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Separation Theorems For Fuzzy Soft normed space

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ABSTRACT

In this paper we discussed some theorems in fuzzy soft normed space , and we have some new results on separation theorems.

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1. Introduction

In 2002 , Maji et.al gave a new concept called fuzzy soft set , After the rontier work of Maji, many investigator have extended this concept in various branches of mathematics and Kharal and Ahmad in [3] introduced new theories like new properties of fuzzy soft set and then in [2] defined the concept of mapping on fuzzy soft classes and studies of fuzzy soft in topological introduced by Tanay and Kandemir [4].Mahanta and Das [5] continued studies . we essentially concerned in theory of fuzzy soft normed spaces and their generalization . In this paper we have studied the continuity and boundedness in fuzzy soft in this structure we prove some separation theorem in fuzzy soft normed space .

2. Preliminaries

Definition 2.1 [1] **:** A pair (f ,E) is called a fuzzy soft set over X , F.S set briefly if f is a mapping given by f : $E \to I^X$. So $\forall\ e\ \in A$, $f(e)$ is a fuzzy subset of X , with membership function $f_e : X \to [0,1]$

In fact , the membership function f_e indicates degree of belongingness of each element of X has the parameter $e \in E$.

Definition 2.2 [7]:Let $\tilde{X}$ be an absolute soft liner over the scalar filed K, suppose $*$ is continuous t-norm ,R(A^*) is the set of all non negative soft real numbers and $SSP(\tilde{X})$ denote the set of all soft points on $\tilde{X}$. A fuzzy sub set Γ on $SSP(\tilde{X})$ x R(A^*) is called fuzzy soft norm on $\tilde{X}$ if and only if for $x_e, y_{e'} \in SSP(\tilde{X})$ and $\tilde{k} \in K$ (where $\tilde{k}$ is a soft scalar) the following conditions hold

1. $\Gamma(\tilde{x}_e, \tilde{t}\,) = 0\ \ \forall\ \tilde{t} \,\tilde{\in}\, \mathbf{R}(A^*) with\ \ \tilde{t}\ \leq 0$

2. $\Gamma(\tilde{x}_e, \tilde{t}\,)\ = 1\ \forall\ \tilde{t} \,\tilde{\in}\, \mathbf{R}(A^*)\ \ with\ \tilde{t} \,\tilde{>}\, \tilde{0}$ if and only if $\tilde{x}_e = \tilde{\theta}_0$

3. $\Gamma(\tilde{k}\ \tilde{x}_e , \tilde{t}) = \Gamma(\tilde{x}_e , \frac{\tilde{t}}{|\tilde{k}|})$ if $\tilde{k} \neq \tilde{0}\ \forall\ \tilde{t} \,\tilde{\in}\, \mathbf{R}(A^*)$, $\tilde{t} \,\tilde{>}\, \tilde{0}$

4. $\Gamma(\tilde{x}_e \oplus \tilde{y}_{e/} , \tilde{t} \oplus \check{s}) \,\tilde{\geq}\, \Gamma(\tilde{x}_e, \tilde{t}) * \Gamma(\tilde{y}_{e/}, \tilde{s})\ \ \forall\ \tilde{t}, \tilde{s}\ \tilde{\in}\ \mathbf{R}(A^*)$ and $x_e , y_{e/} \in SSP(\tilde{X})$

5. $\Gamma(\tilde{x}_e , .)$ is a continuous nondecreasing function of $\mathbf{R}(A^*)$ and $\lim_{\tilde{t}\to\infty} \Gamma(x_e , t) = 1$

The triple $(\tilde{X}, \Gamma , \|.\|)$ will be refered to a fuzzy soft normed linear space.

The above definition includes a fuzzy soft normed space on soft vector space .In our research , we need to define the definition of Fuzzy soft normed space on normal vector space and thus we define as follow .

