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Coefficient estimates for some subclasses of bi-univalent functions related to m-fold symmetry

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Abstract:

The purpose of present paper is to introduce and investigate two new subclasses $\mathcal{N}_{\sum_m}(\tau,\gamma,\alpha)$ and $\mathcal{N}_{\sum_m}(\tau,\gamma,\beta)$ of analytic and m-fold symmetric bi- univalent functions in the open unit disk . Among other results belonging to these subclasses upper coefficients bounds $|a_{m+1}|$ and $|a_{2m+1}|$ are obtained in this study. Certain special cases are also indicated .

Keywords:m-fold symmetry , bi-univalent functions , coefficient estimates.

Mathematics Subject Classification: 30C45.

1. Introduction

Let S denote the family of functions analytic in the open unit disk $U = \{z: z \in \mathbb{C}, |z| < 1\}$

and normalized by the conditions $f(0) = f'(0) - 1 = 0$ and having the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k. \qquad (1)$$

Also let $\mathcal{A}$ denote the subclass of functions in S which are univalent in U .

The Koebe One Quarter Theorem $(e.g., see$ [6]$)$ ensures that the image of U under every

univalent function $f(z) \in S$ contains the disk of radius $1/4$. Thus every univalent function

f has an inverse f^{-1} satisfying

$$f^{-1}(f(z)) = z \quad , \qquad (z \in U)$$

and

$$f(f^{-1}(w)) = w \quad , \quad \left(|w| < r_\circ(f), r_\circ(f) \geq \frac{1}{4}\right)$$

where

$$g(w) = f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \cdots \quad . \quad (2)$$

A function $f \in S$ is said to be bi-univalent in U if both f and f^{-1} are univalent in U .

Let $\sum$ denotes the class of analytic and bi-univalent functions in U . Some examples of functions in class $\sum$ are

$$h_1(z) = \frac{z}{1-z}, \quad h_2(z) = -\log(1-z), \quad h_3(z) = \frac{1}{2}\log\left(\frac{1+z}{1-z}\right), \quad z \in U .$$

For each function $f \in \mathcal{A}$, the function $h(z) = (f(z^m))^{\frac{1}{m}}$, $(z \in U, m \in \mathbb{N})$ is univalent and maps the unit disk U into a region with m-fold symmetry . A function is said to be m-fold symmetric $(see$ [9,10]$)$ if it has the following normalized form :

$$f(z) = z + \sum_{k=1}^{\infty} a_{mk+1} z^{mk+1}, \ (z \in U, m \in \mathbb{N}). \ (3)$$

We denote S_m the class of m-fold symmetric univalent functions in U , which are normalized by the series expansion (3). In fact , the functions in the class $\mathcal{A}$ are one-fold symmetric . Analogous to the concept of m-fold symmetric univalent functions , we here introduced the concept of m-fold symmetric univalent functions , we here introduced the concept of m-fold symmetric bi-univalent functions . Each function $f \in \sum$ generates an m-fold symmetric bi-univalent function for each integer $m \in \mathbb{N}$. Furthermore, for the normalized form of f is given by (3) , they obtained the series expansion for f^{-1} as follows :

$$g(w) = w - a_{m+1}w^{m+1} + [(m+1)a_{m+1}^2 - a_{2m+1}]w^{2m+1} - \left[\frac{1}{2}(m+1)(3m+2)a_{m+1}^2 - (3m+2)a_{m+1}a_{2m+1} + a_{3m+1}\right]w^{3m+1} + \cdots, (4)$$

where $f^{-1} = g$. We denote by $\sum_m$ the class of m-fold symmetric bi-univalent functions in U . It is easily seen that for m=1 , the formula (4) coincides with the formula (2) of the class $\sum$. Some examples of m-fold symmetric bi-univalent functions are given as follows :

$$\left(\frac{z^m}{1-z^m}\right)^{\frac{1}{m}}, \left[\frac{1}{2}log\left(\frac{1+z^m}{1-z^m}\right)\right]^{\frac{1}{m}} \ and$$

$$[-log(1-z^m)]^{\frac{1}{m}}$$

with the corresponding inverse functions

$$\left(\frac{w^m}{1+w^m}\right)^{\frac{1}{m}}, \left(\frac{e^{2w^m}-1}{e^{2w^m}+1}\right)^{\frac{1}{m}} \ and \ \left(\frac{e^{w^m}-1}{e^{w^m}}\right)^{\frac{1}{m}},$$

respectively .

