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Supra $\widehat{\omega}$-regular and Supra ŋ̂-regular Spaces

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ABSTRACT

The aim of this paper is to introduce new forms of the supra regular spaces by using new supra sets which are supra $\widehat{\omega}$-open and supra ŋ̂-open sets, and to introduce new types of supra T_3-spaces by using these supra open sets, we support our work by examples and some facts.

1. Introduction

The concept of supra spaces was introduced at the first time by Mashhour in 1983[3] where he defined it as (Let X be a set, the sub collection μ of $\mathcal{P}(X)$ is called a supra topology on X if $X \in \mu$, and it is closed under the arbitrary union. The pair (X,μ) is called a supra space. Any set $\mathcal{W} \in \mu$ is called supra open set and its complement is supra closed). Also he submitted the definition of supra closure and supra interior for a subset of a supra space. After that many researchers dealt with this space and submitted new concepts in it, in [5] the researcher provided the supra regular spaces and the supra T_3-spaces. And we provided in this research new supra sets which are supra $\widehat{\omega}$-open and supra ŋ̂-open sets, also we introduced supra $\widehat{\omega}$-regular space, supra $\widehat{\omega}^*$-regular space, supra $\widehat{\omega}^{**}$-regular space, supra ŋ̂-regular space, supra ŋ̂*-regular space and supra ŋ̂**-regular space, also we illustrate the relationships between these types.

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2- Types of supra $\widehat{\omega}$-regular and supra ŋ̂-regular spaces.

In this part we will provide the definitions of supra $\widehat{\omega}$-open and supra ŋ̂-open sets, and the definition of the new types of supra regular spaces by using these sets and give the relation between them. Through this paper we will use the abbreviation "su." to express the "supra."

Definition (2.1): 1- A subset $\mathcal{W}$ of a su. space (X,μ) is called su. $\widehat{\omega}$-open (resp. su. ŋ̂-open) set, if for any point x in $\mathcal{W}$ there is $G \in \mu$ with $x \in G$ and G - $\mathcal{W}$ is countable (resp. finite). The complement of $\mathcal{W}$ is called su. $\widehat{\omega}$-closed (resp. su. ŋ̂-closed) set.

2- The su. $\widehat{\omega}$-closure for a subset $\mathcal{W}$ of a su. space (X,μ) is the intersection of all su. $\widehat{\omega}$-closed subsets of X which contain $\mathcal{W}$, and we denote it $by\ cl^{\mu}_{\widehat{\omega}}(\mathcal{W})$. While the su. $\widehat{\omega}$-interior for $\mathcal{W}$ is the union of all su. $\widehat{\omega}$-open subsets of X which contained in $\mathcal{W}$, and we denote it by $Int^{\mu}_{\widehat{\omega}}(\mathcal{W})$.

3- The su. ŋ̂-closure for a subset $\mathcal{W}$ of a su. space (X,μ) is the intersection of all su. ŋ̂-closed subsets of X which contain $\mathcal{W}$, and we denote it by $cl^{\mu}_{\widehat{\omega}}(\mathcal{W})$. While the su. ŋ̂-interior for $\mathcal{W}$ is the union of all su. ŋ̂-open subsets of X which contained in $\mathcal{W}$, and we denote it by $Int^{\mu}_{\hat{ŋ}}(\mathcal{W})$.

Definition (2.2): A space (X,μ) is called-:

1- Su. regular space if for each point $x \in X$ and each su. closed subset $\mathcal{M}$ of X such that $x \notin \mathcal{M}$, there are two disjoint sets $\mathcal{W}_1, \mathcal{W}_2 \in \mu$ in which $x \in \mathcal{W}_1$ and $\mathcal{M} \subseteq \mathcal{W}_2$ [5].

2- Su. $\widehat{\omega}$-regular (resp. su. ŋ̂-regular) space if for each point $x \in X$ and each su. closed subset $\mathcal{M}$ of X such that $x \notin \mathcal{M}$, there are two disjoint su. $\widehat{\omega}$-open (resp. su. ŋ̂-open) sets $\mathcal{W}_1, \mathcal{W}_2$ in X in which $x \in \mathcal{W}_1$ and $\mathcal{M} \subseteq \mathcal{W}_2$.

3- Su. $\widehat{\omega}^*$-regular (resp. su. ŋ̂*-regular) space if for each point $x \in X$ and each su. $\widehat{\omega}$-closed (resp. su. ŋ̂-closed) subset $\mathcal{M}$ of X such that $x \notin \mathcal{M}$, there are two disjoint sets $\mathcal{W}_1, \mathcal{W}_2 \in \mu$ in which $x \in \mathcal{W}_1$ and $\mathcal{M} \subseteq \mathcal{W}_2$.

4- Su. $\widehat{\omega}^{**}$-regular (resp. su. ŋ̂**-regular) space if for each point $x \in X$ and each su. $\widehat{\omega}$-closed (resp. su. ŋ̂-closed) subset $\mathcal{M}$ of X such that $x \notin \mathcal{M}$ there are two disjoint su. $\widehat{\omega}$-open (resp. su. ŋ̂-open) sets $\mathcal{W}_1, \mathcal{W}_2$ in which $x \in \mathcal{W}_1$ and $\mathcal{M} \subseteq \mathcal{W}_2$.

5- Su. T_1-space if for any two distinct points x, y in X there are su. open subsets $\mathcal{W}, \mathcal{B}$ of X in which $x \in \mathcal{W}, y \notin \mathcal{W}$ and $y \in \mathcal{B}, x \notin \mathcal{B}$ [3].

6- A su. T_2-space if for each non-equal points x, y in X, there are disjoint su. open subsets $\mathcal{W}_1, \mathcal{W}_2$ of X in which $x \in \mathcal{W}_1$ and $y \in \mathcal{W}_2$ [3].

7- A su. $\widehat{\omega}T_2$-space if for each non-equal elements x, y in X, there are disjoint su. $\widehat{\omega}$-open subsets $\mathcal{W}_1, \mathcal{W}_2$ of X of which $x \in \mathcal{W}_1$ and $y \in \mathcal{W}_2$.

8- A su. ŋ̂T_2-space if for each non-equal elements x, y in X, there are disjoint su. ŋ̂-open subsets $\mathcal{W}_1, \mathcal{W}_2$ of X in which $x \in \mathcal{W}_1$ and y $\in \mathcal{W}_2$.

Example (2.3): 1- Let X= {1, 2, 3}, μ_X= {∅, X, {1}, {2}, {1, 2}, {1, 3}, {2, 3}}, so (X, μ_X) is su. regular, su. $\widehat{\omega}^{**}$-regular, su. ŋ̂**-regular, su. ŋ̂-regular, su. $\widehat{\omega}$-regular, su. T_2-space, su. T_1-space, su. $\widehat{\omega}T_2$-space and su. ŋ̂T_2-space, but neither su. $\widehat{\omega}^*$ -regular nor su. ŋ̂*-regular space.

2- $(\mathcal{R}, \mu_D)$ is su. $\widehat{\omega}^*$-regular and su. ŋ̂*-regular space.

Remark (2.4): 1- Every su. $\widehat{\omega}^*$-regular space is su. ŋ̂*-regular.

2- There is no relation between su. $\widehat{\omega}^{**}$-regular space and su. ŋ̂**-regular.

3- Every su. ŋ̂-regular space is su. $\widehat{\omega}$-regular.

