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Transportation polytopes and its relation to graph theory

Shatha A.Salman[a] , Amal A.Ibrahim [b]

[a]Applied Science,Department Mathematics and computer applications,University of Technology,Iraq. Email:drshatha.alnajar@gmail.com.

[b]Applied Science,Department Mathematics and computer applications,University of Technology,Iraq.Email:amalibrahim658 @gmail.com.

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ABSTRACT

Transportation polytopes of non- negative $m \times n$ matrices compose of two vectors: a and b which row sums are equal to fixed constant and column sums are equal to different constant. The transportation polytopes are denoted by T (a, b) and these two vectors are called margins. An open problem that the 2-way transportation polytopes are Hamiltonian is proved in this paper with application of optimization.

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1. Introduction

Transportation polytopes are familiar object in operations research and mathematical programming [1, 6, and 16] and statisticians [4, 5] during the 1970's and 1980's the research of the 2-way transportation polytopes was very active. The transportation polytopes and transporting problem has been studied by Yemelichev, Kovalev, and Kratsov [21] and Vlach's survey [19].

Classical transportation problems appear from the transporting goods such as the north -west corner rule which is a method that depend on finding the first reasonable solution of the transportation problem. One possible way to solve the transportation problem is that demand should be equal to the supply. In fact the main reason that led Kantorovich [11], Hitchcock [10], and T. C. Koopmans [17] to look at these problems is to improve the cost of transporting goods. Its considered the first linear programming problem that have been discussed then not so long Birkhoff [3], von Neumann [20], and Motzkin [15] were the main contributors to this subject The prosperity of Aggregate algorithm such as Hungarian method [12,18] depends on the substantial Aggregate structure of the convex polyhedral that illustrate the possible solutions.

Corresponding author Shatha Assaad Salman

Email addresses: drshatha.alnajar@gmail.com

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Transportation polytopes have a relationship with complete bipartite graph. let $K_{m,n}$ be complete bipartite graph with two sets of vertices V_1 and V_2. It can be considered that V_1 is the supply and V_2 is the demand where $e_{i,j}$ is an edge connecting between a vertex in V_1 to vertex in V_2 in $K_{m,n}$

2. Basic concepts

Definition (1), [9]

A graph is a set of point or vertices and set of lines or edges G = (V, E) where V (G) is the set of vertices and E (G) is the set of edges. Each vertex in the graph is connected to each other through an edge as shown in fig 1.

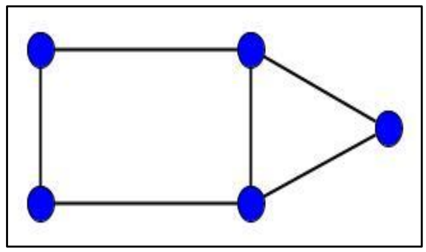


Figure 1. A graph G

Definition (2), [9]

The graph $K_{m,n}$ is the complete bipartite graph consisting of two sets $V_1 = \{v_1, . . . ,v_m\}$ and $V_2 = \{u_1, . . . ,u_n\}$ and having an edge connecting every vertex in V_1 to every vertex in V_2 as shown in fig 2.

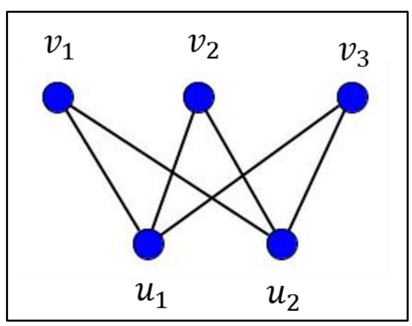


Figure 2. Complete bipartite graph

Definition (3) [9]

A closed walk of length at least 3 is called a cycle such that no repeated edges exist and the only repeated vertex is v_1, such that $v_1 = v_m$. Hamiltonian cycle is a cycle containing every vertex in the graph. A graph G = (V, E) is said to be Hamiltonian if it contains a Hamiltonian cycle as shown in fig 3.

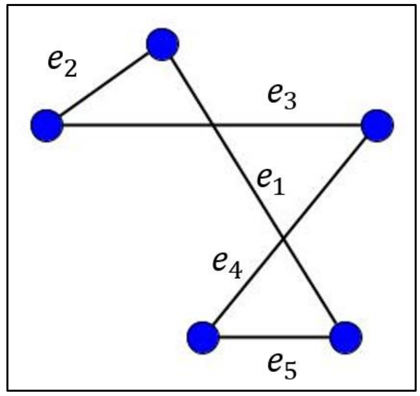


Figure 3. Hamiltonian graph

Definition (4), [9]

A graph G(V, E) is graph consisting of set of vertices V(G), which are connected to each other by a set of edges E(G), all edges are directed from one vertex to another such graph is called directed graph.

