence $(\mathcal{Z}, T_{Exc})$ is $L(\omega\mathcal{C})$-space.

Remark 3.3 Every $L\mathcal{C}$-space is $L(\omega\mathcal{C})$-space.

The following example show that the convers of Remark 3.3 is not hold.

Example 3.4 Let $(\mathcal{R}, T_{cof})$ be a co-finite topology on a real line $\mathcal{R}$, $(\mathcal{R}, T_{cof})$ is $L(\omega\mathcal{C})$-space but not $L\mathcal{C}$-space.

Proposition 3.5 Every subspace of $L(\omega\mathcal{C})$-space is $L(\omega\mathcal{C})$-space.

Proof: Suppose a space $\mathcal{X}$ is $L(\omega\mathcal{C})$ and $\mathcal{G}$ be subspace of $\mathcal{X}$, a subset $\mathcal{M}$ is a Lindelof in $\mathcal{G}$, so $\mathcal{M}$ is Lindelof in $\mathcal{X}$ and then $\mathcal{M}$ is ω-closed in $\mathcal{X}$, from Proposition 2.13, we get $\mathcal{M}$ is ω-closed in $\mathcal{G}$, hence $\mathcal{G}$ is $L(\omega\mathcal{C})$-space

Proposition 3.6 Every $L(\omega\mathcal{C})$-space is ωT_1-space.

Proof: Let $e \in \mathcal{X}$, then $\{e\}$ is Lindelof subset in $\mathcal{X}$, but $\mathcal{X}$ is $L(\omega\mathcal{C})$-space, so $\{e\}$ is ω-closed from Proposition 2.17 , $\mathcal{X}$ is ωT_1-space.

Definition 3.7 A subset $\mathcal{F}$ of a space $\mathcal{X}$ is said to be $\mathcal{F}e$-ω-closed set if $\mathcal{F}$ is the union of countable ω-closed sets.

Definition 3.8 A subset G of a space $\mathcal{X}$ is said to be $\mathfrak{S}_\partial$-ω-open set, if G is the intersection of countably ω-open sets.

Remark 3.9

1. very ω-closed sets is $\mathcal{F}e$-ω-closed set.
2. very ω-open sets is $\mathfrak{S}_\partial$-ω-open set.

Example 3.10 Let $(\mathcal{R}, T_{cof})$ be a co-finite topology on a real line $\mathcal{R}$, a natural numbers $\mathcal{N}$ of $\mathcal{R}$ is not ω-closed but it is $\mathcal{F}e$-ω-closed set. And $\mathcal{R} - \mathcal{N}$ is not ω-open set but it is $\mathfrak{S}_\partial$-ω-open set.

Definition 3.11 [8] A topological space $(\mathcal{X}, T)$ is said to be $\mathcal{P}^\star$-space if any $\mathfrak{S}_\partial$-ω-open subset of $\mathcal{X}$ is ω-open set.

Definition 3.12 A space $\mathcal{X}$ is said to be:

1. -space if any Lindelof $\mathcal{F}e$-ω-closed set is ω-closed set.
2. ωL_2-space if $\mathcal{N}$ is Lindelof in $\mathcal{X}$, then. $Cl_\omega(\mathcal{N})$ is Lindelof.
3. ωL_3-space if for each Lindelof subset $\mathcal{N}$ is $\mathcal{F}e$-ω-closed set.

4. $w\mathrm{L}_4$-space if $\mathcal{N}$ is a Lindelof in a space $\mathcal{X}$ then there is a Lindelof $\mathcal{F}e$-w-closed set $\mathcal{M}$ and $\mathcal{N} \subseteq \mathcal{M} \subseteq Cl_w(\mathcal{N})$.

