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Strongly b star(Sb*) – cleavability(splitability)

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ABSTRACT

A. Poongothai and R. Parimelazhagan[5] introduced some new type of separation axioms and study some of their basic properties. Some implications betweenT_0,T_1 and T_2 axioms are also obtained.

In this paper we studied the concept of cleavability over these spaces: (sb*-T_0, sb*-T_1, sb*-T_2) as following:

1- If $\mathcal{P}$ is a class of topological spaces with certain properties and if X is cleavable over $\mathcal{P}$, then X$\in \mathcal{P}$

2- If $\mathcal{P}$ is a class of topological spaces with certain properties and if Y is cleavable over $\mathcal{P}$, then Y$\in \mathcal{P}$.

1. Introduction

In 1985 Arhangl' Skii [1] introduced different types of cleavability(originally named splitability) as following :
A topological space X is said to be cleavable over a class of spaces $\mathcal{P}$, if for $A \subset X$ there exists a continuous mapping $f: X \to Y \in \mathcal{P}$ such that$f^{-1}f(A) = A$, $f(X)=Y$.
Throughout this paper, X and Y denote the topological spaces (X, τ) and (Y, σ) respectively . Let A be a subset of the space X ,the interior and closure of a set A inXare denoted by $int(A)$ and $cl(A)$ respectively. The complement of A is denoted byA^c.

2-Preliminaries

In this section, we recall some definitions and results which are needed in this paper.
Definition 2.1. [3]

A topological space X is called a T_0 - space if and only if it satisfies the following axiom of Kolmogorov. (T_0) If x and y are distinct points of X, then there exists an open set which contains one of them but not the other.

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Definition 2.2. [3]

A topological space X is a T_1 -space , if and only if it satisfies the following separation axiom of Frechet. (T_1),if x and y are two distinct points of X, then there exist two open sets, one containing x but not y and the other containing y but not x.

Definition2.3 [6]
A topological space X is said to be a T_2 - space or housdorff space , ifand only if for every pair of distinct points x and y of X, there exist two disjoint open sets, one containing x and the other containing y.

Definition2.4 [3]
A subset $A \subseteq X$is said to be Sb*-closed set if cl(int(A) $\subseteq$ U , whenever $A \subseteq$ U and U is b-open in X. The complements of closed sets Sb*-closed set is Sb*- open sets .The family of all sb*-open sets of a space X is denoted by sb*O(X) .

Theorem2.5[5]
Let X be a topological space and A be a subset of X . Then A is Sb*open iff A contains a Sb* open neighbourhood of each of its points .

Definition2.5 [3]
A map f: X⟶ Y is said to be Sb*-open mapif the image of every open set in X is Sb*-open in Y.

Definition2.6 [3]
Let X and Y be topological spaces. A map f: X ⟶ Y is called strongly b* - continuous (sb*- continuous) if the inverse image of every open set in Y is sb* - open in X.

Definition 2.7 [3]

Let X and Y be topological spaces. A map f : X ⟶ Y is called strongly b* -closed (briefly sb* - closed) map if the image of every closed set in X is sb*- closed in Y.

Definition 2.8 [3]
Let X and Y be topological spaces. A map f: (X, τ) ⟶ (Y, σ) is said to be sb* - Irresoluteif the inverse image of every sb* - closed(respectively sb* - open) set in Y is sb* - closed (respectively sb* - open) set in X.

Definition 2.9 [5]
A topological space X is said to be sb*-T_0if for every pair of distinct points x and y of X, there exists a sb*-open set G such that $x \in$ G and $y \notin$G or y$\in$ G and $x \notin$G.

Definition 2.10 [5]
A space X is said to be sb*- T_1if for every pair of distinct points x and y in X, there exist sb* - open sets U and V such that x $\in$ U but y$\notin$U and y$\in$ V but x$\notin$V.

Definition 2.11 [5]
A space X is said to be sb*-T_2 if for every pair of distinct points x and y in X, there are disjoint sb*- open sets U and V in X containing x and y respectively.

3- sb* – cleavability
Definition 3.1
A topological spaces X is said to be sb*- pointwise cleavable over a class of spaces $\mathcal{P}$ if for every point $x \in X$ there exists a sb*- continuous mapping $f : X \to Y \in \mathcal{P}$, such that $f^{-1} f(x) = \{x\}$.

