Journal of AL-Qadisiyah for computer science and mathematics Vol.11 No.2 Year 2019

ISSN (Print): 2074 – 0204 ISSN (Online): 2521 – 3504

Some Properties of Topology Fuzzy Modular Space

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Recived : 27\1\2019 Revised : 12 \ 2 \ 2019 Accepted : 14\3\2019

Available online : 24 /4/2019

Abstract:

In the present paper , the authors have introduced and studied fuzzy modular space. they have investigated some properties of this space in the open and closed balls. Also the authors discussed the convex set and the locally convex in fuzzy modular space. The result obtained are correct and the methods used are interesting .

M.S.C: 03E72 , 46A80 .

Keywords: fuzzy set , modular space , fuzzy modular space.

Journal of AL-Qadisiyah for computer science and mathematics Vol.11 No.2 Year 2019
ISSN (Print): 2074 – 0204 ISSN (Online): 2521 – 3504

1.Introduction

The concept of fuzzy sets was introduced by Zadeh [5] in 1965 and there after several authors applied it to different branches of pure and applied mathematics. The concept of modular space was introduction by S.S. Abed, K .A .Abdul Sada in 2017.The concept of fuzzy modular space was introduced by Young Shen and Wei Chen [7] in 2013 .

Definition(1.1) [4]

A fuzzy set A in X (or a fuzzy subset in X) is a function from X into $I = [0,1]$ that is $A \in I^x$ $A \in I^x$.

De **Definition(1.2)[6]** Let X be a linear space. over F. A function $M: X \to [0, \infty]$ is called modular if:

1. $M(x) = 0 \iff x = 0$,
2. $M(\alpha x) = M(x)$ for $\alpha \in F$ with $|\alpha| = 1$ for all $\alpha \in F$.
3. $M(\alpha x + \beta y) \leq M(x) + M(y)$ iff $\alpha, \beta \geq 0$,for all $x \in X$.

Definition (1.3)[2]

A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is called a continuous t-norm if it satisfies the following

1. $*$ is commutative and associative .
2. $*$ is continuous .
2. $a * b \leq c * d$ whenever $a \leq c$, $b \leq d$ and $a, b, c, d \in [0,1]$.

Three common examples of the continuous t −norm are

1. $a *_M b = min\, [a ,b]$.
2. $a *_p b = a . b$.
3. $a *_L b = Max\, \{a + b - 1,0\}$.

Lemma (1.4): [2]

If the t-norm $*$ is continuous ,then

1. for every $\gamma_1, \gamma_2 \in (0,1)$ with $\gamma_1 > \gamma_2$,there exist $\gamma_3 \in (0,1)$ such that $\gamma_1 * \gamma_3 \geq \gamma_2$.
2. for every $\gamma_4 \in (0,1)$,there exist $\gamma_5 \in (0,1)$ such that $\gamma_5 * \gamma_5 \geq \gamma_4$.

Definition (1.5): [7]

A fuzzy modular space is an ordered triple $(X, \mu, *)$ such that X is a vector space, $*$ is continuous t- norm and μ is a fuzzy set on $X \times (0, \infty)$ satisfying the following condition ,for all $x, y \in X$ and $\alpha, \beta \geq 0$ *with* $\alpha + \beta = 1$:

1. $\mu(x,t) > 0$.
2. $\mu(x,t) = 1$ for all $t > 0$ *if and only if* $x = 0$.
3. $\mu(x,t) = \mu(-x,t)$.
4. $\mu(\alpha x + \beta y, s + t) \geq \mu(x,s) * \mu(y,t)$.
5. $\mu(x,.) : (0,\infty) \to (0,1]$ is continuous.

Generally ,if $(X, \mu, *)$ is fuzzy modular space, we say that $(\mu, *)$ is a fuzzy modular on X.

Moreover, the triple $(X, \mu, *)$ is called β-homogeneous if every $x \in X$,t > 0 and $\lambda \in \mathbb{R} \setminus \{0\}$

$\mu(\lambda x, t) = \mu\left(x, \frac{t}{|\lambda|^\beta}\right)$, where $\beta \in (0,1]$.

Example (1.6): [7]

Let X be a vector space and let ρ be a modular on X. Take t-norm $a * b = a *_M b$. For every t $\in (0, \infty)$ define $\mu(x,t) = t/(t + p(x))$

ISSN (Print): 2074 – 0204 ISSN (Online): 2521 – 3504

for all $x \in X$. Then $(X,\mu,*)$ is a fuzzy modular space .

Remark $(\mathbf{1.7})$: [7]

Note the above conclusion still holds even if The t-norm is replaced by $a * b = a *_p b$ and $a * b = a *_L b$, respectively.