Example 3.13 The usual topology define on the set of real number $\mathcal{R}$, $(\mathcal{R}, T_{\mathcal{U}})$ is an $w\mathrm{L}_2$-space but neither $w\mathrm{L}_3$-space nor $\mathrm{L}(w\mathcal{C})$-space, if L is a Lindelof set in $\mathcal{R}$, since $\mathcal{R}$ is second countable space, then $Cl_w(\mathrm{L})$ is second countable subspace, then $Cl_w(\mathrm{L})$ is a Lindelof, so $\mathcal{R}$ is an $w\mathrm{L}_2$-space but not an $w\mathrm{L}_3$-space, since $\mathbb{Q}^c$ is a Lindelof but not $\mathcal{F}e$-w-closed (since if $\mathbb{Q}^c$ is $\mathcal{F}e$-w-closed then $\mathbb{Q}$ is $\mathfrak{S}_\partial$-w-open set that is $\mathbb{Q} = \bigcap_{i=1}^{\infty} G_i$ where G_i is an w-open set in $\mathcal{R}$, so $\mathbb{Q} \subseteq G_i$ for each i, but the only w-open set containing $\mathbb{Q}$ is $\mathcal{R}$, that is $G_i = \mathcal{R}$ for each i and $\mathbb{Q} = \bigcap_{i=1}^{\infty} G_i = \mathrm{R}$ which is contradiction, also $\mathcal{R}$ is not $\mathrm{L}(w\mathcal{C})$-space (since $\mathbb{Q}^c$ is a Lindelof but not w-closed).

Proposition 3.14 Every $\mathrm{L}(w\mathcal{C})$-space is $w\mathrm{L}_1$ (resp., $w\mathrm{L}_2$, $w\mathrm{L}_3$, $w\mathrm{L}_3$)-space.

Proof: Let $\mathcal{F}$ be a lindelof $\mathcal{F}e$-w-closed subset of a space $\mathcal{X}$, since $\mathcal{X}$ is an $\mathrm{L}(w\mathcal{C})$-space, so $\mathcal{F}$ is w-closed set, hence $\mathcal{X}$ is $w\mathrm{L}_1$.Now, let $\mathcal{A}$ be a lindelof in $\mathcal{X}$,but $\mathcal{X}$ is $\mathrm{L}(w\mathcal{C})$-space, so $\mathcal{A}$ is w-closed and by Proposition 2.7 part (5), $Cl_w(\mathcal{A}) = \mathcal{A}$, hence $Cl_w(\mathcal{A})$ is Lindelof and then $\mathcal{X}$ is $w\mathrm{L}_2$. Let $\mathcal{B}$ be a lindelof set of $\mathcal{X}$, since $\mathcal{X}$ is $\mathrm{L}(w\mathcal{C})$-space, then $\mathcal{B}$ is w-closed, so $\mathcal{B}$ is $\mathcal{F}e$-w-closed set by Remark 3.7 part (1), we get $\mathcal{X}$ is $w\mathrm{L}_3$. Let $\mathcal{D}$ be a lindelod in $\mathcal{X}$, also $\mathcal{X}$ is $\mathrm{L}(w\mathcal{C})$-space, then $\mathcal{D}$ is w-closed, from Remark 3.9 part (1), $\mathcal{D}$ is $\mathcal{F}e$-w-closed set, put $\mathcal{D} = \mathrm{L}$, then $\mathcal{D} \subseteq \mathrm{L} \subseteq Cl_w(\mathcal{D})$, so $\mathcal{X}$ is an $w\mathrm{L}_4$-space.

Proposition 3.15 If a space $\mathcal{X}$ is $w\mathrm{L}_1$ and $w\mathrm{L}_3$-spaces then it is $\mathrm{L}(w\mathcal{C})$-space.

Proof: Let $\mathcal{K}$ be a Lindelof of $\mathcal{X}$, since $\mathcal{X}$ is $w\mathrm{L}_3$-space, so $\mathcal{K}$ is $\mathcal{F}e$-w-closed, also $\mathcal{X}$ is an $w\mathrm{L}_1$-space, hence $\mathcal{K}$ is w-closed that is $\mathcal{X}$ is $\mathrm{L}(w\mathcal{C})$-space.

Proposition 3.16 If a space $\mathcal{X}$ is $w\mathrm{L}_1$ and $w\mathrm{L}_4$-spaces then it is $w\mathrm{L}_2$-space.

Proof: Let $\mathcal{X}$ be an $w\mathrm{L}_4$-space, $\mathcal{N}$ be a Lindelof in $\mathcal{X}$, so there exists a Lindelof $\mathcal{F}e$-w-closed $\mathcal{S}$ with

$\mathcal{N} \subseteq \mathcal{S} \subseteq Cl_w(\mathcal{N})$, since $\mathcal{X}$ is $w\mathrm{L}_1$-space, we get $\mathcal{S}$ is w-closed set and $\mathcal{S} = Cl_w(\mathcal{S})$ by Proposition 2.7 part (5), but $\mathcal{N} \subseteq \mathcal{S}$ then $Cl_w(\mathcal{N}) \subseteq Cl_w(\mathcal{S}) = \mathcal{S}$, so $Cl_w(\mathcal{N}) = \mathcal{S}$, and since $\mathcal{S}$ is Lindelof then $Cl_w(\mathcal{N})$ is Lindelof, therefore $\mathcal{X}$ is $w\mathrm{L}_2$-space.