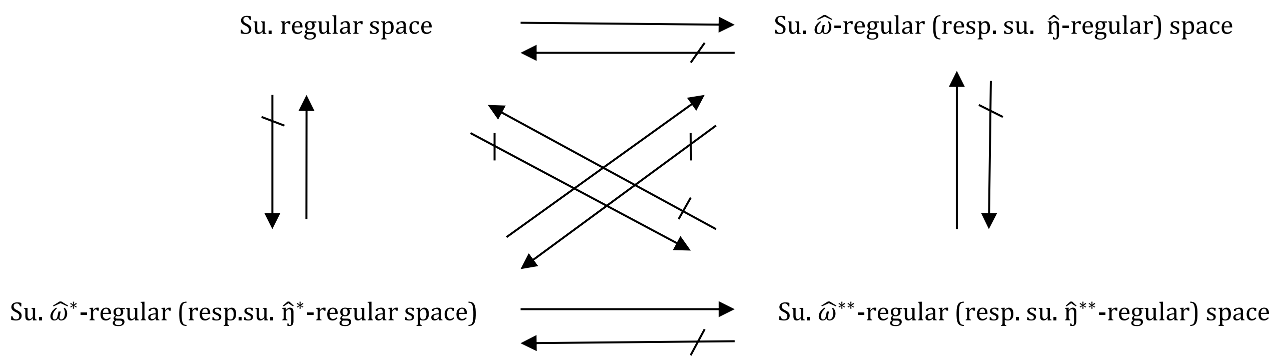
4- If $\mathcal{M}$ is su. closed set in a su. space X, then it is su. $\widehat{\omega}$-closed (resp. su. ŋ̂-closed) set.

5- If $\mathcal{W}$ is su. open set in a su. space X, then it is su. $\widehat{\omega}$-open (resp. su. ŋ̂-open) set.

6- Every topology is su. topology[1].

The converse of each of the above statement is not true.

The next diagram is useful.



Example (2.5): 1- $(\mathcal{R}, \mu_u)$ is su. ῆ*-regular and ῆ**-regular space but neither su. $\widehat{\omega}^{*}$-regular, nor $\widehat{\omega}^{**}$-regular space.

2- Suppose X= {1, 2, 3} and μ = {∅, X, {1}, {2}, {1, 2}}, so (X, μ) is su. $\widehat{\omega}$-regular, su. ῆ-regular, su. ῆ**-regular and su. $\widehat{\omega}^{**}$-regular but not su. regular, not su. ῆ*-regular and not su. $\widehat{\omega}^{*}$-regular.

3- $(\mathcal{Z}, \mu_{ind})$ is su. $\widehat{\omega}^{**}$-regular, su. ῆ-regular, su. $\widehat{\omega}$-regular and su. regular space, while it is not su. ῆ**-regular, not su. ῆ*-regular, and not su. $\widehat{\omega}^{*}$-regular space.

4- $(\mathcal{R}, \mu_{ind})$ is su. regular, su. $\widehat{\omega}$-regular and su. ῆ-regular space but not su. $\widehat{\omega}^{**}$-regular and not su. ῆ**-regular space.

5- $(\mathcal{Z}, \mu_{cof})$ is su. $\widehat{\omega}$-regular, su. $\widehat{\omega}^{**}$-regular space while it is not su. $\widehat{\omega}^{*}$-regular, not su. regular and not su. ῆ-regular.

Theorem (2.6): The space (X, μ) is su. $\widehat{\omega}^{*}$-regular (resp. su. ῆ*-regular) space iff for any element x in X and any su. $\widehat{\omega}$-neighborhood (resp. su. ῆ-neighborhood) $\mathcal{K}$ to x, there is a su. neighborhood $\mathcal{W}$ in X for x with $cl^{\mu}(\mathcal{W}) \subseteq \mathcal{K}$.
Proof: Let $x \in X$ and $\mathcal{K}$ be a su. $\widehat{\omega}$-neighborhood (resp. su. ῆ-neighborhood) to x. So, there exists a su. $\widehat{\omega}$-open (resp. su. ῆ-open) set E in X with $x \in E \subseteq \mathcal{K}$, set $\mathcal{M} = E^c$, hence $\mathcal{M}$ is su. $\widehat{\omega}$-closed (resp. su. ῆ-closed) set in X and $x \notin \mathcal{M}$. But X is su. $\widehat{\omega}^{*}$-regular (resp. su. ῆ*-regular) space, thus there are two disjoint su. open sets $\mathcal{W}, \mathcal{B}$ in X such that $x \in \mathcal{W}, \mathcal{M} \subseteq \mathcal{B}$, then $\mathcal{W}$ is a su. neighborhood to x and $\mathcal{W} \subseteq \mathcal{B}^c$, where $\mathcal{B}^c$ is su. closed set in X. Therefore $cl^{\mu}(\mathcal{W}) \subseteq cl^{\mu}(\mathcal{B}^c) = \mathcal{B}^c \Rightarrow cl^{\mu}(\mathcal{W}) \subseteq \mathcal{B}^c$ .... (1), and since $\mathcal{M} \subseteq \mathcal{B}$ then $\mathcal{B}^c \subseteq \mathcal{M}^c = E \subseteq \mathcal{K}$ .....(2). From (1) & (2) we have, $cl^{\mu}(\mathcal{W}) \subseteq \mathcal{B}^c \subseteq \mathcal{M}^c \subseteq \mathcal{K} \Rightarrow cl^{\mu}(\mathcal{W}) \subseteq \mathcal{K}$. Conversely, suppose $x \in X$ and $\mathcal{M}$ is su. $\widehat{\omega}$-closed (resp. su. ῆ-closed) set in X with $x \notin \mathcal{M}$, so $x \in \mathcal{M}^c$ which is a su. $\widehat{\omega}$-open (resp. su. ῆ-open) set in X, then $\mathcal{M}^c$ is su. $\widehat{\omega}$-neighborhood (resp. su. ῆ-neighborhood) to x, hence there is su. neighbourhood $\mathcal{W}$ in X to x with $cl^{\mu}(\mathcal{W}) \subseteq \mathcal{M}^c$(from hypothesis). Since $\mathcal{W}$ is su. neighborhood for x, so there is a su. open set $\mathcal{W}_1$ in X with $x \in \mathcal{W}_1 \subseteq \mathcal{W}$, from $cl^{\mu}(\mathcal{W}) \subseteq \mathcal{M}^c$ we get $\mathcal{M} \subseteq \left(cl^{\mu}(\mathcal{W})\right)^c$, put $\mathcal{B} = \left(cl^{\mu}(\mathcal{W})\right)^c \Rightarrow \mathcal{B}$ is su. open set of X and $\mathcal{M} \subseteq \mathcal{B}$ and since $\mathcal{W} \cap \mathcal{W}^c = \emptyset$, then $\mathcal{W}_1 \cap \left(cl^{\mu}(\mathcal{W})\right)^c = \emptyset$ (because $\mathcal{W}_1 \subseteq \mathcal{W}$ and because $\mathcal{W} \subseteq cl^{\mu}(\mathcal{W})$, so $\left(cl^{\mu}(\mathcal{W})\right)^c = \mathcal{B} \subseteq \mathcal{W}^c$). So for any point x in X and any su. $\widehat{\omega}$-closed (resp. su. ῆ-closed) set $\mathcal{M}$ in X where $x \notin \mathcal{M}$ there are disjoint su. open sets $\mathcal{W}_1, \left(cl^{\mu}(\mathcal{W})\right)^c$ such that $x \in \mathcal{W}_1$ and $\mathcal{M} \subseteq (cl^{\mu}(\mathcal{W}))^c$ Which implies X is su. $\widehat{\omega}^{*}$-regular (resp. ῆ*-regular) space.
The following proposition can be proved by the same manner.
Proposition (2.7): 1- The su. space (X, μ) is su. regular space iff for any element x in X and any su. neighborhood $\mathcal{K}$ to x, there is a su. neighborhood $\mathcal{W}$ to x with $cl^{\mu}(\mathcal{W}) \subseteq \mathcal{K}$.
2- The su. space (X, μ) is su. $\widehat{\omega}$-regular (resp. su. ῆ-regular) space iff for any element x in X and any su. neighborhood $\mathcal{K}$ to x, there is a su. $\widehat{\omega}$-neighborhood (resp. su. ῆ-neighborhood) $\mathcal{W}$ to x with $cl^{\mu}_{\widehat{\omega}}(\mathcal{W}) \subseteq \mathcal{K}$ (resp. $cl^{\mu}_{ῆ}(\mathcal{W}) \subseteq \mathcal{K}$).

