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Some Results on Symmetric Reverse $\ast$-n-Derivations

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ABSTRACT

In this paper, the commuting and centralizing of symmetric reverse $\ast$-n-derivation on Lie ideal are studied and the commutativity of prime $\ast$-ring with the concept of symmetric reverse $\ast$-n-derivations are proved under certain conditions.

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1. Introduction

Throughout this paper $\mathcal{R}$ will represent an associative ring with center $\mathcal{Z}(\mathcal{R})$. For any $\upsilon, \gamma \in \mathcal{R}$, the commutator $\upsilon\gamma$-$\gamma\upsilon$ was denoted by $[\upsilon, \gamma]$ and the anti-commutator $\upsilon \circ \gamma$ was denoted by $\upsilon\gamma + \gamma\upsilon$ [8]. A ring $\mathcal{R}$ is said to be n-torsion free if na=0 with $a \in \mathcal{R}$ then a=0, where n is nonzero integer [7]. Recall that a ring $\mathcal{R}$ is said to be prime if $a\mathcal{R}b$=0 implies that either a=0 or b=0 for all $a, b \in \mathcal{R}$ [12] and it is semiprime if $a\mathcal{R}a$=0 implies that a=0 for all $a \in \mathcal{R}$ [7]. An additive mapping $\xi:\mathcal{R} \rightarrow \mathcal{R}$ is called a derivation if $\xi(\upsilon\gamma) = \xi(\upsilon)\gamma + \upsilon\xi(\gamma)$ for all $\upsilon, \gamma \in \mathcal{R}$ [11]. In [2] were introduced the concept of reverse derivations; an additive mapping $\xi:\mathcal{R} \rightarrow \mathcal{R}$ is called a reverse derivation if $\xi(\upsilon\gamma)=\xi(\gamma)\upsilon+\gamma\xi(\upsilon)$ for all $\upsilon, \gamma \in \mathcal{R}$. A map $\mathcal{F}: \mathcal{R} \rightarrow \mathcal{R}$ is said to be commuting (resp. centralizing) on $\mathcal{R}$ if $[\mathcal{F}(\upsilon),\upsilon]$=0 (resp. $[\mathcal{F}(\upsilon), \upsilon] \in \mathcal{Z}(\mathcal{R})$) for all $\upsilon \in \mathcal{R}$ [12]. An additive mapping $\upsilon \rightarrow \upsilon^*$ of $\mathcal{R}$ into itself is called an involution if the following conditions are satisfied (i) $(\upsilon\gamma)^* = \gamma^*\upsilon^*$ (ii) $(\upsilon^*)^* = \upsilon$ for all $\upsilon, \gamma \in \mathcal{R}$ [8]. A ring equipped with an involution is known as ring with involution or $*$-ring. Let $\mathcal{R}$ be a $*$-ring. An additive mapping $\xi:\mathcal{R} \rightarrow \mathcal{R}$ is called a $*$-derivation (resp. a reverse $*$-derivation) if $\xi(\upsilon\gamma) = \xi(\upsilon)\gamma^* + \upsilon\xi(\gamma)$ (resp. $\xi(\upsilon\gamma)=\xi(\gamma)\upsilon^*+\gamma\xi(\upsilon)$) for all $\upsilon, \gamma \in \mathcal{R}$ [2]. An additive subgroup $\mathcal{U}$ of $\mathcal{R}$ is called Lie ideal if whenever $u \in \mathcal{U}$, $\mathfrak{r} \in \mathcal{R}$ then $[u, \mathfrak{r}] \in \mathcal{U}$ [7]. A Lie ideal $\mathcal{U}$ of $\mathcal{R}$ is called a square closed Lie ideal of $\mathcal{R}$ if $u^2 \in \mathcal{U}$, for all $u \in \mathcal{U}$ [3]. A square closed Lie ideal $\mathcal{U}$ of $\mathcal{R}$ such that $\mathcal{U} \nsubseteq \mathcal{Z}(\mathcal{R})$ is called an admissible Lie ideal of $\mathcal{R}$ [11]. Relationship between derivations and reverse derivations with examples were given by [13]. Recently there has been a great deal of work done by many authors on commuting and centralizing mappings on prime rings and semiprime rings, see ([4],[5],[6],[9],[10]). In [2] studied the notion of a $*$-derivation of $\mathcal{R}$. Recently [1] defined the concept of $*$-n-derivation in prime $*$-rings and semiprime $*$-rings. Many authors have proved the commutativity of prime and semiprime rings admitting derivation ([11],[3]). In the present paper the commuting and centralizing of symmetric reverse $*$-n-derivation of Lie ideal are studied under certain conditions and on the other hand the commutativity of prime $*$-ring with symmetric reverse $*$-n-derivations that satisfying certain identities and some regarding results have also been discussed.
Throughout this paper consider n is a fixed positive integer.

2. Preliminaries

Some definitions and fundamental facts of symmetric reverse $*$-n-derivations are recalled in this section, which are principals of reverse left $*$-n-derivation.

Proposition (2.1) [8]

Let $\mathcal{R}$ be a ring, then for all $\upsilon, \gamma, z \in \mathcal{R}$ we have

1- $[\upsilon, \gamma z] = \gamma[\upsilon, z] + [\upsilon, \gamma]z$
2- $[\upsilon\gamma, z] = \upsilon[\gamma, z] + [\upsilon, z]\gamma$
3- $\upsilon \circ (\gamma z) = (\upsilon \circ \gamma)z - \gamma[\upsilon, z] = \gamma(\upsilon \circ z) + [\upsilon, \gamma]z$
4- $(\upsilon\gamma) \circ z = \upsilon(\gamma \circ z) - [\upsilon, z]\gamma = (\upsilon \circ z)\gamma + \upsilon[\gamma, z]$

Definition (2.2) [9]

A map $\xi: \mathcal{R}^n \to \mathcal{R}$ is called permuting (or symmetric) if the equation $\xi(\upsilon_1, \upsilon_2, \dots, \upsilon_n) = \xi(\upsilon_{\pi(1)}, \upsilon_{\pi(2)}, \dots, \upsilon_{\pi(n)})$ holds, for all $\upsilon_i \in \mathcal{R}$ and for every permutation $\{\pi(1), \pi(2), \dots, \pi(n)\}$.