Proposition 3.17 Every $w\mathrm{L}_2$-space is $w\mathrm{L}_4$-space, also any $w\mathrm{L}_3$-space is $w\mathrm{L}_4$-space.

Proof: Let G be a Lindelof in a space $\mathcal{X}$, since $\mathcal{X}$ is $w\mathrm{L}_2$, so $Cl_w(G)$ is Lindelof and w-closed set then $Cl_w(G)$ is $\mathcal{F}e$-w-closed and $G \subseteq Cl_w(G) \subseteq Cl_w(G)$, take $\mathcal{F} = Cl_w(G)$, then $G \subseteq \mathcal{F} \subseteq Cl_w(G)$ that is $\mathcal{X}$ is $w\mathrm{L}_4$-space. To prove the second part let $\mathcal{J}$ be a Lindelof set of a space $\mathcal{X}$, and $\mathcal{X}$ is $w\mathrm{L}_3$-space, so $\mathcal{J}$ is $\mathcal{F}e$-w-closed subset in $\mathcal{X}$, take $\mathcal{J} = \mathcal{N}$ and $\mathcal{J} \subseteq \mathcal{J} \subseteq Cl_w(\mathcal{J})$, hence $\mathcal{J} \subseteq \mathcal{N} \subseteq Cl_w(\mathcal{J})$, hence $\mathcal{X}$ is $w\mathrm{L}_4$-space.

Proposition 3.18 Every $\mathcal{P}^*$-space is an $w\mathrm{L}_1$-space.

Proof: Let G be a Lindelof $\mathcal{F}e$-w-closed set of $\mathcal{X}$, but $\mathcal{X}$ is $\mathcal{P}^*$-space and G^c is $\mathfrak{S}_\partial$-w-open set of $\mathcal{X}$, so G is w-closed, therefore $\mathcal{X}$ is $w\mathrm{L}_1$-space.

Definition 3.19 A subset $\mathcal{M}$ of a space $\mathcal{X}$ is said to be w-dense if $Cl_w(\mathcal{M}) = \mathcal{X}$.

Proposition 3.20 Each Lindelof space is an $w\mathrm{L}_2$-space, also each w-dense Lindelof subset of $w\mathrm{L}_2$-space is Lindelof.

Proof: Let $\mathcal{P}$ be a Lindelof set in $\mathcal{X}$, $Cl_w(\mathcal{P})$ is w-closed in $\mathcal{X}$ from Proposition 2.29, $Cl_w(\mathcal{P})$ is

Lindelof, then $\mathcal{X}$ is an $w\mathrm{L}_2$-space. Now, let $\mathcal{F}$ be w-dense Lindelof in $\mathcal{X}$, hence $Cl_w(\mathcal{F}) = \mathcal{X}$ and $\mathcal{X}$ is an $w\mathrm{L}_2$-space , so $Cl_w(\mathcal{F})$ is Lindelof, that is $\mathcal{X}$ Lindelof.

Theorem 3.21 The property $w\mathrm{L}_3$-space is hereditary property.

Proof: Let $\mathcal{A}$ be a subspace of $w\mathrm{L}_3$-space $\mathcal{X}$,and L is a Lindelof subset of $\mathcal{A}$, then $\mathcal{S}$ is Lindelof subset of $\mathcal{X}$, by hypothesis $\mathcal{S}$ is $\mathcal{F}e$-w-closed , so

there is $\{\mathcal{M}_n\}_{n\in I}$ a family of w-closed in $\mathcal{X}$ and $\mathcal{S} = \cup_{n\in I}\mathcal{M}_n$, take $\mathcal{M}_n^* = \mathcal{M}_n \cap \mathcal{A}$, so $\mathcal{M}_n^*$ is w-closed sets in $\mathcal{A}$, for each n, also $\mathcal{S} = \mathcal{S} \cap \mathcal{A} = (\cup_{n\in I}\mathcal{M}_n) \cap \mathcal{A} = \cup_{n\in I}(\mathcal{M}_n \cap \mathcal{A}) = \cup_{n\in I}\mathcal{F}_n^*$, so $\mathcal{S}$ is $\mathcal{F}e$-w-closed in $\mathcal{A}$, hence $\mathcal{A}$ is an $w\mathrm{L}_3$-space.

