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On B^*c −Convergence of Filters

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ABSTRACT

In this paper we introduce and study types of convergence in topological spaces namely, B^*c −convergence of filters by using the concept of B^*c −open sets. Also, some properties of B^*c −cluster points of filters has been studied.

MSC. **54xx**

1.Introduction

The notion of convergence is one of the basic notion in analysis. There are important type of convergence theories used in general topology, which goes back to work of Cartan [3] in 1937, Willard [8] in 1970 and Bourbaki [2] in 1989 , is based on the notion of a filter, while the cannotation of B^*c −open sets was introduced by Karim [4] in 2018, which are dependent on the cannotation of β −open sets and that goes back its work to Abd El-monsef [1] in 1983. Through the previous two concepts we were able to get some theories, properties and important relations between them. We also in this work found some examples of some reverse cases that are generally incorrect.

2.Preliminares

Now, we introduce some elementary concepts which we need in our work.

Definition (2.1):[1]

Let (X, T) be a topological space. Then a subset A of X is said to be

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(i) $\beta-$open set, if $A \subseteq cl\,(int(cl\,A))$.

(ii) $\beta-$closed set, if $int\,(cl\,(int\,A)) \subseteq A$.

The family of all $\beta-$open (resp. $\beta-$closed) sets of a space X denoted by $\beta o(X)$ (resp. $\beta c(X)$).

The complement of $\beta-$open set in a topological space (X, T) is called is $\beta-$closed set.

Theorem (2.2):[1]

Let (X, T) be a topological space. Then:

(i) Every open set is $\beta-$open set in X.

(ii) Every closed set is $\beta-$closed set in X.

Theorem (2.3):[1]

Let (X, T) be a topological space, then the following statements are holds:

(i) The union of $\beta-$open sets is $\beta-$open set.

(ii) The intersection of $\beta-$closed sets is $\beta-$closed set.

Remark (2.4):[4]

(i) The intersection of any two $\beta-$open sets is not $\beta-$open set in general.

(ii) The union of any two $\beta-$ closed sets is not $\beta-$closed set in general.

Definition (2.5):[4]

Let (X, T) be a topological space and $A \subseteq X$. Then a β-open set A is called a B^*c-open set if for all $x \in A$ their exists F closed set such that $x \in F \subseteq A$. A is a B^*c-closed set if A^c is a B^*c-open set, the family of all B^*c-open (resp. B^*c-closed) set subset of a space X will be as always denoted by $B^*co(X)$ (resp. $B^*cc(X)$).

Remark (2.6):

From definition **(2.5)** note that:

(i) Every B^*c-open set is $\beta-$open set.

(ii) Every B^*c-closed set is $\beta-$closed set.

Proposition (2.7):[4]

Let (X, T) be a topological space. Then:

(i) the union family of B^*c-open set is a B^*c-open set.

(ii) the intersection family of B^*c-closed set is a B^*c-closed set.

Remark (2.8):[4]

(i) The intersection of any two B^*c-open sets is not B^*c-open set in general.

(ii) The union of any two B^*c-closed sets is not B^*c-closed set in general.

Theorem (2.9):

Let (X, T) be a topological space and $A \subseteq X$. Then:

(i) $y \in \beta-cl(A)$ if and only if for all $\beta-$open set N and $y \in N$ such that $N \cap A \neq \emptyset$. **[1]**

(ii) $y \in B^*c-cl(A)$ if and only if for all B^*c-open set N and $y \in N$ such that $N \cap A \neq \emptyset$. **[4]**

Theorem (2.10):

Let (X, T) be a topological space and $A \subseteq X$. Then:

(i) A is a $\beta-$closed set if and only if $A = \beta-cl(A)$. **[1]**

(ii) A is a B^*c-closed set if and only if $A = B^*c-cl(A)$.**[4]**

Definition (2.11):[6]

Let (X, T) be a topological space and $x \in X$. A subset N of X is called a $\beta-$neighborhood of x, if there exists V a $\beta-$open set in X such that $x \in V \subseteq N$.

The collection of all $\beta-$ neighborhoods of x always denoted by $N_\beta(x)$.

Definition (2.12):

Let (X, T) be a topological space and $x \in X$. A subset N of X is called a $\mathrm{B}^*\mathrm{c}-$neighborhood of x, if there exists V a $\mathrm{B}^*\mathrm{c}-$open set in X such that $x \in V \subseteq N$.

The collection of all $\mathrm{B}^*\mathrm{c}-$neighborhoods of x always denoted by $\mathrm{N}_{\mathrm{B}^*\mathrm{c}}(x)$.

Definition(2.13):[7]

A function $f: X \to Z$ from a topological space X in to a topological space Z is said to be $\beta-$irresolute if $f^{-1}(A)$ is an $\beta-$open set in X for every $\beta-$open set A in Z.

Theorem (2.14):[7]

A function $f: X \to Z$ from a topological space X in to a topological space Z is $\beta-$irresolute if and only if for each $x \in X$ and each $\beta-$neighborhood W of $f(x)$ in Z, there is an $\beta-$neighborhood N of x in X such that $f(N) \subseteq W$.

Definition (2.15):

Let X and Z be two topological spaces. Then a function $f: X \to Z$ is called $\mathrm{B}^*\mathrm{c}-$irresolute function if for all $\mathrm{B}^*\mathrm{c}-$open set A in Z, then $f^{-1}(A)$ is an $\mathrm{B}^*\mathrm{c}-$open set in X.

Theorem (2.16):

Let X and Z be two topological spaces. Then a function $f: X \to Z$ is called $\mathrm{B}^*\mathrm{c}-$irresolute function if and only if for each $x \in X$ and each $\mathrm{B}^*\mathrm{c}-$neighborhood W of $f(x)$ in Z, there is an $\mathrm{B}^*\mathrm{c}-$neighborhood N of x in X such that $f(N) \subseteq W$.

Proof:

Let $f: X \to Z$ be an $\mathrm{B}^*\mathrm{c}-$irresolute function and W be an $\mathrm{B}^*\mathrm{c}-$neighborhood of $f(x)$ in Z. To prove that there is an $\mathrm{B}^*\mathrm{c}-$neighborhood N of x in X such that $f(N) \subseteq W$. Since f is an $\mathrm{B}^*\mathrm{c}-$irresolute then $f^{-1}(W)$ is an $\mathrm{B}^*\mathrm{c}-$neighborhood of x in X. Let $N = f^{-1}(W)$, then $f(N) = f\left(f^{-1}(W)\right) \subseteq W$ and hence $f(N) \subseteq W$.