Recently , many authors investigated bounds for various subclass of m-fold bi-univalent functions $(see$ [1,2,3,4,5,7,9,12,13,15]$)$ .The aim of the present paper is to introduce the new subclass $\mathcal{N}_{\sum_m}(\tau,\gamma;\alpha)$ and $\mathcal{N}_{\sum_m}(\tau,\gamma;\beta)$ of $\sum_m$ and find estimates on the coefficients $|a_{m+1}|$ and $|a_{2m+1}|$ for functions in each of these new subclass .

In order to prove our main results , we require the following lemma .

Lemma 1.([6]) .If $h \in \mathcal{P}$,then $|c_k| \leq 2$ for each $k \in \mathbb{N}$, where $\mathcal{P}$ is the family of all functions h analytic in U for which

$$Re(h(z)) > 0 \quad , \qquad (z \in U)$$

where

$$h(z) = 1 + c_1 z + c_2 z^2 + \cdots \ . \qquad (z \in U)$$

Definition 1. A function $f(z) \in \sum_m$ given by (3) is said to be in the class $\mathcal{N}_{\sum_m}(\tau,\gamma;\alpha)$ if the following condition are satisfied :

$$\left|arg\left(1 + \frac{1}{\tau}\left[\frac{(1+\gamma)z^2 f''(z) + zf'(z)}{(1+\gamma)zf'(z) - \gamma f(z)} - 1\right]\right)\right| < \frac{\alpha\pi}{2}$$

$$(z \in U) \qquad (5)$$

and

$$\left|arg\left(1+\frac{1}{\tau}\left[\frac{(1+\gamma)w^2g''(w)+wg'(w)}{(1+\gamma)wg'(w)-\gamma g(w)}-1\right]\right)\right|<\frac{\alpha\pi}{2}$$

$(w \in U)$ (6)

$(0 < \alpha \leq 1 \,; \tau \in \mathbb{C} \setminus \{0\} \,; 0 \leq \gamma < 1)$,

where the function $g = f^{-1}$ is given by (4) .

Definition 2. A function $f(z) \in \sum_m$ given by (3) is said to be in the class $\mathcal{N}_{\sum_m}(\tau,\gamma;\beta)$ if the following conditions are satisfied :

$$Re\left(1+\frac{1}{\tau}\left[\frac{(1+\gamma)z^2f''(z)+zf'(z)}{(1+\gamma)zf'(z)-\gamma f(z)}-1\right]\right)>\beta \quad ,$$

$(z \in U)$ (7)

and

$$\mathrm{Re}\left(1+\frac{1}{\tau}\left[\frac{(1+\gamma)w^2g''(w)+wg'(w)}{(1+\gamma)wg'(w)-\gamma g(w)}-1\right]\right)>\beta \quad ,$$

$(w \in U)$ (8)

$(0 \leq \beta < 1 \,; \tau \in \mathbb{C} \setminus \{0\} \,; 0 \leq \gamma < 1)$,

where the function $g = f^{-1}$ is given by (4) .

2.Coefficient Estimates for the Functions Class $\mathcal{N}_{\sum_m}(\tau,\gamma;\alpha)$

We begin this section by finding the estimates on the coefficients $|a_{m+1}|$ *and* $|a_{2m+1}|$ for functions in the class $\mathcal{N}_{\sum_M}(\tau,\gamma;\alpha)$.