Theorem 3.22 Each $\mathcal{F}e$-w-closed subset of $w\mathrm{L}_1$(resp., $w\mathrm{L}_2$, $w\mathrm{L}_4$)-space is $w\mathrm{L}_1$(resp., $w\mathrm{L}_2$, $w\mathrm{L}_4$)-space.

Proof: Let $\mathcal{X}$ be $w\mathrm{L}_1$-space, $\mathcal{A}$ be $\mathcal{F}e$-w-closed set in $\mathcal{X}$, to show $\mathcal{A}$ is an $w\mathrm{L}_1$-space, let $\mathcal{J}$ be a Lindelof $\mathcal{F}e$-w-closed set in $\mathcal{A}$ that is there exist a collection $\{\mathcal{S}_i^*\}_{i\in J}$ of w-closed sets in $\mathcal{A}$ with $\mathcal{J} = \cup_{i\in J}\mathcal{S}_i^*$, let $\mathcal{S}_i^* = \mathcal{S}_i \cap \mathcal{A}$, whenever $\mathcal{S}_i$ is an w-closed in $\mathcal{X}$ for all i, so $\mathcal{J} = \cup_{i\in J}(\mathcal{A} \cap \mathcal{S}_i) = \mathcal{A} \cap (\cup_{i\in J}\mathcal{S}_i) = (\cup_{j\in I}\mathcal{V}_j) \cap (\cup_{i\in J}\mathcal{S}_i)$,where $\mathcal{V}_j$ is an w-closed subset of $\mathcal{X}$, since $\mathcal{A}$ is an $\mathcal{F}e$-w-closed, so $\mathcal{J} = \cup_{i,j\in J}(\mathcal{S}_i \cap \mathcal{V}_j)$, L is a Lindelof $\mathcal{F}e$-w-closed closed in $\mathcal{A}$ therefore $\mathcal{A}$ is $w\mathrm{L}_1$-space .To show that the property $w\mathrm{L}_2$-space is hereditary on -$\mathcal{F}e$-w-closed, let $\mathcal{K}$ be a Lindelof in $\mathcal{A}$, hence $\mathcal{K}$ is Lindelof in $\mathcal{X}$, also $\mathcal{X}$ is $w\mathrm{L}_2$-space, so $Cl_w(\mathcal{K})$ Lindelof ,
$Cl_{w\ in\ \mathcal{A}}(\mathcal{K})=$
$Cl_{w\ in\ \mathcal{X}}(\mathcal{K}) \cap \mathcal{A} = Cl_{w\ in\ \mathcal{X}}(\mathcal{K}) \cap (\cup_{i\in I}\mathcal{F}_i)$,
where $\mathcal{F}_i$ is an w-closed in $\mathcal{A}$ which is $\mathcal{F}e$-w-closed, $Cl_{w\ in\ \mathcal{A}}(\mathcal{K}) = \cup_{i\in I}(Cl_{w\ in\ \mathcal{X}}(\mathcal{K}) \cap \mathcal{F}_i)$ is Lindelof (since a countable union of Lindelof subset is Lindelof), then $Cl_{w\ in\ \mathcal{A}}(\mathcal{K})$ is Lindelof, so $\mathcal{A}$ is an $w\mathrm{L}_2$-space. To show that the property $w\mathrm{L}_4$-space is hereditary on $\mathcal{F}e$-w-closed, let $\mathcal{A}$ be $\mathcal{F}e$-w-closed in $w\mathrm{L}_4$-space $\mathcal{X}$, to show $\mathcal{A}$ is $w\mathrm{L}_4$-space, let L be a Lindelof subset in $\mathcal{A}$ and hence it is Lindelof in $\mathcal{X}$, so there is a Lindelof $\mathcal{F}e$-w-closed $\mathcal{F}$ in $\mathcal{X}$ with $\mathrm{L} \subseteq \mathcal{F} \subseteq Cl_{w\ in\ \mathcal{X}}(\mathrm{L})$, put $\mathcal{H} = \mathcal{F} \cap \mathcal{A}$ and $\mathcal{A} = \cup_{i\in I}\mathcal{C}_i$, where $\mathcal{C}_i$ is w-closed in $\mathcal{A}$, to show $\mathcal{H}$ is a Lindelof $\mathcal{F}e$-w-closed in $\mathcal{A}$ since
$\mathcal{H} = \mathcal{F} \cap \mathcal{A} = \mathcal{F} \cap (\cup_{i\in I}\mathcal{C}_i) = \cup_{i\in I}(\mathcal{F} \cap \mathcal{C}_i)$ and $(\mathcal{F} \cap \mathcal{C}_i)$ is w-closed in $\mathcal{F}$ which is Lindelof , then for all i, $(\mathcal{F} \cap \mathcal{C}_i)$ is Lindelof and so $\mathcal{H} = \cup_{i\in I}(\mathcal{F} \cap \mathcal{C}_i)$ is Lindelof (since a countable

