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Influence of Heat Transfer on Magnetohydrodynamics Oscillatory Flow for Carreau-Yasuda Fluid Through a Porous Medium

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ABSTRACT

This paper, we have examined the influence of transfer the heat on the magnetohydrodynamics (MHD) oscillatory flow of Carreau-Yasuda fluid through porous medium for two types of geometries "Poiseuille flow and Couette flow". We have solved the problem by using the perturbation technique in terms of the Weissenberg number (We), discussed and analyzed the numerical and arithmetical results of the effect of several parameters, namely The number of Darcy, the number of Reynold, the number of Peclet, the parameter of magnetic, the number of Grashof, the number of Weissenberg, frequency of the oscillation and radiation parameter for the velocity and temperature that are effective on the fluid movement by using MATHEMATICA program using illustrations.

MSC. 76Wxx, 76Sxx.

1 . Introduction

The flow of electrically oriented fluid has a lot of applications and this science deal with many branches. In astronomy, it helps to understand what happens in the sun, such as rotating solar spots, what happens inside other stars through their life cycle, and geology. The resulting magnetic and mechanical properties, and this science is also looking at generating electricity directly from hot gases evaporated ionizing generators that rely on this magnetic movement. It is also looking at tracking what happens in nuclear fusion by putting high electromagnetic energy on a mixture of deuterium and tritium in the laboratory to imitate what is happening inside the sun and in nuclear reactors using molten sodium molten metal. To reduce it in an area far from the walls of the container by magnetic

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fields, so that the temperature and pressure can be increased to values close to the corresponding values with in the stars and so on.

The effect of heat transfer on magnetohydrodynamics oscillatory flow of Williamson fluid in a channelwas discussed by Wissam and Dheia [7]. Nigam and Singh [9], have studied the effect of heat transfer on laminar flow among parallel flakes under the impact of transverse magnetic field. Attia and Kotb [3], have studied the heat transfer with MHD flow of viscous fluid among two parallel flakes. Massias was discuss the hydromagnetic free convection flow during a porous medium among two parallel plates [10]. Mustafa [8], have analyzed the thermal radiation effect on un-steady magnetohydrodynamics free convection flow past a vertical plate with temperature relied on viscosity. The un-steady transfer of heat to magnetohydrodynamics oscillatory flow through a porous medium under slip condition have studied by Hamza et al [5]. Moreover, the fluids of Newtonian are less appropriate than the fluids of non-Newtonian in many practical applications. There are many examples of such fluids include ketchup, shampoo, cosmetic products, lubricants, polymers, mud, blood at low shear rate and many others. All the non-Newtonian fluids (in terms of their various characteristics), unlike the viscous fluids, cannot be portrayed by a single constitutive relationship. Hence, many models of Non-Newtonian fluids are suggested in the literature.

The development of Poiseuille flow of the yield-stress fluid was discussed by Al-Khatib and Wilson [2]. Frigaard and Ryan [4], have analyzed the flow of a viscous-plastic fluid in a channel of slowly varying width. Kavita et al. [6], have studied the effect of heat transfer on magnetohydrodynamics oscillatory flow of Jeffrey fluid in a channel. The effect of heat transfer on the MHD oscillatory flow of Carreau fluid with variable viscosity model during porous medium studied by Al-Khafajy [1].

We consider a mathematical model to study the influence of heat transfer on magnetohydrodynamics oscillatory in the flow of Carreau-Yasuda fluid through a porous medium. The numerical solutions perturbation technique for the two kinds of flow Poiseuille flow and Couette flow are addressed. We discussed the pertinent parameters that appear in the problem during the graphs.

2. Mathematical Formulation

Let us consider the flow of a Carreau-Yasuda fluid in a channel of breadth h qualify the effects of magnetic field and radioactive heat transfer as described in Figure 1. We supposed that the fluid has very small electromagnetic force produced and the electrical conductivity is small. We are considering Cartesian coordinate system such that $(u(y), 0,0)$ is the velocity vector in which u is the x-component of velocity and y is orthogonal to the x-axis.

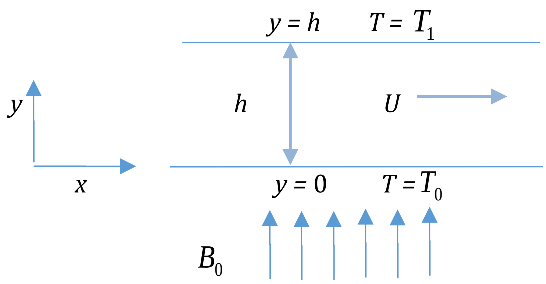

Figure 1. Geometry of the problem

The fundamental equation for Carreau-Yasuda fluid is [10]:

$$\boldsymbol{S} = -\bar{p}\boldsymbol{I} + \tau \tag{1}$$

$$\bar{\tau} = \mu_\infty + (\mu_0 - \mu_\infty)[1 + (\Gamma\bar{\dot{\gamma}})^b]^{\frac{n-1}{b}}\bar{\dot{\gamma}} \tag{2}$$

where $\bar{p}$ is the pressure, I is the unit tensor, $\bar{\tau}$ is the extra stress tensor, Γ is the time constant, μ_∞ and μ_0 are the infinite shear rate viscosity and zero shear rate viscosity, then $\dot{\gamma}$ is defined as:

$$\dot{\gamma} = \sqrt{\frac{1}{2}\sum_i \sum_j \dot{\gamma}_{ij}\dot{\gamma}_{ji}} = \sqrt{\frac{1}{2}\Pi} \tag{3}$$

Here $\prod$ is the second invariant strain tensor. We consider the fundamental Eq. (2), the case for which $\Gamma\dot{\gamma} < 1$, and $\mu_\infty = 0$. We can write the component of extra stress tensor according to follows as:

$$\bar{\tau} = \mu_0[1 + \left(\frac{n-1}{b}\right)\Gamma^b \bar{\dot{\gamma}}^b] \quad \bar{\dot{\gamma}} \tag{4}$$

