Journal of Al-Qadisiyah for Computer Science and Mathematics Vol.11(4) 2019 , pp Math.13–22

University Of AL-Qadisiyah

Available online at www.qu.edu.iq/journalcm

JOURNAL OF AL-QADISIYAH FOR COMPUTER SCIENCE AND MATHEMATICS

ISSN:2521-3504(online) ISSN:2074-0204(print)

Influence of Magnetohydrodynamics Oscillatory Flow for Carreau Fluid Through Regularly Channel With Varying Temperature

Hayder K. Mohammed [a] , Wissam S. Khudair [b], Raheem J. Mezher [c] and Qassim A. Shakir [d]

[a] *Department of Mathematics, Ministry of Education, Directorate of Education Qadisiyah, Diwaneyah, Iraq.* Email: Header.Kadim@gimal.com

[b] *Department of Mathematics, Ministry of Education, Directorate of Education Babylon, Babylon, Iraq. Email:* wissamhsse12@gmail.com

[c] *Department of Mathematics, Ministry of Education, Directorate of Education Qadisiyah, Diwaneyah, Iraq. Email :raheemjebur1983@gmail.com*

[d] *Department of Mathematics, Faculty of Computer Science and Information Technology, University of Al-Qadisiyah, Diwaneyah, Iraq.*

E. mail: qassim4000@gmail.com

ARTICLE INFO

Article history:
Received: 06/08/2019
Rrevised form: 22 /08/2019
Accepted : 29/08/2019
Available online: 16 /09/2019

Keywords:

Carreau fluid, Heat transfer. Magnetohydrodynamics (MHD), Oscillatory flow,

ABSTRACT

This paper investigates the influence of magnetohydrodynamics oscillatory flow for Carreau fluid through regularly channel with varying temperature for two types of geometries "Poiseuille flow and Couette flow". The fluid is assumed to be non-Newtonian, namely Carreau fluid. The governing equations are solved analytically by the perturbation method. The study is intended to calculate the solution for the small number of Weissenberg number ($We << 1$) to get clear forms for velocity field by assisting the (MATHEMATICA-11) program to obtain the numerical results and illustrations. The physical features of Darcy number, Reynolds number, Peclet number, magnetic parameter, Grashof number and radiation parameter are discussed simultaneously through presenting graphical discussion. The velocity and temperature fields are discussed with different values of involved parameter with the help of graphs.

MSC2010: 76A05, 76Wxx.

DOI : 10.29304/jqcm.2019.11.4.614

1 . Introduction

The studies of laminar flow of non-Newtonian fluid have received much attention because it has many applications in science and engineering technology. Fluids differ in their viscosity, which may depend on deformation rate and some fluids have elastic character in nature, which is known as non-Newtonian fluids. Existing literatures indicate that many researchers investigated heat and mass transfer characteristics of non-Newtonian fluids (Nigam and Singh, 1960) [1], (Kavita and others, 2012)[8].

Flow through a porous medium, under the influence of temperature variations, is one of the most important contemporary topics because it finds great applications in geophysics and technology. The study of the flow of sedimentary liquids is of

Corresponding author Hayder K. Mohammed

Email addresses: *Header.Kadim@gimal.com*

Communicated by Qusuay Hatim Egaar

practical importance, especially flow through packed beds, sedimentation, environmental pollution, and central filtering of particles. (Frigaard and Ryan, 2004)[7] studied the flow of blood through the veins and arteries. Recently, the requirements of modern technology have stimulated interest in fluid flow studies, involving the interaction of many phenomena (Hamza and others, 2011)[5].
(Raptis and others, 1982) [3] studied the effect of heat transfer on magnetohydrodynamics oscillatory flow of Jeffrey fluid in a channel, and have investigated when variable viscosity. (Al-Khatib and Wilson, 2001) [6] have study the heat transfer to magneto hydro dynamics oscillatory flow during a porous medium in slip form.
(Khudair and Al-Khafajy, 2018) [9] have investigated the flow for Williamson fluid for two kinds of geometries "Couette flow and Poiseuille flow" in an inclined channel. influence of heat transfer on magnetohydrodynamics for the oscillatory flow of Williamson fluid with model for two kinds of geometries "Poiseuille flow and Couette flow" through a porous medium channel.
Have made an analytical examination on magnetohydrodynamics boundary layer slip flow in a porous medium over a stretching surface with temperature (Attia and Kotb, 1996) [2]. (Mostafa, 2009) [4] have discussed the effect of chemical reaction effects on magnetohydrodynamics free convection flow in an irregular channel with porous medium.
The study considers a mathematical model for the influence of magneto hydrodynamics oscillatory flow for Carreau fluid through regularly channel with varying temperature.