union of Lindelof subset is Lindelof), to show $\mathcal{H}$ is $\mathcal{F}e$-w-closed $\mathcal{F}$ in $\mathcal{A}$, let $\mathcal{F} = \cup_{j\in I}\mathcal{N}_j$, where $\mathcal{N}_j$ is an w-closed in $\mathcal{X}$, so
$\mathcal{H} = \mathcal{F} \cap \mathcal{A} = (\cup_{j\in I}\mathcal{N}_j) \cap (\cup_{i\in I}\mathcal{C}_i) = \cup_{i,j\in I}(\mathcal{N}_j \cap \mathcal{C}_i) = \cup_{n\in I}\mathcal{S}_n$
, where $\mathcal{S}_n = \mathcal{N}_j \cap \mathcal{C}_i \subseteq \mathcal{C}_i \subseteq \mathcal{A}$ is w-closed in $\mathcal{A}$, so $\mathcal{H} = \cup_{n\in I}\mathcal{S}_n$ is $\mathcal{F}e$-w-closed in $\mathcal{A}$, therefore $\mathrm{L} \subseteq \mathcal{H} \subseteq Cl_{w\ in\ \mathcal{A}}(\mathrm{L})$, so $\mathcal{A}$ is $w\mathrm{L}_4$-space.

Proposition 3.23 Let $(\mathcal{X},\mathcal{T})$ be an $\mathcal{W}$-Hausdorff space, a space $\mathcal{X}$ is an $w\mathrm{L}_1$ and $w\mathrm{L}_2$-spaces iff $\mathcal{X}$ is an is $\mathrm{L}\,(w\mathcal{C})$-space.

Proof: Let $\mathcal{K}$ be a Lindelof in $\mathcal{X}$, let $x \notin \mathcal{K}$, so for any $y \in \mathcal{K}$ there exists an w-open sets $\mathcal{V}_y$ containing y, with $x \in Cl_w(\mathcal{V}_y)$, now $\{\mathcal{V}_y : y \in \mathcal{K}\}$ is cover of $\mathcal{K}$ and $\mathcal{K}$ is Lindelof, so there exists a countable set $\mathcal{C} \subseteq \mathcal{K}$ such that $\mathcal{K} \subseteq \cup\{\mathcal{V}_y : y \in \mathcal{C}\} \subseteq \cup\{Cl_w(\mathcal{V}_y) : y \in \mathcal{C}\}$, for each $y \in \mathcal{C}$, $\mathcal{K} \cap Cl_w(\mathcal{V}_y)$ is Lindelof by Proposition 2.23 and $Cl_w(\mathcal{K} \cap Cl_w(\mathcal{V}_y))$ is Lindelof since $\mathcal{X}$ is an $w\mathrm{L}_2$-space, also if $\mathcal{N} = \cup\{Cl_w(\mathcal{K} \cap Cl_w(\mathcal{V}_y)) : y \in \mathcal{C}\}$, then $\mathcal{N}$ is Lindelof $\mathcal{F}e$-w-closed and since $\mathcal{X}$ is $w\mathrm{L}_1$-space, so $\mathcal{N}$ is an w-closed and $x \notin \mathcal{N}$, thus $x \notin Cl_w(\mathcal{K})$, this show that $\mathcal{K}$ is w-closed subset of $\mathcal{X}$.