Definition 2.3 :Let X be a vector space over the scalar filed K , suppose $*$ is continuous t-norm , and. A fuzzy sub set E on X x $(0,\infty)$ is called fuzzy soft norm on X if and only if for $x_e , y_{e'} \in X$ and $k \in K$ the following condition hold

1) $E(x_e , t) = 0\ \ \forall\, t \leq 0$

2) $E(x_e , t) = 1\ \forall\, t \geq 0$ if and only if $x_e = \theta_0$

3) $E(k\, x_e , t) = E(x_e , \frac{t}{|k|})$ if $k \neq 0\ \forall\ t > 0$

4) $E(x_e \oplus x_{e'} , t \oplus s) \geq E(x_e , t) * E(y_{e'} , s)\ \forall\, t , s > 0$ and $x_e , y_e \in X$

5) $E(x_e , .)$ is continuous function and $\lim_{t\to\infty} E(x_e , t) = 1$

The triple (X, E , ‖.‖) will be refered to a fuzzy soft normed space

Remark 2.4 :

1) For any $r_1 , r_2 \in (0 , 1)$ with $r_1 > r_2$, there exist $r_3 \in (0 ,1)$ such that

$$r_1 * r_3 \geq r_2$$

2) For any $r_4 \in (0 ,1)$, there exist $r_5 \in (0 ,1)$, such that
$r_5 * r_5 \geq r_4$

Definition(2.5)[7]: Let $(\tilde{X},\Gamma, \|.\|)$ be a fuzzy soft normed linear space and $\tilde{t} \,\tilde{>}\, \tilde{0}$ be a soft real number . We define an open ball , a closed ball and sphere with center at $\tilde{x}_{e_1}$ and radius r as follows

$B(\tilde{x}_{e_1} , r , \tilde{t}) = \{\tilde{y}_{e_2} \in SSP(\tilde{X}) : \Gamma(\tilde{x}_{e_1} - \tilde{y}_{e_2} , \tilde{t}) \,\tilde{>}\, 1 - r\}$

$\bar{B}(\tilde{x}_{e_1} , r , \tilde{t}) = \{\tilde{y}_{e_2} \in SSP(\tilde{X}) : \Gamma(\tilde{x}_{e_1} - \tilde{y}_{e_2} , \tilde{t}) \,\tilde{\geq}\, 1 - r\}$

$S(\tilde{x}_{e_1} , r , \tilde{t}) = \{\tilde{y}_{e_2} \in SSP(\tilde{X}) : \Gamma(\tilde{x}_{e_1} - \tilde{y}_{e_2} , \tilde{t}) = 1 - r\}$

$SFS(B(\tilde{x}_{e_1} , r , \tilde{t}))$, $SFS(\bar{B}(\tilde{x}_{e_1} , r , \tilde{t}))$ and $SFS(S(\tilde{x}_{e_1} , r , \tilde{t}))$ are called fuzzy soft open ball, fuzzy soft closed ball ,fuzzy soft sphere respectively with centerx_e and radius r.

The above definition includes fuzzy soft open ball , fuzzy soft closed ball and sphere in fuzzy soft normed linear space .In our research , we need to define the definitions in fuzzy soft normed space .

Definition 2. 6 : let (X , E, ‖.‖) be a fuzzy soft normed space and $t > 0$ we define an open ball , a closed ball and sphere with center at x_e and radius $\propto$ as follows

$B(x_{e1} , r , t) = \{y_{e2} \in X: E(x_{e1} - y_{e2} , t) > 1 - r\}$

$\bar{B}(x_{e1}, r, t) = \{y_{e2} \in X: E(x_{e1} - y_{e2}, t) \geq 1 - r\}$

$S(x_{e1}, r, t) = \{y_{e2} \in X: E(x_{e1} - y_{e2}, t) = 1 - r\}$

SFS(B(x_{e1} , r , t)) , SFS(B(x_{e1} , r , t)) and SFS(S(x_{e1} , r , t)) are called fuzzy soft open ball, fussy soft closed ball , fuzzy soft sphere respectively with center x_e and radius r

Definition 2.7 : Let (X , E , ||. ||) be a fuzzy soft normed . A sub set A of X is said to be open set , if for all $x \in A$, *there eixest* $r \in (0,1)$, $t \in (0, \infty)$ such that $B(x_{e1}, r, t) \subset A$

Theorem 2.8: In fuzzy soft normed space. Then the intersection finite numbers of open sets is open .