2. Mathematical Formulation

Consider the flow of a Carreau fluid in the channel of breadth l qualify the effects of magnetic field and Radioactive heat transference as described in (Fig.1). We supposed that the fluid have very small electromagnetic force produced and the electrical conductivity is small. Cartesian coordinates system such that, $(v(y),0,0)$ is the velocity vector in which v is the x-component of velocity and y is orthogonal to x-axis.

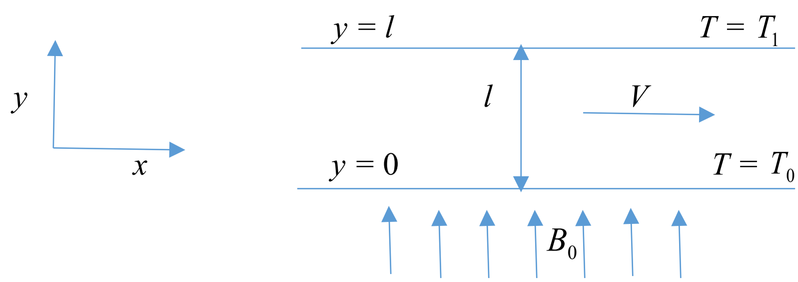

Figure.1 Graph of the problem

The fundamental equation for Carreau fluid is (Nadeem ,2014) [10] :

$$\boldsymbol{S} = -\bar{p}\boldsymbol{I} + \tau \qquad (1)$$

$$\bar{\tau} = [\mu_\infty + (\mu_0 - \mu_\infty)((1 + \Gamma\bar{\dot{\gamma}})^2)^{\frac{n-1}{2}}]A^* \qquad (2)$$

In which $\bar{p}$ is the pressure, $\boldsymbol{I}$ is the unit tensor, $\bar{\tau}$ is the extra stress tensor, Γ is the time constant, μ_∞ and μ_0 are the infinite and zero shear rate viscosity, then $\dot{\gamma}$ is defined as :

$$\dot{\gamma} = \sqrt{\frac{1}{2}\sum_i \sum_j \dot{\gamma}_{ij}\dot{\gamma}_{ji}} = \sqrt{\frac{1}{2}\prod} \text{ and } \prod = tr(A^*)^2, A^* = \Delta\bar{V} + (\Delta\bar{V})^T \qquad (3)$$

Here $\prod$ is the second invariant strain tensor. We consider the fundamental Eq. (2), the case for which $\Gamma\dot{\gamma} < 1$, and $\mu_\infty = 0$. We can write the component of extra stress tensor according to follows as :

$$\bar{\tau} = \mu_0[1 + (\frac{n-1}{2})\Gamma^2\dot{\gamma}^2]A^* \qquad (4)$$

The equations of momentum and energy governing such a flow, subjugate to the Boussinesq approximation, are :

$$\rho\frac{\partial\bar{v}}{\partial\bar{t}} = -\frac{\partial\bar{p}}{\partial\bar{x}} + \frac{\partial\bar{\tau}_{\bar{x}\bar{x}}}{\partial\bar{x}} + \frac{\partial\bar{\tau}_{\bar{x}\bar{y}}}{\partial\bar{y}} + \frac{\partial\bar{\tau}_{\bar{x}\bar{z}}}{\partial\bar{z}} + \rho g\beta(T - T_0) - \sigma B_0^2\bar{v} - \frac{\mu_0}{k}\bar{v} \qquad (5)$$

$$\rho\frac{\partial T}{\partial \bar{t}}=\frac{K}{C_p}\frac{\partial^2 T}{\partial \bar{y}^2}-\frac{1}{C_p}\frac{\partial q}{\partial y} \qquad (6)$$

The temperatures at the walls of the chan nel are given as:

$$T=T_0 \text{ at } \bar{y}=0\,, \text{ and } T=T_1 \text{ at } \bar{y}=l. \qquad (7)$$

In which $\bar{v}$ is the axial velocity, T is a fluid temperature, B_0 is a magnetic field strength, ρ is a fluid density, σ is a conductivity of the fluid, β is a coefficient of volume amplification due to temperature, g is an hastening due to gravity, k is a permeability, c_p is a specific heat at constant pressure, K is a thermal conductivity and q is a radioactive heat flux.