4-Locally $\mathbf{L}\,(w\mathcal{C})$-space

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In this part we define Locally $L(\mathcal{wC})$-space, and some theorems, properties about Locally $L(\mathcal{wC})$-space which is generalized of $L(\mathcal{wC})$-space.

Definition 4.1 A space $\mathcal{X}$ is said to be Locally $L(\mathcal{wC})$-space if each point has a neighborhood which is an $L(\mathcal{wC})$-subspace.

Clearly any Locally $L\mathcal{C}$-space is Locally $L(\mathcal{wC})$-space.The following example refers the invers direction is not hold.

Example 4.2 Let $(\mathcal{Z}, T_{Exc})$ be an Excluded point topology on the integer numbers $\mathcal{Z}$, where $T_{Exc} = \{\mathcal{U} \subseteq \mathcal{X}, x_0 \notin \mathcal{U}$, for some $x_0 \in \mathcal{X}\} \cup \{\mathcal{X}\}$. Let $x_0 = 6$, since for each $y \in \mathcal{Z}$, since $\{y\}$ is finite, so $\{y\}$ is countable and then it is Lindelof in $\mathcal{X}$, also it is $\mathcal{w}$-closed with $y \neq 6$. So $(\mathcal{Z}, T_{Exc})$ is Locally $L(\mathcal{wC})$-space, but $\{y\}$ is not closed, hence this space is not Locally $L\mathcal{C}$-space.

Definition 4.3 A space $\mathcal{X}$ is called $\mathcal{w}L(\mathcal{wC})$-space if for each $\mathcal{w}$-Lindelof set in $\mathcal{X}$ is $\mathcal{w}$-closed.

Definition 4.4 A space $\mathcal{X}$ is said to be Locally $\mathcal{w}L(\mathcal{wC})$-space if each point has a neighborhood which is an $\mathcal{w}L(\mathcal{wC})$-subspace.

Remark 4.5 Each $\mathcal{w}L(\mathcal{wC})$-space is Locally $\mathcal{w}L(\mathcal{wC})$-space.

Theorem 4.6 A space $\mathcal{X}$ is $L(\mathcal{wC})$-space iff every point in it has clopen set containing x which is $L(\mathcal{wC})$-space.

Proof: Let $\mathcal{X}$ be $L(\mathcal{wC})$-space, so for any $x \in \mathcal{X}$, $\mathcal{X}$ it self is clopen that is $L(\mathcal{wC})$-space.Converse direction, let $\mathcal{D}$ be a Lindelof in $\mathcal{X}$ and $x \notin \mathcal{D}$. Choose a clopen $\mathcal{W}_x$ containing x such that $\mathcal{W}_x$ is $L(\mathcal{wC})$-subspace, if $\mathcal{D} \cap \mathcal{W}_x = \emptyset$, so it is Lindelof also if $\mathcal{D} \cap \mathcal{W}_x \neq \emptyset$, it is Lindelof in the subspace $\mathcal{W}_x$ by Proposition 2.26, therefore $\mathcal{D} \cap \mathcal{W}_x$ is $\mathcal{w}$-closed in $\mathcal{W}_x$ also $\mathcal{w}$-closed in $\mathcal{X}$, hence $\mathcal{W}_x - (\mathcal{D} \cap \mathcal{W}_x) = \mathcal{W}_x - \mathcal{D}$ is $\mathcal{w}$-open in $\mathcal{X}$, so $\mathcal{D}$ is $\mathcal{w}$-open in $\mathcal{X}$, hence $\mathcal{X}$ is $L(\mathcal{wC})$-space.

Corollary 4.7 A space $\mathcal{X}$ is $L(\mathcal{wC})$-space iff every point in it has $\mathcal{w}$-clopen set containing x which is $L(\mathcal{wC})$-space.

Proof: From Theorem 4.6, and Remark 2.3, we get the first direction. To prove the second direction, from Theorem 4.6 and Proposition 2.26, a space $\mathcal{X}$ is $L(\mathcal{wC})$-space.

Corollary 4.8 A discrete Locally $L(\mathcal{wC})$-space is $L(\mathcal{wC})$-space.

Theorem 4.9 Any Locally $L(\mathcal{wC})$-space is $\mathcal{w}T_1$-space.