Definition (2.3) [9]

A map $\delta: \mathcal{R} \to \mathcal{R}$ is define as $\delta(\upsilon) = \Omega(\upsilon, \upsilon, \dots, \upsilon)$ for all $\upsilon \in \mathcal{R}$, where $\Omega: \mathcal{R}^n \to \mathcal{R}$ is called the trace of the symmetric mapping Ω.

It is clear that the trace function δ is an odd function if n is an odd number and is an even function if n is an even number.

Note (2.4) [9]

Let δ be a trace of an n-additive symmetric map $\delta: \mathcal{R}^n \to \mathcal{R}$, then δ satisfies the relation $\delta(\upsilon+\gamma) = \delta(\upsilon) + \delta(\gamma) + \sum_{k=1}^{n-1} \binom{n}{k} h_k(\upsilon, \gamma)$ for all $\upsilon, \gamma \in \mathcal{R}$ such that $h_k(\upsilon, \gamma) = \Omega(\upsilon, \upsilon, \dots, \upsilon, \gamma, \gamma, \dots, \gamma)$ where υ appears $(n-k)$-times and γ appear k-times and $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

Definition (2.5) [9]

An n-additive mapping $\xi: \mathcal{R}^n \to \mathcal{R}$ is said to be a symmetric $*$-n-derivation if the following equations are identical:

$\xi(\upsilon_1\gamma, \upsilon_2, \dots, \upsilon_n) = \xi(\upsilon_1, \upsilon_2, \dots, \upsilon_n)\gamma^* + \upsilon_1\xi(\gamma, \upsilon_2, \dots, \upsilon_n)$

$\xi(\upsilon_1, \upsilon_2\gamma, \dots, \upsilon_n) = \xi(\upsilon_1, \upsilon_2, \dots, \upsilon_n)\gamma^* + \upsilon_2\xi(\upsilon_1, \gamma, \dots, \upsilon_n)$

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$\xi(\upsilon_1, \upsilon_2, \dots, \upsilon_n\gamma) = \xi(\upsilon_1, \upsilon_2, \dots, \upsilon_n)\gamma^* + \upsilon_n\xi(\upsilon_1, \upsilon_2, \dots, \gamma)$, for all $\upsilon_1, \gamma, \upsilon_2, \dots, \upsilon_n \in \mathcal{R}$.

Definition (2.6) [15]

An n-additive symmetric mapping $\xi: \mathcal{R}^n \to \mathcal{R}$ is said to be a symmetric reverse $*$-n-derivation if

$\xi(\upsilon_1\gamma, \upsilon_2, \dots, \upsilon_n) = \xi(\gamma, \upsilon_2, \dots, \upsilon_n)\upsilon_1{}^* + \gamma\xi(\upsilon_1, \upsilon_2, \dots, \upsilon_n)$

$\xi(\upsilon_1, \upsilon_2\gamma, \dots, \upsilon_n) = \xi(\upsilon_1, \gamma, \dots, \upsilon_n)\upsilon_2{}^* + \gamma\xi(\upsilon_1, \upsilon_2, \dots, \upsilon_n)$

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$\xi(\upsilon_1, \upsilon_2, \dots, \upsilon_n\gamma) = \xi(\upsilon_1, \upsilon_2, \dots, \gamma)\upsilon_n{}^* + \gamma\xi(\upsilon_1, \upsilon_2, \dots, \upsilon_n)$, for all $\upsilon_1\gamma, \upsilon_2, \dots, \upsilon_n \in \mathcal{R}$.

Example (2.7):

Consider $\mathcal{R}=\left\{\begin{pmatrix}0 & a & b\\ 0 & 0 & c\\ 0 & 0 & 0\end{pmatrix} \mid a,b,c\in\mathbb{C}\right\}$, where $\mathbb{C}$ is a ring of complex numbers and $\mathcal{R}$ is a non-commutative ring under the usual addition and multiplication of matrices. A map $\xi:\mathcal{R}^n\to\mathcal{R}$ is define by ξ $\left(\begin{pmatrix}0 & a_1 & b_1\\ 0 & 0 & c_1\\ 0 & 0 & 0\end{pmatrix},\begin{pmatrix}0 & a_2 & b_2\\ 0 & 0 & c_2\\ 0 & 0 & 0\end{pmatrix},\ldots,\begin{pmatrix}0 & a_n & b_n\\ 0 & 0 & c_n\\ 0 & 0 & 0\end{pmatrix}\right)=\begin{pmatrix}0 & 0 & c_1c_2\ldots c_n\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$, for all

$\begin{pmatrix}0 & a_1 & b_1\\ 0 & 0 & c_1\\ 0 & 0 & 0\end{pmatrix},\begin{pmatrix}0 & a_2 & b_2\\ 0 & 0 & c_2\\ 0 & 0 & 0\end{pmatrix},\ldots,\begin{pmatrix}0 & a_n & b_n\\ 0 & 0 & c_n\\ 0 & 0 & 0\end{pmatrix}\in\mathcal{R}$.

And $*$ is defined by $\begin{pmatrix}0 & a & b\\ 0 & 0 & c\\ 0 & 0 & 0\end{pmatrix}^*=\begin{pmatrix}0 & c & b\\ 0 & 0 & a\\ 0 & 0 & 0\end{pmatrix}$. Then, ξ is a symmetric reverse $*$-n-derivations.

Lemma (2.8) [11]: Let $\mathcal{R}$ be a prime ring and $\xi:\mathcal{R}\to\mathcal{R}$ be a derivation such that $a\in\mathcal{R}$. If $a\xi(\upsilon)=0$ holds for all $\upsilon\in\mathcal{R}$, then either $a=0$ or $\xi=0$.

Lemma (2.9) [14]: Let $\mathcal{R}$ be a $n!$-torsion free ring and $\lambda\gamma_1+\lambda^2\gamma_2+\ldots+\lambda^n\gamma_n=0$ where $\gamma_1,\gamma_2,\ldots,\gamma_n\in\mathcal{R}$ with $\lambda=1,2,\ldots,n$. Then $\gamma_i=0$, for all $i=1,2,\ldots,n$.