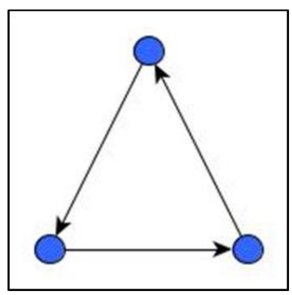


Figure 4. Directed graph

Definition (5), [9]

A graph with no cycles is called acyclic as shown in fig 5.

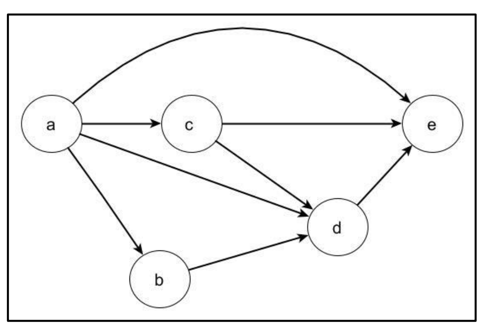


Figure 5. Cyclic graph

Definition (6), [9]

Let G = (V, E) be acyclic graph. If G has more than one component, then G is called a forest. other wise is called a tree as shown in fig 6.

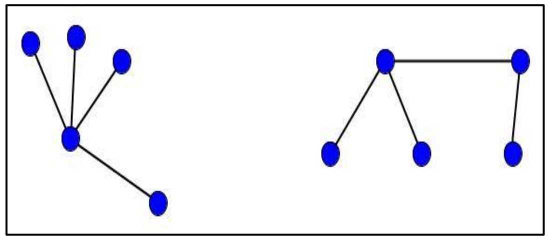


Figure 6. Forest graph

Definition (7), [9]

Let G = (V, E) be a graph. If H = (V`, E`) is acyclic subgraph of G such that V = V` then H is called a spanning forest of G. If H has exactly one component, then H is called a spanning tree.

Definition (8), [13]

Given two vectors of positive entries a = $(a_1, \ldots, a_m)$ and b = $(b_1, \ldots, b_n)$ whose coordinates sum to a fixed number, the transportation polytope denoted by T (a, b), is the set of all m × n non-negative matrices in which row i has sum a_i and column j has sum b_j. We call T (a, b) transportation polytope of order m × n.

Example 1: Let a = $(5, 5, 1)^T$ and b = $(2, 7, 2)^T$ the transportation polytopes has the vertices.

$$m_1 = \begin{pmatrix} 2 & 2 & 1 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{matrix} \longrightarrow & 5 \\ \longrightarrow & 5 \\ \longrightarrow & 1 \end{matrix} \quad m_2 = \begin{pmatrix} 2 & 3 & 0 \\ 0 & 4 & 1 \\ 0 & 0 & 1 \end{pmatrix}, \quad m_3 = \begin{pmatrix} 1 & 3 & 1 \\ 0 & 4 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad m_4 = \begin{pmatrix} 1 & 4 & 0 \\ 1 & 3 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{matrix} \downarrow & \downarrow & \downarrow \\ 2 & 7 & 1 \end{matrix}$$

$$m_5 = \begin{pmatrix} 0 & 3 & 2 \\ 1 & 4 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad m_6 = \begin{pmatrix} 1 & 3 & 1 \\ 1 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad m_7 = \begin{pmatrix} 0 & 4 & 1 \\ 1 & 3 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

Definition (9), [13]

The auxiliary graph is subgraph of $K_{m,n}$ with edge set $\{\boldsymbol{e_{i,j}} \mid m(i,j) > 0\}$.where m is a vertex of T (a, b) and m (i, j) is the number of units from the ith row to the jth column.