Proof: Suppose $\mathcal{X}$ is not $\mathcal{w}T_1$-space, that is there is $n, m \in \mathcal{X}$, with every $\mathcal{w}$-open set contain n also contain m, let $\mathcal{U}$ be an $\mathcal{w}$-open neighborhood of m such that $(\mathcal{U}, T_{\mathcal{U}})$ is $L(\mathcal{wC})$-space, since $\mathcal{X}$ is Locally $L(\mathcal{wC})$-space, so $(\mathcal{U}, T_{\mathcal{U}})$ is $\mathcal{w}T_1$-space, by Proposition 3.6, also from Proposition 2.17, $\{n\}$ is $\mathcal{w}$-closed in $\mathcal{U}$, then $\mathcal{U} - \{n\}$ is $\mathcal{w}$-open in $\mathcal{U}$ and $\mathcal{U}$ is $\mathcal{w}$-open in $\mathcal{X}$, $\mathcal{U} - \{n\}$ is $\mathcal{w}$-open in $\mathcal{X}$, by Proposition 2.12, but $m \in \mathcal{U} - \{n\}$ and $n \notin \mathcal{U} - \{n\}$, this is contradiction, hence $\mathcal{X}$ is $\mathcal{w}T_1$-space.

Proposition 4.10 Let $f : (\mathcal{X}, T) \longrightarrow (\mathcal{Y}, T')$ be a bijective open function, if $\mathcal{X}$ is Locally $L\mathcal{C}$-space then $\mathcal{Y}$ is Locally $L(\mathcal{wC})$-space.

Proof: Let $\mathcal{X}$ be a is Locally $L\mathcal{C}$-space, so for any $m \in \mathcal{X}$, there exists a neighborhood $\mathcal{M}$ of m such that $\mathcal{M}$ is $L\mathcal{C}$-subspace, but $\mathcal{M}$ is a neighborhood, then there is open set $\mathcal{H}$ such that $m \in \mathcal{H} \subseteq \mathcal{M}$, so $n = f(m) \in f(\mathcal{H}) \subseteq f(\mathcal{M}) \subseteq \mathcal{Y}$, therefore $\mathcal{Y}$ is Locally $L(\mathcal{wC})$-space since for any $n \in \mathcal{Y}$, there is an open neighborhood $f(\mathcal{M})$, (since f is open), $\mathcal{M}$ is $L\mathcal{C}$-space and by hereditary property of $L\mathcal{C}$-space, we have $\mathcal{H}$ is $L\mathcal{C}$-space, then $f(\mathcal{H})$ is $L\mathcal{C}$-subspace of $\mathcal{Y}$ from Proposition 2.33, then $f(\mathcal{H})$ is $L(\mathcal{wC})$-subspace by Remark 3.3, hence $\mathcal{Y}$ is Locally $L(\mathcal{wC})$-space.

Proposition 4.11 Let $f:(\mathcal{X},T) \longrightarrow (\mathcal{Y},T')$ be a bijectiv ω-open function. If $\mathcal{H}$ is ω-Lindelof subset of $\mathcal{Y}$, then $f^{-1}(\mathcal{H})$ is Lindelof subset of $\mathcal{X}$.

Proof: Let $\mathcal{H}$ is ω-Lindelof subset of $\mathcal{Y}$, let $\{\mathcal{W}_\alpha\}_{\alpha\in\Delta}$ be an open cover of $f^{-1}(\mathcal{H})$ in $\mathcal{X}$, that is $f^{-1}(\mathcal{H})\subseteq\cup_{\alpha\in\Delta}\mathcal{W}_\alpha$,so $\mathcal{H} = f(f^{-1}(\mathcal{H}))\subseteq f(\cup_{\alpha\in\Delta}\mathcal{W}_\alpha) = \cup_{\alpha\in\Delta} f(\mathcal{W}_\alpha)$, since f is surjective, but f is ω-open, so $f(\mathcal{W}_\alpha)$ is ω-open for each $\alpha \in \Delta$, and $\mathcal{H}$ is ω-Lindelof subset of $\mathcal{Y}$, hence $\mathcal{H}\subseteq \cup_{\alpha\in\Delta'} f(\mathcal{W}_\alpha)$, Δ' is a countable subset of Δ,
also
$f^{-1}(\mathcal{H})\subseteq f^{-1}(\cup_{\alpha\in\Delta'} f(\mathcal{W}_\alpha)) = \cup_{\alpha\in\Delta'} f^{-1}f(\mathcal{W}_\alpha)) = \cup_{\alpha\in\Delta'} \mathcal{W}_\alpha$
, therefore $f^{-1}(\mathcal{H})$ is Lindelof in $\mathcal{X}$.