Definition 3.2
A topological spaces X is said to be sb* Irresolute - pointwise cleavable over a class of spaces $\mathcal{P}$ if for every point $x \in X$ there exists a sb* - Irresolute - continuous mapping $f : X \to Y \in \mathcal{P}$, such that $f^{-1} f(x) = \{x\}$.

Remark3.1

By a sb*- -open(closed) pointwise cleavable ,we mean that the sb*-(Irresolute) continuous function $f: X \longrightarrow Y \in \rho$is an bijective and open(closed) respectively.

Theorem 3.1[3]

If a map $f: X \longrightarrow Y \in \rho$is continuous , then it is sb* - continuous but not conversely.

Example 3.1

Let$X = \{a, b\}$, $\tau = \{X, \phi, \{a\}, \{b\}\}$, $Y = \{p, q\}$, $\sigma = \{Y, \phi, \{p\}, \{q\}\}$, let

$f: (X, \tau) \longrightarrow (Y, \sigma)$is a sb*-continuous map defined by $f(a) = p$, $f(b) = q$

To show that$X$ sb*- pointwise cleavableover$Y$as follows :

Clearly $f$is sb* - continuous function.

Now, $a \in X$, $f(a) = \{p\}$, $f^{-1}f(a) = f^{-1}\{p\} = \{a\}$, also$b \in X$,, $f(b) = \{q\}$, $f^{-1}f(b) = f^{-1}\{q\} = \{b\}$,then$X$is sb*- pointwise cleavable over Y.

Proposition 3.1

Let X be a sb* - irresolute pointwise cleavable over a class of sb*-T_0 spaces $\mathcal{P}$, then X $\in \mathcal{P}$.

Proof:

Let $x \in X$, then there exist sb*T_0- space$Y$ and sb* irresolute continuous mapping $f: X \longrightarrow Y \in \rho$, such that $f^{-1}f(x) = \{x\}$.This implies that for every $y \in X$ with $x \neq y$,we have $f(x) \neq f(y)$, since Y is a sb* T_0-space, so there exists a sb*-open set G in Y contains one of the two points but not the other.Let $f(x) \in G, f(y) \notin G$, then $f^{-1}f(x) \in f^{-1}(G)$,$f^{-1}f(y) \notin f^{-1}(G)$. This implies that $x \in f^{-1}(G)$and$y \notin f^{-1}(G)$,since f is a sb*irresolute a continuous, so $f^{-1}(G)$ is a sb*-open set in X. Therefore X is sb* T_0- space .

Theorem 3.1.[5]

Every subspace of a sb*-T_0. space is sb*-T_0.

Proposition 3.2

Let X be a sb* T_0–space is a sb* - irresolute pointwise cleavable over a class $\mathcal{P}$, of space Y, then Y is sb* T_0–space ,hence Y $\in \mathcal{P}$

Proof:

Let $y \in Y$, then there exists a sb*-irresolute continuous mapping $f: X \longrightarrow Y \in \rho$ such that $f^{-1}f\{f^{-1}(y)\} = f^{-1}(y)$. This implies that for every $x \in Y$ with $x \neq y$,we have $f^{-1}(x) \neq f^{-1}(y)$,since X is a sb*-T_0-space , so there exists a sb*-open setsU contains one of the two points but not the other .Let $f^{-1}(y) \in U$ and $f^{-1}(x) \notin U$, then $ff^{-1}(y) \in f(U)$ and$ff^{-1}(x) \notin f(U)$. This implies that y $\in f(U)$ and $x \notin f(U)$ .Therefore Y is sb* T_0 -space , hence Y $\in \mathcal{P}$

Proposition 3.3

Let X be a sb* - irresolute pointwisecleavable over a class of sb*-T_1 spaces $\mathcal{P}$, then X $\in \mathcal{P}$.