The equations of the momentum and energy governing such a flow, subjugate to the Bossiness approximation, are:

$$\rho\frac{\partial \bar{u}}{\partial \bar{t}} = -\frac{\partial \bar{p}}{\partial \bar{x}} + \frac{\partial \bar{\tau}_{\bar{x}\bar{x}}}{\partial \bar{x}} + \frac{\partial \bar{\tau}_{\bar{x}\bar{y}}}{\partial \bar{y}} + \rho g\beta(T - T_0) - \sigma B_0^2\bar{u} - \frac{\mu_0}{k}\bar{u} \tag{5}$$

$$\rho\frac{\partial T}{\partial \bar{t}} = \frac{K}{C_p}\frac{\partial^2 T}{\partial \bar{y}^2} - \frac{1}{C_p}\frac{\partial q}{\partial y} \tag{6}$$

The temperatures at the walls of the channel are given as:

$$T = T_0 \text{ at } \bar{y} = 0 \text{, and } T = T_1 \text{ at } \bar{y} = h. \tag{7}$$

where $\bar{u}$ is the axial velocity, T is a fluid temperature, B_0 is a magnetic field strength, ρ is a fluid density, σ is a conductivity of the fluid, β is a coefficient of volume amplification due to temperature, g is a hastening due to gravity, k is a permeability, c_p is a specific heat at constant pressure, K is a thermal conductivity and q is a radioactive heat flux.

Following Vinvent et al. [12], it is supposed that the fluid is visually thin with a relatively low density and the radioactive heat flux is given by:

$$\frac{\partial q}{\partial y} = 4\eta^2(T_0 - T) \tag{8}$$

Here η is the mean radiation absorption coefficient.

Non-dimensional parameters are:

$$\left.\begin{array}{l} x=\frac{\bar{x}}{h}, y=\frac{\bar{y}}{h}, u=\frac{\bar{u}}{U}, \theta=\frac{T-T_0}{T_1-T_0},\ t=\frac{\bar{t}U}{h}, \\ We=\frac{\Gamma U}{h}, \tau_{xx}=\frac{h}{\mu_0 U}\bar{\tau}_{\overline{xx}}, \tau_{xy}=\frac{h}{\mu_0 U}\bar{\tau}_{\overline{xy}}, \dot{\gamma}=\frac{h}{U}\bar{\dot{\gamma}} \\ , Pe=\frac{\rho h U c_p}{K}, N^2=\frac{4\eta^2 h^2}{K}, Gr=\frac{\rho g\beta h^2(T_1-T_0)}{\mu U} \\ , p=\frac{\bar{p}h}{\mu U}, Re=\frac{\rho h U}{\mu}, Da=\frac{k}{h^2}, M^2=\frac{\sigma B_0^2 h^2}{\mu} \end{array}\right\} \quad (9)$$

Where (U) is the mean flow velocity, (Da) is Darcy number, (Re) is Reynolds number, (Gr) is Grashof number, (M) is magnetic parameter, (Pe) is the Peclet number and (N) is the radiation parameter.

Substituting (8) and (9) into equations (5), (6) and (7), we obtain

$$\rho\frac{U\partial u}{\frac{h}{U}\partial t}=-\frac{\frac{\mu_0 U}{h}\partial p}{h\partial x}+\frac{\frac{\mu_0 U}{h}\partial\tau_{xy}}{h\partial y}+\rho g\beta(T_1-T_0)\theta-\sigma B_0^2 U u-\frac{\mu_0 U}{k}u \quad (10)$$

$$\rho\ \frac{\partial(\theta(T_1-T_0)+T_0))}{\frac{h}{U}\partial t}=\frac{k}{C_P}\left[\frac{\partial^2(\theta(T_1-T_0)+T_0))}{h^2\partial y^2}-\frac{1}{k}4\eta^2(T_0-T)\right] \quad (11)$$

Where

$$\tau_{xx}=0, \tau_{yy}=0, \tau_{xy}=\tau_{yx}=\mu_0[1+\left(\frac{n-1}{b}\right)\Gamma^b\dot{\gamma}^b](u_y)$$

The following are the non-dimensional boundary conditions corresponding to the temperature equation:

$$\theta(0)=0\ ,\ \theta(1)=1 \quad (12)$$

We get the following non-dimensional equations:

$$\frac{\partial p}{\partial x}=-Re\frac{\partial u}{\partial t}+\frac{\partial}{\partial y}\left[\frac{\partial u}{\partial y}+\left(\frac{n-1}{b}\right)We^b\left(\frac{\partial u}{\partial y}\right)^{b+1}\right]+Gr\theta_0-(M^2+\frac{1}{Da})u \quad (13)$$

$$Pe\ \frac{\partial\theta}{\partial t}=\frac{\partial^2\theta}{\partial y^2}+N^2\theta \quad (14)$$

To solve the temperature equation (14) with condition (12),

Let $\theta(y,t)=\theta_0(y)e^{i\omega t}$ (15)

The frequency of the oscillation denoted by ω.

Substituting equation (15) into equation (14), we have

$$\frac{\partial^2\theta_0}{\partial y^2}+(N^2-i\omega Pe)\theta_0=0 \quad (16)$$

The solution of equation (16) with condition (12) is $\theta_0(y) = \csc(\varphi)\sin(\varphi y)$, where $\varphi = \sqrt{N^2 - i\omega Pe}$. Therefore

$$\theta(y,t) = \csc(\varphi)\sin(\varphi y)e^{i\omega t} \qquad (17)$$

The calculated of equation (13) have been solved in the next parts for two kinds of boundary conditions "Poiseuille flow and Couette flow"

3. Solution of the Problem

(A) Poiseuille flow

We suppose that the rigid flakes are at $y = 0$ and $y = h$ are at rest. Therefore

$\bar{u} = 0$ at $\bar{y} = 0$, and $\bar{u} = 0$ at $\bar{y} = h$.