Example 2: using $\boldsymbol{m_1}$ in example 1 the auxiliary graph is

E $(\boldsymbol{m_1})$ = $\{e_{11}, e_{12}, e_{13}, e_{22}, e_{33}\}$

Then the auxiliary graph is

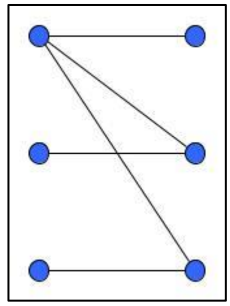


Figure 7. Auxiliary graph

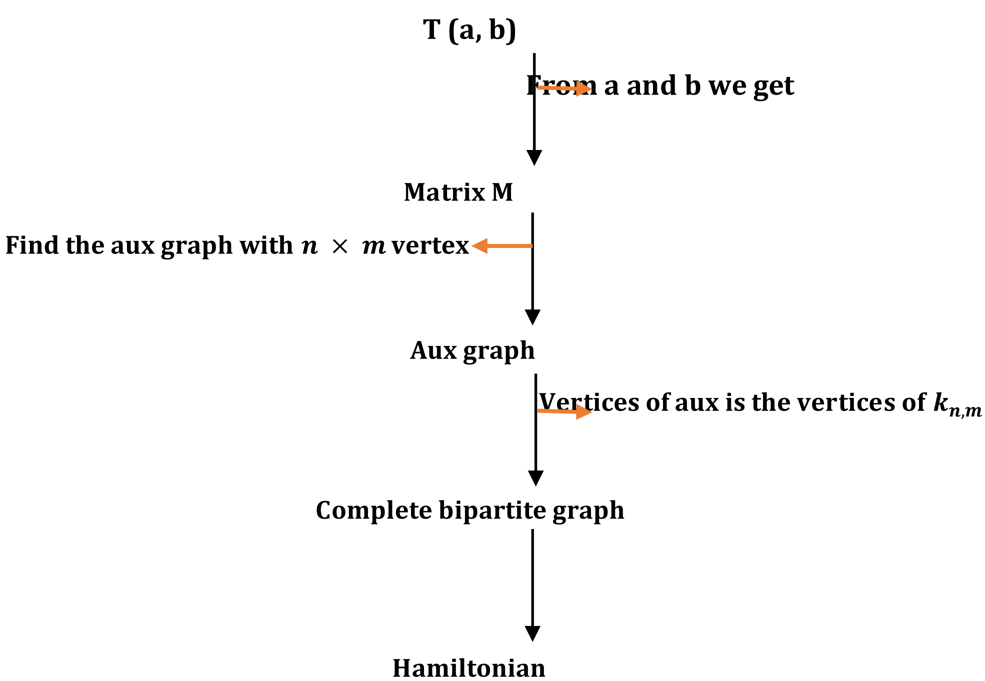
Now an open problem is proved about the properties of transportation polytope that given as a theorem

Theorem

The graph formed by the vertices and edges of any 2_way transportation polytope is Hamiltonian. If $m = n$, where m and n denoted the number of rows and columns for the transportation polytopes.

Let M = (m_{ij}) be a matrix of size $n \times m$ that belong to the transportation polytope T (a, b). Create an auxiliary graph with $n \times m$ node and edges {(i , j) : m_{ij} > 0 }, where the matrix M is a vertex of the transportation polytope T(a ,b), if and only if the auxiliary graph forms a spanning forest in the complete bipartite graph $k_{n,m}$. From the definition of bipartite graph every cycle in bipartite graph is even and alternates between vertices from V_1 and V_2 . Since Hamilton cycle uses all the vertices in V_1 and V_2 , then m=$|V_1|$=$|V_2|$=n. suppose that $k_{n,n}$ has partite sets V_1 =$\{v_{1,}..., v_n\}$ and V_2 =$\{w_1, ..., w_n\}$, since $v_i w_j$ is an edge of $k_{n,n}$ for every 1≤ i , j ≤ n then $v_1, w_1, v_2, w_2, ..., v_n, w_n$ is a Hamiltonian cycle .

That is 2-way transportation polytope is Hamiltonian.



Example 3: Let a, b = (1, 1, 2) then the T (a, b) has the vertices.

$$m_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} , \ m_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix} , \ m_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$$m_3 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} , \ m_4 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix} , m_5 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$$m_6 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$T_i = \text{aux}\ (m_i)$

$T_0 = \{e_{11} , e_{22} , e_{33}\} , T_1 = \{e_{12} , e_{21} , e_{33}\} , \ T_2 = \{e_{11} , e_{23} , e_{32}, e_{33}\}$

$T_3 = \{e_{13} , e_{22} , e_{31}, e_{33}\} , T_4 = \{e_{12} , e_{23} , e_{31}, e_{33}\}$

$T_5 = \{e_{13} , e_{21} , e_{32}, e_{33}\} , \ T_6 = \{e_{13} , e_{23} , e_{31}, e_{32}\}$.