Conversely, to prove that $f: X \to Z$ is an $\mathrm{B}^*\mathrm{c}-$irresolute function. Let W be an $\mathrm{B}^*\mathrm{c}-$open set in Z. To prove that $f^{-1}(W)$ is an $\mathrm{B}^*\mathrm{c}-$open set in X. Let $x \in f^{-1}(W)$, then $f(x) \in W$. Thus W is an $\mathrm{B}^*\mathrm{c}-$neighborhood of $f(x)$. By hypothesis there is an $\mathrm{B}^*\mathrm{c}-$neighborhood N_x of x such that $f(N_x) \subseteq W \Longrightarrow N_x \subseteq f^{-1}(W)$, for all $x \in f^{-1}(W)$. Thus there exists an $\mathrm{B}^*\mathrm{c}-$open set U_x of x such that $U_x \subseteq N_x \subseteq f^{-1}(W)$, for all $x \in f^{-1}(W)$, thus $\bigcup_{x \in f^{-1}(W)} U_x \subseteq f^{-1}(W)$.

Since $f^{-1}(W) = \bigcup_{x \in f^{-1}(W)} \{x\} \subseteq \bigcup_{x \in f^{-1}(W)} U_x$.Thus $f^{-1}(W) = \bigcup_{x \in f^{-1}(W)} U_x$, therefore $f^{-1}(W)$ is an $\mathrm{B}^*\mathrm{c}-$open set in Z. Hence f is an $\mathrm{B}^*\mathrm{c}-$irresolute function.

3. On $\mathrm{B}^*\mathrm{c}-$Convergence of Filters

In this section we introduce a new type of convergence namely, $\mathrm{B}^*\mathrm{c}-$convergence of filters. Also, we give examples and theorems about this subject.

Definition (3.1):[8]

Let Σ be a non – empty collection of a non – empty subset of a non – empty set X. We say that Σ is a filter on X if :

(i) $\mathrm{F}_1, \mathrm{F}_2 \in \Sigma$, then $\mathrm{F}_1 \cap \mathrm{F}_2 \in \Sigma$.

(ii) $\mathrm{F}_1 \in \Sigma$ and $\mathrm{F}_1 \subseteq \mathrm{F}_2$ then $\mathrm{F}_2 \in \Sigma$.

Definition (3.2):[5]

A sub collection $\Sigma_\circ$ of a filter Σ on a non– empty set X is called afilter base if and only if each element of Σ contains some element of $\Sigma_\circ$.i.e. each $\mathrm{F} \in \Sigma$ there is $\mathrm{F}_\circ \in \Sigma_\circ$ such that $\mathrm{F}_\circ \subseteq \mathrm{F}$.

Remark (3.3):[5]

If $\Sigma_\circ$ is a filter base for a filter Σ on a non – empty set X. Then $\Sigma = \{\mathrm{F} \subseteq \mathrm{X} : \mathrm{F}_\circ \subseteq \mathrm{F}, \text{for some } \mathrm{F}_\circ \in \Sigma_\circ\}$ is called filter generated by $\Sigma_\circ$.

Theorem (3.4):[5]

Let $\mathrm{X} \neq \emptyset$ and let $\Sigma_\circ$ be a non – empty collection of a non – empty subset of X and $\Sigma = \{\mathrm{A} \subseteq \mathrm{X}, \mathrm{B} \subseteq \mathrm{A} \text{ for some } \mathrm{B} \in \Sigma_\circ\}$. Then Σ is a filter on X if and only if for all $\mathrm{C}, \mathrm{D} \in \Sigma_\circ$, there exists $\mathrm{F} \in \Sigma_\circ$ such that $\mathrm{F} \subseteq \mathrm{C} \cap \mathrm{D}$.

Theorem (3.5):[8]

Let $X \neq \emptyset$ and $\emptyset \neq Y \subseteq X$, if $\Sigma_\circ$ is a filter base on Y, then $\Sigma_\circ$ is a filter base of a filter on X.

Definition (3.6):[5]

A filter Σ on a topological space (X, T) is said to be convergent to a point $y \in X$ (written $\Sigma \to y$) if and only if $N(y) \subseteq \Sigma$. The point $y \in X$ is called a limit point of Σ, also, we say that $y \in X$ is a cluster point of Σ and it is denoted by $(\Sigma \propto y)$ if and only if $F \cap V \neq \emptyset$, for all $F \in \Sigma$ and $V \in N(y)$.

Remark (3.7):[5]

Let f be a function from a topological space X in to a topological space Z, then:

(i) If Σ is a filter on X.Then $f(\Sigma)$ is a filter on Z having for a base the sets $f(F)$, $F \in \Sigma$.

(ii) If $\Sigma_\circ$ is a filter base on X. Then $f(\Sigma_\circ)$ is a filter base on Z.

Theorem (3.8):[8]

Let f be a function from a topological X in to a topological space Z, $y \in X$. Then f is continuous function if and only if whenever $\Sigma \to y$ in X, then $f(\Sigma) \to f(y)$ in Z.

Definition (3.9):

(i) A filter Σ on a topological space (X, T) is said to be β −convergent to a point $y \in X$ (written $\Sigma \overset{\beta}{\to} y$) if and only if $N_\beta(y) \subseteq \Sigma$.

Also, a filter Σ on a topological space (X, T) has $y \in X$ as β −cluster point (written $\overset{\beta}{\Sigma \propto y}$) if and only if $F \in \Sigma$ meets each $V \in N_\beta(y)$.

(ii) A filter Σ on a topological space (X, T) is said to be B^*c −convergent to a point $y \in X$ (written $\Sigma \xrightarrow{B^*c} y$) if and only if $N_{B^*c}(y) \subseteq \Sigma$.

Also, a filter Σ on a topological space (X, T) has $y \in X$ as B^*c −cluster point (written $\overset{B^*c}{\Sigma \propto y}$) if and only if $F \in \Sigma$ meets each $V \in N_{B^*c}(y)$.