3- The su. space (X,μ) is su. $\widehat{\omega}^{**}$-regular (resp. su. $\hat{\eta}^{**}$-regular) space iff for any element x in X and any su. $\widehat{\omega}$-neighborhood (resp. su. $\hat{\eta}$-neighborhood) $\mathcal{K}$ to x, there is a su. $\widehat{\omega}$-neighborhood (resp. su. $\hat{\eta}$-neighborhood) $\mathcal{W}$ to x with $cl^{\mu}_{\widehat{\omega}}(\mathcal{W}) \subseteq \mathcal{K}$ (resp. $cl^{\mu}_{\hat{\eta}}(\mathcal{W}) \subseteq \mathcal{K}$).

Proposition (2.8): If $\mathcal{W}$ is a su. $\widehat{\omega}$-open set in a su. space X, and Y is a su. sub space of X, then $\mathcal{W} \cap Y$ is a su. $\widehat{\omega}$-open set in Y.

Proof: Consider $x \in \mathcal{W} \cap Y$, so $x \in \mathcal{W}$ and $x \in Y$, hence there is a su. open set G in X with $x \in G$ and G-$\mathcal{W}$ is countable set. Since $[(G\text{-}\mathcal{W}) \cap Y] \subseteq (G\text{-}\mathcal{W})$, so $(G\text{-}\mathcal{W}) \cap Y$ is also countable, but $(G\text{-}\mathcal{W}) \cap Y = (G \cap Y)\text{-}(\mathcal{W} \cap Y)$, hence $(G \cap Y)$ - $(\mathcal{W} \cap Y)$ is countable, therefore $\mathcal{W} \cap Y$ is su. $\widehat{\omega}$-open set in the su. sub space Y.

Corollary (2.9): If $\mathcal{W}$ is a su. $\widehat{\omega}$-closed set in a su. space X, and Y is a su. sub space of X, then $\mathcal{W} \cap Y$ is a su. $\widehat{\omega}$-closed set in Y.

Theorem (2.10): A su. space (X,μ) is su. $\widehat{\omega}^{*}$-regular (resp. su. $\hat{\eta}^{*}$-regular) space, if any element x in X has a su. closed neighborhood which is a su. $\widehat{\omega}^{*}$-regular (resp. su. $\hat{\eta}^{*}$-regular) sub space of X.

Proof: Take $\mathcal{K}$ as a su. $\widehat{\omega}$-neighborhood (resp. su. $\hat{\eta}$-neighborhood) for an element x in X, by hypothesis there is a su. closed neighborhood $\mathcal{H}$ to x which is a su. $\widehat{\omega}^{*}$-regular (resp. $\hat{\eta}^{*}$-regular) sub space of X. Suppose $\mu_{\mathcal{H}}$ denote the relative su. topology on $\mathcal{H}$, so $\mathcal{K} \cap \mathcal{H}$ is a su. $\widehat{\omega}$-neighborhood (resp. su. $\hat{\eta}$-neighborhood) of x in $\mathcal{H}$, but $\mathcal{H}$ is su. $\widehat{\omega}^{*}$-regular (resp. su. $\hat{\eta}^{*}$-regular) space, hence from theorem (2.6) there is su. closed neighborhood $\mathcal{W}$ to x in $\mathcal{H}$ with $\mathcal{W} \subseteq \mathcal{K} \cap \mathcal{H} \subseteq \mathcal{K}$, also, since $\mathcal{W}$ is su. closed set in $\mathcal{H}$, so there is a su. closed set $\mathcal{V}$ in X such that $\mathcal{W} = \mathcal{V} \cap \mathcal{H}$. But $\mathcal{V}, \mathcal{H}$ are su. closed sets in X, so $\mathcal{W}$ is su. closed neighbourhood to x in X in which $\mathcal{W} \subseteq \mathcal{K}$, therefore (X,μ) is a su. $\widehat{\omega}^{*}$-regular (resp. su. $\hat{\eta}^{*}$-regular) space.

Proposition (2.11): 1- A su. space (X,μ) is su. regular space, if any element x in X has a su. closed neighborhood which is a su. regular sub space of the su. space X.

2- A su. space (X,μ) is su. $\widehat{\omega}$-regular (resp. su. $\hat{\eta}$-regular) space, if any element x in X has a su. closed neighborhood which is a su. $\widehat{\omega}$-regular (resp. su. $\hat{\eta}$-regular) sub space of the su. space X.

3- A su. space (X,μ) is su. $\widehat{\omega}^{**}$-regular (resp. su. $\hat{\eta}^{**}$-regular) space, if any element x in X has a su. closed neighborhood which is a su. $\widehat{\omega}^{**}$-regular (resp. su. $\hat{\eta}^{**}$-regular) sub space of the su. space X.

Remark (2.12): 1- Every su. open set is su. $\widehat{\omega}$-open (resp. su. $\hat{\eta}$-open) set.

2- Every su. closed set is su. $\widehat{\omega}$-closed (resp. su. $\hat{\eta}$-closed) set.

Theorem (2.13): The property of being su. $\widehat{\omega}^{*}$-regular (resp. su. $\hat{\eta}^{*}$- regular) space is a su. hereditary property.

Proof: Let Y be a su. sub space of a su. $\widehat{\omega}^{*}$-regular (resp. su. $\hat{\eta}^{*}$-regular) space X, take $\mathcal{M}$ as a su. $\widehat{\omega}$-closed (resp. su. $\hat{\eta}$-closed) set in Y and q as any element in Y such that $q \notin \mathcal{M}$, so there is a su. $\widehat{\omega}$-closed (resp. su. $\hat{\eta}$-closed) set $\mathcal{M}'$ in X in which $\mathcal{M} = \mathcal{M}' \cap Y$ (corollary (2.9)), it is clear that $q \notin \mathcal{M}'$, since if not, then $q \in \mathcal{M}' \cap Y = \mathcal{M}$ C!, so $q \in \mathcal{M}'$. But X is su. $\widehat{\omega}^{*}$-regular (resp. su. $\hat{\eta}^{*}$-regular), hence there are two su. open sets $\mathcal{W}, \mathcal{B}$ in X with $q \in \mathcal{W}$, $\mathcal{M}' \subseteq \mathcal{B}$, and $\mathcal{W} \cap \mathcal{B} = \emptyset$, thus $\mathcal{W} \cap Y, \mathcal{B} \cap Y$ are su. open sets in Y, in which $q \in \mathcal{W} \cap Y$ and $\mathcal{M}' \cap Y = \mathcal{M} \subseteq \mathcal{B} \cap Y$ and $(\mathcal{W} \cap Y) \cap (\mathcal{B} \cap Y) = (\mathcal{W} \cap \mathcal{B}) \cap Y = \emptyset \cap Y = \emptyset$, therefore Y is su. $\widehat{\omega}^{*}$-regular (resp. su. $\hat{\eta}^{*}$-regular) space.

Proposition (2.14): 1- The property of being su. $\widehat{\omega}$-regular (resp. su. $\hat{\eta}$-regular) space is a su. hereditary property.

2- The property of being su. regular space is a su. hereditary property [5].

3- The property of being su. $\widehat{\omega}^{**}$-regular (resp. su. $\hat{\eta}^{**}$-regular) space is a su. hereditary property.

Definition (2.15): The function $f : (X, \mu_X) \longrightarrow (Y, \mu_Y)$ is called -:

1- Su*. continuous function if $f^{-1}(\mathcal{W})$ is su. open (resp. su. closed) set in X for each su. open (resp. su. closed) set $\mathcal{W}$ in Y [3].

2- Su*. $\widehat{\omega}$-continuous function if $f^{-1}(\mathcal{W})$ is su. $\widehat{\omega}$-open (resp. su. $\widehat{\omega}$-closed) set in X for each su. open (resp. su. closed) set $\mathcal{W}$ in Y

3- Su*. $\hat{\eta}$-continuous function if $f^{-1}(\mathcal{W})$ is su. $\hat{\eta}$-open (resp. su. $\hat{\eta}$-closed) set in X for each su. open (resp. su. closed) set $\mathcal{W}$ in Y.