Proof : Let (X , E , ||. ||) be a fuzzy normed space and let

$\{B_i : i = 1,2, \dots, n\}$ be a finite collection of open sets in the fuzzy soft normed

space , let $H = \cap \{B_i : i = 1,2, \dots, n\}$ To prove that H is an open set.

let $x \in H$ *then* $x \in B_i$, $\forall i = 1,2, \dots, n$

Since B_i open set $\forall i$ *then there exiset* $r_i \in (0,1)$ *and* $t_i > 0$

Such that $B(x, r_i, t_i) \subset B_i$, $i = 1, \dots, n$

Let $t_k = max\{t_1, t_2, \dots, t_n\}$ *and* $r_k = min\{r_1, r_2, \dots, r_n\}$

Then $B(x, r_k, t_k) \subset B_i$ *for all* $i = 1,2,3, \dots, n$

Then $B(x, r_k, t_k) \subset \cap B_i$

Then $B(x, r_k, t_k) \subset H$

$\therefore$ *H is open set*

Theorem 2.9 : In fuzzy soft normed space, the union of an arbitrary

collection of open sets is open .

Proof: Let (X , E , ||. ||)be fuzzy soft normed space and let $\{G_\lambda : \lambda \in \Lambda\}$ be an

arbitrary collection of open sets in X. let $G = \cup \{G_\lambda : \lambda \in \Lambda\}$ we must to

prove G is open, now let $x \in G$ then $x \in G_\lambda$ for some $\lambda \in \Lambda$ since G_λ is

open set then there exist $r \in (0,1), t > 0$ such that $B(x_{e1}, r, t) \subset G_\lambda$

and since $G_\lambda \subset G$ Then $B(x_{e1}, r, t) \subset G$ *then* G is open set

Theorem 2.10 : Let (X , E, ||. ||) be fuzzy soft normed space if A is open set in

a vector space X and $B \subset X$ then $A + B$ is open set in X.

Proof : Let $x \in X$ and $a \in A$ since A is open set then there exist

$r \in (0,1)$such that $B(x_{e1}, r, t) \subset A$ *then* $B(x_{e1}, r, t) + x \subset A + x$

then $B(a_e + x, r, t) \subset A + x$

then $A + x$ is open set in X for all $x \in X$

and since $A + B = \cup \{A + b : b \in B\}$

then $A+B$ is open set in X

Theorem 2.11: Every open ball in fuzzy soft normed space $(X, \mathrm{E}, \|.\|)$ is open set .

Proof : Let $B(x_{e1}, r, t)$ be an open ball and $y \in B(x, r, t)$ implies that

$\mathrm{E}(x-y,t) > 1-r \ldots . (*)$

Then there exist $t_0 \in (0,t)$, the relation $(*)$ is true .So for $t_0 \in (0,t)$.

$\mathrm{E}(x-y,t_0) > 1-r$

Let $r_0 = \mathrm{E}(x-y,t_0) > 1-r$ since $r_0 > 1-r$ we can find $0 < s < 1$,such that

$r_0 > 1-s > 1-r$

Now for a given r_0 and s such that $r_0 > 1-s$ we can find r_1 ,

$0 < r_1 < 1$, such that $r_0 * r_1 \geq 1-s$

Now consider the ball $B(y, 1-r_1, t-t_0)$ we claim that

$B(y, 1-r_1, t-t_0) \subset B(x_{e1}, r, t)$

Let $z \in B(y, 1-r_1, t-t_0)$ then $E(y-z, t-t_0) > r_1$ therefore

$\mathrm{E}(x-z,t) \geq \mathrm{E}(x-y+y-z, t-t_0+t_0)$

$\geq \mathrm{E}(x-y,t_0) * \mathrm{E}(y-z,t-t_0)$

$> r_0 * r_1 \geq 1-s > 1-r$

Therefore $z \in B(x,r,t)$ and hence $B(y, 1-r_1, t-t_0) \subset B(x,r,t)$

Definition2.12 [7]: Let $(\tilde{X}, \Gamma, \|.\|)$ be a fuzzy soft normed linear space , then :

a) A sequence $\{\tilde{x}_{ej}^{n}\}$of soft vectors in fuzzy soft normed linear space .

Then the sequence is converges to $\tilde{x}_{ej}^{0}$ with respect to fuzzy soft norm Γ

If $\Gamma(\tilde{x}_{ej}^{n} - \tilde{x}_{ej}^{0}, \tilde{t}) \gtrapprox 1-\alpha$ for every $n \geq n_0$ and $\alpha \in (0,1)$ where n_0 is positive integer and $\tilde{t} \gtrsim \tilde{0}$

Or $\lim_{n\to\infty} \Gamma(\tilde{x}_{ej}^{n} - \tilde{x}_{ej}^{0}, \mathrm{t}) = 1$ *as* $\tilde{t} \to \infty$