Lemma (2.10) [9]: Let $\mathcal{R}$ be a $n!$-torsion free ring and $\lambda\gamma_1+\lambda^2\gamma_2+\ldots+\lambda^n\gamma_n\in\mathcal{Z}(\mathcal{R})$ where $\gamma_1,\gamma_2,\ldots,\gamma_n\in\mathcal{R}$ with $\lambda=1,2,\ldots,n$. Then $\gamma_i\in\mathcal{Z}$, for all $i=1,2,\ldots,n$.

3. The Main Results

The commuting and centralizing of symmetric reverse $*$-n-derivations are studied and investigate the commutativity of prime $*$-ring with symmetric reverse $*$- n-derivations that satisfying certain conditions to obtain main results.
In the following results, $\mathcal{U}$ assumed as an admissible Lie ideal of $n!$-torsion free ring $\mathcal{R}$ with $n\geq 2$.

Theorem (3.1): Let $\mathcal{R}$ be a prime $*$-ring and $\Omega:\mathcal{U}^n\to\mathcal{R}$ be a symmetric reverse $*$-n-derivation associated with involution. If the trace δ of Ω satisfies $[\delta(\upsilon),\upsilon^*]=0$, for all $\upsilon\in\mathcal{U}$ then $\Omega(\upsilon_1,\upsilon_2,\ldots,\upsilon_n)=0$, for all $\upsilon_i\in\mathcal{U}$, $i=1,2,\ldots,n$.

Proof:

$[\delta(\upsilon),\upsilon^*]=0,\quad\forall\upsilon\in\mathcal{U}$ … (1)

Substituting $\upsilon=\upsilon+\mu\gamma$ in equation (1) and using it and let $\mu(1\leq\mu\leq n)$ be any integer, to obtain

$0=[\delta(\upsilon+\mu\gamma),\upsilon^*+\mu\gamma^*)]$

$=[\delta(\upsilon)+\delta(\mu\gamma)+\sum_{s=1}^{n-1}C_sf_s(\upsilon,\mu\gamma),\ \upsilon^*+\mu\gamma^*]$

$=\mu\{[\delta(\upsilon),\gamma^*]+[c_1f_1(\upsilon,\gamma),\upsilon^*]\}+\mu^2\{[c_2f_2(\upsilon,\gamma),\upsilon^*]+[c_1f_1(\upsilon,\gamma),\gamma^*]\}+\ldots+\mu^n\{[\delta(\gamma),\upsilon^*]+[c_{n-1}f_{n-1}(\upsilon,\gamma),\gamma^*]\}$ … (2)

Applying lemma (2.9) to equation (2), to get

$[\delta(\upsilon),\gamma^*]+[c_1f_1(\upsilon,\gamma),\upsilon^*]=0$ … (3)

Replacing $\gamma=2\upsilon\gamma$ in equation (3) then

$0=[\delta(\upsilon),(2\upsilon\gamma)^*]+[c_1f_1(\upsilon,2\upsilon\gamma),\upsilon^*]$

$=[\delta(\upsilon),\gamma^*]\upsilon^*+c_1[f_1(\upsilon,\gamma),\upsilon^*]\upsilon^*+c_1[\gamma,\upsilon^*]\delta(\upsilon)+c_1\gamma[\delta(\upsilon),\upsilon^*]$

$=\{[\delta(\upsilon),\gamma^*]+c_1[f_1(\upsilon,\gamma),\upsilon^*]\}\upsilon^*+c_1[\gamma,\upsilon^*]\delta(\upsilon)$

By using equation (3) with the equation above to obtain

$c_1[\gamma, \upsilon^*]\delta(\upsilon)$=0

Using n!-torsion freeness, to get

$[\gamma, \upsilon^*]\delta(\upsilon)$=0, $\forall\ \upsilon, \gamma \in \mathcal{U}$ ... (4)

Replacing γ =2 γw in equation (4) and using it, for all $w \in \mathcal{U}$ then

0=$[2\gamma w, \upsilon^*]\delta(\upsilon)$

=$[\gamma, \upsilon^*]w\delta(\upsilon)$ ... (5)

By using lemma (2.8), that $\gamma \to [\gamma, \alpha^*(\upsilon)]$ is a derivation on $\mathcal{U}$.

Then $\delta(\upsilon)$=0 ... (6)

Now, for each value l=1,2,..., n , let us denote

$T_l(\upsilon)=\Omega(\upsilon, \upsilon, \dots, \upsilon_{l+1}, \upsilon_{l+2}, \dots, \upsilon_n)$, where $\upsilon, \upsilon_i \in \mathcal{U}$, $i=l+1, l+2, \dots, n$.

$T_n(\upsilon)$=δ(υ)=0 ... (7)

Let $\eta(1 \leq \eta \leq n)$ be any positive integer. From equation (7) to have

$0=T_n(\eta\upsilon+\upsilon_n)=T_n(\upsilon_n)+T_n(\eta\upsilon)+\sum_{l=1}^{n-1}\eta^l C_l T_l(\upsilon)=\delta(\upsilon_n)+\eta^n\delta(\upsilon)+\sum_{l=1}^{n-1}\eta^l C_l T_l(\upsilon)$

$=\sum_{l=1}^{n-1}\eta^l C_l T_l(\upsilon)=\eta^1 C_1 T_1(\upsilon)+\eta^2 C_2 T_2(\upsilon)+\dots+\eta^{n-1}C_{n-1}T_{n-1}(\upsilon)$ ... (8)

Applying lemma (2.9) to equation (8) then

$c_1 T_1(\upsilon)$=0 then $T_1(\upsilon)$=0 which implies that $\Omega(\upsilon, \upsilon_2, \upsilon_3, \dots, \upsilon_n)$=0

$c_2 T_2(\upsilon)$=0 then $T_2(\upsilon)$=0 which implies that $\Omega(\upsilon, \upsilon, \upsilon_3, \dots, \upsilon_n)$=0

$c_{n-1}T_{n-1}(\upsilon)$=0 then $T_{n-1}(\upsilon)$=0 which implies that $\Omega(\upsilon, \upsilon, \upsilon, \dots, \upsilon_n)$=0