4- Strongly su*. $\widehat{\omega}$-continuous function if $f^{-1}(\mathcal{W})$ is su. open (resp. su. closed) set in X for each su. $\widehat{\omega}$-open (resp. su. $\widehat{\omega}$-closed) set $\mathcal{W}$ in Y.

5- Strongly su*. $\hat{\eta}$-continuous function if $f^{-1}(\mathcal{W})$ is su. open (resp. su. closed) set in X for each su. $\hat{\eta}$-open (resp. $\hat{\eta}$-closed) set $\mathcal{W}$ in Y.

6- Su*. $\widehat{\omega}$-irresolute function if $f^{-1}(\mathcal{W})$ is su. $\widehat{\omega}$-open (resp. su. $\widehat{\omega}$-closed) set in X for each su. $\widehat{\omega}$-open (resp. su. $\widehat{\omega}$-closed) set Y.

7- Su*. ŋ̂-irresolute function if f^{-1} $(\mathcal{W})$ is su. ŋ̂-open (resp. su. ŋ̂-closed) set in X for each su. ŋ̂-open (resp. su. ŋ̂-closed) set $\mathcal{W}$ in Y.

8- Su*. closed (resp. su*. open) function, if $f(\mathcal{B})$ is su. closed (resp. su. open) set in Y, for any su. closed (resp. su. open) set $\mathcal{B}$ in X [6].
9-Su*. $\widehat{\omega}$-closed (resp. su*. $\widehat{\omega}$-open) function, if $f(\mathcal{B})$ is su. $\widehat{\omega}$-closed (resp. su. $\widehat{\omega}$-open) set in Y, for any su. closed (resp. su. open) set $\mathcal{B}$ in X.
10- Totally su*. $\widehat{\omega}$-closed (resp. totally su*. $\widehat{\omega}$-open) function, if $f(\mathcal{B})$ is su. closed (resp. su. open) set in Y, for any su. $\widehat{\omega}$-closed (resp. su. $\widehat{\omega}$-open) set $\mathcal{B}$ in X.
11- Strongly su*. $\widehat{\omega}$-closed (resp. strongly su*. $\widehat{\omega}$-open) function, if $f(\mathcal{B})$ is su. $\widehat{\omega}$-closed (resp. su. $\widehat{\omega}$-open) set in Y, for any su. $\widehat{\omega}$-closed (resp. su. $\widehat{\omega}$-open) set $\mathcal{B}$ in X.
12- Su*. ŋ̂-closed (resp. su*. ŋ̂-open) function, if $f(\mathcal{B})$ is su. ŋ̂-closed (resp. su. ŋ̂-open) set in Y, for any su. closed (resp. su. open) set $\mathcal{B}$ in X.
13- Totally su*. ŋ̂-closed (resp. totally su*. ŋ̂-open) function, if $f(\mathcal{B})$ is su. closed (resp. su. open) set in Y, for any su. ŋ̂-closed (resp. su. ŋ̂-open) set $\mathcal{B}$ in X. .
14- Strongly su*. ŋ̂-closed (resp. strongly su*. ŋ̂-open) function, if $f(\mathcal{B})$ is su. ŋ̂-closed (resp. su. ŋ̂-open) set in Y, for any su. ŋ̂-closed (resp. su. ŋ̂-open) set $\mathcal{B}$ in X.

Example (2.16): 1- Let X=Y={1, 2, 3}, μ_X= {∅, X, {1}, {3}, {1, 3}, {2, 3}, {1, 2}} and μ_Y={∅, Y, {3}, {1, 2}}, so $f: X \longrightarrow Y$ defined as f(1)=2, f(2)=1, f(3)=3 is su*. continuous, su*. $\widehat{\omega}$-continuous, su*. ŋ̂-continuous, su*. $\widehat{\omega}$-irresolute, su*. ŋ̂-irresolute function, but not strongly su*. $\widehat{\omega}$-continuous and not strongly su*. ŋ̂-continuous function, since {1} is su. $\widehat{\omega}$-open and su. ŋ̂-open set in Y but f^{-1}({1}) = {2} is not su. open set in X.
2- $f: (\mathcal{R}, \mu_{ind}) \longrightarrow (\mathcal{R}, \mu_D)$ is strongly su*. $\widehat{\omega}$-continuous function, where f is a constant function.
3- $f: (X, \mu_D) \longrightarrow (Y, \mu_Y)$ is strongly su*. ŋ̂-continuous function.
4- A function $f: (X, \mu_X) \longrightarrow (Y, \mu_Y)$, where X= {1, 2}, μ_X= {∅, X,{1}}, Y= {1, 2, 3, 4} and μ_Y={∅, Y, {1}, {2}, {3}, {1, 2}, {2, 3}, {1, 3}, {1, 4}, {2, 4}, {1, 2, 3}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}, {3, 4}}such that f(1)=1 and f(2)=2, then f is su*. closed, su*. open, su*. $\widehat{\omega}$-closed, su*. $\widehat{\omega}$-open, totally su*. $\widehat{\omega}$-closed, totally su*. $\widehat{\omega}$-open, strongly su*. $\widehat{\omega}$-closed, strongly su*. $\widehat{\omega}$-open, su*. ŋ̂-closed, su*. ŋ̂-open, totally su*. ŋ̂-closed, totally su*. ŋ̂-open, strongly su*. ŋ̂-closed, strongly su*. ŋ̂-open function.
Definition (2.17): If $f: (X, \mu_X) \longrightarrow (Y, \mu_Y)$ is bijective and each of f and $f^{-1}$are-:

1- Su*. continuous, then f is called su*. homeomorphism function.
2- Su*. $\widehat{\omega}$-continuous, then f is called su*. $\widehat{\omega}$-homeomorphism function.
3- Su*. ŋ̂-continuous, then f is called su*. ŋ̂-homeomorphism function.
4- Su*. $\widehat{\omega}$-irresolute, then f is called su*. $\widehat{\omega}^*$-homeomorphism function.
5- Su*. ŋ̂-irresolute, then f is called su*. ŋ̂*-homeomorphism function.
6- Strongly su*. $\widehat{\omega}$-continuous, then f is called su*. $\widehat{\omega}^{**}$-homeomorphism function.
7- Strongly su*. ŋ̂-continuous, then f is called su*. ŋ̂**-homeomorphism function.

Definition (2.18): If $f: (X, \mu_X) \longrightarrow (Y, \mu_Y)$ is bijective and if f is-:
1- Su*. continuous and su*. open (or su*. closed) function, then it is su*. homeomorphism function.
2- Su*. $\widehat{\omega}$-continuous and su*. $\widehat{\omega}$-open (or su*. $\widehat{\omega}$-closed) function, then it is su*. $\widehat{\omega}$-homeomorphism function [7].
3- Su*. ŋ̂-continuous and su*. ŋ̂-open (or su*. ŋ̂-closed) function, then it is su*. ŋ̂-homeomorphism function.
4- Su*. $\widehat{\omega}$-irresolute and strongly su*. $\widehat{\omega}$-open (or strongly su*. $\widehat{\omega}$-closed) function, then it is su*. $\widehat{\omega}^*$-homeomorphism function.
5- Su*. ŋ̂-irresolute and strongly su*. ŋ̂-open (or strongly su*. ŋ̂-closed) function, then it is su*. ŋ̂*-homeomorphism function.
6- Strongly su*. $\widehat{\omega}$-continuous and totally su*. $\widehat{\omega}$-open (or totally su*. $\widehat{\omega}$-closed) function, then it is su*. $\widehat{\omega}^{**}$-homeomorphism function.