The non-dimensional boundary conditions are:

$$u(0) = 0\,,\ u(1) = 0 \qquad (18)$$

To solve the momentum equation (13), let

$$-\frac{dp}{dx} = \lambda e^{i\omega t} \qquad (19)$$

$$u(y,t) = u_0(y)e^{i\omega t} \qquad (20)$$

where λ is a real constant.

Substituting equation (19) and equation (20) into equation (13), we have

$$-\lambda = -i\omega Re u_0 + \frac{\partial^2 u_0}{\partial y^2} + \left(\frac{(n-1)(b+1)}{b}\right)We^b e^{ib\omega t}\left(\frac{\partial u_0}{\partial y}\right)^b \frac{\partial^2 u_0}{\partial y^2} + Gr\theta_0 - (M^2 + \frac{1}{Da})u_0 \qquad (21)$$

Equation (21) is non-linear and difficult to get an exact solution. So for waning (We), the boundary value problem is agreeing to an easy analytic solution .In this case the equation can be solved. Nevertheless, we suggest the perturbation technique to solve the problem. Accordingly, we write:

$$u_0 = u_{00} + We^b u_{01} + O(We^{2b}) \qquad (22)$$

Substituting Eq. (22) in Eq. (21) with boundary conditions from equation (18), then we equality the powers of (We), we obtain:

i - Zeros-order system

$$\frac{\partial^2 u_{00}}{\partial y^2} - \left(M^2 + i\omega Re + \frac{1}{Da}\right)u_{00} = -(\lambda + Gr\theta_0) \quad (23)$$

The associated boundary conditions are:

$$u_{00}(0) = u_{00}(1) = 0 \qquad (24)$$

***ii* – First - order system**

$$\frac{\partial^2 u_{01}}{\partial y^2} - \left(M^2 + i\omega Re + \frac{1}{Da}\right) u_{01} = -(b+1)\left(\frac{n-1}{b}\right) e^{bi\omega t} \left(\frac{\partial u_{00}}{\partial y}\right)^b \left(\frac{\partial^2 u_{00}}{\partial y^2}\right) \quad (25)$$

The associated boundary conditions are:

$$u_{01}(0) = u_{01}(1) = 0 \quad (26)$$

***iii* – Zeros - order solution**

The solution of Eq. (23) subset to the associate boundary conditions from equation (24) is:

$$u_{00} = \frac{B}{A\left(1+e^{\sqrt{A}}\right)}\left(1 + e^{\sqrt{A}y} - e^{\sqrt{A}y} - e^{\sqrt{A}(1-y)}\right) \quad (27)$$

***iv* – First - order solution**

The solution of Eq. (25) subset to the associate boundary conditions in equation (26) is long.

where $A = \left(M^2 + i\omega Re + \frac{1}{Da}\right)$ *and* $B = (\lambda + Gr\theta_0)$

(B) Couette flow

The lower flake is fixed and the upper plate is locomotion with the velocity U_h, the boundary conditions for the Couette flow problem as defined:

$$u(0) = 0 \ , \ u(1) = U_0 \quad (28)$$

By same the governing equations in Poiseuille flow Eq. (21). The solution, in this case, has been calculated by the perturbation technique and the results have been discussed during graphs.

4. Results and Discussion

We discuss the influence of heat transfer on the Magnetohydrodynamics oscillatory flow of Carreau-Yasuda fluid through a porous medium for Poiseuille flow and Couette flow in some results through the graphical illustrations. Numerical assessments of analytical results and some of the graphically significant results are presented in Figures (2-16). We used the (MATHEMATICA-11) program to find the numerical results and illustrations. The momentum equation is resolved by using perturbation technique and all the results are discussed graphically. The velocity profile of Poiseuille flow is shown in Figures (2-7). Figure 2 shows the velocity profile u decreases with the increasing (Da) *and* (M).Figure 3 illustrates the influence (Gr) *and* (λ) on the velocity profiles function u vs. y.It is found by the increasing (Gr) *and*(λ) the velocity profiles function u increase.Figure 4 shows that velocity profile u decreases by the increasing influence(N) and(ω).Figure 5 shows that velocity profiles decreases with the increase of the parameters (Re) *and* (Pe). Figure 6 illustrates the influence (We)*and*(n) on the velocity profiles function u vs. y. It is found by the increasing (We) *and*(n) the velocity profiles function u decreases at y$<$ 0.5, while it increases when y > 0.5 .Figure 7 shows that velocity profiles increases with the increase of the parameters(Gr)*and* (b).The velocity profile of Couette flow is shown in Figures (8-13). It is noted that by the decreasing each of parameters $(Pe), (Da), (N), (Re), (M)$and (ω) the velocity profile u increases, while u increases by the increasing $(Gr), (n), (We), (b)$and(λ).Based on equation (17), Figure 14 show that influence of(N) on the temperature function(θ).The temperature increases with the increase in(N).Figure 15 we observed that the influence(Pe) in temperature (θ) by the increasing (Pe) then (θ) decreases.Figure 16 show us that with the increasing of (ω) the temperature (θ) decreases.