Hence from above we have $T_{n-1}(\upsilon)$=0 ... (9)

Again let $\tau(1 \leq \tau \leq n-1)$ be any positive integer. Then from equation (9) to get

$0=T_{n-1}(\tau\upsilon+\upsilon_{n-1})=T_{n-1}(\tau\upsilon)+T_{n-1}(\upsilon_{n-1})+\sum_{t=1}^{n-2}\tau^t C_t T_t(\upsilon)$

$=\tau^1 C_1 T_1(\upsilon)+\tau^2 C_2 T_2(\upsilon)+\dots+\tau^{n-2}C_{n-2}T_{n-2}(\upsilon)$ ... (10)

Again applying lemma (2.9) to equation (10) to get

$\Omega(\upsilon, \upsilon, \dots, \upsilon, \upsilon_{n-1}, \upsilon_n)=T_{n-2}(\upsilon)$=0 ... (11)

Continuing the above process, finally we obtain $T_1(\upsilon)$=0, then

$\Omega(\upsilon_1, \upsilon_2, \upsilon_3, \dots, \upsilon_{n-1}, \upsilon_n)$=0 ... (12)

Replacing $\upsilon_1=2\upsilon_1 p_1$, where $p_1 \in \mathcal{U}$ in equation (12) to get

$0=\Omega(2\upsilon_1 p_1, \upsilon_2, \upsilon_3, \dots, \upsilon_{n-1}, \upsilon_n)=\alpha(p_1)\ \Omega(\upsilon_1, \upsilon_2, \upsilon_3, \dots, \upsilon_{n-1}, \upsilon_n)+\upsilon_1\Omega(p_1, \upsilon_2, \upsilon_3, \dots, \upsilon_{n-1}, \upsilon_n)=\upsilon_1\Omega(p_1, \upsilon_2, \upsilon_3, \dots, \upsilon_{n-1}, \upsilon_n)$ ... (13)

Applying lemma (2.8) to equation (13) then

$\Omega(p_1, \upsilon_2, \upsilon_3, \ldots, \upsilon_{n-1}, \upsilon_n)$=0, $\forall p_1, \upsilon_i \in \mathcal{U}$.

Replacing $\upsilon_2 = \upsilon_2 p_2$, $p_2 \in \mathcal{U}$ in equation (13) to obtain

0=Ω $(p_1, \upsilon_2 p_2, \upsilon_3, \ldots, \upsilon_{n-1}, \upsilon_n) = \alpha(p_2)\,\Omega(p_1, \upsilon_2, \upsilon_3, \ldots, \upsilon_{n-1}, \upsilon_n) + \upsilon_2\Omega(p_1, p_2, \ldots, \upsilon_{n-1}, \upsilon_n) = \upsilon_2\Omega(p_1, p_2, \ldots, \upsilon_{n-1}, \upsilon_n) = \Omega(p_1, p_2, \ldots, \upsilon_{n-1}, \upsilon_n), \forall p_1, p_2, \upsilon_i \in \mathcal{U}$

Repeating the above process we finally obtain $\Omega(p_1, p_2, \ldots, p_{n-1}, p_n) = 0, \forall p_i \in \mathcal{U}$.

Theorem (3.2): Let $\mathcal{R}$ be a semiprime $*$-ring and $\Omega: \mathcal{U}^n \to \mathcal{R}$ be a symmetric reverse $*$-n-derivation associated with involution. If the trace δ of Ω such that δ is commuting on $\mathcal{U}$ and $[\delta(\upsilon), \upsilon^*] \in \mathcal{Z}(\mathcal{R})$, then $[\delta(\upsilon), \upsilon^*]$=0 for all $\upsilon \in \mathcal{U}$.

Proof:

$[\delta(\upsilon), \upsilon^*] \in \mathcal{Z}(\mathcal{R}), \quad \forall\, \upsilon \in \mathcal{U}$ ... (1)

Substituting υ=υ+$\mu\gamma$ in equation (1) and using it and let $\mu(1 \le \mu \le n)$ be any integer, then

$\mathcal{Z}(\mathcal{R}) \ni [\delta(\upsilon + \mu\gamma), \upsilon^* + \mu\gamma^*]$

$=[\delta(\upsilon) + \delta(\mu\gamma) + \sum_{s=1}^{n-1} C_s f_s(\upsilon, \mu\gamma),\ \upsilon^* + \mu\gamma^*]$

$= [\delta(\upsilon), \upsilon^*] + \mu\{[\delta(\upsilon), \gamma^*] + [c_1 f_1(\upsilon, \gamma), \upsilon^*]\} + \mu^2\{[c_2 f_2(\upsilon, \gamma), \upsilon^*] + [c_1 f_1(\upsilon, \gamma), \gamma^*]\} + \cdots + \mu^n\{[\delta(\gamma), \upsilon^*] + [c_{n-1} f_{n-1}(\upsilon, y), \gamma^*]\} + \mu^{n+1}[\delta(\gamma), \gamma^*]$ ... (2)

Commuting equation (2) with $\delta(\upsilon)$ to get

$\big[[\delta(\upsilon), \upsilon^*], \delta(\upsilon)\big] + \mu\big\{\big[[\delta(\upsilon), \gamma^*] + [c_1 f_1(\upsilon, \gamma), \upsilon^*], \delta(\upsilon)\big]\big\} + \mu^2\{[[c_2 f_2(\upsilon, \gamma), \upsilon^*]$
$[c_1 f_1(\upsilon, \gamma), \gamma^*], \delta(\upsilon)]\}+\ldots+\mu^n\{[[\delta(\gamma), \upsilon^*]+[c_{n-1} f_{n-1}(\upsilon, \gamma), \gamma^*], \delta(\upsilon)]\}+\mu^{n+1}\big[[\delta(\gamma), \gamma^*], \delta(\upsilon)\big]=0$ ... (3)

Applying lemma (2.9) to equation (3) to have

$[[\delta(\upsilon), \gamma^*], \delta(\upsilon)] + [[c_1 f_1(\upsilon, \gamma), \upsilon^*], \delta(\upsilon)]=0$ ... (4)