Theorem 4.12 Let $f:(\mathcal{X},T) \longrightarrow (\mathcal{Y},T')$ be a bijectiv ω-open function. If $\mathcal{X}$ is $\mathrm{L}\mathcal{C}$-space then $\mathcal{Y}$ is $\omega\mathrm{L}(\omega\mathcal{C})$-space.

Proof: Let $\mathcal{F}$ be ω-Lindelof subset of $\mathcal{Y}$, then $f^{-1}(\mathcal{F})$ is Lindelof set in $\mathcal{X}$ by Proposition 4.11 , but $\mathcal{X}$ is $\mathrm{L}\mathcal{C}$-space, so is $(f^{-1}(\mathcal{F}))^c$ is open set in $\mathcal{X}$, also f is ω-open function, hence $f((f^{-1}(\mathcal{F}))^c) = f(\mathcal{X} - f^{-1}(\mathcal{F})$ $= f(\mathcal{X}) - f\big(f^{-1}(\mathcal{F})\big) = \mathcal{Y} - \mathcal{F} = \mathcal{F}^c$ is ω-open in $\mathcal{Y}$, so $\mathcal{F}$ is ω-closed in $\mathcal{Y}$, therefore $\mathcal{Y}$ is $\omega\mathrm{L}(\omega\mathcal{C})$-space.

Corollary 4.13 Let $f:(\mathcal{X},T) \longrightarrow (\mathcal{Y},T')$ be a bijectiv ω-open function. If $\mathcal{X}$ is $\mathrm{L}\mathcal{C}$-space then $\mathcal{Y}$ is Locally $\omega\mathrm{L}(\omega\mathcal{C})$-space.

Proof: By Theorem 4.12 and Remark 4.5, we get $\mathcal{Y}$ is Locally $\omega\mathrm{L}(\omega\mathcal{C})$-space.

Theorem 4.14 Every subspace of Locally $\mathrm{L}(\omega\mathcal{C})$-space is Locally $\mathrm{L}(\omega\mathcal{C})$-space.

Proof: Let $\mathcal{H}$ be a subspace of a space $\mathcal{X}$, and $c \in \mathcal{P}$, so c has $\mathrm{L}(\omega\mathcal{C})$ a neighborhood in $\mathcal{X}$, hence there is open set $\mathcal{U}$ in $\mathcal{X}$ such that $c \in \mathcal{U}$ and $\mathcal{U}$ is $\mathrm{L}(\omega\mathcal{C})$-subspace in $\mathcal{X}$, let $\mathcal{V} = \mathcal{H} \cap \mathcal{U}$, then $\mathcal{V}\subseteq\mathcal{U}$, so $\mathcal{V}$ is $\mathrm{L}(\omega\mathcal{C})$-subspace from Proposition 3.5, since $c \in \mathcal{U}$ and $c \in \mathcal{H}$ then $c \in \mathcal{H} \cap \mathcal{U}$, hence

$c \in \mathcal{V} = \mathcal{D} \cap \mathcal{U}$, so $(\mathcal{H},T_{\mathcal{H}})$ is Locally $\mathrm{L}(\omega\mathcal{C})$-subspace of $\mathcal{X}$.

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فضاءات-$L(\mathcal{wC})$ والصيغ *الضعيفه لها*

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المستخلص:

في هدا البحث قدمنا تعميم جديد لفضاء $\mathrm{L}\mathcal{C}$ - وهو فضاء $\mathrm{L}(\mathcal{wC})$- وفضاء $\mathcal{w}\mathrm{L}(\mathcal{wC})$ - وكدلك صيغ ضعيفة لفضاء $\mathrm{L}(\mathcal{wC})$ - وهي فضاء $\mathcal{w}\mathrm{L}_i$- حيث ان $(i = 1,2,3,4)$. وبالإضافة الى دلك قدمنا علاقات بين تلك الانواع ودرسنا الصفة الوراثية لكل نوع.