Example (1.8): [7]

Let $X = \mathbb{R}$, take t-norm $a * b = a *_M b$. For every $x, y \in X$ and $t \in (0,\infty)$, we define

If we take $\alpha = \sqrt{2}/2, \beta = 1 - \frac{\sqrt{2}}{2}, x \neq y$, and $x, y \in Z$, then we know $\alpha x + \beta y \in \mathbb{R} \setminus Z$. Thus, for all $t, s > 0$, we have $\mu(\alpha x + \beta y, t + s) = 1/4$.

But

$\mu(x,t) *_M \mu(y,s) = \min\{\mu(x,t), \mu(y,s)\} = 1/2$. Obviously $(\mu, *_M)$ is not a fuzzy modular on X

Example (1.9)[7]

Let $X = \mathbb{R}, \rho$ is amodular on X, which is defined by $\rho(x) = |x|^{\beta}$, where $\beta \in (0,1]$ .Take t-norm $a * b = a *_p b$. for every $t \in (0,\infty)$, we define

$$\mu(x,t) = \frac{1}{e^{\rho(x)/t}}$$

for all $x \in X$.Then $(X, \mu, *)$ is β-homogenous fuzzy modular space.

Definition (1.10) : [1]

A fuzzy metric space is an ordered triple $(X, M, *)$ such that X is a nonempty set, $*$ is a continuous t-norm, and M is a fuzzy set on $X \times X \times (0,\infty)$ satisfying the following conditions :

1. 0 .
2. 1 $if\ and\ only\ if\ x = y$.
3. $M(y,x,t)$.
4. $M(y,z,t) \le M(x,z.t+s)$.
5. $M(x,y,\cdot): (0,\infty) \to (0,1]$ is continuous .

Theorem $(\mathbf{1.11})$:

Every fuzzy modular space is fuzzy metric space .

Proof :

Let $(X,\mu,*)$ be a fuzzy modular space, defined $M: X \times X \times (0,\infty) \to [0,1]$ as follows:

$M(x,y,t) = \mu(x-y,t) \quad \forall\, x, y \in X$

1. Let $x, y \in X$

$\Longrightarrow x - y \in X \Longrightarrow \mu(x-y,t) > 0 \Longrightarrow M(x,y,t) > 0$

2. uppose $M(x-y,t) = 1 \Leftrightarrow 1 = M(x,y,t)$

$= \mu(x-y,t) \Leftrightarrow x - y = 0 \Leftrightarrow x = y$

3. Let $x, y \in X\ and\ t > 0$

$M(x,y,t) = \mu(x-y,t) = \mu(y-x,t)$

4.

4. Let $x, y, z \in X\ and\ s, t > 0$

$M(x,y,t) * \mu(y,z,t) = \mu(x-y,t) * \mu(y-z,s)$

$\le \mu\big((x-y)+(y-z), t+s\big) = \mu(x-z,t+s)$

$= M(x,z,t+s)$

5.$M(x,y,\cdot) = \mu(x-y,\cdot): (0,\infty) \to [0,1]$is continuous.

4

$Then (X, M, *)$ is fuzzy metric space .

Theorem $(\mathbf{1.12})$: [7]

If $(X,\mu,*)$ is a fuzzy modular space, then $\mu(x,.)$ is nondecreasing for all $x \in X$.

Definition (1.13): [7]

Let $(X,\mu,*)$ be a fuzzy modular space .we define the open ball $B(x,r,t)$ and the closed ball $B[x,r,t]$ with center $x \in X$ and radius $0 < r < 1$ as follows: For $t > 0$

$B(x,r,t) = \{y \in X: \mu(x-y,t) > 1-r\}$ open balls

$B[x,r,t] = \{y \in X: \mu(x-y,t) \geq 1-r\}$ closed balls

Theorem (1:14)
If $(x,\mu,*)$ is a β- homogenous fuzzy modular space then $B(x,r,t) \subset B(x,r,\frac{t}{2})$.

Proof:

Let $B(x,r_{11},t)$ and $B(x,r_2,t)$ be open balls with the same center $x \in X$ and $t > 0$ with the radius $0 < r_1 < 1$ and $0 < r_2 < 1$, respectively.

Then we either have :

$B(x,r_1,t) \subset B(x,r_2,t)$ or $B(x,r_2,t) \subset B(x,r_1,t)$.

Let $x \in X$ and $t > 0$ .consider the open ball $B(x,r_1,t)$ and $B(x,r_2,t)$ with $0 < r_1 < 1$, $0 < r_2 < 1$.If r_1=r_2 ,then the Theorem holds.