Replacing γ=$2\upsilon^2$ in equation (4) to get

0=$[[\delta(\upsilon), (2\upsilon^2)^*], \delta(\upsilon)]$+ $[[c_1 f_1(\upsilon, 2\upsilon^2), \upsilon^*], \delta(\upsilon)]$

$= [[\delta(\upsilon), \upsilon^*], \delta(\upsilon)]\upsilon^* + [\delta(\upsilon), \upsilon^*][\upsilon^*, \delta(\upsilon)] + [\upsilon^*, \delta(\upsilon)][\delta(\upsilon), \upsilon^*] + \upsilon^*\big[[\delta(\upsilon), \upsilon^*], \delta(\upsilon)\big] + c_1[[\delta(\upsilon), \upsilon^*], \delta(\upsilon)]\upsilon^* + c_1[\delta(\upsilon), \upsilon^*][\upsilon^*, \delta(\upsilon)]+c_1[[\upsilon, \upsilon^*], \delta(\upsilon)]\delta(\upsilon)+c_1[\upsilon, \upsilon^*][\delta(\upsilon), \delta(\upsilon)]+c_1[\upsilon, \delta(\upsilon)][\delta(\upsilon), \upsilon^*]+c_1\upsilon[[\delta(\upsilon), \upsilon^*], \delta(\upsilon)]$

$= -(c_1+2)[\delta(\upsilon), \upsilon^*]^2+c_1[[\upsilon, \upsilon^*], \delta(\upsilon)]\,\delta(\upsilon)$

$=-(c_1+2)[\delta(\upsilon), \upsilon^*]^2+[[\upsilon, \delta(\upsilon)], \upsilon^*]\delta(\upsilon)$

$=(c_1 + 2)[\delta(\upsilon), \upsilon^*]^2$ ... (5)

Commuting equation (2) with υ^* and using lemma (2.9) to get

0=$[[\delta(\upsilon), \gamma^*], \upsilon^*] + [c_1 f_1(\upsilon, \gamma), \upsilon^*], \upsilon^*]$ ... (6)

Replacing γ =$2\upsilon\gamma$ in equation (6) to obtain

0=$\big[[\delta(\upsilon), (2\upsilon\gamma)^*], \upsilon^*\big]$+$[[c_1 f_1(\upsilon, 2\upsilon\gamma), \upsilon^*], \upsilon^*]$

$= \big[[\delta(\upsilon), \gamma^*], \upsilon^*\big]\upsilon^*+[\gamma^*, \upsilon^*]\,[\delta(\upsilon), \upsilon^*]+\gamma^*\,\big[[\delta(\upsilon), \upsilon^*], \upsilon^*\big]+c_1[[f_1(\upsilon, \gamma), \upsilon^*], \upsilon^*]\upsilon^*+c_1[[\gamma, \upsilon^*], \upsilon^*]\delta(\upsilon)+ c_1[\gamma, \upsilon^*][\delta(\upsilon), \upsilon^*]+ c_1[\gamma, \upsilon^*][\delta(x), \upsilon^*]+c_1\gamma[[\delta(\upsilon), \upsilon^*], \upsilon^*]$

$= \{\big[[\delta(\upsilon), \gamma^*], \upsilon^*\big]+c_1\big[[f_1(\upsilon, \gamma), \upsilon^*], \upsilon^*\big]\}\upsilon^*+[\gamma^*, \upsilon^*][\delta(\upsilon), \upsilon^*]+c_1[[\gamma, \upsilon^*], \upsilon^*]\delta(\upsilon)+2c_1[\gamma, \upsilon^*][\delta(\upsilon), \upsilon^*]$

By using equation (6) with above equation to get

$[\gamma^*, \upsilon^*][\delta(\upsilon), \upsilon^*]+ c_1\left[[\gamma, \upsilon^*], \upsilon^*\right]\delta(\upsilon)+2c_1[\gamma, \upsilon^*][\delta(\upsilon), \upsilon^*]=0$ ... (7)

Replacing $\gamma= \delta(\upsilon)[\delta(\upsilon), \upsilon^*]$ in equation (7), to have

$0=[[\delta(\upsilon), \upsilon^*]\delta(\upsilon), \upsilon^*][\delta(\upsilon), \upsilon^*]+c_1\left[[\delta(\upsilon)[\delta(\upsilon), \upsilon^*], \upsilon^*], \upsilon^*\right]\delta(\upsilon)+ 2c_1[\delta(\upsilon)[\delta(\upsilon), \upsilon^*], \upsilon^*][\delta(\upsilon), \upsilon^*]$

$=(2c_1 + 1)[\delta(\upsilon), \upsilon^*]^3$ ... (8)

$=(2c_1 + 1)[\delta(\upsilon), \upsilon^*]^2\mathcal{U}\ (2c_1+1)[\delta(\upsilon), \upsilon^*]^2$

Since $\mathcal{R}$ is a semiprime, then

$(2c_1 + 1)[\delta(\upsilon), \upsilon^*]^2=0$, for all $\upsilon \in \mathcal{U}$ ... (9)

Combining equations (5) and (9) to get

$[\delta(\upsilon), \upsilon^*]^2=0$, for all $\upsilon \in \mathcal{U}$.

As the center of the semiprime ring contains no non-zero nilpotent elements, then $[\delta(\upsilon), \upsilon^*]=0$, for all $\upsilon \in \mathcal{U}$.

Theorem (3.3): Let $\mathcal{R}$ be a prime $*$-ring and $\Omega: \mathcal{U}^n \to \mathcal{R}$ be a non-zero symmetric reverse $*$-n-derivation associated with involution. If the trace δ of Ω is commuting on $\mathcal{U}$ and $[\delta(\upsilon), \upsilon^*] \in \mathcal{Z}(\mathcal{R})$ for all $\upsilon \in \mathcal{U}$, then $\mathcal{U}$ must be commutative.

Proof:

Suppose that $\mathcal{U}$ is anon commutative prime ring. From Theorem (3.2) we have $[\delta(\upsilon), \upsilon^*]=0$ for all $\upsilon \in \mathcal{U}$. And from Theorem (3.1) we have $\Omega=0$ which it contradiction, hence $\mathcal{U}$ is commutative prime ring.