Following (Vinvent and others, 1968) [11], it is supposed that the fluid is visually thin with a relatively low density and the radioactive heat flux is given by:

$$\frac{\partial q}{\partial y}=4b^2(T_0-T) \qquad (8)$$

(b) is radiation absorption coefficient.

Non-dimensional parameters are (Khudair and Al-Khafajy, 2018) [9] :

$$\left.\begin{array}{l} v=\frac{\bar{v}}{V}, x=\frac{\bar{x}}{l}\ , y=\frac{\bar{y}}{l}\ , \theta=\frac{T-T_0}{T_1-T_0}\ , t=\frac{\bar{t}V}{l}\ , p=\frac{\bar{p}h}{\mu V}\ , M^2=\frac{\sigma B_0^2 h^2}{\mu}\ , Da=\frac{k}{l^2}, Gr=\frac{\rho g\beta l^2(T-T_0)}{\mu V} \\ Re=\frac{\rho l V}{\mu}\ , Pe=\frac{\rho l V c_p}{K}\ , N^2=\frac{4b^2 l^2}{K}\ , \tau_{xx}=\frac{l}{\mu_0 V}\bar{\tau}_{\overline{xx}}, \tau_{xy}=\frac{l}{\mu_0 V}\bar{\tau}_{\overline{xy}}, \tau_{xz}=\frac{l}{\mu_0 V}\bar{\tau}_{\overline{xz}}, \dot{\gamma}=\frac{l}{V}\bar{\dot{\gamma}} \end{array}\right\} \qquad (9)$$

Where V is the mean flow velocity, Darcy number (Da), Reynolds number (Re), Peclet number (Pe), magnetic parameter (M), Grashof number (Gr) and radiation parameter (N) .

Substituting equations (8) and (9) into equations (5) - (7), we obtain :

$$\rho\frac{V\partial v}{\frac{l}{V}\partial t}=-\frac{\frac{\mu_0 V}{l}\partial p}{l\partial x}+\frac{\frac{\mu_0 V}{l}\partial\tau_{xx}}{l\partial x}+\frac{\frac{\mu_0 V}{l}\partial\tau_{xy}}{l\partial y}+\frac{\frac{\mu_0 V}{l}\partial\tau_{xz}}{l\partial z}+\rho g\beta(T_1-T_0)\theta-\sigma B_0^2 V v-\frac{\mu_0 V}{k}v \qquad (10)$$

$$\rho\ \frac{\partial(\theta(T_1-T_0)+T_0))}{\frac{l}{V}\partial t}=\frac{k}{C_P}\left[\frac{\partial^2(\theta(T_1-T_0)+T_0))}{l^2\partial y^2}-\frac{1}{k}4b^2(T_0-T)\right] \qquad (11)$$

where $\tau_{xx}=0\,,\ \tau_{xy}=\mu_0\left[\left(1+(\frac{n-1}{2})(\mathrm{We})^2\dot{\gamma}^2\right)\right]\frac{\partial v}{\partial y}\,,\ \tau_{xz}=0\,.$

The following are the non-dimensional boundary conditions corresponding to the temperature equation:

$$\theta(0)=0\ ,\ \theta(1)=1 \qquad (12)$$

Finally, we get the following non-dimensional equations:

$$Re\frac{\partial v}{\partial t}=-\frac{dp}{\partial x}+\frac{\partial}{\partial y}\left[(1+(\frac{n-1}{2})(\mathrm{We})^2\left(\frac{\partial v}{\partial y}\right)^2)\frac{\partial v}{\partial y}\right]+Gr\theta-\left(M^2+\frac{1}{Da}\right)v \qquad (13)$$

$$\rho\frac{\partial\theta}{\partial t}=\frac{\partial^2\theta}{\partial y^2}+N^2\theta \qquad (14)$$

To solve the temperature equation (14) with boundary conditions (12), let

$$\theta(y,t)=\theta_f(y)e^{i\omega t} \qquad (15)$$

where ω is the frequency of the oscillation.