Theorem (3.10):

A filter Σ on a topological space (X, T) has $y \in X$ as β −cluster point if and only if $y \in \cap \beta - cl(F)$, for all $F \in \Sigma$.

Proof:

$\overset{\beta}{\Sigma \propto y} \Leftrightarrow$ for all $V \in N_\beta(y)$ and for all $F \in \Sigma$, $F \cap V \neq \emptyset$.

$\Leftrightarrow y \in \beta - cl(F)$, for all $F \in \Sigma$.

$\Leftrightarrow y \in \cap \beta - cl(F)$.

Theorem (3.11):

A filter Σ on a topological space (X, T) has $y \in X$ as B^*c −cluster point if and only if $y \in \cap B^*c - cl(F)$, for all $F \in \Sigma$.

Proof:

$\overset{B^*c}{\Sigma \propto y} \Leftrightarrow$ for all $V \in N_{B^*c}(y)$ and for all $F \in \Sigma$, $F \cap V \neq \emptyset$

$\Leftrightarrow y \in B^*c - cl(F)$, for all $F \in \Sigma$.

$\Leftrightarrow y \in \cap B^*c - cl(F)$.

Theorem (3.12):

Let Σ be a filter on a topological space (X, T) and $y \in X$, then:

(i) If $\Sigma \overset{\beta}{\to} y$ ($\overset{\beta}{\Sigma \propto y}$) then $\Sigma \to y$ $(\Sigma \propto y)$.

(ii) If $\Sigma \xrightarrow{\beta} y$ ($\overset{\beta}{\Sigma \propto y}$) then $\Sigma \xrightarrow{B^*c} y$ ($\overset{B^*c}{\Sigma \propto y}$).

Proof:

(i) let $\Sigma \xrightarrow{\beta} y$, then $N_\beta(y) \subseteq \Sigma$. Since every open set is β −open set by theorem **(2.2)**. Thus $N(y) \subseteq \Sigma$, there fore $\Sigma \to y$.

Now, let $\overset{\beta}{\Sigma \propto y}$, then for all $F \in \Sigma$ and for all $V \in N_\beta(y)$, $F \cap V \neq \emptyset$. Since every open set is β −open set by theorem **(2.2)**. Thus for all $F \in \Sigma$ and for all $V \in N(y)$, $F \cap V \neq \emptyset$. Therefore $\Sigma \propto y$.

(ii) let $\Sigma \xrightarrow{\beta} y$, then $N_\beta(y) \subseteq \Sigma$. Since every B^*c − open set is β −open set by remark **(2.6)**. Thus $N_{B^*c}(y) \subseteq \Sigma$, there fore $\Sigma \xrightarrow{B^*c} y$. Now, let $\overset{\beta}{\Sigma \propto y}$, then for all $F \in \Sigma$ and for all $V \in N_\beta(y)$, $F \cap V \neq \emptyset$. Since every B^*c −open set is β −open set by remark **(2.6)**. Thus for all $F \in \Sigma$ and for all $V \in N_{B^*c}(y)$, $F \cap V \neq \emptyset$. Therefore $\overset{B^*c}{\Sigma \propto y}$.

The converse of theorem **(3.12)** is not true in general. The following example showing that

Example (3.13):

(i) Let $X = \{a, b, c\}$ with $T = \{\emptyset, X, \{a, b\}\}$, then $\beta o(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$, let $\Sigma = \{X, \{a, b\}\}$ be a filter on X, since $N(a) = \{X, \{a, b\}\}$, then $N(a) \subseteq \Sigma$, thus Σ converges to a. Since $N_\beta(a) = \{X, \{a\}, \{a, b\}, \{a, c\}\}$, then $N_\beta(a) \not\subset \Sigma$, therefore Σ is not β −converges to a.

Also, $B^*co(X) = \{\emptyset, X\}$, since $N_{B^*c}(a) = \{X\}$, then $N_{B^*c}(a) \subseteq \Sigma$, thus Σ B^*c −converges to a. But $N_\beta(a) \not\subset \Sigma$, therefore Σ is not β −converges to a.

(ii) Let $X = \{a, b, c, d\}$ with $T = T_{ind}$, then
$\beta o(X) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\},$

$\{a, c, d\}, \{b, c, d\}\}$, let $\Sigma = \{X, \{a\}, \{a, b\}, \{a, b, d\}\}$ be a filter on X, since $N(d) = \{X\} \Longrightarrow \Sigma \propto d.$
Since $N_\beta(d) = \{X, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$, thus Σ is not β −cluster to d.
Since $\{d\} \cap \{a, b\} = \emptyset$.

Also, $B^*co(X) = \{\emptyset, X\}$, since $N_{B^*c}(d) = \{X\} \Longrightarrow \overset{B^*c}{\Sigma \propto d}$. But Σ is not β −cluster to d. Since $\{d\} \cap \{a\} = \emptyset$.

Remark (3.14):

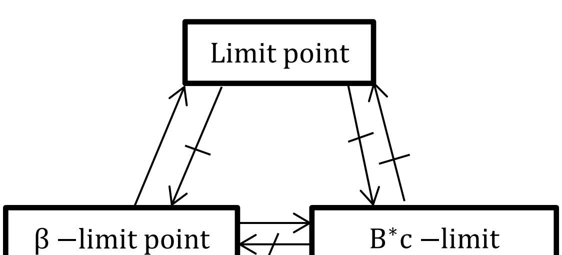
Let (X, T) be a topological space and let Σ be a filter on X. Then the limit point and B^*c −limit point of the filter Σ are independent in general. Also, the cluster point and B^*c −cluster point of the filter Σ are independent in general.