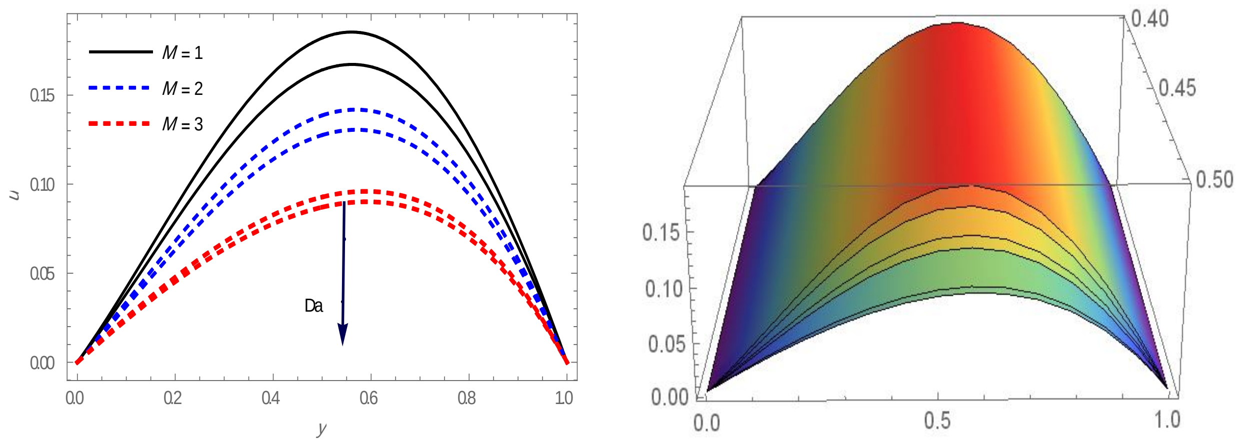

Figure 2 "Poiseuille flow"; Velocity profile for Da and M $with\ b = 3, n = 3, N = 2, \mathrm{Gr} = 1, \omega = 1, \mathrm{Re} = 1, Pe = 1\ ,\ \lambda = 1, \mathrm{We} = 0.1, t = 0.5.$

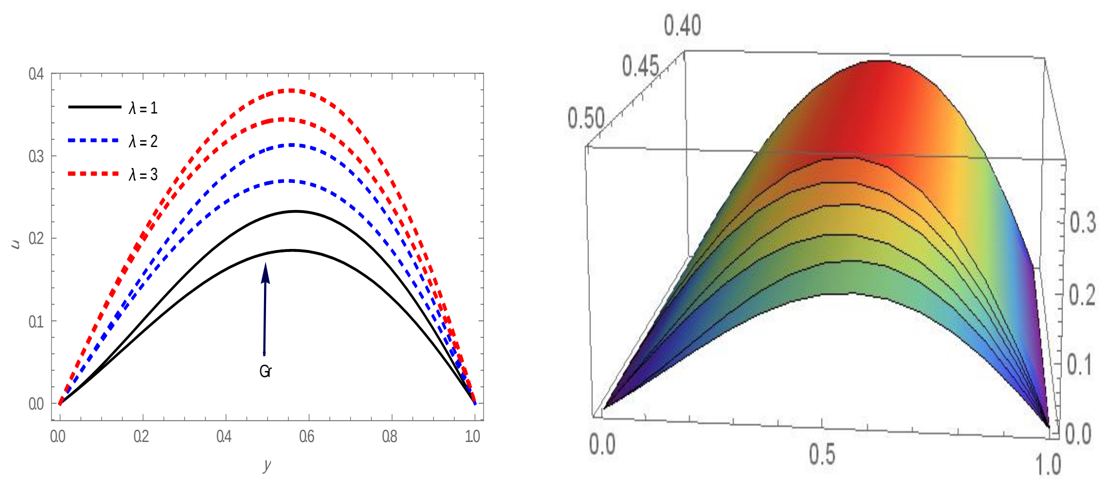

Figure 3 "Poiseuille flow"; Velocity profile for Gr and λ $with\ b = 3, n = 3, N = 2, \mathrm{Da} = 0.8, \omega = 1, \mathrm{Re} = 1, Pe = 1, M = 1, \mathrm{We} = 0.1, t = 0.5.$

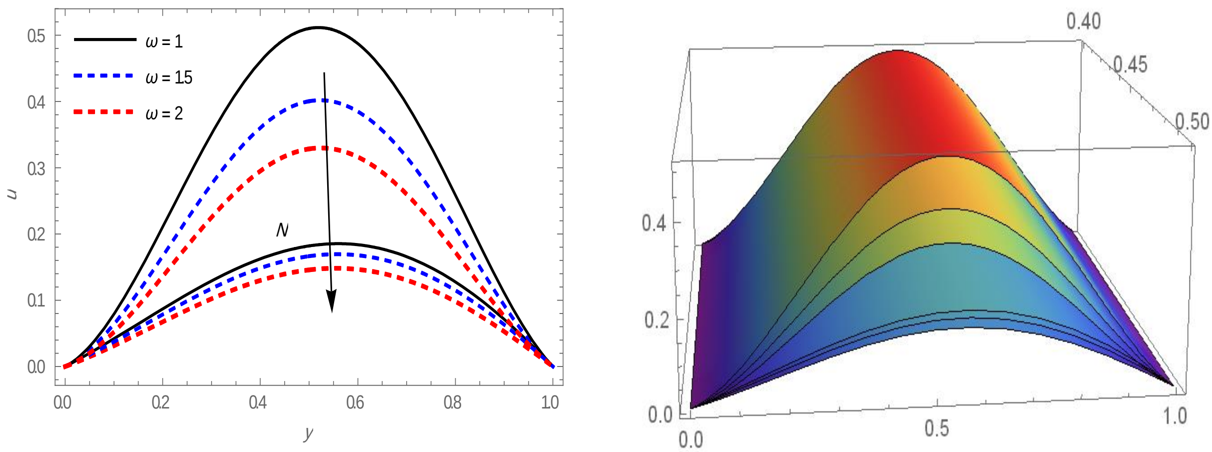

Figure 4 "Poiseuille flow"; Velocity profile for N and ω $with\ b=3, n=3, M=1, \mathrm{Gr}=1, Da=0.8, \mathrm{Re}=1, Pe=1, \lambda=1, \mathrm{We}=0.1, t=0.5.$