Substituting the equation (15) into the equation (14), we have

$$\frac{\partial^2\theta_f}{\partial y^2}+(N^2-i\omega Pe)\theta_f=0 \qquad (16)$$

The solution of equation (16) with boundary conditions (12) is $\theta_f(y) = \csc(\varphi)\sin(\varphi)$, where $\varphi = \sqrt{N^2 - i\omega Pe}$. Therefore

$$\theta(y,t) = \csc(\varphi)\sin(\varphi)e^{i\omega t} \tag{17}$$

The calculated of equation (13) have been solution in the next parts for two kinds of boundary conditions "Poiseuille flow and Couette flow".

III. SOLUTION OF THE PROBLEM

(i) Poiseuille flow

In this status we suppose that the rigid flakes at $y = 0$ and $y = l$ are at rest. Therefore

$\bar{v} = 0$ at $\bar{y} = 0$, and $\bar{v} = 0$ at $\bar{y} = l$.

The non-dimensional boundary conditions are :

$$v(0) = 0,\ v(1) = 0. \tag{18}$$

To solve the momentum equation (13) , let

$$-\frac{dp}{\partial x} = \lambda e^{i\omega t} \tag{19}$$

$$v(y,t) = v_f(y)e^{i\omega t} \tag{20}$$

where λ is a real constant.

Substituting the equations (19) and (20) into the equation (13), we have :

$$Re\frac{\partial}{\partial t}(v_f(y)e^{i\omega t}) = \lambda e^{i\omega t} + \frac{\partial}{\partial y}\left[(1 + (\frac{n-1}{2})(\mathrm{We})^2\left(\frac{\partial}{\partial y}(v_f(y)e^{i\omega t})\right)^2)\frac{\partial}{\partial y}(v_f(y)e^{i\omega t})\right](v_f(y)e^{i\omega t}) + Gr\theta_0 - \left(M^2 + \frac{1}{Da}\right)(v_f(y)e^{i\omega t}) \tag{21}$$

Equation (21) is non-linear and it is difficult to get an exact solution. So for waning (We), the boundary value problem is agreeing to an easy analytical solution. In this case the equation can be solved. Nevertheless, we suggest a small Γ and used the perturbation technique to solve the problem. Accordingly, we write :

$$v_f = v_{00} + We^2 v_{02} + O(We^4) \tag{22}$$

Substituting equation (22) in equation (21) with boundary conditions (18), then we equality the powers of (We), we obtain :

A - Zeros-order system (We^0)

$$\frac{\partial v_{00}}{\partial y^2} - \left(M^2 + Rei\omega + \frac{1}{Da}\right)v_{00} = -(\lambda + Gr\theta_f) \tag{23}$$

The associated boundary conditions are:

$$v_{00}(0) = v_{00}(1) = 0 \tag{24}$$

B - Second -order system (We^2)

$$\frac{\partial v_{01}}{\partial y^2} - \left(M^2 + Rei\omega + \frac{1}{Da}\right)v_{01} = \frac{-3(n-1)}{2}(\frac{\partial v_{00}}{\partial y})^2(\frac{\partial^2 v_{00}}{\partial y^2})e^{i\omega t} \tag{25}$$

The associated boundary conditions are:

$$v_{01}(0) = v_{01}(1) = 0 \tag{26}$$

Finally, the perturbation solutions up to second order for v_f is given by

$$v_f = v_{00} + We^2 v_{02} + O(We^4)$$

Therefore, the fluid velocity is given as:

$$v(y,t) = v_f(y)e^{i\omega t} \tag{31}$$

(ii) Couette flow

The upper flake is locomotion and the lower flake is fixed with the velocity V_h. The boundary conditions for the Couette flow problem defined as:

$$v(0) = 0 \ , \ v(1) = V_0 \tag{32}$$

We have same defined as the governing equation in Poiseuille flow equation (21). The solution in this case has been calculated by the perturbation technique and the results have been discussed during graphs.

IV. RESULTS AND DISCUSSION

We are discussed influence of magnetohydrodynamics oscillatory flow for Carreau fluid through regularly channel with varying temperature for Poiseuille flow and Couette flow in some results during the graphical illustrations. Numerical assessments of analytical results and some of the graphically significant results are presented in Figure (2-14).