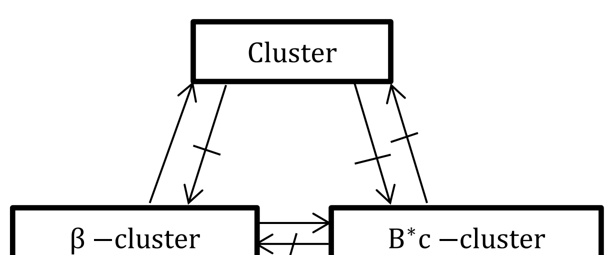
Example (3.15):

(i) Let $X = \{a, b, c\}$ with $T = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$, then $\beta o(X) = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{a, b\}, \{b, c\}\}$ and
$B^*co(X) = \{\emptyset, X, \{a, b\}, \{b, c\}\}$. Let $\Sigma = \{X, \{b\}, \{a, b\}\}$ be a filter on X. Since $N(b) = \{X\}$, thus Σ converges to b.
But Σ is not B^*c −converges to b, since $N_{B^*c}(b) = \{X, \{a, b\}, \{b, c$and $N_{B^*c}(b) \not\subset \Sigma$.
Also, since $N_{B^*c}(a) = \{X, \{a, b\}\}$, thus Σ B^*c −converges to a. But Σ is not converges to a,
since $N(a) = \{X, \{a\}, \{a, b\}, \{a, c\}\}$ and $N(a) \not\subset \Sigma$.

(ii) In **(i)** note that, since $N_{B^*c}(c) = \{X, \{b, c\}\}$, thus Σ B^*c −cluster to c. But Σ is not cluster to c,
since $N(c) = \{X, \{c\}, \{a, c\}, \{b, c\}\}$ and $\{c\} \cap \{b\} = \emptyset$. Also, let $\Sigma = \{X, \{a\}\}$ be a filter on X. Note that, since
$N(b) = \{X\}$, thus Σ cluster to b. But Σ is not B^*c −cluster to b, since $N_{B^*c}(b) = \{X, \{a, b\}, \{b, c\}\}$ and $\{a\} \cap \{b, c\} = \emptyset$.

The following two diagrams showing the relation of among type points.





$diagram(1)$ $diagram(2)$

Theorem (3.16):

Let Σ be a filter on a topological space (X , T) and $y \in X$, then:

(i) If $\Sigma \xrightarrow{\beta} y$ then $\Sigma \overset{\beta}{\propto} y$.

(ii) If $\Sigma \xrightarrow{B^*c} y$ then $\Sigma \overset{B^*c}{\propto} y$.

Proof:

(i) let $\Sigma \xrightarrow{\beta} y$, then $N_\beta(y) \subseteq \Sigma$, thus $F \cap V \in \Sigma$, for all $V \in N_\beta(y)$ and for all $F \in \Sigma$. Since Σ is a filter on X, thus $F \cap V \neq \emptyset$ for all $V \in N_\beta(y)$ and for all $F \in \Sigma$, Therefore $\Sigma \overset{\beta}{\propto} y$.

(ii) let $\Sigma \xrightarrow{B^*c} y$, then $N_{B^*c}(y) \subseteq \Sigma$, thus $F \cap V \in \Sigma$, for all $V \in N_{B^*c}(y)$ and for all $F \in \Sigma$. Since Σ is a filter on X, thus $F \cap V \neq \emptyset$ for all $V \in N_{B^*c}(y)$ and for all $F \in \Sigma$, Therefore $\Sigma \overset{B^*c}{\propto} y$.

The converse of theorem **(3.16)** is not true in general. The following example showing that

Example (3.17):

(i) Let $(\mathbb{R}, T_U)$ be the usual topological space where $\mathbb{R}$ be the set of all real numbers and $\Sigma = \{A \subseteq \mathbb{R}: [-1,1] \subseteq A\}$ be a filter on $\mathbb{R}$, then $\Sigma \overset{\beta}{\propto} 0$ but Σ is not β −converges to 0. Since $(-1,1) \in N_\beta(0)$, but $(-1,1) \notin \Sigma$.

(ii) In example **(3.15)**, let $\Sigma = \{X, \{a, c\}\}$ be a filter on X. Note that, since $N_{B^*c}(a) = \{X, \{a, b\}\}$, then $\Sigma \overset{B^*c}{\propto} a$ but Σ is not B^*c −converges to a. Since $\{a, b\} \in N_{B^*c}(a)$, but $\{a, b\} \notin \Sigma$.

Theorem (3.18):

Let Σ be a filter on a topological space (X , T) and $y \in X$, then:

(i) If $\Sigma \xrightarrow{\beta} y$, then every filter finer than Σ also β −converges to y.

(ii) If $\Sigma \xrightarrow{B^*c} y$, then every filter finer than Σ also B^*c −converges to y.

Proof:

It is a Obvious.

The converse of theorem **(3.18)** is not true in general. The following example showing that

Example (3.19):

(i) Let $X = \{a , b\}$ with $T = \{\emptyset, X, \{a\}\}$,then $\beta o(X) = \{\emptyset, X, \{a\}\}$. Let $\Sigma' = \{X, \{a\}\}$ and $\Sigma = \{X\}$. Since $N_\beta(a) = \{X, \{a\}\}$, then $N_\beta(a) \subseteq \Sigma'$, thus Σ' is β −converges to a. But $\Sigma \subseteq \Sigma'$ and Σ is not β −converges to a, since $N_\beta(a) \not\subset \Sigma$.

(ii) Let $X = \{a , b, c\}$ with $T = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$, then $\beta o(X) = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{a, b\}, \{b, c\}\}$ and $B^*co(X) = \{\emptyset, X, \{a, b\}, \{b, c\}\}$. Let $\Sigma' = \{X, \{a, b\}\}$ and $\Sigma = \{X\}$. Since $N_{B^*c}(a) = \{X, \{a, b\}\}$, then $N_{B^*c}(a) \subseteq \Sigma'$, thus Σ' is B^*c −converges to a. But $\Sigma \subseteq \Sigma'$ and Σ is not B^*c −converges to a, since $N_{B^*c}(a) \not\subset \Sigma$.

Definition (3.20):

(i) A filter base $\Sigma_\circ$ on a topological space (X , T) is said to be β −convergence to $y \in X$ (written $\Sigma_\circ \xrightarrow{\beta} y$) if and only if the filter generated by $\Sigma_\circ$ β −convergent to y. Also, we say that a filter base $\Sigma_\circ$ has $y \in X$ as β −cluster point (written $\Sigma_\circ \overset{\beta}{\propto} y$) if and only if each $F_\circ \in \Sigma_\circ$ meets each $V \in N_\beta(y)$.