Similarly if $\lim_{n\to\infty} \Delta(\tilde{x}_{ej}^{n} - \tilde{x}_{ej}^{0}, \tilde{\mathrm{t}}) = 1$ as $\tilde{t} \to \infty$,then $\{\tilde{x}_{ej}^{n}\}$ is convergent sequence in fuzzy soft metric space $(\tilde{X}, \Delta, *)$

b) A sequence$\{\tilde{x}_{ej}^{n}\}$of soft vectors in fuzzy soft normed linear space is said to be Cauchy sequence if $\Gamma(\tilde{x}_{ej}^{n} - \tilde{\mathrm{x}}_{\mathrm{ej}}^{0}, \tilde{\mathrm{t}}) \gtrapprox 1-\alpha$

for every $n, m \geq n_0$ and $\alpha \in (0,1]$ where n_0 is positive integer and $\tilde{t} \gtrapprox \tilde{0}$

Or $\lim_{n,m\to\infty} \Gamma(X_{ej}^{n} - X_{ej}^{m}, \tilde{\mathrm{t}}) = 1$ as $\tilde{t} \to \infty$. then $\{\tilde{x}_{e_j}^{n}\}$is Cauchy sequence in fuzzy soft metric space $(\tilde{X}, \Delta, *)$.

The above definition includes fuzzy soft convergent sequence , fuzzy soft Cauchy sequence in fuzzy soft normed linear space .In our research , we need to define the definitions in fuzzy soft normed space .

Definition 2.13 : Let (X , E , ‖. ‖) be a fuzzy soft normed space **,** then :

c) A sequence $\{x_{ej}^{n}\}$in X is said to be converges to x_{ej}^{0} in X if for each

If $(X_{ej}^{n}$- X_{ej}^{0},t) $\geq 1-\alpha$ for every $n \geq n_0$ and $\alpha \in (0,1)$ where n_0 is positive integer and $t > 0$

Or $\lim_{n\to\infty} E(X_{ej}^{n} - X_{ej}^{0}, t) = 1 \; as \; t \to \infty$

d) A sequence $\{x_{ej}^{n}\}$in X is said to be Cauchy if $E(X_{ej}^{n} - X_{ej}^{0}, t) \geq 1-\alpha$

for every $n , m \geq n_0$ and $\alpha \in (0,1]$ where n_0 is positive integer and $t \geq 0$

Or $\lim_{n,m\to\infty} E\,(X_{ej}^{n} - X_{ej}^{m}, t) = 1$ as $t \to \infty$

3. The Main Results

Definition 3.1 [6] **:** Let X be a vector space F. The function P:$X \to R$ is called Sub-linear functional on X if

1) $P(x+y) \leq P(x) + P(y) for\ all\ x , y \in X$
2) $P(\lambda x) = \boldsymbol{\lambda}\, P(x) for\ all\ x \in X\ and\ for\ all\ \lambda \geq 0$

Theorem 3.2 :let $(X, E , \|.\|)$ be a fuzzy soft normed space, we further

assume that $(\boldsymbol{x_i})\ a * a = a \quad \forall\ a \in [0,1]$.

Define $P_\alpha(x) = inf\{t : E(x,t) > \alpha\}$, $\alpha \in (0,1)$, $t \in (0,\infty)$ then

$\{P_\alpha : \alpha \in (0,1)\}$ is sub-linear function on X.

Proof :

1-$P_\alpha(x) + P_\alpha(y) = inf\{t > 0 : E(x,t) > \alpha\} + inf\{s > 0 : E(y,s) > \alpha\}$

$t , s \in (0,\infty) \quad and \quad \alpha \in (0,1)$

$$= inf\{t+s : E(x,t) > \alpha , E(y,s) > \alpha\}$$

$$= inf\{t+s : E(x,t) * E(y,s) > \alpha * \alpha\}$$

$$\geq inf\{t+s : E(x+y, t+s) > \alpha\}$$

$$= P_\alpha(x+y)$$

2- If $c = 0\ then\ P_\alpha(cx) = P_\alpha(0) = inf\{t : E(0,t) > \alpha\}$

Since $\|(0,t)\| = 1 > \alpha \quad \forall\, t \in (0,\infty) \quad and \quad \forall \quad \alpha \in (0,1)$

Then $inf\{(0,\infty) : E(0,t) > \alpha\} = 0 = c\, P_\alpha(x)$

If $c \neq 0\ then\ P_\alpha(cx) = inf\{s : E(cx,s) > \alpha\}$

$= inf\left\{s : E\left(x, \frac{s}{|c|}\right) > \alpha\right\}$

Let $t = s/|c|$ then $P_\alpha(cx) = inf\{|c|t : E(x,t) > \alpha\}$

$= |c|\, \inf\{t : E(x,t) > \alpha\}$

$= |c| P_\alpha(x) = cP_\alpha(x)$ (since c> $0\ then\ |c| = c$)

$\therefore P_\alpha$is sub-linear functional .