Next, we assume that $r_1 \neq r_2$. we may Assume without loss of generality that

$0 < r_1 < r_2 < 1$

Now let $y \in B(x,r_1,t) \Longrightarrow \mu(x-y,t) < r_1 < r_2$

Hence $y \in B(x,r_2,t)$. This shows that $B(x,r_1,t) \subset B(x,r_2,t)$. By assuming that $0 < r_2 < r_1 < 1$, we can similary to show $B(x,r_2,t) \subset B(x,r_1,t)$.

Definition (1.15):

Let $(X,\mu,*)$ be a fuzzy modular space .A subset A of X is said to be open set, if for all $x \in A \; \exists \; r \in (0,1), t \in (0,\infty)$ such that $B(x,r,t) \subset A$

Theorem (1.16): [7]
Let $(X,\mu,*)$ be a β-homogenous fuzzy modular space .Every μ.ball $B(x,r,t)$ in $(X,\mu,*)$ is a μ-open set.

Theorem (1.17):

The intersection number of open sets in fuzzy modular space is open sets

Proof:

Let $(X,\mu,*)$be a fuzzy modular space and let $\{G_i$:i=1,2,…m} be a finite collection of open set in the fuzzy modular space. Let

H=$\cap \{G_i , i = 1,2,\ldots,m\}$

To prove That H is an open set

let $x \in H \Longrightarrow x \in G_i \quad \forall_i$=1,2,…m

|Since G_i open set : $\forall_i \quad \Longrightarrow \exists r_i \in (0,1)$ and

$t_i > 0 > \ni B(x,r_i,t_i) \subset G_i$

Let $t_k = \max\{t_1,t_2,\ldots,t_n\}$ and

$r_k = \min\{r_1,r_2,\ldots,r_n\}$

$\Longrightarrow B(x,r_k,t_k\} \subset G_i$

$\Longrightarrow B(x,r_k,t_k) \subset \cap G_i \Longrightarrow B(x,r_k,t_k) \subset H$

Then H is open set

Theorem (1.18) The union of an arbitrary collection of open set in fuzzy modular space are open sets

Proof:

Let $(X,\mu,*)$ be a fuzzy modular space and let $\{\gamma_\lambda : \lambda \in \wedge\}$ be an arbitrary collection of open sets in X . Let $G = \cup\{\gamma_\lambda : \lambda \in \wedge\}$ We must to prove G is open

Let $X \in G \implies X \in \gamma_\lambda$ for some $\lambda \in \wedge$

Since γ_λ is open set

$\Longrightarrow$there exist $r \in (0,1)$ such that $B(x,r,t) \subset \gamma_\lambda$

Since $\gamma_\lambda \subset G \Longrightarrow$Then $B(x,r,t) \subset G$

$\Longrightarrow G$ is open set .

Theorem (1.19):

Let $(X,\mu,*)$be afuzzy modular space if C and D are open sets in a vector space X then $C + D$ is open set in X .

Proof:

Let $x \in X$and a $\in C$,since A is open set then there exist $r \in (0,1)$ such that

$B(a,r,t) \subset C$,

then $B(a,r,t) + x \subset C + x$

$\Longrightarrow B(a + x, r, t) \subset C + x \Longrightarrow C + x$ is open set in X for all $x \in X$

Since $C + D = \cup\{C + d : d \in D\}$

Then $C + D$ is open set in X.

Theorem (1.20):

Every single set in fuzzy modular space is closed set .

Proof:

Let X be a fuzzy modular space

Let $B = \{x\}$ be a set in X ,to prove B is closed set

Let $y \in A^c \Longrightarrow y \neq X$

$\mu(y - x, t) > 0$ (since X is fuzzy modular space)

$X \notin B(y,r,t) = \{a \in X : \mu(a - y, t) > 1 - r\}$

$B \cap B(y,r,t) = \emptyset \Longrightarrow B(y,r,t) \subseteq A^c$

Then $y \in B(y,r,t) \subseteq B^c \Longrightarrow y$ is interior point

Then B^c is open set

Then B is closed set .

Corollary (1.21) Every finite set in fuzzy modular space is closed set.

Definition (1.22):
A subset U of X is said to be a neighborhood of $x \in X$ in $(X,\mu,*)$ if there exist $r \in (0,1)$ and $t \in (0,\infty)$ such that $B(x,r,t) \subset U$.

Definition (1.23):[2]

A subset A of a vector space X over F is called convex set if

$\alpha A + (1-\alpha)A \subset A$ for all $0 \leq \alpha \leq 1$.

Theorem (1.24):

Every open and closed balls in fuzzy modular space are convex sets.

Proof:

Let $y_1, y_2 \in B(x,r,t)$ such that $r \in (0,1), t > 0$,

$\mu(x - y_1, t) > 1 - r$ and $\mu(x - y_2, t) > 1 - r$.