(ii) A filter base $\Sigma_\circ$ on a topological space (X , T) is said to be B^*c −convergence to $y \in X$ (written $\Sigma_\circ \xrightarrow{B^*c} y$) if and only if the filter generated by $\Sigma_\circ$ B^*c −convergent to y. Also, we say that a filter base $\Sigma_\circ$ has $y \in X$ as B^*c −cluster point (written $\Sigma_\circ \overset{\beta}{\propto} y$) if and only if each $F_\circ \in \Sigma_\circ$ meets each $V \in N_{B^*c}(y)$.

Definition (3.21):

Let $\Sigma_\circ$ be a filter base on a topological space (X, T) and $y \in X$. Then:

(i) A point y is said to be $\beta-$adherent point of $\Sigma_\circ$ if $y \in \beta-cl(F_\circ)$, for every $F_\circ \in \Sigma_\circ$.

(ii) A point y is said to be $\beta-$accumulation point of $\Sigma_\circ$ if $y \in \bigcap \beta-cl(F_\circ)$, for every $F_\circ \in \Sigma_\circ$.

(iii) A point y is said to be B^*c-adherent point of $\Sigma_\circ$ if $y \in B^*c-cl(F_\circ)$, for every $F_\circ \in \Sigma_\circ$.

(iv) A point y is said to be B^*c-accumulation point of $\Sigma_\circ$ if $y \in \bigcap B^*c-cl(F_\circ)$, for every $F_\circ \in \Sigma_\circ$.

Theorem (3.22):

A filter base $\Sigma_\circ$ on a topological space (X, T) is $\beta-$convergence to a point $y \in X$ if and only if for each $V \in N_\beta(y)$, there is $F_\circ \in \Sigma_\circ$ such that $F_\circ \subseteq V$.

Proof:

Given $\Sigma_\circ \xrightarrow{\beta} y$, then a filter generated by $\Sigma_\circ$ and $\Sigma \xrightarrow{\beta} y$. Then $N_\beta(y) \subseteq \Sigma$, hence for each $V \in N_\beta(y), V \in \Sigma$ thus there is $F_\circ \in \Sigma_\circ$ such that $F_\circ \subseteq V$.

Conversely, to prove that $\Sigma_\circ \xrightarrow{\beta} y$ i.e. , Σ be a filter on X generated by $\Sigma_\circ$ with $\Sigma \xrightarrow{\beta} y$. Let $V \in N_\beta(y)$ then by hypothesis, there is $F_\circ \in \Sigma_\circ$ such that $F_\circ \subseteq V$. Since Σ is a filter on X, then $V \in \Sigma$. Hence $V \in \Sigma$ and $N_\beta(y) \subseteq \Sigma$, therefore $\Sigma_\circ \xrightarrow{\beta} y$.

Theorem (3.23):

A filter base $\Sigma_\circ$ on a topological space (X, T) is B^*c-convergence to a point $y \in X$ if and only if for each $V \in N_{B^*c}(y)$, there is $F_\circ \in \Sigma_\circ$ such that $F_\circ \subseteq V$.

Proof:

Given $\Sigma_\circ \xrightarrow{B^*c} y$, then a filter generated by $\Sigma_\circ$ and $\Sigma \xrightarrow{B^*c} y$. Then $N_{B^*c}(y) \subseteq \Sigma$, hence for each $V \in N_{B^*c}(y), V \in \Sigma$ thus there is $F_\circ \in \Sigma_\circ$ such that $F_\circ \subseteq V$.

Conversely, to prove that $\Sigma_\circ \xrightarrow{B^*c} y$ i.e. , Σ be a filter on X generated by $\Sigma_\circ$ with $\Sigma \xrightarrow{B^*c} y$. Let $V \in N_{B^*c}(y)$ then by hypothesis, there is $F_\circ \in \Sigma_\circ$ such that $F_\circ \subseteq V$ .Since Σ is a filter on X, then $V \in \Sigma$. Hence $V \in \Sigma$ and $N_{B^*c}(y) \subseteq \Sigma$, therefore $\Sigma_\circ \xrightarrow{B^*c} y$.

Theorem (3.24):

A filter Σ on a topological space (X, T) has $y \in X$ as a $\beta-$cluster point if and only if there is a filter Σ' finer than Σ which $\beta-$convergence to y.

Proof:

Suppose that $\Sigma \overset{\beta}{\propto} y$, then by definition **(3.20.i)** each $F \in \Sigma$ meets each $V \in N_\beta(y)$. Then $\Sigma'_\circ = \{F \cap V : F \in \Sigma, V \in N_\beta(y)\}$ is a filter base for some filter Σ' which is finer than Σ and $\beta-$convergence to y.

Conversely, give $\Sigma \subseteq \Sigma'$ and $\Sigma' \xrightarrow{\beta} y$, then $\Sigma' \xrightarrow{\beta} y$ and $N_\beta(y) \subseteq \Sigma'$. Hence each $F \in \Sigma$ and each $V \in N_\beta(y)$ belong to Σ'. Since Σ' is a filter, then $F \cap V \neq \emptyset$.

Theorem (3.25):

A filter Σ on a topological space (X, T) has $y \in X$ as a B^*c-cluster point if and only if there is a filter Σ' finer than Σ which B^*c-convergence to y.

Proof:

Suppose that $\Sigma \overset{B^*c}{\propto} y$, then by definition **(3.20.ii)** each $F \in \Sigma$ meets each $V \in N_{B^*c}(y)$. Then $\Sigma'_\circ = \{F \cap V : F \in \Sigma, V \in N_{B^*c}(y)\}$ is a filter base for some filter Σ' which is finer than Σ and B^*c-convergence to y.

Conversely, give $\Sigma \subseteq \Sigma'$ and $\Sigma' \overset{B^*c}{\longrightarrow} y$, then $\Sigma' \overset{B^*c}{\longrightarrow} y$ and $N_{B^*c}(y) \subseteq \Sigma'$. Hence each $F \in \Sigma$ and each $V \in N_{B^*c}(y)$ belong to Σ'. Since Σ' is a filter, then $F \cap V \neq \emptyset$.

Theorem (3.26):

Let (X, T) be a topological space and $A \subseteq X$, $y \in X$. Then $y \in \beta - cl(A)$ if and only if there is a filter Σ on X such that $A \in \Sigma$ and $\Sigma \overset{\beta}{\to} y$.