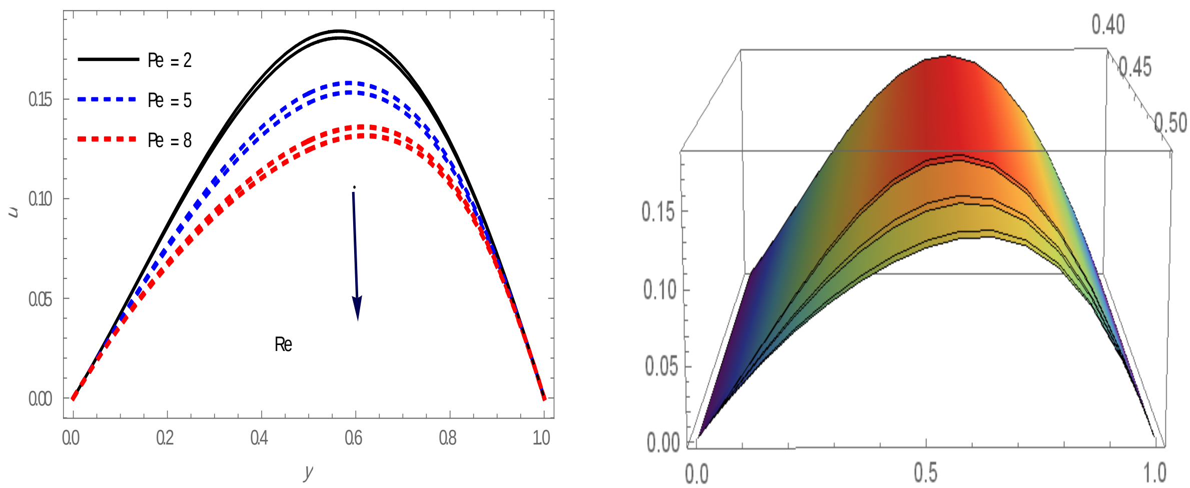

Figure 5 "Poiseuille flow"; Velocity profile for Re and Pe $with\ b=3, n=3, N=2, \mathrm{Gr}=1, \omega=1, \mathrm{M}=1, Da=0.8, \lambda=1, \mathrm{We}=0.1, t=0.5.$

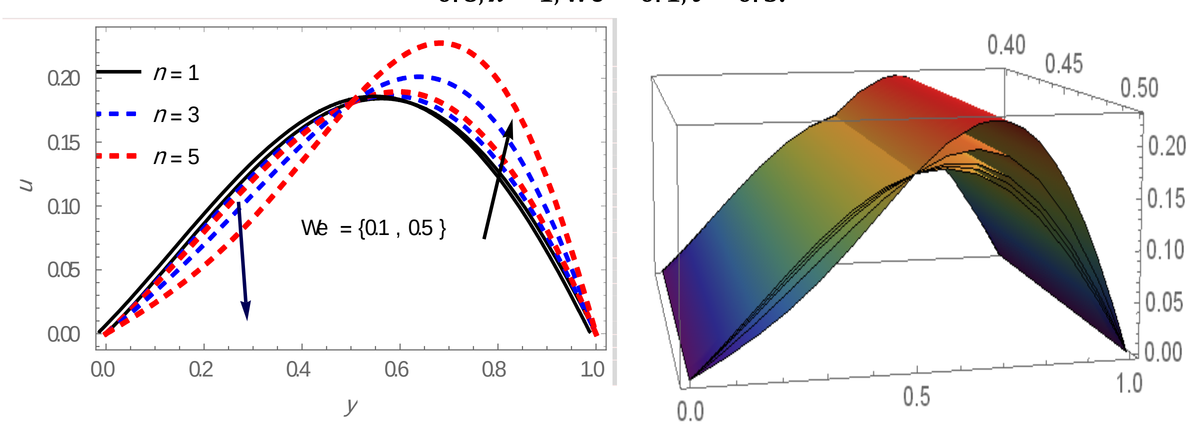

Figure 6 "Poiseuille flow"; Velocity profile for $We\ and\ n$ $with\ b=1,\ N=2, M=1, \mathrm{Gr}=1, \omega=1, \mathrm{Re}=1, Pe=2, \lambda=1, \mathrm{Da}=0.8, t=0.5.$

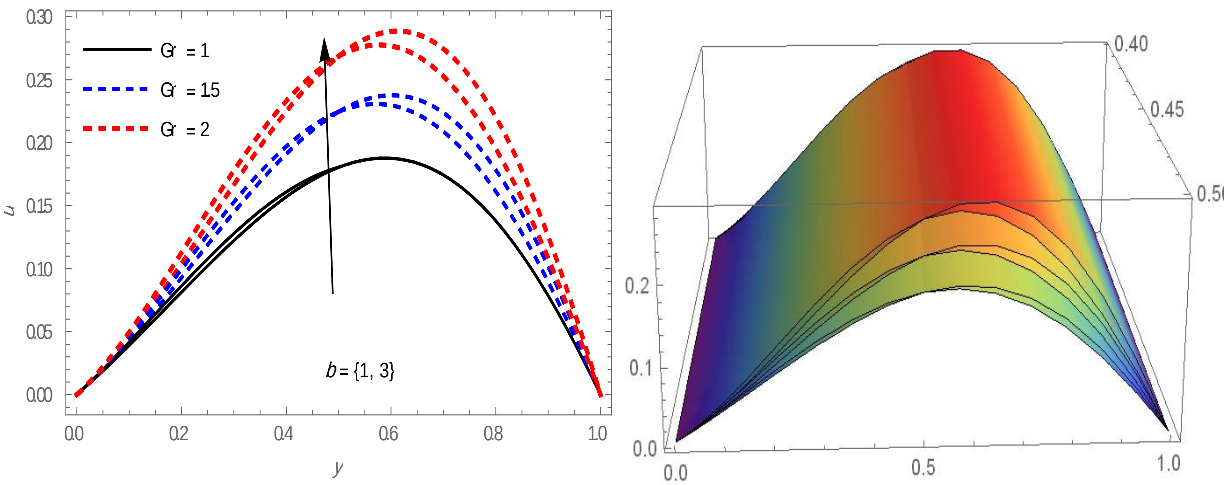

Figure 7 "Poiseuille flow"; Velocity profile for b and Gr $with$ $n = 2,\ N = 2, M = 1, \mathrm{We} = 0.3, \omega = 1, \mathrm{Re} = 1, Pe = 2, \lambda = 1, \mathrm{Da} = 0.8, t = 0.5$