Now ,we have to prove $\alpha y_1 + (1-\alpha)y_2 \in B(x,r,t)$

$\mu(x - (\alpha y_1 + (1-\alpha)y_2, t)$

$=\mu(x - \alpha y_1 + (1-\alpha)x - (1-\alpha)y_2, t)$

$= \mu(\alpha(x - y_1), t) + \mu\big((1-\alpha)(x - y_2), t\big)$

$= \mu(x - y_1, t) * \mu(x - y_2, t)$

$> 1 - r * 1 - r - r$

$= 1 - r$

Journal of AL-Qadisiyah for computer science and mathematics Vol.11 No.2 Year 2019

ISSN (Print): 2074 – 0204 ISSN (Online): 2521 – 3504

Then $B(x,r,t)$ is convex set.

similary, we can prove $B[x,r,t]$ is convex set.

Theorem (1.25):

Let X be a vector space . if A is convex set in fuzzy modular space then $\overline{A}$ is convex set.

proof :

Let $x, y \in \overline{A}$,$0 \leq \lambda \leq 1 \Longrightarrow \exists\ a, b \in A$ such that $\mu(x-a) < r, \mu(x-b) < r$

Since A is convex$\Rightarrow \alpha a + (1-\alpha)b \in A$

$\alpha x + (1-\alpha)y - (\alpha a + (1-\alpha)b)$

$= \alpha(x-a) + (1-\alpha)(y-b)$

$\mu\big(\alpha x + (1-\alpha)y - (\alpha a + (1-\alpha)b)\big)$

$\leq \alpha\, \mu(x-a) + (1-\alpha)\mu(y-b)$

$< \alpha r + (1-\alpha)r = r$

$\Longrightarrow \alpha x + (1-\alpha)y \in \overline{A} \Longrightarrow \overline{A}$ is convex set.

Lemma (1.26):

Let $(X, \mu, *)$ be afuzzy modular space and $A \subset X$, if for any $x \in \overline{A}$, then there exist a sequence $\{x_n\}$ in A such that $\lim_{n\to\infty} \mu(x_n, x, t) = 1$ for all $t > 0$.

Proof:

Suppose $x \in \overline{A}$

Since $\overline{A} = A \cup A^{`} \Longrightarrow x \in A$ and$x \in A^{`}$

If $x \in A$, *the* $\{x, x, \dots\dots\} \to x$

If $x \in A^{`}$ and $x \notin A \Longrightarrow \forall\, n \in Z^+ , r = \frac{1}{n}$,

$B(x,r,t) - \{x\} \cap A \neq \emptyset$

$x_n \in B(x,r,t) \cap A \Longrightarrow x_n \in A \in A$

$\Longrightarrow \mu(x_n - x, t) > 1 - \frac{1}{n}$

Then $B(x, \frac{1}{n}, t)$ contains $x_n \in A$ and $x_n \to x$ since $\lim_{n\to\infty} \mu(x_n - x, t) = 1$

Conversely :Suppose the condition is true ,to prove $X \in A^{`}$

If $\{x_n\}$ in A and $x_n \to x$ then $x \in A$

If $x \in A$

Let G be an open set contain $X \Longrightarrow \exists\, r > 0, B(x,r,t) \subseteq G$

Since $\{x_n\}$ in A such that $x_n \to x$ as $n \to \infty$

$\Longrightarrow \mu(x_n - x, t) > 1 - r$

Then $x_n \in G \Longrightarrow G - \{x\} \cap A \neq \emptyset$

Then $x \in \overline{A}$.

Definition (1.27):

A fuzzy modular space $(X, \mu, *)$ is called a locally convex if there is a local base β at 0 in X such that every member of β are convex sets.

Example (1.28) :

Every fuzzy modular space is locally convex

Solution: Let $(X, \mu, *)$ be afuzzy modular space Let $\beta = \{B(r, 0, t) : r > 0\}$,

where $B(r, 0, t) = \{x \in X : \mu(x, t) > 1 - r\}$,

Let G be an open set in X,then G is the union of open balls ,so $0 \in B(r, 0, t) \subset G$ for some $r > 0$, then β is alocal base at 0 in$X$

Since every open ball is convex set , then $B(x, 0, t)$ is convex set for all $r > 0$, then β is a convex local base at 0 in X

Therefore $(X, \mu, *)$ is locally convex space.

ISSN (Print): 2074 – 0204 ISSN (Online): 2521 – 3504

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Journal of AL-Qadisiyah for computer science and mathematics Vol.11 No.2 Year 2019

ISSN (Print): 2074 – 0204 ISSN (Online): 2521 – 3504

بعض خصائص فضاء الوحدات الضبابي التبولوجي

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