Proof:

If $y \in \beta - cl(A)$, then $A \cap V \neq \emptyset$ for all $V \in N_\beta(y)$. Then $\Sigma_\circ = \{A \cap V : V \in N_\beta(y)\}$ is a filter base for some filter Σ. The resulting filter contains A and $\Sigma \overset{\beta}{\to} y$.

Conversely, If $A \in \Sigma$ and $\Sigma \overset{\beta}{\to} y$, then $N_\beta(y) \subseteq \Sigma$. Since Σ is a filter and $A \cap V \neq \emptyset$ for all $V \in N_\beta(y)$.
Thus $y \in \beta - cl(A)$.

Theorem (3.27):

Let (X, T) be a topological space and $A \subseteq X$, $y \in X$. Then $y \in B^*c - cl(A)$ if and only if there is a filter Σ on X such that $A \in \Sigma$ and $\Sigma \overset{B^*c}{\longrightarrow} y$.

Proof:

If $y \in B^*c - cl(A)$, then $A \cap V \neq \emptyset$ for all $V \in N_{B^*c}(y)$. Then $\Sigma_\circ = \{A \cap V : V \in N_{B^*c}(y)\}$ is a filter base for some filter Σ. The resulting filter contains A and $\Sigma \overset{B^*c}{\longrightarrow} y$.

Conversely, If $A \in \Sigma$ and $\Sigma \overset{B^*c}{\longrightarrow} y$, then $N_\beta(y) \subseteq \Sigma$. Since Σ is a filter and $A \cap V \neq \emptyset$ for all $V \in N_{B^*c}(y)$.
Thus $y \in B^*c - cl(A)$.

Corollary (3.28):

Let (X, T) be a topological space and $A \subseteq X$, $y \in X$. Then:

(i) $y \in \beta - cl(A)$ if and only if there is a filter base $\Sigma_\circ$ on X such that $A \in \Sigma_\circ$ and $\Sigma \overset{\beta}{\to} y$.

(ii) $y \in B^*c - cl(A)$ if and only if there is a filter base $\Sigma_\circ$ on X such that $A \in \Sigma_\circ$ and $\Sigma \overset{B^*c}{\longrightarrow} y$.

Theorem (3.29):

Let $f : X \to Z$ be a function and Σ is a filter Σ on X, $y \in X$. Then f is β −irresolute continuous if and only if whenever $\Sigma \overset{\beta}{\to} y$ in X, then $f(\Sigma) \overset{\beta}{\to} f(y)$ in Z.

Proof:

Suppose that f is β −irresolute continuous function and $\Sigma \overset{\beta}{\to} y$. To prove $f(\Sigma) \overset{\beta}{\to} f(y)$ in Z. Let $W \in N_\beta(f(y))$, since f is β −irresolute continuous, then there is $V \in N_\beta(y)$ such that $f(V) \subseteq W$. Since $\Sigma \overset{\beta}{\to} y$, then $V \in \Sigma$. But $W \in f(\Sigma)$, thus $f(\Sigma) \overset{\beta}{\to} f(y)$.

Conversely, suppose that the condition is holds, to prove that f is β −irresolute continuous. Let $\Sigma = \{V : V \in N_\beta(y)\}$ is a filter on X and $\Sigma \overset{\beta}{\to} y$. By hypothesis $f(\Sigma) \overset{\beta}{\to} f(y)$ for each $W \in N_\beta(f(y))$, we have $W \in f(\Sigma)$. There is $V \in N_\beta(y)$ such that $f(V) \subseteq W$.Thus f is β −irresolute continuous function.

Theorem (3.30):

Let $f : X \to Z$ be a function and Σ is a filter Σ on X, $y \in X$. Then f is B^*c −irresolute continuous if and only if whenever $\Sigma \overset{B^*c}{\longrightarrow} y$ in X, then $f(\Sigma) \overset{B^*c}{\longrightarrow} f(y)$ in Z.

Proof:

Suppose that f is β −irresolute continuous function and $\Sigma \xrightarrow{B^*c} y$. To prove $f(\Sigma) \xrightarrow{B^*c} f(y)$ in Z. Let $W \in N_{B^*c}(f(y))$, since f is B^*c −irresolute continuous, then there is $V \in N_{B^*c}(y)$ such that $f(V) \subseteq W$. Since $\Sigma \xrightarrow{B^*c} y$, then $V \in \Sigma$. But $W \in f(\Sigma)$, thus $f(\Sigma) \xrightarrow{B^*c} f(y)$.

Conversely, suppose that the condition is holds, to prove that f is B^*c −irresolute continuous. Let $\Sigma = \{V: V \in N_{B^*c}(y)\}$ is a filter on X and $\Sigma \xrightarrow{B^*c} y$. By hypothesis $f(\Sigma) \xrightarrow{B^*c} f(y)$ for each $W \in N_{B^*c}(f(y))$, we have $W \in f(\Sigma)$. There is $V \in N_{B^*c}(y)$ such that $f(V) \subseteq W$. Thus f is B^*c −irresolute continuous function.

Theorem (3.31):

Let (X , T) be a topological space and $A \subseteq X$. A point $y \in X$ is β −limit point of A if and only if $A - \{y\}$ belongs to some filter Σ which β −convergence to y.

Proof:

Suppose that y is β −limit point of A, then $A - \{y\} \cap V \neq \emptyset$ for all $V \in N_\beta(y)$. $\Sigma_\circ = \{A - \{y\} \cap V: V \in N_\beta(y)\}$ is a filter base for some filter Σ. The resulting filter contains $A - \{y\}$ and $\Sigma \xrightarrow{\beta} y$.

Conversely, If $A - \{y\} \in \Sigma$ with $\Sigma \xrightarrow{\beta} y$, then $A - \{y\} \in \Sigma$ and $N_\beta(y) \subseteq \Sigma$. Since Σ is a filter. Then $A - \{y\} \cap V \neq \emptyset$ for all $V \in N_\beta(y)$. Hence y is β −limit point of a set A.

Theorem (3.32):

Let (X , T) be a topological space and $A \subseteq X$. A point $y \in X$ is B^*c −limit point of A if and only if $A - \{y\}$ belongs to some filter Σ which B^*c −convergence to y.