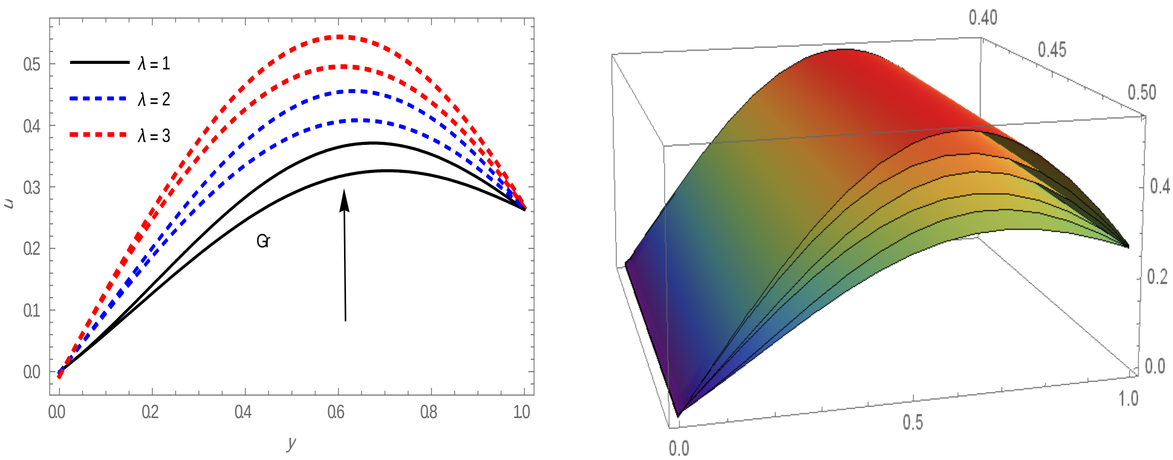

Figure 8 "Couette flow"; Velocity profile for Gr and λ with $b = 3, n = 3, M = 1, N = 2, \omega = 1, Da = 0.8, \mathrm{Re} = 1, Pe = 1, \mathrm{We} = 0.1, U_0 = 0.3, t = 0.5.$

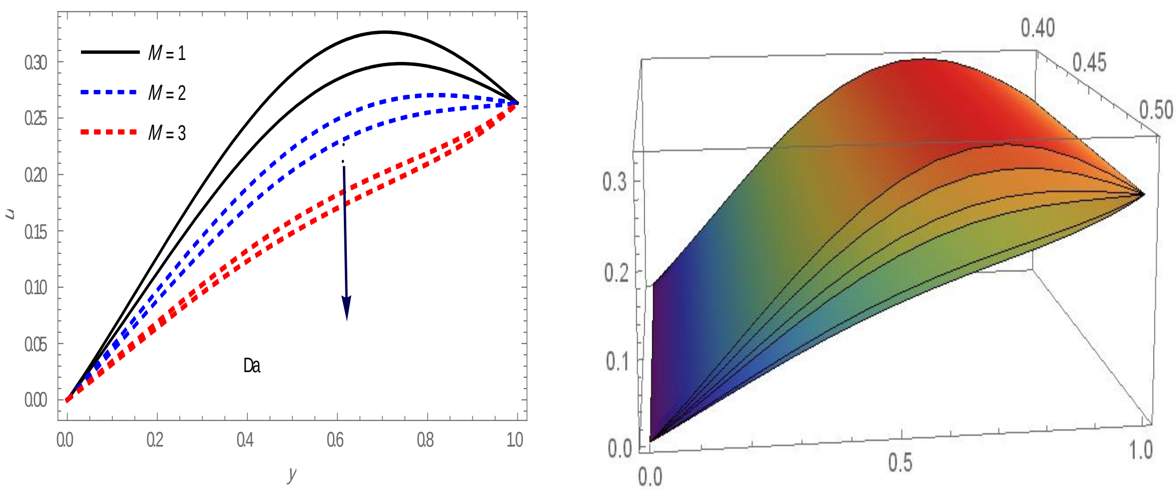

Figure 9 "Couette flow"; Velocity profile for Da and M with $b = 3, n = 3, \lambda = 1, N = 2, \omega = 1, Gr = 1, \mathrm{Re} = 1, Pe = 1, \mathrm{We} = 0.1, U_0 = 0.3, t = 0.5.$

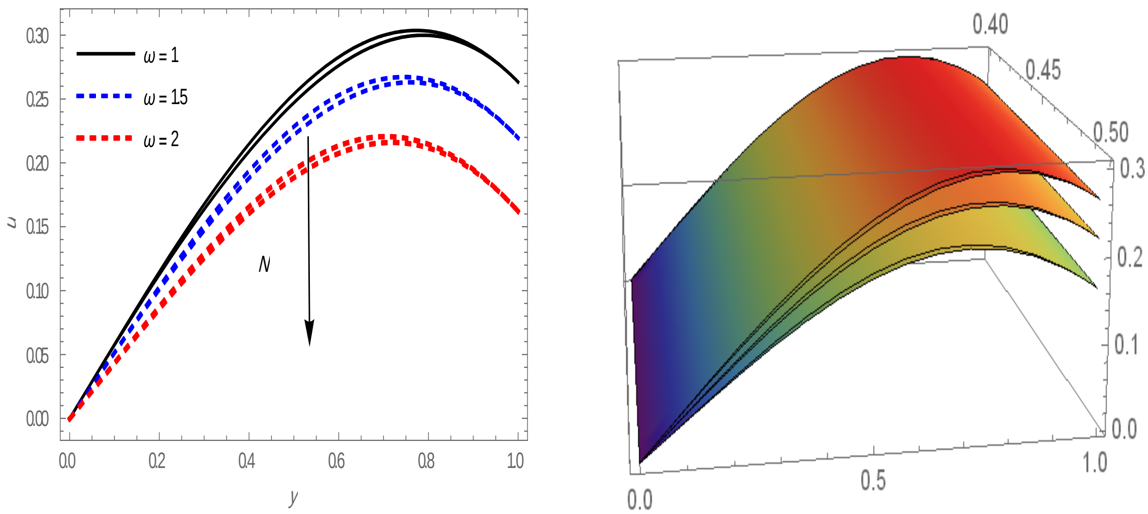

Figure 10 "Couette flow"; Velocity profile for N and ω with $b = 3, n = 3, \lambda = 1, M = 1, \text{Da} = 0.8, Gr = 1, \text{Re} = 1, Pe = 1, \text{We} = 0.1, U_0 = 0.3, t = 0.5$.