We used the MATHEMATICA program to find the numerical results and illustrations. The momentum equation is resolved by using " perturbation technique " and all the results are discussed graphically.

The velocity profile of Poiseuille flow is shown during Figure (2-6). Figure.2 illustrates the influence Da and M on the velocity profiles function v vs. y. It is found by the increasing Da the velocity profiles function v increases, while v decreases with increasing M. Figure.3 show that velocity profile v rising up by the increasing influence of both the parameters Gr and λ. Figure.4 we observed that v increases by the increasing influence of both the parameters Re and Pe. Figure.5 show the velocity profile v increases by the increasing N, and show that by the increasing ω the velocity profile v decreases.

The fluid velocity starts to be constant at the walls and increasing, as fixed by the boundary conditions. Figure.6 show that velocity profiles increases with the increasing of the parameters We when $0.45 < y < 1$, while v decreases by the increasing of We when $0 < y < 0.45$. The velocity profile of Couette flow is shown during Figure (7–11). It is noted that by the increasing Each of parameters Re, Pe, Gr, Da, N and λ the velocity profile v increases, while v decreases by the increasing We, M and ω .

Based on Eq. (17), figure.12 show that influence of N on the temperature function θ. The temperature increases by the increase in N. Figure.13 we observed that the influence Pe in temperature θ by the increasing Pe then θ increases. Figure.14 show as that by the increasing of ω the temperature θ decreases .

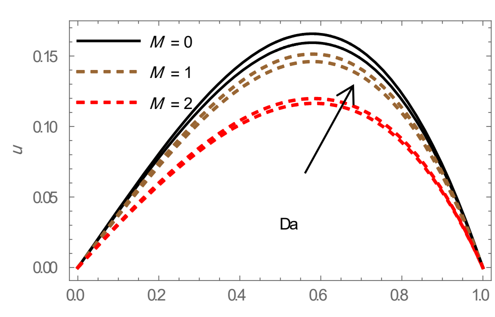

Figure. 2 Velocity profile for Da and M with

$\omega = 1, n = 1, N = 1, \mathrm{Gr} = 1, \mathrm{Re} = 1, Pe = 1, \lambda = 1, \mathrm{We} = 0.05, t = 0.5$ in Poiseuille flow.

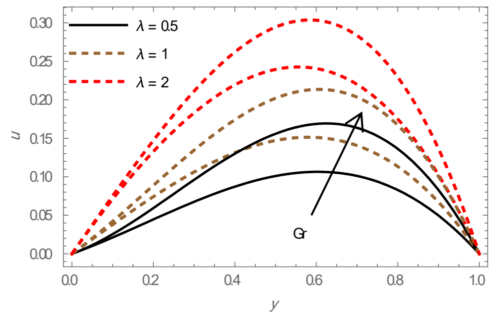

Figure. 3 Velocity profile for λ and Gr with

$\omega = 1, n = 1, N = 1, M = 1, \mathrm{Re} = 1, Pe = 1, Da = 0.8, \mathrm{We} = 0.05, t = 0.5$ in Poiseuille flow.

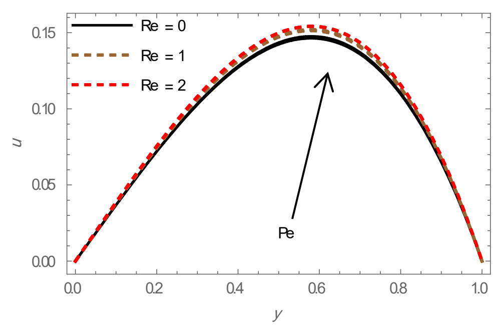

Figure. 4 Velocity profile for Re and Pe with

$\omega = 1, n = 1, N = 1, M = 1, \lambda = 1, Gr = 1, Da = 0.8, \mathrm{We} = 0.05, t = 0.5$ in Poiseuille flow.