Proof:

Suppose that y is B^*c −limit point of A, then $A - \{y\} \cap V \neq \emptyset$ for all $V \in N_{B^*c}(y)$. $\Sigma_\circ = \{A - \{y\} \cap V: V \in N_{B^*c}(y)\}$ is a filter base for some filter Σ. The resulting filter contains $A - \{y\}$ and $\Sigma \xrightarrow{B^*c} y$.

Conversely, If $A - \{y\} \in \Sigma$ with $\Sigma \xrightarrow{B^*c} y$, then $A - \{y\} \in \Sigma$ and $N_{B^*c}(y) \subseteq \Sigma$. Since Σ is a filter. Then $A - \{y\} \cap V \neq \emptyset$ for all $V \in N_{B^*c}(y)$. Hence y is B^*c −limit point of a set A.

Definition (3.33):[8]

Let $\{h_a: a \in D\}$ be a net in a topological space (X , T), Σ is a filter generated by a filter base $\Sigma_\circ$ consisting of the sets $A_{a_\circ} = \{h_a: a \geq a_\circ, a_\circ \in D\}$ is called a filter generated by $\{h_a: a \in D\}$. i.e. , $\Sigma_\circ = \{A_{a_\circ} \subseteq X: h_a \textit{ is eventually in } A_{a_\circ}\}$ is a filter base, Σ is a filter on X and it is called a filter associated with the net $\{h_a: a \in D\}$.

Theorem (3.34):

A net $\{h_a: a \in D\}$ in a topological space (X , T) is β −convergence to $y \in X$ if and only if a filter Σ generated by $\{h_a: a \in D\}$ is β −convergent to y.

Proof:

A net $\{h_a: a \in D\}$ is β −convergent to $y \in X$ if and only if each $V \in N_\beta(y)$ contains a tail of $\{h_a: a \in D\}$, since the tails of $\{h_a: a \in D\}$ are a base for a filter generated by $\{h_a: a \in D\}$, the result follows.

Theorem (3.35):

A net $\{h_a: a \in D\}$ in a topological space (X , T) is B^*c −convergence to $y \in X$ if and only if a filter Σ generated by $\{h_a: a \in D\}$ is B^*c −convergent to y.

Proof:

A net $\{h_a: a \in D\}$ is B^*c −convergent to $y \in X$ if and only if each $V \in N_{B^*c}(y)$ contains a tail of $\{h_a: a \in D\}$, since the tails of $\{h_a: a \in D\}$ are a base for a filter generated by $\{h_a: a \in D\}$, the result follows.

Definition (3.36):[8]

Let $\Sigma_\circ$ be a filter base on a topological space (X , T), for all $F_1, F_2 \in \Sigma_\circ$, we put $F_1 \geq F_2$ if and only if $F_1 \subseteq F_2$, then $(\Sigma_\circ, \geq)$ is a directed set. For all $F \in \Sigma_\circ$, define $h: \Sigma_\circ \to \cup F, F \in \Sigma_\circ$ such that for all $F \in \Sigma_\circ$ take (fixed) $h_F \in F$ such that $h(F) = h_F$. Thus $\{h_F: F \in \Sigma_\circ\}$ is a net in X and it is called a net associated with a filter base $\Sigma_\circ$.

Theorem (3.37):

Let $\{h_F: F \in \Sigma_\circ\}$ be a net associated with a filter base $\Sigma_\circ$ on a topological space (X , T) and y ∈ X. If $\Sigma_\circ \xrightarrow{\beta} y$, then $h_F \xrightarrow{\beta} y$.

Proof:

Let $\Sigma_\circ \xrightarrow{\beta} y$ and $V \in N_\beta(y)$. Thus there is $F_\circ \in \Sigma_\circ$ such that $F_\circ \subseteq V$, then $h_{F_\circ} \in V$, so $h_F \in V$ for all $F \geq F_\circ$. Therefore $h_F \xrightarrow{\beta} y$.

Example (3.38):

Let X = {1 , 2 , 3} and T = {∅ , X , {1}} be a topology on X. Put $\Sigma_\circ = \{1, 3\}$ and $\Sigma = \{\{1, 3\}, X\}$, $N_\beta(1) = \{X, \{1\}, \{1, 2\}, \{1, 3\}\}$. Define $h: \Sigma_\circ \to \{1, 3\}$ by $h(\{1, 3\}) = 1$, then h is a net in X. Thus $h \xrightarrow{\beta} 1$, but $\Sigma_\circ$ does not β −convergence to 1, since $\{1\} \in N_\beta(1)$ but $\{1\} \notin \Sigma$.

Theorem (3.39):

Let $\{h_F: F \in \Sigma_\circ\}$ be a net associated with a filter base $\Sigma_\circ$ on a topological space (X , T) and y ∈ X. If $\Sigma_\circ \xrightarrow{B^*c} y$, then $h_F \xrightarrow{B^*c} y$.

Proof:

Let $\Sigma_\circ \xrightarrow{B^*c} y$ and $V \in N_{B^*c}(y)$. Thus there is $F_\circ \in \Sigma_\circ$ such that $F_\circ \subseteq V$, then $h_{F_\circ} \in V$, so $h_F \in V$ for all $F \geq F_\circ$. Therefore $h_F \xrightarrow{B^*c} y$.

Example (3.40):

Let X = {1 , 2 , 3} and T = {∅ , X , {1}, {3}, {1,3}} be a topology on X. Put $\Sigma_\circ = \{1, 2\}$ and $\Sigma = \{\{1, 2\}, X\}$, $N_{B^*c}(2) = \{X, \{1, 2\}, \{2, 3\}\}$. Define $h: \Sigma_\circ \to \{1, 2\}$ by $h(\{1, 2\}) = 2$, then h is a net in X. Thus $h \xrightarrow{B^*c} 2$, but $\Sigma_\circ$ does not B^*c −convergence to 2, since $\{2, 3\} \in N_{B^*c}(2)$, but $\{2, 3\} \notin \Sigma$.

Definition (3.41):[8]

Let $\Sigma_\circ$ be a filter base on a topological space (X , T). Put $D = \{(y, F): y \in F, F \in \Sigma_\circ\}$, $(D, \geq)$ is directed set by the relation, $(y_1, F_1) \geq (y_2, F_2)$ if and only if $F_1 \subseteq F_2$, so define a function $h: D \to X$, by $h(a) = h_a \in X$, where $a = (y, F)$. Then $\{h_a: a \in D\}$ is called the canonical net (net based) of $\Sigma_\circ$.