Theorem 2.1 Let $f(z) \in \mathcal{N}_{\sum_m}(\tau,\gamma;\alpha)(0 < \alpha \leq 1;\ \tau \in \mathbb{C} \setminus \{0\}, 0 \leq \gamma < 1)$ be of the form (3) . Then

$$|a_{m+1}| \leq \frac{2\alpha|\tau|}{\sqrt{|[2m((2m+2m\gamma+1)(m+1)-(m+m\gamma+1)^2)-(\alpha-1)m^2(m+m\gamma+1)]|}} \quad (9)$$

and

$$|a_{2m+1}| \leq \frac{2\alpha^2|\tau|^2(m+1)}{m^2(m+m\gamma+1)^2}+\frac{\alpha|\tau|}{m(2m+2m\gamma+1)} \qquad (10)$$

Proof. It follows from (5) and (6) that

$$1+\frac{1}{\tau}\left[\frac{(1+\gamma)z^2f''(z)+zf'(z)}{(1+\gamma)zf'(z)-\gamma f(z)}-1\right]=[p(z)]^\alpha \quad (11)$$

and

$$1+\frac{1}{\tau}\left[\frac{(1+\gamma)w^2g''(w)+wg'(w)}{(1+\gamma)wg'(w)-\gamma g(w)}-1\right]=[q(w)]^\alpha, (12)$$

where the functions $p(z)$ and $q(w)$ are in $\mathcal{P}$ and have the following series representations:

$$p(z) = 1 + p_m z^m + p_{2m} z^{2m} + p_{3m} z^{3m} + \cdots \quad (13)$$

and

$$q(w) = 1 + q_m w^m + q_{2m} w^{2m} + q_{3m} w^{3m} + \cdots. \quad (14)$$

Now ,equating the coefficients in (11) and (12) ,we obtain

$$\frac{m(m+m\gamma+1)a_{m+1}}{\tau} = \alpha p_m \,, \qquad (15)$$

$$\frac{(2m(2m+2m\gamma+1)a_{2m+1}-m(m+m\gamma+1)^2a_{m+1}^2)}{\tau} = \alpha p_{2m}+\frac{\alpha(\alpha-1)}{2}p_m^2 \,,$$

(16)

and

$$\frac{-m(m+m\gamma+1)a_{m+1}}{\tau} = \alpha q_m$$

(17) $$\frac{(2m(2m+2m\gamma+1)[(m+1)a_{m+1}^2-a_{2m+1}]-m(m+m\gamma+1)^2a_{m+1}^2)}{\tau} = \alpha q_{2m}+\frac{\alpha(\alpha-1)}{2}q_m^2 \,. \qquad (18)$$

From (15)and (17) ,we find

$$p_m = -q_m (19)$$

and

$$2\frac{m^2(m+m\gamma+1)^2a_{m+1}^2}{\tau^2} = \alpha^2(p_m^2+q_m^2) \,. \quad (20)$$

From (16),(18) and (20), we get

$$\frac{((2m+2m\gamma+1)(m+1)-(m+m\gamma+1)^2)2ma_{m+1}^2}{\tau}$$

$$= \alpha(p_{2m}+q_{2m})+\frac{\alpha(\alpha-1)}{2}(p_m^2+q_m^2)$$

$$= \alpha(p_{2m}+q_{2m})+\frac{(\alpha-1)m^2(m+m\gamma+1)}{\alpha\tau^2}a_{m+1}^2 \,. \qquad (21)$$

Therefore ,we have

$$a_{m+1}^2 = \frac{\alpha^2\tau^2(p_{2m}+q_{2m})}{[2m((2m+2m\gamma+1)(m+1)-(m+m\gamma+1)^2)-(\alpha-1)m^2(m+m\gamma+1)]} \,. \quad (22)$$

Applying Lemma 1 for the coefficients p_{2m} and q_{2m} , we have

$$|\mathrm{a}_{m+1}| \leq \frac{2\alpha|t|}{\sqrt{|[2m((2m+2m\gamma+1)(m+1)-(m+m\gamma+1)^2)-(\alpha-1)m^2(m+m\gamma+1)]|}} . (23)$$

This gives the desired bound for $|a_{m+1}|$ as asserted in (9). In order to find the bound on $|a_{2m+1}|$, by subtracting (18) from (16), we get

$$\frac{2m[(2m+2m\gamma+1)a_{2m+1}-(2m+2m\gamma+1)(m+1)a_{m+1}^2]}{\tau} = \alpha(p_{2m}-q_{2m})+\frac{\alpha(\alpha-1)}{2}(p_m^2-q_m^2). \quad (24)$$