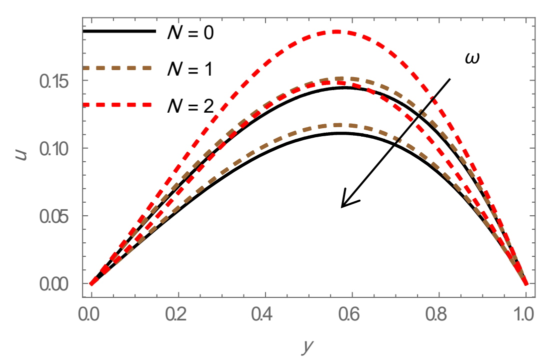

Figure. 5 Velocity profile for ω and N with

$Re = 1, n = 1, Pe = 1, M = 1, \lambda = 1, Gr = 1, Da = 0.8, \mathrm{We} = 0.05, t = 0.5$ in Poiseuille flow.

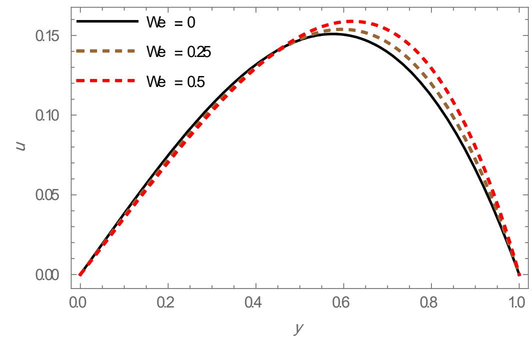

Figure. 6 Velocity profile for We with

$\omega = 1, n = 1, N = 1, Re = 1, Pe = 1, M = 1, \lambda = 1, Gr = 1, Da = 0.8, t = 0.5$ in Poiseuille flow.

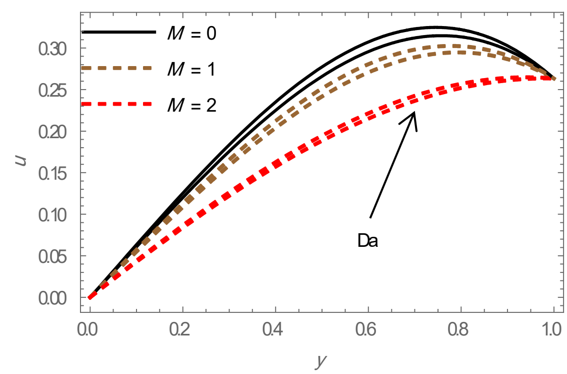

Figure. 7 Velocity profile for M and Da with

$\omega = 1, n = 1, N = 1, \mathrm{Gr} = 1, \mathrm{Re} = 1, Pe = 1, \lambda = 1, \mathrm{We} = 0.05, V_0 = 0.3, t = 0.5$ in Couette flow.

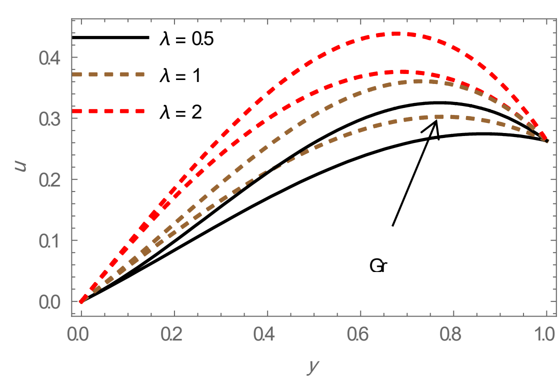

Figure. 8 Velocity profile for λ and Gr with

$\omega = 1, n = 1, N = 1, M = 1, \mathrm{Re} = 1, Pe = 1, Da = 0.8, \mathrm{We} = 0.05, V_0 = 0.3, t = 0.5$ in Couette flow.

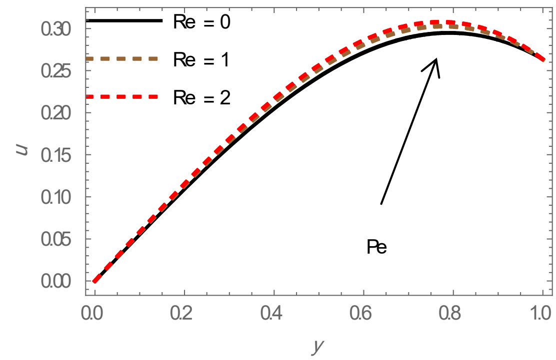

Figure. 9 Velocity profile for Re and Pe with

$\omega = 1, n = 1, N = 1, M = 1, \lambda = 1, Gr = 1, Da = 0.8, \mathrm{We} = 0.05, V_0 = 0.3, t = 0.5$ in Couette flow.