Theorem (3.42):

A filter base $\Sigma_\circ$ on a topological space (X , T) is β −convergence to a point y ∈ X if and only if the canonical net of $\Sigma_\circ$ is β −convergence to y.

Proof:

Let $\Sigma_\circ \xrightarrow{\beta} y$ and $V \in N_\beta(y)$, then there is $F_\circ \in \Sigma_\circ$ such that $F_\circ \subseteq V$. Since $F_\circ \neq \emptyset$, there is $y_\circ \in F_\circ$. Pick $a_\circ = (y_\circ, F_\circ)$, then $h_a \in V$ for all $a \geq a_\circ$. Therefore $h_a \xrightarrow{\beta} y$.

Conversely, let $h_a \xrightarrow{\beta} y$ and $V \in N_\beta(y)$, there is $a_\circ \in D$ such that $h_a \in V$ for all $a \geq a_\circ$. Thus there is $F_\circ \in \Sigma_\circ$ and $y_\circ \in F_\circ$ such that $a_\circ = (y_\circ, F_\circ)$. To prove $F_\circ \subseteq V$, let $y_\circ \in F_\circ$. Then $a = (y, F) \geq (y_\circ, F_\circ) = a_\circ$, thus $h_a \in V$. Hence $F_\circ \subseteq V$, therefore $\Sigma_\circ \xrightarrow{\beta} y$.

Theorem (3.43):

A filter base $\Sigma_\circ$ on a topological space (X , T) is B^*c −convergence to a point y ∈ X if and only if the canonical net of $\Sigma_\circ$ is B^*c −convergence to y.

Proof:

Let $\Sigma_\circ \xrightarrow{B^*c} y$ and $V \in N_{B^*c}(y)$, then there is $F_\circ \in \Sigma_\circ$ such that $F_\circ \subseteq V$. Since $F_\circ \neq \emptyset$, there is $y_\circ \in F_\circ$. Pick $a_\circ = (y_\circ, F_\circ)$, then $h_a \in V$ for all $a \geq a_\circ$. Therefore $h_a \xrightarrow{B^*c} y$.

Conversely, Let $h_a \overset{B^*c}{\longrightarrow} y$ and $V \in N_{B^*c}(y)$, there is $a_\circ \in D$ such that $h_a \in V$ for all $a \geq a_\circ$. Thus there is $F_\circ \in \Sigma_\circ$ and $y_\circ \in F_\circ$ such that $a_\circ = (y_\circ , F_\circ)$. To prove $F_\circ \subseteq V$, let $y_\circ \in F_\circ$. Then $a = (y , F) \geq (y_\circ , F_\circ) = a_\circ$, thus $h_a \in V$. Hence $F_\circ \subseteq V$, therefore $\Sigma_\circ \overset{B^*c}{\longrightarrow} y$.

Corollary (3.44):

A filter base $\Sigma_\circ$ on a topological space (X , T) has $y \in X$ as a :

(i) $\beta -$cluster point if and only if the canonical net on $\Sigma_\circ$ has y as a $\beta -$cluster point.

(ii) $B^*c -$cluster point if and only if the canonical net on $\Sigma_\circ$ has y as a $B^*c -$cluster point.

Theorem (3.45):

A topological space (X , T) is $\beta -$Hausdorff space if and only if every $\beta -$convergent filter in X has a unique $\beta -$limit point.

Proof:

Let (X , T) be a $\beta -$Hausdorff space and Σ be a filter on X such that $\Sigma \overset{\beta}{\rightarrow} y$ and $\Sigma \overset{\beta}{\rightarrow} z$ with $y \neq z$. Since X is a $\beta -$Hausdorff space, then there is $V \in N_\beta(y)$ and $W \in N_\beta(z)$ such that $V \cap W = \emptyset$. Since $\Sigma \overset{\beta}{\rightarrow} y$ then $N_\beta(y) \subseteq \Sigma$ and $\Sigma \overset{\beta}{\rightarrow} z$ then $N_\beta(z) \subseteq \Sigma$. Since Σ be a filter, then $V \cap W \neq \emptyset$. This is a contradiction, hence the result follows.

Conversely, to prove that X is a $\beta -$Hausdorff space. Suppose not, then there are $y, z \in X$ with $y \neq z$ such that for all $V \in N_\beta(y)$ and for all $W \in N_\beta(z)$, $V \cap W \neq \emptyset$. Then $\Sigma_\circ = \{V \cap W : V \in N_\beta(y) \text{ and } W \in N_\beta(z)\}$ is a filter base for some filter Σ. The resulting filter $\beta -$convergence at y and z. This is a contradiction, thus X is a $\beta -$Hausdorff space.

Theorem (3.46):

A topological space (X , T) is $B^*c -$Hausdorff space if and only if every $B^*c -$convergent filter in X has a unique $B^*c -$limit point.

Proof:

Let (X , T) be a $B^*c -$Hausdorff space and Σ be a filter on X such that $\Sigma \overset{B^*c}{\longrightarrow} y$ and $\Sigma \overset{B^*c}{\longrightarrow} z$ with $y \neq z$. Since X is a $B^*c -$ Hausdorff space, then there is $V \in N_{B^*c}(y)$ and $W \in N_{B^*c}(z)$ such that $V \cap W = \emptyset$. Since $\Sigma \overset{B^*c}{\longrightarrow} y$ then $N_{B^*c}(y) \subseteq \Sigma$ and $\Sigma \overset{B^*c}{\longrightarrow} z$ then $N_{B^*c}(z) \subseteq \Sigma$. Since Σ be a filter, then $V \cap W \neq \emptyset$. This is a contradiction, hence the result follows.

Conversely, to prove that X is a $B^*c -$Hausdorff space. Suppose not, then there are $y, z \in X$ with $y \neq z$ such that for all $V \in N_{B^*c}(y)$ and for all $W \in N_{B^*c}(z)$, $V \cap W \neq \emptyset$. Then $\Sigma_\circ = \{V \cap W : V \in N_{B^*c}(y) \text{ and } W \in N_{B^*c}(z)\}$ is a filter base for some filter Σ. The resulting filter $\beta -$ convergence at y and z This is a contradiction, thus X is a $B^*c -$Hausdorff space.

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