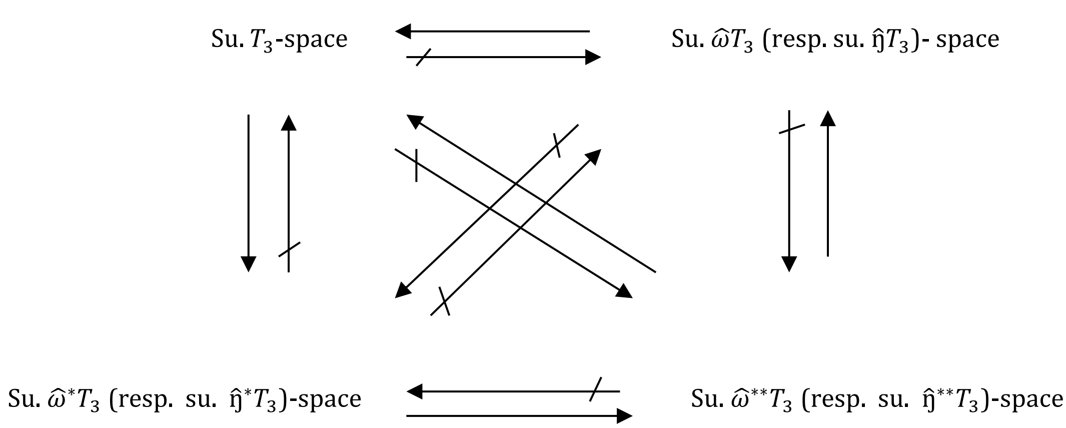
7- Strongly su*. ŋ̂-continuous and totally su*. ŋ̂-open (or totally su*. ŋ̂-closed) function, then it is su*. $\hat{ŋ}^{**}$-homeomorphism function.

Lemma (2.19) [2]: Consider $f: X \longrightarrow Y$ is a surjective function, then-:
1- The image of any finite set is finite.
2- The image of any countable set is countable.
Lemma (2.20): Suppose $f:(X,\mu_X) \longrightarrow (Y,\mu_X)$ is su*. homeomorphism function, then the image of any su. $\widehat{\omega}$-open (resp. su. ŋ̂-open) set in X is su. $\widehat{\omega}$-open (resp. su. ŋ̂-open) set in Y.
Proof: Let $\mathcal{H}$ be a su. $\widehat{\omega}$-open (resp. su. ŋ̂-open) set in X, and $y \in f(\mathcal{H})$, so there is $x \in X$ such that $f(x) = y$ (because f is onto), since $y \in f(\mathcal{H})$, then $x = f^{-1}(y) \in f^{-1}(f(\mathcal{H})) = \mathcal{H}$ (f is one to one), hence $x \in \mathcal{H}$ which is su. $\widehat{\omega}$-open (resp. su. ŋ̂-open) set, thus there is a su. open set $\mathcal{W}$ in X in which $x \in \mathcal{W}$ and $\mathcal{W}$-$\mathcal{H}$ is countable (resp. finite), so $f(\mathcal{W}$-$\mathcal{H})$ is countable set in Y (by lemma (2.19)), but $f(\mathcal{W}$-$\mathcal{H}) = f(\mathcal{W})$-$f(\mathcal{H})$, and since $f(\mathcal{W})$ is su. open set in Y (because f is su*. open function), and $x \in \mathcal{W}$, then $f(x) = y \in f(\mathcal{W})$, therefore $f(\mathcal{H})$ is su. $\widehat{\omega}$-open (resp. su. ŋ̂-open) set in Y.
Theorem (2.21): The property of a su. space being su. $\widehat{\omega}$-regular (resp. su. ŋ̂-regular) space is a su. topological property.
Proof: Suppose X is a su. $\widehat{\omega}$-regular (resp. su. ŋ̂-regular) space, and f is a su*. homeomorphism function from X into a su. space Y, let $\mathcal{M}$ be a su. closed set in Y and q be any point in Y in which $q \notin \mathcal{M}$, so there is a point $p \in X$ such that $f(p) = q$. We have $f^{-1}(\mathcal{M})$ is su. closed set in X (because f is su*. continuous function) and $f^{-1}(q) = p \notin f^{-1}(\mathcal{M})$, but X is su. $\widehat{\omega}$-regular (resp. su. ŋ̂-regular) space, so there are su. $\widehat{\omega}$-open (resp. su. ŋ̂-open) sets $\mathcal{W}, \mathcal{B}$ in X where $p \in \mathcal{W}, \mathcal{M} \subseteq \mathcal{B}$, and $\mathcal{W} \cap \mathcal{B} = \emptyset$. So $f(p) = q \in f(\mathcal{W})$ and $f(\mathcal{M}) \subseteq f(\mathcal{B})$ where $f(\mathcal{W}), f(\mathcal{B})$ are su. $\widehat{\omega}$-open (resp. ŋ̂-open) sets in Y (by lemma (2.19)), also $f(\mathcal{W}) \cap f(\mathcal{B}) = f(\mathcal{W} \cap \mathcal{B}) = f(\emptyset) = \emptyset$, hence Y is su. $\widehat{\omega}$-regular (resp. su. ŋ̂-regular), therefore the property of su. space being su. $\widehat{\omega}$-regular (resp. su. ŋ̂-regular) space is a su. topological property.
Theorem (2.22): Suppose f is su*. $\widehat{\omega}^{**}$-homeomorphism (resp. su*. $\hat{ŋ}^{**}$-homeomorphism) function from a su. $\widehat{\omega}^{*}$-regular (resp. su. $\hat{ŋ}^{*}$-regular) space (X,μ_X) into a su. space (Y,μ_Y). Hence Y is su. $\widehat{\omega}^{*}$-regular (resp. su. $\hat{ŋ}^{*}$-regular) space too.
Proof: Let $\mathcal{M}$ be a su. $\widehat{\omega}$-closed (resp. su. ŋ̂-closed) set in the space Y and let q be any element in Y with $q \notin \mathcal{M}$, so there is an element $p \in X$ in which $f(p) = q$ (because f is bijective). $f^{-1}(\mathcal{M})$ is su. closed set in X (because f is strongly su*. $\widehat{\omega}$-continuous (resp. strongly su*. ŋ̂-continuous) function), and then it is su. $\widehat{\omega}$-closed (su. ŋ̂-closed) set (by remark (2.4)). Since $q \notin \mathcal{M}$, accordingly $f^{-1}(q) = p \notin f^{-1}(\mathcal{M})$. But X is su. $\widehat{\omega}^{*}$-regular (resp. su. $\hat{ŋ}^{*}$-regular) space, so there are su. open sets $\mathcal{W}, \mathcal{B}$ in X containing $p, f^{-1}(\mathcal{M})$ respectively and $\mathcal{W} \cap \mathcal{B} = \emptyset$. then $f(f^{-1}(\mathcal{M})) \subseteq f(\mathcal{B})$ and since f is onto, thus $\mathcal{M} \subseteq f(\mathcal{B})$, also $f(p) = q \in f(\mathcal{W})$, where $f(\mathcal{B}), f(\mathcal{W})$ are su. open sets in Y (because f is totally su*. $\widehat{\omega}$-open (resp. totally su*. ŋ̂-open) function and $\mathcal{B}, \mathcal{W}$ are su. $\widehat{\omega}$-open (resp. su. ŋ̂-open) sets in X by remark (2.4)), and $f(\mathcal{W}) \cap f(\mathcal{B}) = f(\mathcal{W} \cap \mathcal{B}) = f(\emptyset) = \emptyset$, therefore Y is su. $\widehat{\omega}^{*}$-regular (resp. su. $\hat{ŋ}^{*}$-regular) space.
Proposition (2.23): 1- Suppose f is su*. homeomorphism function from a su. regular space (X,μ_X) into a su. space (Y,μ_Y). Hence Y is su. regular space too.
2- The property of a su. space being su. $\widehat{\omega}^{*}$-regular (resp. su. $\hat{ŋ}^{*}$-regular) space is a topological property.
3- The property of a su. space being su. $\widehat{\omega}^{**}$-regular (resp. su. $\hat{ŋ}^{**}$-regular) space is a topological property.
4- Suppose f is su*. $\widehat{\omega}^{**}$-homeomorphism function from a su. $\widehat{\omega}$-regular space (X,μ_X) into a su. space (Y,μ_Y). Hence Y is su. $\widehat{\omega}$-regular space too.
5- Suppose f is su*. $\hat{ŋ}^{**}$-homeomorphism function from a su. ŋ̂-regular space (X,μ_X) into a su. space (Y,μ_Y). Hence Y is su. ŋ̂-regular space too.
6- Suppose f is su*. $\widehat{\omega}^{*}$-homeomorphism function from a su*. $\widehat{\omega}^{**}$-regular space (X,μ_X) into a su. space (Y,μ_Y). Hence Y is su. $\widehat{\omega}^{**}$-regular space too.
7- Suppose f is su*. $\hat{ŋ}^{*}$-homeomorphism function from a su. $\hat{ŋ}^{**}$-regular space (X,μ_X) into a su. space (Y,μ_Y). Hence Y is su. $\hat{ŋ}^{**}$-regular space too.