Proof:

Let $x \in X$, then there exist asb * -T_1-space Y and a sb* - irresolute- continuous mapping $f: X \rightarrow Y \in \mathcal{P}$ such that $f^{-1}f(x) = \{x\}$.This implies that for every y $\in X$ with $x \neq y$, we have $f(x) \neq f(y)$. Since Y is sb*-T_1space , so there exist two sb*-- open sets U and V such that $f(x) \in U$, $f(y) \notin U$ and $f(y) \in V$, $f(x) \notin V$, then $f^{-1}f(x) \in f^{-1}(U)$, $f^{-1}f(y) \notin f^{-1}(U)$ and $f^{-1}f(y) \in f^{-1}f(V)$, $f^{-1}f(x) \notin f^{-1}(V)$. This implies that $x \in f^{-1}(U)$, y$\notin f^{-1}(U)$ and y$\in f^{-1}(V)$, $x \notin f^{1-}(V)$ .By a sb* - irresolute - continuity of f , $f^{-1}(U)$, $f^{-1}(V)$ are sb*- open sub sets in X . Then$X$ is sb*-T_1 spaces , henceX $\in \mathcal{P}$.

Proposition 3.4

Let X be a sb* - pointwisecleavable over a class of T_1 - spaces $\mathcal{P}$, then X is sb*- T_1- space

Proof:

Let $x \in X$, then there exist a T_1- space Y and a sb*- continuous mapping $f: X \rightarrow Y \in \mathcal{P}$ such that $f^{-1}f(x) = \{x\}$. This implies mapping $f: X \rightarrow Y \in \mathcal{P}$ such that $f^{-1}f(x) = \{x\}$, $f^{-1}f(x) = \{x\}$. This implies that for every$x^* \in X$ with $x \neq x^*$, we have $f(x) \neq f(x^*)$. Since Y is T_1 -space , so there exist two open sets G and H such that $f(x) \in G$, $f(x^*) \notin G$ and $f(x^*) \in H$, $f(x) \notin H$, then $f^{-1}f(x) \in f^{-1}(G)$, $f^{-1}f(x^*) \notin f^{-1}(G)$ and $f^{-1}f(x^*) \in f^{-1}(H)$,$f^{-1}f(x) \notin f^{-1}(H)$. This implies that $x \in f^{-1}(H)$, $x^* \notin f^{-1}(G)$ and $x^* \in f^{-1}(H)$, $x \notin f^{1-}(H)$ .By a sb* - continuity of$f$, then $f^{-1}(G)$, $f^{-1}(H)$ are sb*- open sub sets in X . Thus X is sb*- T_1- space, hence X $\in \mathcal{P}$.

Proposition 3.5

Let$X$ be sb* T_1–space is a sb* - open pointwise cleavable over a class of spaces $\mathcal{P}$, then Y is sb* T_1–space , thus $Y \in \mathcal{P}$.

Proof:

Let$y \in Y$, then there exist a sb* T_1-space X and sb* - open continuousmapping$f$:$X \to Y \in \mathcal{P}$, such that $ff^{-1}\{f^{-1}(y)\} = f^{-1}(y)$. This implies that for every $x \in Y$ with $y \neq x$, we have $f^{-1}(y) \neq f^{-1}(x)$. Since X is sb* T_1-space, so there exist two sb* -open sets V and W , such that $f^{-1}(y) \in V$, $f^{-1}(x) \notin V$ and $f^{-1}(x) \in W$, $f^{-1}(y) \notin W$. Then $ff^{-1}(y) \in f(V)$, $ff^{-1}(x) \notin f(V)$ and $ff^{-1}(x) \in f(W)$, $ff^{-1}(y) \notin f(W)$. This implies that $y \in f(V)$, $x \notin f(V)$ and $x \in f(W)$, $y \notin f(W)$, since f is a sb* open , so $f(V)$, $f(W)$ are open sb* sets of Y , then Y is sb* T_1–space .Therefore $Y \in \mathcal{P}$.

Proposition 3.6

Let X be sb*- T_2 - space is a sb* - open pointwise cleavable over a class $\mathcal{P}$ of spaces Y, then Y is sb*- T_2 – space , thus $Y \in \mathcal{P}$.