It follows from(19) and (24) that

$$a_{2m+1} = \frac{\alpha^2\tau^2(p_m^2+q_m^2)(m+1)}{4m(m+m\gamma+1)^2}+\frac{\alpha\tau(p_{2m}-q_{2m})}{4m(2m+2m\gamma+1)} .(25)$$

Applying Lemma 1 once again for the coefficients p_m, p_{2m}, q_m and q_{2m} ,we readily obtain

$$|a_{2m+1}| \le \frac{2\alpha^2|\tau|^2(m+1)}{m^2(m+m\gamma+1)^2} + \frac{\alpha|\tau|}{m(2m+2m\gamma+1)}. \quad (26)$$

3.Coefficient Bounds for the Functions Class $\mathcal{N}_{\Sigma_m}(\tau,\gamma;\beta)$

This section is devoted to find the estimates on the coefficients $|a_{2m+1}|$ *and* $|a_{m+1}|$ for functions in the class $\mathcal{N}_{\Sigma_m}(\tau,\gamma;\beta)$.

Theorem 3.1 Let $f(z) \in \mathcal{N}_{\Sigma_m}(\tau,\gamma;\beta)(0 \le \beta < 1\,; \tau \in \mathbb{C}\setminus\{0\}, 0 \le \gamma < 1)$ be of the form (3).

Then

$$|a_{m+1}| \le \sqrt{\frac{2|\tau|(1-\beta)}{m((2m+2m\gamma+1)(m+1)-(m+m\gamma+1)^2)}} \quad (27)$$

and

$$|a_{2m+1}| \le \frac{4|\tau|^2(1-\beta)^2(m+1)}{m^2(m+m\gamma+1)^2} + \frac{2|\tau|(1-\beta)}{m(2m+2m\gamma+1)} (28)$$

Proof. It follows from (7) and (8) that there exist , $p, q \in \mathcal{P}$ such that

$$1 + \frac{1}{\tau}\left[\frac{(1+\gamma)z^2f''(z)+zf'(z)}{(1+\gamma)zf'(z)-\gamma f(z)} - 1\right] = \beta + (1-\beta)p(z) \quad (29)$$

and

$$1 + \frac{1}{\tau}\left[\frac{(1+\gamma)w^2g''(w)+wg'(w)}{(1+\gamma)wg'(w)-\gamma g(w)}\right] = \beta + (1-\beta)q(w), \quad (30)$$

where $p(z)$ *and* $q(z)$ have the forms (13) and (14) , respectively . By suitably comparing coefficients in (29) and (30), we get

$$\frac{m(m+m\gamma+1)a_{m+1}}{\tau} = (1-\beta)p_m\,, \quad (31)$$

$$\frac{(2m(2m+2m\gamma+1)a_{2m+1} - m(m+m\gamma+1)^2a_{m+1}^2)}{\tau} = (1-\beta)p_{2m}\,, \quad (32)$$

$$\frac{-m(m+m\gamma+1)a_{m+1}}{\tau} = (1-\beta)q_m\,, \quad (33)$$

$$\frac{(2m(2m+2m\gamma+1)[(m+1)a_{m+1}^2 - a_{2m+1}] - m(m+m\gamma+1)^2a_{m+1}^2)}{\tau} = (1-\beta)q_{2m}\,. \quad (34)$$

From (31) and (33) , we find

$$p_m = -q_m (35)$$

and

$$\frac{2m^2(m+m\gamma+1)^2a_{m+1}^2}{\tau^2} = (1-\beta)^2(p_m^2 + q_m^2)\,. \quad (36)$$

Adding (32) and (34) ,we have

$$\frac{\left((2m+2m\gamma+1)(m+1)-(m+m\gamma+1)^2\right)2ma_{m+1}^2}{\tau} = (1-\beta)(p_{2m} + q_{2m})\,. (37)$$

Applying Lemma 1 , we obtain

$$|a_{m+1}| \le \sqrt{\frac{2|\tau|(1-\beta)}{m((2m+2m\gamma+1)(m+1)-(m+m\gamma+1)^2)}}\,.$$

This is the bound on $|a_{m+1}|$ asserted in (27) .