Theorem (3.4): Let $\mathcal{R}$ be a semiprime $*$-ring. If $\mathcal{R}$ admits a symmetric reverse $*$- n-derivation ξ of $\mathcal{R}$, then ξ is a maps from $\mathcal{R}$ to $\mathcal{Z}(\mathcal{R})$.

Proof: By hypothesis

$\xi(\upsilon\gamma, \upsilon_2, \dots, \upsilon_n)=\xi(\gamma, \upsilon_2, \dots, \upsilon_n)\ \upsilon^*+ \gamma\xi(\upsilon, \upsilon_2, \dots, \upsilon_n)$ ... (1)

Let $\gamma = \gamma z$ in equation (1) to get

$\xi(\upsilon\gamma z, \upsilon_2, \dots, \upsilon_n)=\xi(\gamma z, \upsilon_2, \dots, \upsilon_n)\ \upsilon^*+ \gamma z\xi(\upsilon, \upsilon_2, \dots, \upsilon_n)$

$=\xi(z, \upsilon_2, \dots, \upsilon_n)\ \gamma^*\ \upsilon^*+ z\xi(\gamma, \upsilon_2, \dots, \upsilon_n)\ \upsilon^*+ \gamma z\xi(\upsilon, \upsilon_2, \dots, \upsilon_n)$, for all $\upsilon, \gamma, z, \upsilon_2, \dots, \upsilon_n \in \mathcal{R}$. ... (2)

Also, $\xi(\upsilon\gamma z, \upsilon_2, \dots, \upsilon_n)=\xi(z, \upsilon_2, \dots, \upsilon_n)\ \gamma^*\ \upsilon^*+ z\xi(\upsilon\gamma, \upsilon_2, \dots, \upsilon_n)$

$=\xi(z, \upsilon_2, \dots, \upsilon_n)\ \gamma^*\ \upsilon^*+ z\xi(\gamma, \upsilon_2, \dots, \upsilon_n)\ \upsilon^*+ z\gamma\xi(\upsilon, \upsilon_2, \dots, \upsilon_n)$ ... (3)

Comparing equations (2) and (3) to have

$[\gamma, z]\xi(\upsilon, \upsilon_2, \dots, \upsilon_n)=0$ ... (4)

Replacing $\gamma = \xi(\upsilon, \upsilon_2, \dots, \upsilon_n)\gamma$ in equation (4) and using it then

$[\xi(\upsilon, \upsilon_2, \dots, \upsilon_n), z]\gamma\xi(\upsilon, \upsilon_2, \dots, \upsilon_n)=0$ ... (5)

Let $\gamma=\gamma z$ in equation (5) to have

$[\xi(\upsilon,\upsilon_2,\dots,\upsilon_n),z]\gamma z\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)$=0 ... (6)

Now, multiplying equation (5) from the right side by z, to have

$[\xi(\upsilon,\upsilon_2,\dots,\upsilon_n),z]\gamma\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)z$=0 ... (7)

Comparing equations (6) and (7) then

$[\xi(\upsilon,\upsilon_2,\dots,\upsilon_n),z]\gamma[\xi(\upsilon,\upsilon_2,\dots,\upsilon_n),z]$=0, hence $[\xi(\upsilon,\upsilon_2,\dots,\upsilon_n),z]\mathcal{R}\,[\xi(\upsilon,\upsilon_2,\dots,\upsilon_n),z]$=0. Since $\mathcal{R}$ is semiprime then $[\xi(\upsilon,\upsilon_2,\dots,\upsilon_n),z]$=0 for all $\upsilon,z,\upsilon_2,\dots,\upsilon_n\in\mathcal{R}$ and then ξ is a map from $\mathcal{R}$ into $\mathcal{Z}(\mathcal{R})$.

Theorem (3.5): Let $\mathcal{R}$ be a prime $*$-ring. If $\mathcal{R}$ admits a symmetric reverse $*$-n-derivation ξ of $\mathcal{R}$ such that $\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)\neq\upsilon$ and $\xi(\upsilon\gamma,\upsilon_2,\dots,\upsilon_n)=\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$ for all $\upsilon,\gamma,\upsilon_2,\dots,\upsilon_n\in\mathcal{R}$ then ξ =0.

Proof: By hypothesis

$\xi(\upsilon\gamma,\upsilon_2,\dots,\upsilon_n)$=$\xi(\gamma,\upsilon_2,\dots,\upsilon_n)\upsilon^*$+ $\gamma\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)$= $\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)\,\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$ ... (1)

Let $\gamma=z\gamma$ in equation (1) to get

$\xi(z,\upsilon_2,\dots,\upsilon_n)\xi(\gamma,\upsilon_2,\dots,\upsilon_n)\upsilon^*$ + $z\gamma\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)$ = $\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)$ $\xi(z,\upsilon_2,\dots,\upsilon_n)\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$ = $\xi(\upsilon z,\upsilon_2,\dots,\upsilon_n)\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$={$\xi(z,\upsilon_2,\dots,\upsilon_n)\upsilon^*$+$z\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)$}$\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$}

This implies that

$\xi(z,\upsilon_2,\dots,\upsilon_n)[\xi(\gamma,\upsilon_2,\dots,\upsilon_n),\upsilon^*]$+$z\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)\big(\gamma-\xi(\gamma,\upsilon_2,\dots,\upsilon_n)\big)$=0

By theorem (3.4) the above equation becomes

$z\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)\big(\gamma-\xi(\gamma,\upsilon_2,\dots,\upsilon_n)\big)$=0

Hence,$\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)z\big(\gamma-\xi(\gamma,\upsilon_2,\dots,\upsilon_n)\big)$=0. We can written as $\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)\mathcal{R}\,\big(\gamma-\xi(\gamma,\upsilon_2,\dots,\upsilon_n)\big)$=0. Since $\mathcal{R}$ is prime then either $\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)$=0 or $\big(\gamma-\xi(\gamma,\upsilon_2,\dots,\upsilon_n)\big)$=0, but $\xi(\gamma,\upsilon_2,\dots,\upsilon_n)\neq\gamma$, then $\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)$=0 for all $\upsilon,\upsilon_2,\dots,\upsilon_n\in\mathcal{R}$.