3- Types of Su. T_3-spaces.

In this part we will introduce su. $\widehat{\omega}T_3$-space, su. $\widehat{\omega}^*T_3$-space, su. $\widehat{\omega}^{**}T_3$-space, su. ŋ̂T_3-space, su. ŋ̂$^*T_3$-space, and su. ŋ̂$^{**}T_3$-space. And we will illustrate the relationship between them.

Definition (3.1): In case a su. space (X, μ) is-:
1- Su. regular space and su. T_1-space, then it is su. T_3-space [5].
2- Su. $\widehat{\omega}$-regular space and su. T_1-space, then it is su. $\widehat{\omega}T_3$-space.
3- Su. ŋ̂-regular space and su. T_1 -space, then it is su. ŋ̂T_3 -space.
4- Su. $\widehat{\omega}$-regular space and su. T_1-space, then it is su. $\widehat{\omega}^*T_3$-space.
5- Su. ŋ̂*-regular space and su. T_1 -space, then it is su. ŋ̂$^*T_3$ -space.
6- Su. $\widehat{\omega}^{**}$-regular space and su. T_1-space, then it is su. $\widehat{\omega}^{**}T_3$-space.
7- Su. ŋ̂**-regular space and su. T_1 -space, then it is su. ŋ̂$^{**}T_3$ -space.
Example (3.2): 1- $(\mathcal{R}, \mu_D)$ is su. $\widehat{\omega}^*T_3$ and ŋ̂$^*T_3$-space.
2- X= {1, 2, 3}, μ= {∅, X, {1}, {2, 3}, {2}, {1, 2}, {1, 3}} is su. T_3-space, su. $\widehat{\omega}T_3$-space, su. ŋ̂T_3-space, su. $\widehat{\omega}^{**}T_3$-space, su. ŋ̂$^{**}T_3$-space, su. $\widehat{\omega}^{**}$-regular and su. ŋ̂**-regular space.
Remark (3.3): 1- If X is a su. T_3-space, then it is a su. regular.
2- If X is a su. regular space, then it is not necessary su. T_2-space.
3- If X is a su. T_2-space need not su. regular.
4- If X is a su. T_2-space, so it is su. T_1-space.
5- Every singleton subset $\{x\}$ of su. T_1-space is su. closed set [3].
The following scheme is helpful.



Example (3.4): 1- X = {a, b, c, d, e}, μ= {∅, X, {a}, {e}, {a, b}, {a, e}, {a, b, e}, {c, d, e}, {b, c, d, e}, {a, c, d, e}, {a, b, c, d}} is su. regular, but neither su. T_3-regular nor su. T_2-space.
2- X= {a, b, c, d}, μ= {∅, X, {a}, {c}, {a, c}, {a, b}, {a, d}, {c, d}, {b, d}, {a, b, c}, {a, b, d}, {b, c, d}, {a, c, d}} is su. T_2-space and su. T_1-space but neither su. regular nor su. T_3-space.
3- X= {a, b, c}, μ= {∅, X, {a}, {a, b}, {a, c}, {b, c}} is su. T_1-space, su. $\widehat{\omega}^{**}T_3$-space, su. ŋ̂$^{**}T_3$-space, su. $\widehat{\omega}T_3$-space, su. ŋ̂T_3-space, su. $\widehat{\omega}$-regular and su. ŋ̂-regular space but not su. regular, not su. T_3-regular, not su. $\widehat{\omega}^*T_3$-space, not su. ŋ̂$^*T_3$-space, and not su. T_2-space.
4- X= {a, b, c}, μ= {∅, X, {a}, {b}, {a, b}, {a, c}, {b, c}} is su. T_3-space, su. ŋ̂$^{**}T_3$-space, $\widehat{\omega}T_2$-space and ŋ̂T_2-space but neither su. $\widehat{\omega}^*T_3$-space nor su. ŋ̂$^*T_3$-space.
Proposition (3.5): Each su. topology finer than su. T_2 is also su. T_2.

Proof: Let $a \neq b$ be two elements in a su. space X and μ, μ^* are two su. topologies defined on X, where μ^* is finer than μ, and μ is a su. T_2-topology on X, so there are disjoint sets $\mathcal{W}_1, \mathcal{W}_2 \in \mu$ containing a, b respectively, since $\mu \subseteq \mu^*$, hence $\mathcal{W}_1, \mathcal{W}_2 \in \mu^*$ too, then μ^* is a su. T_2-topology on X.

Proposition (3.6): 1- Each su. topology finer than su. $\widehat{\omega}T_2$ is also su. $\widehat{\omega}T_2$.

2- Each su. topology finer than su. $\hat{\eta}T_2$ is also su. $\hat{\eta}T_2$.

Proof: As in proposition (2.25).

Proposition (3.7): Every su. T_3-space is su. T_2-space.

Proof: Let (X, μ) be a su. T_3-space and let x, y be any distinct points in X. We have X is su. T_1-space (from definition of su. T_3-space), so $\{x\}$ is su. closed set (by remark (3.3)) and $y \notin \{x\}$, since X is su. regular space so there are su. open sets $\mathcal{W}, \mathcal{B}$ such that $\{x\} \subseteq \mathcal{W}$, $y \in \mathcal{B}$ and $\mathcal{W} \cap \mathcal{B} = \emptyset$, thus $x \in \mathcal{W}$, then x, y belong respectively to disjoint su. open sets $\mathcal{W}$ and $\mathcal{B}$, therefore X is su. T_2-space.

Proposition (3.8): 1- Every su. $\widehat{\omega}T_3$-space is su. ωT_2-space.

2- Every su. $\hat{\eta}T_3$-space is su. $\hat{\eta}T_2$-space.

3- Every su. $\widehat{\omega}^*T_3$-space is su. $\widehat{\omega}T_2$-space.

4- Every su. $\hat{\eta}^*T_3$-space is su. $\hat{\eta}T_2$-space.

5- Every su. $\widehat{\omega}^{**}T_3$-space is su. $\widehat{\omega}T_2$-space.

6- Every su. $\hat{\eta}^{**}T_3$-space is su. $\hat{\eta}T_2$-space.

Proof: As in proposition (3.7).

Example (3.9): 1- X= {a, b, c, d}, μ= {$\emptyset$, X, {a}, {c}, {a, c}, {a, b}, {a, d}, {c, d}, {b, d}, {a, b, c}, {a, b, d}, {b, c, d}, {a, c, d}}, so (X, μ) is su. T_2-space but not su. T_3-space, not su. $\widehat{\omega}^*T_3$-space and not su. $\hat{\eta}^*T_3$-space.

Lemma (3.10): If $\{U_i\}_{i=1}^{n}$ is a finite collection of su. $\widehat{\omega}$-open (resp. su. $\hat{\eta}$-open) sets, so their union is also su. $\widehat{\omega}$-open (su. $\hat{\eta}$-open) set.