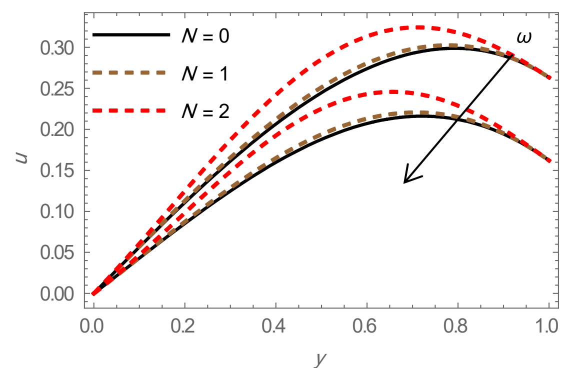

Figure. 10 Velocity profile for ω and N with

$Re = 1, n = 1, Pe = 1, M = 1, \lambda = 1, Gr = 1, Da = 0.8, \mathrm{We} = 0.05, V_0 = 0.3, t = 0.5$ in Couette flow.

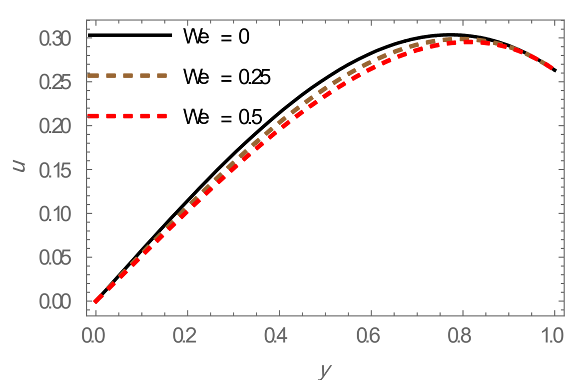

Figure. 11 Velocity profile for We with

$\omega = 1, n = 1, N = 1, Re = 1, Pe = 1, M = 1, \lambda = 1, Gr = 1, Da = 0.8, V_0 = 0.3, t = 0.5$ in Couette flow.

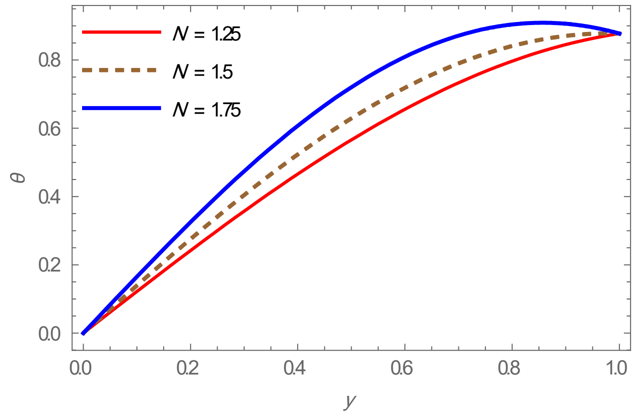

Figure. 12 Influence of N on Temperature θ for $\omega = 1, Pe = 0.7, t = 0.5$

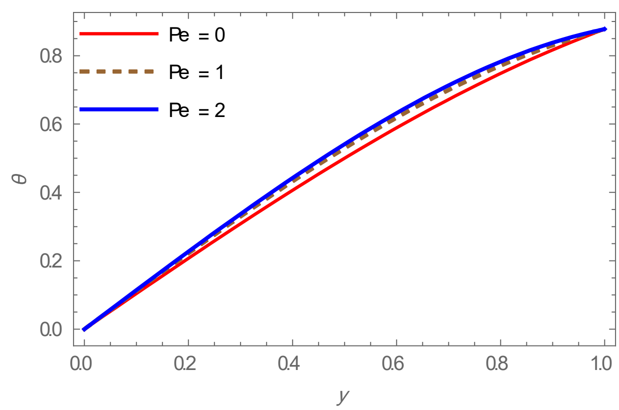

Figure. 13 Influence of Pe on Temperature θ for $t = 0.5, N = 1, \omega = 1$.