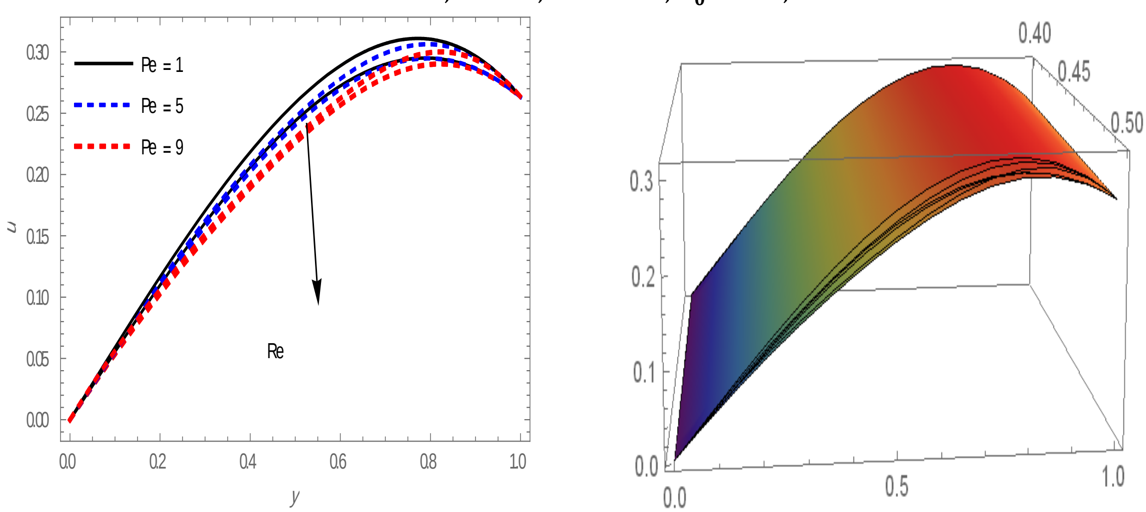

Figure 11 "Couette flow"; Velocity profile for Re and Pe with $b = 3, n = 3, \lambda = 1, N = 1, \omega = 1, Gr = 1, \text{M} = 1, Da = 0.8, \text{We} = 0.1, U_0 = 0.3, t = 0.5$.

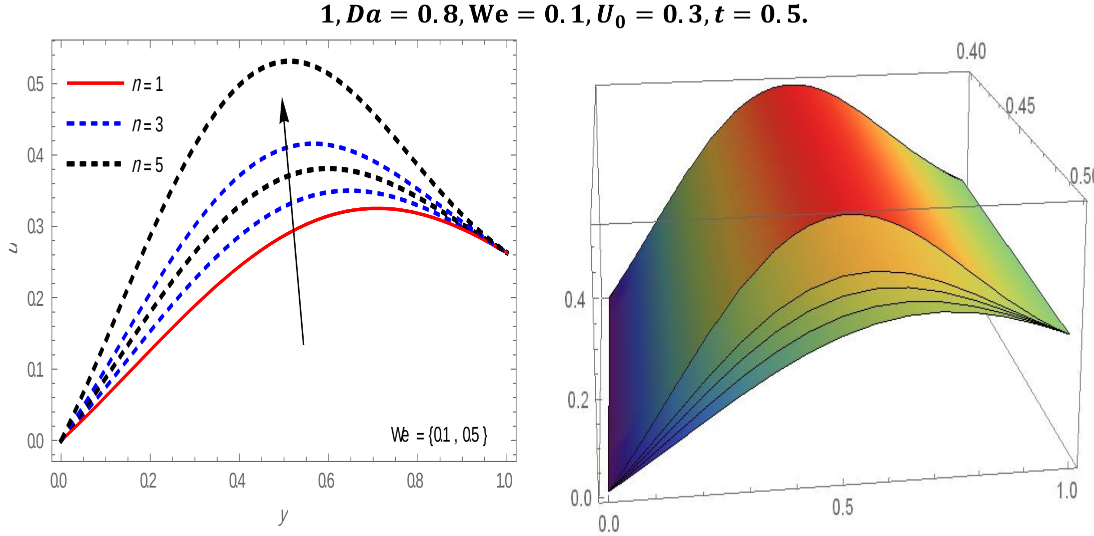

Figure 12 "Couette flow"; Velocity profile for We and n with $b = 1, \lambda = 1, N = 2, \omega = 1, Gr = 1, \text{Re} = 1, Pe = 2, \text{M} = 1, Da = 0.8, U_0 = 0.3, t = 0.5$.