Proof: Let $x \in \bigcup_{i=1}^{n} U_i \Rightarrow x \in U_{\alpha_i}$ for some $\alpha_i \in I \Rightarrow$ there is $G \in \mu$, with $x \in G$ and $G\text{-}U_{\alpha_i}$ is countable (resp. finite), but $G\text{-}\bigcup_{i=1}^{n} U_i \subseteq G\text{-}U_{\alpha_i} \Rightarrow G\text{-}\bigcup_{i=1}^{n} U_i$ is countable (resp. finite) $\Rightarrow \bigcup_{i=1}^{n} U_i$ is su. $\widehat{\omega}$-open (resp. su. $\hat{\eta}$-open) set .

Theorem (3.11) [3]: The property of a space being su. T_1-space is a su. hereditary and su. topologically property.

Theorem (3.12): 1- The property of a space being su. $\widehat{\omega}T_3$-space (resp. $\hat{\eta}T_3$-space) is a su. hereditary and su. topologically property.

2- The property of a space being su. $\widehat{\omega}^*T_3$-space (resp. $\hat{\eta}^*T_3$-space) is a su. hereditary and su. topologically property.

3- The property of a space being su. $\widehat{\omega}^{**}T_3$-space (resp. $\hat{\eta}^{**}T_3$-space) is a su. hereditary and su. topologically property.

Proof: It is clear.

Theorem (3.14): A su. space X is a su. $\widehat{\omega}$-regular (resp. su. $\hat{\eta}$-regular) space, iff for any $x \in X$ and for any su. open set $\mathcal{U}$ in X containing x, there is a su. $\widehat{\omega}$-open (resp. su. $\hat{\eta}$-open) set $\mathcal{V}$ in X such that $x \in \mathcal{V} \subseteq cl^{\mu}_{\widehat{\omega}}(\mathcal{V}) \subseteq \mathcal{U}$.

Proof: Suppose X is su. $\widehat{\omega}$-regular (resp. su. $\hat{\eta}$-regular) space, and let $\mathcal{U}$ be a su. open set in X such that $x \in \mathcal{U}$, then $\mathcal{U}^c$ is su. closed set in X does not containing x. But X is su. $\widehat{\omega}$-regular (resp. su. $\hat{\eta}$-regular) space, so there are two su. $\widehat{\omega}$-open (resp. su. $\hat{\eta}$-open) sets $\mathcal{V}, \mathcal{W}$ such that $x \in \mathcal{V}, \mathcal{U}^c \subseteq \mathcal{W}$ and $\mathcal{V} \cap \mathcal{W} = \emptyset$, so $\mathcal{V} \subseteq \mathcal{W}^c$, thus $cl^{\mu}_{\widehat{\omega}}(\mathcal{V}) \subseteq cl^{\mu}_{\widehat{\omega}}(\mathcal{W}^c) = \mathcal{W}^c$ (resp. $cl^{\mu}_{\hat{\eta}}(\mathcal{V}) \subseteq cl^{\mu}_{\hat{\eta}}(\mathcal{W}^c) = \mathcal{W}^c$...... (1) (since $\mathcal{W}^c$ is su. $\widehat{\omega}$-closed (resp. su. $\hat{\eta}$-closed)) set, and since $\mathcal{U}^c \subseteq \mathcal{W}$, then $\mathcal{W}^c \subseteq \mathcal{U}$... (2), from (1) and (2) we get $x \in \mathcal{V} \subseteq cl^{\mu}_{\widehat{\omega}}(\mathcal{V}) \subseteq \mathcal{W}^c \subseteq \mathcal{U}$ (resp. $x \in \mathcal{V} \subseteq cl^{\mu}_{\hat{\eta}}(\mathcal{V}) \subseteq \mathcal{W}^c \subseteq \mathcal{U}$), which means $x \in \mathcal{V} \subseteq cl^{\mu}_{\widehat{\omega}}(\mathcal{V}) \subseteq \mathcal{U}$ (resp. $x \in \mathcal{V} \subseteq cl^{\mu}_{\hat{\eta}}(\mathcal{V}) \subseteq \mathcal{U}$). Conversely, let $\mathcal{M}$ be a su. closed set in X and x be any point in X such that $x \in \mathcal{M}$, so $\mathcal{M}^c$ is su. open set in X does not containing x, put $\mathcal{M}^c = \mathcal{U}$, then there is a su. $\widehat{\omega}$-open (resp. su. $\hat{\eta}$-open) set $\mathcal{V}$ in X in which $x \in \mathcal{V} \subseteq cl^{\mu}_{\widehat{\omega}}(\mathcal{V}) \subseteq \mathcal{U}$ (resp. $x \in \mathcal{V} \subseteq cl^{\mu}_{\hat{\eta}}(\mathcal{V}) \subseteq \mathcal{U}$), thus $cl^{\mu}_{\widehat{\omega}}(\mathcal{V}) \subseteq \mathcal{U}$, hence $\mathcal{U}^c = \mathcal{M} \subseteq \left(cl^{\mu}_{\widehat{\omega}}(\mathcal{V})\right)^c \subseteq X$ (resp. $\mathcal{U}^c = \mathcal{M} \subseteq \left(cl^{\mu}_{\hat{\eta}}(\mathcal{V})\right)^c \subseteq X$ (since $\mathcal{V}$ is su. $\widehat{\omega}$-open (resp. su. $\hat{\eta}$-open), so $cl^{\mu}_{\widehat{\omega}}(\mathcal{V})$ (resp. $cl^{\mu}_{\hat{\eta}}(\mathcal{V})$) is su. $\widehat{\omega}$-closed (resp. su. $\hat{\eta}$-closed) set), since $V \subseteq cl^{\mu}_{\widehat{\omega}}(\mathcal{V})$ (resp.$\mathcal{V} \subseteq cl^{\mu}_{\hat{\eta}}(\mathcal{V})$), which implies $\mathcal{V} \cap (cl^{\mu}_{\widehat{\omega}}(\mathcal{V}))^c = \emptyset$ (resp. $\mathcal{V} \cap \left(cl^{\mu}_{\hat{\eta}}(\mathcal{V})\right)^c = \emptyset$). Therefore, there are two su. $\widehat{\omega}$-open (resp. su. $\hat{\eta}$-open) sets $\mathcal{V}, \left(cl^{\mu}_{\widehat{\omega}}(\mathcal{V})\right)^c$ [resp. $\mathcal{V}, (cl^{\mu}_{\hat{\eta}}(\mathcal{V}))^c$] in X such that $x \in \mathcal{V}, \mathcal{M} \subseteq \left(cl^{\mu}_{\widehat{\omega}}(\mathcal{V})\right)^c$ (resp. $\mathcal{M} \subseteq \left(cl^{\mu}_{\hat{\eta}}(\mathcal{V})\right)^c$, and $\mathcal{V} \cap (cl^{\mu}_{\widehat{\omega}}(\mathcal{V}))^c = \emptyset$ (resp. $\mathcal{V} \cap cl^{\mu}_{\hat{\eta}}(\mathcal{V}))^c = \emptyset$, then X is su. $\widehat{\omega}$-regular (resp. su. $\hat{\eta}$-regular) space.

Proposition (3.14): A su. space X is -:

1- A su. regular space, iff for any $x \in X$ and for any su. open set $\mathcal{U}$ in X containing x, there is a su. open set $\mathcal{V}$ in X such that $x \in \mathcal{V} \subseteq cl(\mathcal{V}) \subseteq \mathcal{U}$.

2- A su. $\widehat{\omega}^*$-regular (resp. su. $\hat{\eta}^*$-regular) space, iff for any $x \in X$ and for any su. $\widehat{\omega}$-open (resp. su. $\hat{\eta}$-open) set $\mathcal{U}$ in X containing x, there is a su. open set $\mathcal{V}$ in X such that $x \in \mathcal{V} \subseteq cl\,(\mathcal{V}) \subseteq \mathcal{U}$.