Theorem 3.3 : If A is open set in fuzzy soft normed space $(X , E , \|.\|)$

satisfying the condition (x_i) then $A = \{x \in X : P_\alpha(x) < t\}$

Proof : Let B= $\{x \in X : P_\alpha(x) < t\}$, Let $x \in A$ $then\ there\ eixest\ r \in (0,1), t \in (0,\infty)\ such\ that\ B(x,r,t) \subset A$

$then\ B(x,r,t) = \{y \in X : E(x-y,t) > 1-r\} \subset A$

if $y = 0 \in X$ (since X vector space)

$then\ B(x,r,t) = \{0 \in X : E(x,t) > 1-r\} \subset A$

Let $\alpha \leq 1-r$ $then\ \alpha \in (0,1)$

$then\ E(x,t) > \alpha$

$\therefore P_\alpha(x) < t$

$\therefore x \in B$

If $y \neq 0$

$E(x,t) = E(x-y+y, t_1+t_2)\ such\ that\ t_1+t_2 = t$

$\geq E(x-y,t_1) * E(y,t_2)$

$> 1-r * E(y,t_2)$

Since $y \neq 0$ then $E(y,t_2) \neq 1$ and $E(y,t_2) > 0$

Then $E(y,t_2) \in (0,1)$

Let $r' = E(y,t_2)\ then\ r' \in (0,1)$

$then\ E(x,t) > 1-r * r' = \alpha \in (0,1)\ such\ that\ \alpha = 1-r * r'$

$\therefore A \subset B$

$\Leftarrow let\ x \in B$

then $P_\alpha(x) < t$

$then\ E(x,t) > \alpha,\ \alpha \in (0,1)$

Let r =1- α

then $r \in (0,1)$

then $1-r = \alpha$

$E(x,t) > 1-r = E(x-0,t) > 1-r$

then $B(x,r,t) \subset A$

$\therefore x \in A$

$\therefore A = \{x \in X : P_\alpha(x) < t\}$

Theorem 3.4: Let $(X, E, \|.\|)$ be a fuzzy Soft normed space satisfying the

condition (x_i), and $x_0 \in X$, if A is an open set such that $x_0 \notin A$, then there exists $f \in X^*$such that $f(x_0) = 1$ and $f(x) < 1\ \forall x \in A$.

Proof : If $0 \in A$,since $x_0 \notin A$, then $x_0 \neq 0$.

Let $P_\alpha : X \to R$ such that $P_\alpha(x) = inf\{t > 0 : E(x,t) > \alpha , \alpha \in (0,1)\}$ $\forall x \in X$

$then$ P_α is sub-linear and $P_\alpha(x) \geq 0$ $\forall x \in X$

Let $M = [x_0]$ $then$ $M = \{tx_0 , t \in R\}$ $then$ M is sub space of X

Define $g : M \longrightarrow R \ni g(tx_0) = t$

$then$ $g \in M'$ and $g(x) \leq P_\alpha(x)$ $\forall x \in M$

By Hahn Banach theorem $\exists f \in X' \ni f(x) = g(x)$ $\forall x \in M$

then $f(x) \leq P_\alpha(x)$ $\forall x \in X$.

$Since$ $x_0 = 1.x_0 \in M$ $then$ $f(x_0) = g(x_0) = g(1.x_0) = 1$

and since A is open set $then$ $A = \{x \in X \ni P_\alpha(x) < t\}$

If $t = 1$ $then$ $f(x) \leq P_\alpha(x) < 1$ $\forall x \in A$.

We must to prove f is continuous that's equivalent to prove f is bounded

Let $x \in X$ $then$ $f(x) \leq P_\alpha(x)$

if $x \in A$ $then$ $P_\alpha(x) < 1$ $then$ $f(x) < 1$

if $x \in -A$ $then$ $-x \in A$ $then$ $f(-x) < 1$ $then$ $f(x) > -1$

$\therefore -1 < f(x) < 1$ $\forall x \in D = A \cap -A$

$\therefore f$ is bounded on the set D and D is open set $,0 \in D$

$then$ f is bounded .