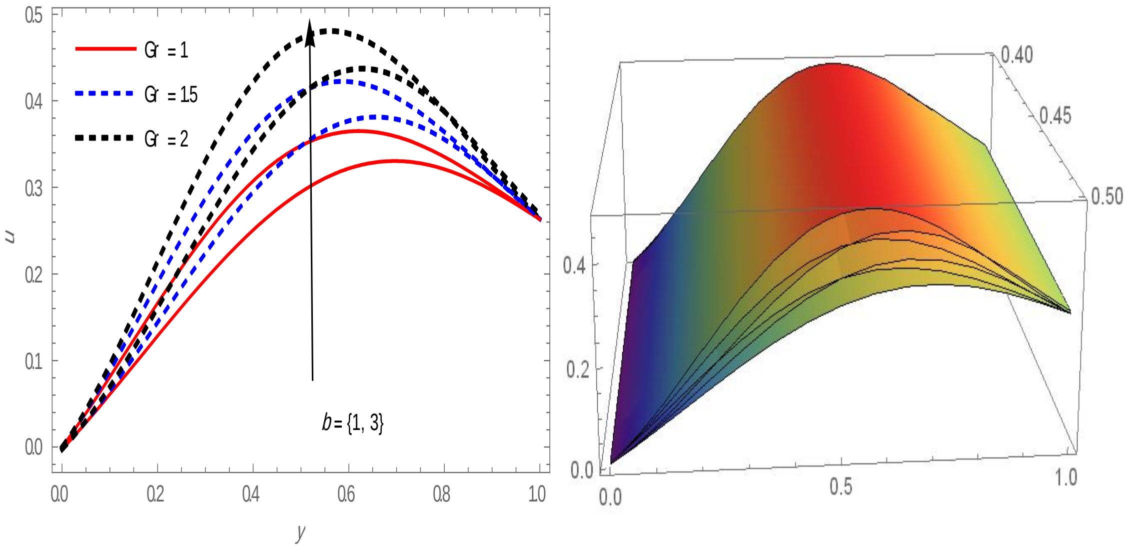

Figure 13 "Couette flow"; Velocity profile for b and Gr with $We = 0.1$, $\lambda = 1$, $N = 2$, $\omega = 1$, $n = 2$, $\mathrm{Re} = 1$, $Pe = 2$, $\mathrm{M} = 1$, $Da = 0.8$, $U_0 = 0.3$, $t = 0.5$.

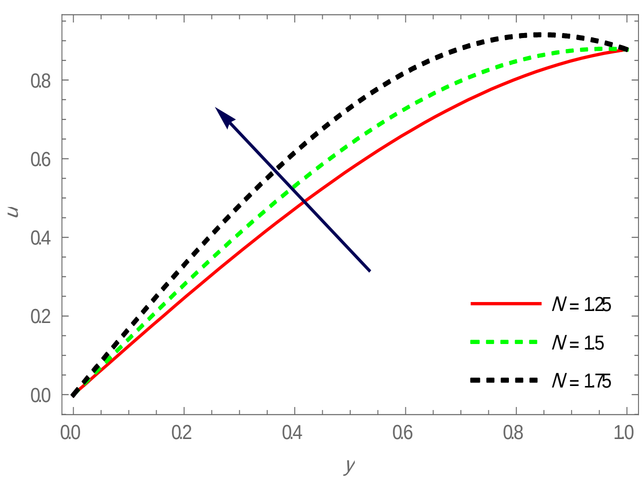

Figure 14 Influence of N on Temperature θ for $\omega = 0, \mathrm{Pe} = 1, \mathrm{t} = 0.5$

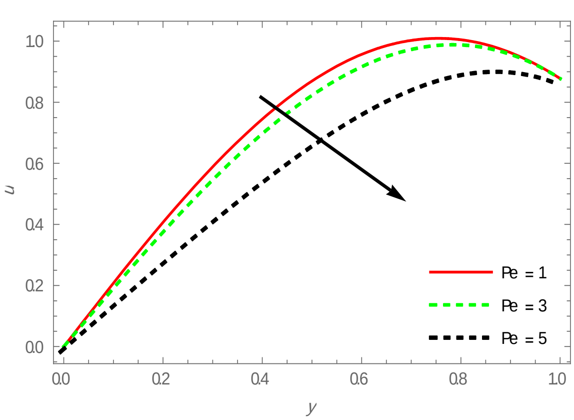

Figure 15 Influence of Pe on Temperature θ for $\mathrm{t} = 0.5, \mathrm{N} = 1.25$,

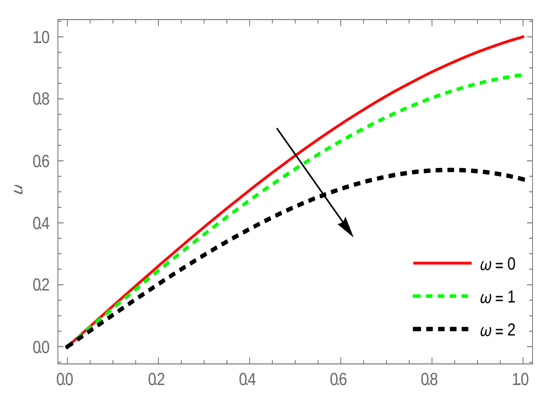

Figure 16 Influence of ω on Temperature θ for $\mathrm{t} = 0.5, \mathrm{N} = 1.25, \mathrm{Pe} = 1$

5. Concluding Remarks

We discuss the influence of heat transfer on magnetohydrodynamics oscillatory flow of Carreau-Yasuda fluid through a porous medium. The perturbation technique for the two kinds of flow Poiseuille flow and Couette flow. We found the velocity and temperature are analytical. We used different values to finding the results of pertinent parameters, namely Darcy number (Da), Reynold number (Re), Peclet number (Pe), magnetic parameter (M), Grashof number (Gr), Weissenberg number (We), frequency of the oscillation (ω) and radiation parameter (N) for the velocity and temperature.

The key points are:

- The velocity profiles were increased by the increasing λ , b and Gr for both the Poiseuille and Couette flow.
- The velocity profiles were increased by the increasing n ,and We in the Couette flow ,while in Poiseuille flow by the increasing (We) and(n) the velocity profiles function u decreases at y$<$ 0.5 and it increases when y $>$ 0.5.
- The velocity profiles decrease with the increasing Pe, Re, ω, Da, N and M for both the Poiseuille and Couette flow.
- We show that by increasing N the temperature increasing θ and the temperature θ decreases with the increasing Pe and ω.

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