Proof:

Let$y_1 \in Y$, then there exist a sb*- T_2- space X and a sb* open continuous mapping $f: X \to Y \in \mathcal{P}$ such that $f^{-1}f(f^{-1}(y)) = f^{-1}(y)$. Thisimplies that for every $y_2 \in Y$, with $y_1 \neq y_2$, we have $f^{-1}(y_1) \neq f^{-1}(y_2)$,so there existx_1 , $x_2$in X,such that$x_1 = f^{-1}(y_1)$, $x_2 = f^{-1}(y_2)$ with $x_1 \neq x_2$, Since X is sb*- T_2 , so there exist two sb* open sets G , Hsuch that $f^{-1}(y_1) \in G$, $f^{-1}(y_2) \in H$ and $G \cap H = \emptyset$, then $ff^{-1}(y_1) \in f(G)$, $ff^{-1}(y_2) \in f(H)$. Since f is sb* open , then $f(G)$, $f(H)$ are sb* open sets of Y and$y_1 \in f(G)$,$y_2 \in f(H)$ and $f(G) \cap f(H) = f(G \cap H) = f(\emptyset) = \emptyset$. Thus Y is sb*- T_2- space ,then $Y \in \mathcal{P}$.

Proposition 3.7

Let X be sb* - open pointwise cleavable over a class of sb*- T_2–spaces $\mathcal{P}$, then $X \in \mathcal{P}$

Proof:

Let $x \in X$, then there exist a sb*- T_2 space Y and a sb*- continuous mapping $f: X \to Y \in \mathcal{P}$ such that $f^{-1}f(x) = \{x\}$. This implies that for every $y \in Y$ with $x \neq y$,we have $f(x) \neq f(y)$. Since Y is sb*- T_2 , so there exist two sb*open sets U and V such that $f(x) \in U$, $f(y) \in V$ and $U \cap V = \emptyset$, then $f^{-1}f(x) \in f^{-1}(U)$, $f^{-1}f(y) \in f^{-1}(V)$, this implies that $x \in f^{-1}(U)$, $y \in f^{-1}(V)$, since f is sb*- continuous , so $f^{-1}(U)$, $f^{-1}(V)$ are sb* open sets of$X$ and $f^{-1}(U) \cap f^{-1}(V) = f^{-1}(U \cap V) = f^{-1}(\emptyset) = \emptyset$. Thus X is sb*- T_2- space , then $X \in \mathcal{P}$.

4-conclusion:

In this paper we have studied and proved these cases:

1) If $\mathcal{P}$ is a class of (sb*- T_0 , sb*- T_1) spaces with certain properties and if X is a sb* - irresolute pointwisecleavable over $\mathcal{P}$, then $X \in \mathcal{P}$,also if $\mathcal{P}$ is a class of (sb*- T_0، sb*- T_1) spaces with certain properties and if X is a sb* - irresolute pointwisecleavable over $\mathcal{P}$, then $Y \in \mathcal{P}$.

2) If $\mathcal{P}$ is a class of (sb*- T_1، sb*- T_2) spaces with certain properties and if X is point wise sb^* – cleavable over $\mathcal{P}$, then $X \in \mathcal{P}$, also if $\mathcal{P}$ is a class of sb^*-T_1 spaces with certain properties and if X is a sb* - irresolute pointwisecleavable over $\mathcal{P}$, then $X \in \mathcal{P}$.

3) If $\mathcal{P}$ is a class of (sb^*-T_1،sb^*-T_2) spaces with certain properties and if X is point wise sb^*cleavable over $\mathcal{P}$,then Y is (sb^*-T_1،sb^*-T_2) respectively , then$Y \in \mathcal{P}$.

References

[1] Arhangel'skii, A.V, A general concept of cleavability of topological spaces over a class of spaces, Abstracts Tirasp. Symp. ·Stiinca, Ki·sinev, (1985), 8-10.

[2] A. Poongothai and R. Parimelazhagan , sb* - closed sets in topological spaces, Int. Journal of Math.Analysis, 6, 47(2012), 2325-2333.

[3] A. Poongothai and R. Parimelazhagan , Strongly b* - continuous functions in Topological spaces, International Journal of Computer Applications, Volume 58, No.14(2012), 8 - 11.

[4] A. Poongothai and R. Parimelazhagan , sb* - irresolute maps and homeomorphisms in Topological spaces, Wulfenia Journal, Vol 20, No. 4(2013). 164 A.

[5] A. Poongothai and R. Parimelazhagan , sb* - Separation axioms, International Journal of Mathematics and Soft Computing, Vol.5, No.2 (2015), 155 - 164.

[6] D. Andrijevic, On b-open sets, Mat. Vesink, 48(1996), 59 - 64.

[7] D. Andrijevic, Semi-preopen sets, Mat. Vesnik, 38(1)(1986),24-32.