Theorem (3.6): Let $\mathcal{R}$ be a prime $*$-ring. If $\mathcal{R}$ admits a reverse $*$-n-derivation ξ of $\mathcal{R}$ such that$\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)\neq\upsilon^*$and $\xi(\upsilon\gamma,\upsilon_2,\dots,\upsilon_n)$=$\xi(\gamma,\upsilon_2,\dots,\upsilon_n)\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)$ for all $\upsilon,\gamma,\upsilon_2,\dots,\upsilon_n\in\mathcal{R}$ then ξ =0.

Proof: By hypothesis

$\xi(\upsilon\gamma,\upsilon_2,\dots,\upsilon_n)$=$\xi(\gamma,\upsilon_2,\dots,\upsilon_n)\upsilon^*$+ $\gamma\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)$=$\xi(\gamma,\upsilon_2,\dots,\upsilon_n)\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)$ ... (1)

Replacing υ =$\upsilon\gamma$ in equation (1) to get

$\xi(\gamma,\upsilon_2,\dots,\upsilon_n)\gamma^*\upsilon^*+\gamma\xi(\gamma,\upsilon_2,\dots,\upsilon_n)\,\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)=\xi(\gamma,\upsilon_2,\dots,\upsilon_n)\,\xi(\upsilon\gamma,\upsilon_2,\dots,\upsilon_n)=\xi(\gamma,\upsilon_2,\dots,\upsilon_n)\,\{\xi(\gamma,\upsilon_2,\dots,\upsilon_n)\upsilon^*+\gamma\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)\}$

By theorem (3.4) then

$\xi(\gamma,\upsilon_2,\dots,\upsilon_n)\,\gamma^*\upsilon^*-\xi(\gamma,\upsilon_2,\dots,\upsilon_n)\xi(\gamma,\upsilon_2,\dots,\upsilon_n)\upsilon^*$=0

$\xi(\gamma,\upsilon_2,\dots,\upsilon_n)(\gamma^*-\xi(\gamma,\upsilon_2,\dots,\upsilon_n))\upsilon^*$=0

Hence $\xi(\gamma,\upsilon_2,\dots,\upsilon_n)\upsilon^*(\gamma^*-\xi(\gamma,\upsilon_2,\dots,\upsilon_n))$=0

We can written as $\xi(\gamma,\upsilon_2,\dots,\upsilon_n)\mathcal{R}\,(\gamma^*-\xi(\gamma,\upsilon_2,\dots,\upsilon_n))$=0. Since $\mathcal{R}$ is prime then either $\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$=0 or ($\gamma^*-\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$)=0, but $\xi(\gamma,\upsilon_2,\dots,\upsilon_n)\neq\gamma^*$, then we have that $\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$=0 for all $\gamma,\upsilon_2,\dots,\upsilon_n\in\mathcal{R}$.

Theorem (3.7): Let $\mathcal{R}$ be a prime $*$-ring and $a\in\mathcal{R}$. If $\mathcal{R}$ admits a symmetric reverse $*$-n-derivation ξ of $\mathcal{R}$ and $[\xi(\upsilon,\upsilon_2,\dots,\upsilon_n),a]$=0, then $\xi(a)$=0 or $a\in\mathcal{Z}(\mathcal{R})$.

Proof: By hypothesis

$[\xi(\upsilon\gamma,\upsilon_2,\dots,\upsilon_n),a]=0,\quad$ for all $\upsilon,\gamma,\upsilon_2,\dots,\upsilon_n\in\mathcal{R}$ ... (1)

That is

$[\xi(\gamma,\upsilon_2,\dots,\upsilon_n)\upsilon^*+\gamma\xi(\upsilon,\upsilon_2,\dots,\upsilon_n),a]=0$

Hence, $\xi(\gamma,\upsilon_2,\dots,\upsilon_n)[\upsilon^*,a]+[\gamma,a]\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)=0$ ... (2)

Replacing $\gamma=a$ and $\upsilon^*=\upsilon$ in equation (2) to get

$\xi(a,\upsilon_2,\dots,\upsilon_n)[\upsilon,a]=0$ ... (3)

Replacing $\upsilon=\upsilon\gamma$ in equation (3) and using it then

$\xi(a,\upsilon_2,\dots,\upsilon_n)\upsilon[\gamma,a]=0$

This implies that $\xi(a,\upsilon_2,\dots,\upsilon_n)\mathcal{R}[\gamma,a]=0$. Since $\mathcal{R}$ is prime then $\xi d(a,\upsilon_2,\dots,\upsilon_n)=0$ for all $a,\upsilon_2,\dots,\upsilon_n\in\mathcal{R}$ or $a\in\mathcal{Z}(\mathcal{R})$.

Theorem (3.8): Let $\mathcal{R}$ be a semiprime $*$-ring. If $\mathcal{R}$ admits a symmetric reverse $*$-n-derivation d of $\mathcal{R}$ then $[\xi(\upsilon,\upsilon_2,\dots,\upsilon_n),z]=0$ for all $\upsilon,z,\upsilon_2,\dots,\upsilon_n\in\mathcal{R}$.

Proof: By hypothesis

$\xi(\upsilon\gamma,\upsilon_2,\dots,\upsilon_n)=\xi(\gamma,\upsilon_2,\dots,\upsilon_n)\upsilon^*+\gamma\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)$ ... (1)

Substituting $\upsilon=\upsilon z$ in equation (1) to get

$\xi(\upsilon z\gamma,\upsilon_2,\dots,\upsilon_n)=\xi(\gamma,\upsilon_2,\dots,\upsilon_n)(\upsilon z)^*+\gamma\xi(\upsilon z,\upsilon_2,\dots,\upsilon_n)$

$=\xi(\gamma,\upsilon_2,\dots,\upsilon_n)z^*\upsilon^*+\gamma\xi(z,\upsilon_2,\dots,\upsilon_n)\upsilon^*+\gamma z\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)$ ... (2)

Also $\xi(\upsilon z\gamma,\upsilon_2,\dots,\upsilon_n)=\xi(z\gamma,\upsilon_2,\dots,\upsilon_n)\upsilon^*+z\gamma\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)$