In order to find the bound on $|a_{2m+1}|$,by subtracting (34) form (32) ,we get

$$\frac{2m[(2m+2m\gamma+1)a_{2m+1} - (2m+2m\gamma+1)(m+1)a_{m+1}^2]}{\tau} = (1-\beta)(p_{2m} - q_{2m}) \quad (38)$$

Or ,equivalently ,

$$a_{2m+1} = \frac{2m(2m+2m\gamma+1)(m+1)a_{m+1}^2}{2m(2m+2m\gamma+1)} + \frac{\tau(1-\beta)(p_{2m}-q_{2m})}{2m(2m+2m\gamma+1)} (39)$$

It follows from (35) and (36) that

$$a_{2m+1} = \frac{\tau^2(1-\beta)^2(m+1)(p_m^2+q_m^2)}{2m^2(m+m\gamma+1)^2} + \frac{\tau(1-\beta)(p_{2m}-q_{2m})}{2m(2m+2m\gamma+1)}\,. (40)$$

Applying lemma 1 once again for the coefficients p_m, p_{2m}, q_m *and* q_{2m} , we easily obtain

$$|a_{2m+1}| \le \frac{4|\tau|^2(1-\beta)^2(m+1)}{m^2(m+m\gamma+1)^2} + \frac{2|\tau|(1-\beta)}{m(2m+2m\gamma+1)}\,. \quad (41)$$

4.Corollaries and Consequencess

For one-fold symmetric bi-univalent functions and $\tau = 1$, Theorem 2.1 and Theorem 3.1 reduce to Corollary 1 and Corollary 2 , respectively , which were proven very recently by Frasin[8]($see\ also$ [11]).

Corollary 4 . Let $f(z) \in \mathcal{N}_{\Sigma}(\alpha,\gamma)(0 < \alpha \le 1; 0 \le \gamma < 1)$ be of the form (1) .

Then

$$|a_2| \le \frac{2\alpha}{\sqrt{2(3-\alpha)-\gamma(\gamma+\alpha-1)}} (42)$$

and

$$|a_3| \le \frac{4\alpha^2}{(2+\gamma)^2} + \frac{\alpha}{(3+2\gamma)}\,. \quad (43)$$

Corollary 5 . Let $f(z) \in \mathcal{N}_{\Sigma}(\beta,\gamma)(0 < \alpha \le 1; 0 \le \gamma < 1)$ be of the form (1) .

Then

$$|a_2| \leq \sqrt{\frac{2(1-\beta)}{(2+2\gamma+\gamma^2)}} \quad (44)$$

and

$$|a_3| \leq \frac{8(1-\beta)^2}{(2+\gamma)^2} + \frac{2(1-\beta)}{(3+2\gamma)} (45)$$

The classes $\mathcal{N}_{\Sigma}(\alpha,\gamma)$ *and* $\mathcal{N}_{\Sigma}(\beta,\gamma)$are defined in the following way :

Definition 3 . A function $f(z) \in \Sigma$ given by (1) is said to be in the class$\mathcal{N}_{\Sigma}$ if the following conditions are satisfied :

$$\left|arg\left(\frac{(1+\gamma)z^2f''(z)+zf'(z)}{(1+\gamma)zf'(z)-\gamma f(z)}\right)\right| < \frac{\alpha\pi}{2}\,(z \in U)(46)$$

And

$$\left|arg\left(\frac{(1+\gamma)w^2g''(w)+wg'(w)}{(1+\gamma)wg'(w)-\gamma g(w)}\right)\right| < \frac{\alpha\pi}{2}\,(w \in U)(47)$$

$(0 < \alpha \leq 1 ; 0 \leq \gamma < 1)$,

where the function $g = f^{-1}$ is given by (2) .