If $0 \notin A$, take $A_1 = A - x_0$, $x_0 \in A$ $then$ $0 \in A_1$

And can prove same that last method .

Theorem 3.5: Let A and B be disjoint ,nonempty, convex set in a fuzzy soft normed space $(X, E, \|.\|)$.If A is open set in X ,then there is $f \in X^*$and $\lambda \in R$ such that

$$f(x) < \lambda \leq f(y) \text{ for all } x \in A \text{ and } y \in B.$$

Proof : let $x_0 = b - a$ where $a \in A$, $b \in B$.

Let $D = A - B + x_0$ $then$ $D = (A - a) - (B - b)$.

Since A, B convex $then$ D is convex and since A is open then D is open

$\because a \in A$ $then$ $0 = a - a \in A - a$

$0 = b - b \in B - b$

$then$ $0 \in D$

we must to prove $x_0 \notin D$.

Let $x_0 \in D$ $then$ $x_0 \in A - B + x_0$ $then$ $0 = x_0 - x_0 \in A - B$

then $0 = (b - a) - (b - a) \in A - B$, $a \in A$ and $b \in B$

$0 = (a - a) - (b - b)$ $then$ $a = b$ $then$ $a \in A \cap B$ and $b \in A \cap B$

$then$ $A \cap B \neq \emptyset$ that's contradiction .

By Theorem(3.4) there exist $f \in X^*$ $\ni f(x_0) = 1$ and $f(x) < 1$ for all $x \in D$ now for all $a \in A$ and all $b \in B$

$a - b + x_0 \in D$ $then$ $f(a - b + x_0) < 1$

then $f(a) - f(b) + f(x_0) < 1$

then $f(a) < f(b)$.

Set $\lambda = \sup\{f(a) : a \in A\}$, then it holds that

$f(a) \leq \lambda \leq f(b)$ $\forall a \in A , b \in B$.

To show that the first is strict , assume there is $a' \in A$ such that

$f(a') = \lambda$, take $k \in X$ such that $f(k) \neq 0$.

Since A open set then $a' + k \in A$, then $\lambda \geq f(a' + k) = f(a') + f(k)$,

which is contradiction (since $f(k) \neq 0$)

then $f(a) < \lambda \leq f(b)$ $\forall a \in A, b \in B$.

Theorem 3.6: Let $(X, E, \|.\|)$ be a locally convex fuzzy soft normed space and A , B convex , nonempty, and disjoint subsets of X, if A is compact , B is closed ,then there exists a continuous linear functional $f \in X^*$and $\lambda \in R$ such that $f(a) < \lambda < f(b)$ $\forall a \in A$, $b \in B$

Proof : let V is a neighborhood of 0 then

$C = A + V$ is open , convex set , and still disjoint from B

Now by theorem (3.5) there exists $f \in X^*$and $\alpha \in R$ such that

$f(c) < \alpha \leq f(b)$ $\forall c \in C$, $b \in B$.

Since f is continuous and A is compact ,$c = a + v$ ($a \in A , v \in V$), then

$f(a + v) < \alpha \leq f(b) \Longrightarrow f(a) + f(v) < \alpha \leq f(b)$

$f(a) < \alpha - f(v) \leq f(b)$.

Put $\lambda = \alpha - f(v) < \alpha$

$f(a) < \lambda < f(b)$ $\forall a \in A$, $b \in B$

Theorem (3.7): Let A is convex , closed subset of a fuzzy soft normed space $(X, E , \|.\|)$ and let $x_0 \in X$, $x_0 \notin A$. Then there exist $f \in X^*$ and $\lambda \in R$ such that $f(x_0) < \lambda \leq f(x)$ $\forall$ $x \in A$

Proof : Since $x_0 \notin A$ then put $r = M_d(x_0, A, t)$ $then$ $r \in (0,1)$

since $B(x_0, r, t)$ is open ball with center x_0 and radius r then $B(x_0, r, t)$ is open and convex set and $B(x_0, r, t) \cap A = \emptyset$

By Theorem (3.5) there exists $f \in X^*$ and $\lambda \in R$ such that

$f(y) < \lambda \leq f(x) \quad \forall\, x \in A\,, y \in B(x_0, r, t)$

In particular $f(x_0) < \lambda \leq f(x)$.

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