$=\xi(\gamma,\upsilon_2,\dots,\upsilon_n)z^*\upsilon^*+\gamma\xi(z,\upsilon_2,\dots,\upsilon_n)\upsilon^*+z\gamma\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)$ ... (3)

Comparing equations (2) and (3) to get

$[\gamma,z]\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)=0$ ... (4)

Replacing $\gamma=\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)\gamma$ in equation (4) and using it then

$[\xi(\upsilon,\upsilon_2,\dots,\upsilon_n),z]\gamma\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)=0$ ... (5)

Let $\gamma=\gamma z$ in equation (5) to have

$[\xi(\upsilon,\upsilon_2,\dots,\upsilon_n),z]\gamma z\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)=0$ ... (6)

Now, multiplying equation (5) from the right side by z to have

$[\xi(\upsilon,\upsilon_2,\dots,\upsilon_n),z]\gamma\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)z=0$ ... (7)

Comparing equations (6) and (7) then

$[\xi(\upsilon,\upsilon_2,\dots,\upsilon_n),z]\gamma[\xi(\upsilon,\upsilon_2,\dots,\upsilon_n),z]$=0

Hence, $[\xi(\upsilon,\upsilon_2,\dots,\upsilon_n),z]\mathcal{R}\,[\xi(\upsilon,\upsilon_2,\dots,\upsilon_n),z]$ =0. Since $\mathcal{R}$ is semiprime then $[\xi(\upsilon,\upsilon_2,\dots,\upsilon_n),z]$ =0, for all $\upsilon,z,\upsilon_2,\dots,\upsilon_n\in\mathcal{R}$.

Theorem (3.9): Let $\mathcal{R}$ be a prime $*$-ring. If $\mathcal{R}$ admits a symmetric reverse $*$-n-derivation ξ of $\mathcal{R}$ such that $\xi([\upsilon,\gamma],\upsilon_2,\dots,\upsilon_n)$=0 for all $\upsilon,\gamma,\upsilon_2,\dots,\upsilon_n\in\mathcal{R}$ then ξ=0 or $\mathcal{R}$ is commutative.

Proof: By hypothesis

$\xi([\upsilon,\gamma],\upsilon_2,\dots,\upsilon_n)$=0 ... (1)

Let $\upsilon=\gamma\upsilon$ in equation (1) and using it then

$[\upsilon,\gamma]\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$=0 ... (2)

Replacing $\upsilon=\upsilon z$ in equation (2) to have

$[\upsilon,\gamma]z\xi(\gamma,\upsilon_2,\dots,\upsilon_n)+\upsilon[z,\gamma]\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$=0

By using equation (2) the above equation becomes

$[\upsilon,\gamma]z\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$=0

This implies that $[\upsilon,\gamma]\mathcal{R}\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$=0. Since $\mathcal{R}$ is prime then $[\upsilon,\gamma]$=0 and that means $\mathcal{R}$ is commutative, or $\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$=0 for all $\gamma,\upsilon_2,\dots,\upsilon_n\in\mathcal{R}$.

Theorem (3.10): Let $\mathcal{R}$ be a prime $*$-ring. If $\mathcal{R}$ admits a symmetric reverse $*$-n-derivation ξ of $\mathcal{R}$ such that $\xi((\upsilon\circ\gamma),\upsilon_2,\dots,\upsilon_n)$=0 for all $\upsilon,\gamma,\upsilon_2,\dots,\upsilon_n\in\mathcal{R}$ then ξ=0 or $\mathcal{R}$ is commutative.

Proof: By hypothesis

$\xi((\upsilon\circ\gamma),\upsilon_2,\dots,\upsilon_n)$=0 ... (1)

Let $\upsilon=\gamma\upsilon$ in equation (1) and using it then

$(\upsilon\circ\gamma)\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$=0 ... (2)

Replacing υ=$s\upsilon$ in equation (2) to have

$(s\circ\gamma)\upsilon\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$=0

Hence, $(s\circ\gamma)\mathcal{R}\,\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$=. Since $\mathcal{R}$ is prime then $(s\circ\gamma)$=0, replace s=sz to get $s[z,\gamma]$=0. Now let s=υs then $\upsilon s[z,\gamma]$=0, that $\upsilon\mathcal{R}\,[z,\gamma]$=0 for $0\neq\upsilon\in\mathcal{R}$ and since $\mathcal{R}$ is prime then $\mathcal{R}$ is commutative, or $\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$=0 for all $\gamma,\upsilon_2,\dots,\upsilon_n\in\mathcal{R}$.

Theorem (3.11): Let $\mathcal{R}$ be a prime $*$-ring. If $\mathcal{R}$ admits a symmetric reverse $*$-n-derivation ξ of $\mathcal{R}$ such that $\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)\circ\gamma$=0 for all $\upsilon,\gamma,\upsilon_2,\dots,\upsilon_n\in\mathcal{R}$ then ξ=0 or $\mathcal{R}$ is commutative.

Proof: By hypothesis

$\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)\circ\gamma$=0 ... (1)

Replacing υ=$z\upsilon$ in equation (1) and using it then

$\xi(\upsilon,\upsilon_2,\dots,\upsilon_n)[z^*,\gamma]-[\upsilon,\gamma]\xi(z,\upsilon_2,\dots,\upsilon_n)$=0 ... (2)

Let $\upsilon = \gamma$ and $z^{*} = z$ in equation (2) to get

$\xi(\gamma, \upsilon_2, \dots, \upsilon_n)[z, \gamma]=0$ … (3)

Replacing $z=z\upsilon$ in equation (3) and using it then

$\xi(\gamma, \upsilon_2, \dots, \upsilon_n)z[\upsilon, \gamma]=0$, for all $\upsilon, \gamma, z, \upsilon_2, \dots, \upsilon_n \in \mathcal{R}$

This implies that $\xi(\gamma, \upsilon_2, \dots, \upsilon_n)\mathcal{R}\,[\upsilon, \gamma]=0$, since $\mathcal{R}$ is prime then $\xi(\gamma, \upsilon_2, \dots, \upsilon_n)=0$ for all $\gamma, \upsilon_2, \dots, \upsilon_n \in \mathcal{R}$ or $\mathcal{R}$ is commutative.

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