3- A su. $\widehat{\omega}^{**}$-regular (resp. su. $\hat{\eta}^{**}$-regular) space, iff for any $x \in X$ and for any su. $\widehat{\omega}$-open (resp. su. $\hat{\eta}$-open) set $\mathcal{U}$ in X containing x, there is a su. $\widehat{\omega}$-open (resp. su. $\hat{\eta}$-open) set $\mathcal{V}$ in X such that $x \in \mathcal{V} \subseteq cl^{\mu}_{\widehat{\omega}}(\mathcal{V}) \subseteq \mathcal{U}$ (resp. $x \in \mathcal{V} \subseteq cl^{\mu}_{\hat{\eta}}\,(\mathcal{V}) \subseteq \mathcal{U}$).

Definition (3.15): A function $f: X \longrightarrow Y$ is called:-

1- Perfectly su*. continuous if the inverse image of any su. open (resp. su. closed) set in Y is a su. clopen set in X [4].

2- Perfectly su*. $\widehat{\omega}$-continuous if the inverse image of any su. $\widehat{\omega}$-open (resp. su. $\widehat{\omega}$-closed) set in Y is a su. clopen set in X.

3- Totally su*. $\widehat{\omega}$-continuous if the inverse image of any su. open (resp. su. closed) set in Y is a su. $\widehat{\omega}$-clopen set in X.

4- Totally su*. $\hat{\eta}$-continuous if the inverse image of any su. open (resp. su. closed) set in Y is a su. $\hat{\eta}$-clopen set in X.

5- Perfectly su*. $\hat{\eta}$-continuous if the inverse image of any su. $\hat{\eta}$-open (resp. su. $\hat{\eta}$-closed) set in Y is a su. clopen set in X.

6- Perfectly su*. $\widehat{\omega}$-irresolute if the inverse image of any su. $\widehat{\omega}$-open (resp. su. $\widehat{\omega}$-closed) set in Y is a su. $\widehat{\omega}$-clopen set in X.

7- Perfectly su*. $\hat{\eta}$-irresolute if the inverse image of any su. $\hat{\eta}$-open (resp. su. $\hat{\eta}$-closed) set in Y is a su. $\hat{\eta}$-clopen set X.

Example (3.16): Let X=Y={1, 2, 3}, μ_X= {∅, X, {1}, {3}, {1, 3}, {2, 3}, {1, 2}}and μ_Y={∅,Y, {3}, {1, 2}}, so $f: X \longrightarrow Y$ defined as f(1)=2, f(2)=1, f(3)=3 is perfectly su*. continuous, perfectly su*. $\hat{\eta}$-irresolute, perfectly su*. $\widehat{\omega}$-irresolute, totally su*. $\widehat{\omega}$-continuous, and totally su*. $\hat{\eta}$-continuous, but not perfectly su*. $\hat{\eta}$-continuous, and not perfectly su*. $\widehat{\omega}$-continuous function.

Theorem (3.17): Consider $f: X \longrightarrow Y$ is surjective, su*. $\widehat{\omega}$-open (resp. su*. $\hat{\eta}$-open) and perfectly su*. continuous function (or su*. continuous) function, if X is su. regular space then Y is su. $\widehat{\omega}$-regular (resp. su. $\hat{\eta}$-regular) space.

Proof: let $y \in Y$ and $\mathcal{M}$ is su. closed set in Y, there exists $x \in X$ such that $f(x) = y$ (since f is surjective) and since f is perfectly su*. continuous, then $f^{-1}\,(\mathcal{M})$ is su. clopen set in X, and then it is su. closed set in X. Since $y \notin \mathcal{M}$, so $f^{-1}\,(y) = x \notin f^{-1}\,(\mathcal{M})$, but X is su. regular space, hence there are su. open sets $U, \mathcal{V}$ in which $x \in U, f^{-1}(\mathcal{M}) \subseteq \mathcal{V}$and $U \cap \mathcal{V}$=∅, thus $f(x) = y \in f(U), f\,(f^{-1}\,(\mathcal{M})) = \mathcal{M} \subseteq f(\mathcal{V})$ and $f(U) \cap f(\mathcal{V}) = f(U \cap \mathcal{V}) = f(\emptyset)$ =∅, but f is su*. $\widehat{\omega}$-open (resp. su*. $\hat{\eta}$-open) function, hence each of $f(U), f(\mathcal{V})$ is su. $\widehat{\omega}$-open (resp. su. $\hat{\eta}$-open) set in Y, therefore Y is su. $\widehat{\omega}$-regular (resp. su. $\hat{\eta}$-regular) space.

Proposition (3.18): 1- Consider $f: X \longrightarrow Y$ is surjective, su*. open and perfectly su*. continuous function, if X is su. regular space then Y is su. regular space.

2- Consider $f: X \longrightarrow Y$ is surjective, su*. open and perfectly su*. $\widehat{\omega}$-continuous (resp. perfectly su*. $\hat{\eta}$-continuous) function, if X is su. regular space then Y is su. $\widehat{\omega}^*$-regular (resp. su. $\hat{\eta}^*$-regular) space.

3- Consider $f: X \longrightarrow Y$ is surjective, su*. open and perfectly su*. $\widehat{\omega}$-continuous (resp. perfectly su*. $\hat{\eta}$-continuous) function, if X is su. regular space then Y is su. $\widehat{\omega}^*$-regular (resp. su. $\hat{\eta}^*$-regular) space.

4- Consider $f: X \longrightarrow Y$ is surjective, su*. $\widehat{\omega}$-open (resp. su*. $\hat{\eta}$-open), and perfectly su*. $\widehat{\omega}$-continuous (resp. perfectly su*. $\hat{\eta}$-continuous) function, if X is su. regular space then Y is su. $\widehat{\omega}^{**}$-regular (resp. su. $\hat{\eta}^{**}$-regular) space.

5- Consider $f: X \longrightarrow Y$ is injective, su*. closed and totally su*. $\widehat{\omega}$-continuous (resp. totally su*. $\hat{\eta}$-continuous) function, if Y is su. regular space then X is su. $\widehat{\omega}$-regular (resp. su. $\hat{\eta}$-regular) space.

6- Consider $f: X \longrightarrow Y$ is injective, su*. closed and perfectly su*. continuous function, if Y is su. regular space then Y is su. regular space.

7- Consider $f: X \longrightarrow Y$ is injective, su*. $\widehat{\omega}$-closed (resp. su*. $\hat{\eta}$-closed) and perfectly su*. continuous function, if Y is su. regular space then X is su. $\widehat{\omega}^*$-regular (resp. su. $\hat{\eta}^*$-regular) space.

8- Consider $f: X \longrightarrow Y$ is injective, su*. $\widehat{\omega}$-closed (resp. su*. ŋ̂-closed) and perfectly su*. $\widehat{\omega}$-continuous (resp. perfectly su*. ŋ̂-continuous) function, if Y is su. regular space then X is su. $\widehat{\omega}^{**}$-regular (resp. su. ŋ̂**-regular) space.□

References

[1] B.K.Mahmoud, On supra – Separation Axioms for Supra Topological Spaces. Tikrit Journal of Pure Science, 22(2) (2017),117-120.

[2] Sharma, J. N., Topology. Krishna Prakashsan Mandir, 1990.

[3] Mashhour, A. S., Allam, A. A., Mahmoud, F. S., and Khedr, F. H. On Supra Topological Spaces. Indian Journal of Pure and Applied Mathematics, 14(4) 1983, 502-510.

[4] P. M. Trinita, and I. Arockiarani, "Some Stronger Forms of $g^{\mu b}$–continuous Functions, Vol. 1, IOSR Journal of Engineering (IOSRJEN), (2011), 111-117.

[5] Tareq Al-Shami. Some Results Related to Supra Topological Spaces. Journal of Advanced Studies in Topology. (2016) Nov. 7(4), 283-294.

[6] Abo-Elhamayel, Mohamed, and Tareq M. Al-Shami. "Supra homeomorphism in supra topological ordered spaces." Facta Universitatis, Series: Mathematics and Informatics 31.5 (2016): 1091-1106.

[7] Taha H. Jassim, Rana B. Yasseen and Nabeel E. Arif. On Some Separation Axioms of Supra Topological Spaces. Tikrit Journal of Pure Science. 2008. 13(2), 59-62.