Definition 4 . A function $f(z) \in \Sigma$ given by (1) is said to be in the class$\mathcal{N}_{\Sigma}(\beta,\gamma)$ if the following conditions are satisfied :

$$Re\left(\frac{(1+\gamma)z^2f''(z)+zf'(z)}{(1+\gamma)zf'(z)-\gamma f(z)}\right) > \beta \quad (z \in U) \qquad (48)$$

And

$$Re\left(\frac{(1+\gamma)w^2g''(w)+wg'(w)}{(1+\gamma)wg'(w)-\gamma g(w)}\right) > \beta \quad (w \in U) \quad (49)$$

$$(0 \leq \beta < 1 ; 0 \leq \gamma < 1) ,$$

where the function $g = f^{-1}$ is given by (2) .

If we set $\gamma = 0$ and $\tau = 1$ in Theorem2. 1 and Theorem 3.1 , then the classes $\mathcal{N}_{\Sigma_m}(\tau,\gamma;\alpha)$ and $\mathcal{N}_{\Sigma_m}(\tau,\gamma;\beta)$ reduce to the classes $\mathcal{N}_{\Sigma m}^{\alpha}$ and $S_{\Sigma m}^{\beta}$ investigated recently by Srivastava et al. [11]and thus, we obtain the following corollaries:

Corollary 6 . Let $f(z) \in \mathcal{N}_{\Sigma m}^{\alpha}\,(0 < \alpha \leq 1)$ be of the form (3) .Then

$$|a_{m+1}| \leq \frac{2\alpha}{\sqrt{[m(2m+1)(m+1)-m(m+1)^2+m^2(m+1)^2(\alpha-1)]}} \qquad (50)$$

and

$$|a_{2m+1}| \leq \frac{\alpha}{m(2m+1)} + \frac{2\alpha(m+1)}{m^3(m+1)^2} . \quad (51)$$

Corollary 7 . Let $f(z) \in \mathcal{N}_{\Sigma m}^{\alpha}\,(0 \leq \beta \leq 1)$ be of the form (4) . Then

$$|a_{m+1}| \leq \sqrt{\frac{2(1-\beta)}{[m(2m+1)(m+1)-m(m+1)^2]}}(52)$$

and

$$|a_{2m+1}| \leq \frac{(1-\beta)}{m(2m+1)} + \frac{2(1-\beta)^2(m+1)}{m^3(m+1)^2}(53)$$

The classes $\mathcal{N}_{\Sigma m}^{\alpha}$ *and* $\mathcal{N}_{\Sigma m}^{\beta}$ are respectively defined as follows :

Definition 5 . A function $f(z) \in \sum_m$ given by (3) is said to be in the class $\mathcal{N}_{\Sigma m}^{\alpha}$ if the following conditions are satisfied :

$$\left|arg\left\{\frac{z^2f''(z)}{zf'(z)} + 1\right\}\right| < \frac{\alpha\pi}{2}\,(z \in U)(54)$$

and

$$\left|arg\left\{\frac{w^2g''(w)}{wg'(w)} + 1\right\}\right| < \frac{\alpha\pi}{2} \quad , \quad (w \in U)(55)$$

and where the function g is given by (4) .

Definition 6 . A function $f(z) \in \sum_m$ given by (3) is said to be in the class $\mathcal{N}_{\Sigma m}^{\beta}$ if the following

conditions are satisfied :

$$Re\left\{\frac{z^2f''(z)}{zf'(z)} + 1\right\} > \beta \quad (z \in U)(56)$$

and

$$Re\left\{\frac{w^2g''(w)}{wg'(w)} + 1\right\} > \beta \;\; (w \in U) . \;\; (57)$$

$(0 \leq \beta < 1)$

And where the function g is given by (4) .

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مخمنات المعامل لبعض الاصناف الجزئية للدوال ثنائية التكافؤ المرتبطة بالطوية m– التناظرية

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المستخلص:

الغرض من البحث الحالي هو ان نقدم ونتحرى عن صنفين جزئيين جديدين $\mathcal{N}_{\sum_m}(\tau,\gamma,\alpha)$ و $\mathcal{N}_{\sum_m}(\tau,\gamma,\beta)$ من الدوال ثنائية التكافؤ المتناظرة ذات الطوية m– والتحليلية في قرص الوحدة المفتوح ومن بين النتائج الاخرى لهذه الاصناف الجزئية حدود المعاملات العليا ($|a_{m+1}|, |a_{2m+1}|$)تم الحصول عليها في هذه الدراسة .