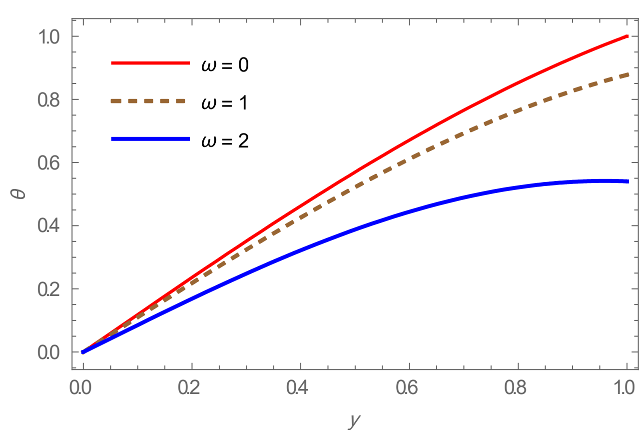

Figure. 14 Influence of ω on Temperature θ for $t = 0.5, N = 1, Pe = 0.7$

V. CONCLUSION AND REMARKS

We discussion the influence of magnetohydrodynamics oscillatory flow for Carreau fluid through regularly channel with varying temperature. We found the velocity and temperature are analytically.

We used different values to finding the results of pertinent parameters namely for the velocity and temperature $(Re, Pe\ , N, Da\ , Gr, \lambda, M, \omega, We)$. The key point are:

- The velocity profiles increases by the increasing $, Pe\ , N, Da\ ,Gr$ and λ for both the Poiseuille and Couette flow.
- The velocity profiles decreases by the increasing ω and M for both the Poiseuille and Couette flow.
- The velocity profiles increases by the increasing of the parameters We when $0.45 < y < 1$, while v decreases with increasing of We when $0 < y < 0.45$, for Poiseuille flow. The velocity profiles decreases with the increasing of the parameters We, for Couette flow.
- We show that by the increases N and Pe the temperature increasing θ and the temperature θ decreases by the increasing ω**.**

REFERENCES

[1] S.D. Nigam and S.N Singh, "Heat-transfer by laminar flow between parallel plates under the action of transverse magnetic field". Q. J. Mech. Appl. Math., vol.13, pp.85-87, (1960).

[2] H.A. Attia and N.A.Kotb, "MHD flow between two parallel plates with heat transfer". Acta Mechanic , vol.117, pp. 215-220, (1996).

[3] A. Raptis, C. Massias, and G. Tzivanidis., "Hydromagnetic free convection flow during a porous medium between two parallel plates MHD", Phys. Lett., 90A,pp. 288-289, (1982).

[4] A.A. Mostafa., "Thermal radiation effect on unsteady MHD free convection flow past a vertical plate with temperature dependent viscosity". The Canadian journal of chemical engineering, 87(1), pp. 171-181, (2009).

[5] M. M. Hamza, B.Y Isah, H. Usman., "Unsteady heat-transfer to MHD oscillatory flow during a porous medium under slip condition". International Journal of Computer Applications, 33(4), pp.12-17, (2011).

[6] M.A. Al Khatib, S.D.Wilson, "The development of Poiseuille flow of yield – stress fluid". J. Non-Newtonian Fluid Mech., 100, pp.1-8, (2001).

[7] I.A Frigaard, D.P. Ryan, "Flow of a visco-plastic fluid in a canal of slowly varying width'". J. Non-Newtonian Fluid Mech.,123, pp.67-83, (2004).

[8] K. Kavita, K.R Prasad, B.A.Kumari, "Influence of heat-transfer on MHD oscillatory flow of Jeffrey fluid in a channel". Advances in Applied Science Research, 3(4),pp. 2312-2325, (2012).

[9] W.S.Khudair and D.G.S.Al-khafajy," Influence of Heat Transfer on MHD Oscillatory Flow for Williamson Fluid with Variable Viscosity Through a Porous Medium", International Journal of Fluid Mechanics & Thermal Sciences., 4(1), pp.11-17, (2018).

[10] M. Y. Malik, I. Zehra, and S. Nadeem.," Flows of Carreau fluid with pressure dependent viscosity in a variable porous medium : Application of polymer melt", Alexandria Eng. J., 53(2), pp. 427-435, (2014).

[11] W.G. Vinvent, A.C.L. Cogley, and E.S. Giles," Differential approximation for radiative heat-transfer in non-linear Eq.s-grey gas near equilibrium", American Institute of Aeronautics and Astronautics, pp. 551-553, (1968)