T is spanning forest

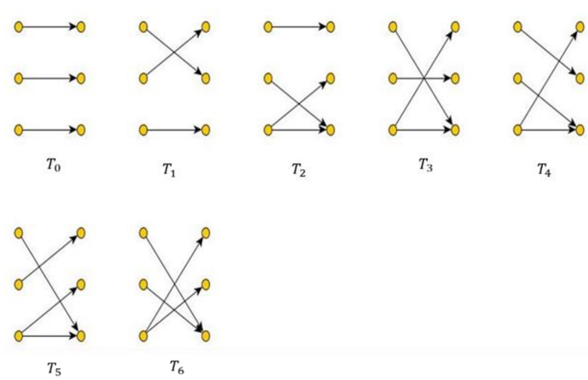


Figure 8. A spanning forest

3. Application [1]

Now an application is given to consolation the concept of the proven theorem.

1-**The optimization program**

The optimization program is used in heavy load situations. If the number of jobs is low, there is no need for the optimization program or its use at a certain time point, the stacker crane control program must determine which of the jobs will be determined later. In addition, the stacker crane may either perform some jobs or be in a state of stillness. If the stacker crane is idle, the generated job will be executed immediately. The administration will not accept waiting for stacker crane times. If there are other jobs, the new generated job will be placed in a list of all jobs.an optimization process is called minimizing the unloaded moves between them. What should not happen is that the stacker crane is not active because the optimization process has not yet ended so a three-phase process has been implemented which can be interrupted during implementation and still ensures a possible solution.

2-Optimization process

Suppose a sequence S = $\{w_1, \ldots, w_n\}$ of n jobs that must be performed has been obtained and the job that is performed is w_1 and the newly created job is w_{new}

Suppose $w_{new} \notin$ S.

Phase 1: Run a simple insertion heuristic

Job w_{new} is inserted between k and $k+1$, where

K=$min_{l=1,\ldots,n} = \{c_{w_l w_{new}} + c_{w_{new} w_{l+1}} - c_{w_l w_{l+1}}\}$

(Assume that $c_{(i,n+1)} = 0 \ \forall i = 1, \ldots, n$).

Here we just scan through the current sequence S and try to insert job w_{new} as

Cheaply as possible.

Phase 2: Run a more sophisticated heuristic

The possibility to use any of the available heuristics for the ATSP,

For example, the farthest insertion heuristic.

Phase 3: Solve the problem to optimality

This is done using the branch&bound code of Fischetti and Toth [14] that solves the instances arising here in a reasonable amount of time.

3-Computational results

The optimization process was integrated into a simulation model and the results obtained from the simulation model were used to compare curriculum used with and the old strategy that is when the stacker crane is in idle mode because the process of optimization is not over yet which leads to delayed performance of the job In addition, there is no restricted amount of computing time for optimization process in the simulation model. All AHHP problems have been resolved with the third phase of optimization process. The following table contains data from the actual life of the week that was used to verify the efficiency of the simulation model.

The Key of the Table is

J: Number of transportation tasks (jobs).

K: Unloaded travel time of the stacker crane with the old priority based

Rule (in seconds).

S: Unloaded travel time of the stacker crane with the optimization process

(In seconds).

I. %: Improvement in %, calculated by $\frac{(uTr-P)-(uTr-O)}{(uTr-P)}.100$

M: Maximal number of jobs at the same time.

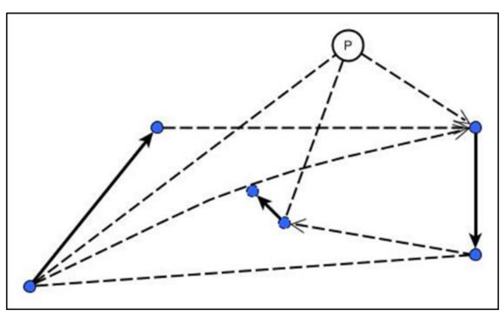
A: Average number of jobs at the same time.

Table1: Minimizing the unloaded travel times

	J	K	S	I.%	M	A
1	416	8599	8325	3.18	6	2.31
2	421	8655	8141	5.93	8	1.94
3	405	8238	7956	3.42	6	2.27
4	398	8017	7634	4.77	8	1.93
5	447	9411	8951	4.88	8	2.13

Analysis of the data showed that the mentioned week had a low production volume, adding that the stacker crane was not subjected to any major faults and therefore the results in the improvements of the unloaded movements shown in the Column (I %) range from 3% to 6% were disappointing in column (A) There were only two jobs at the same time so there was no need for optimize.

Transportation tasks



−−> Possible unloaded moves (P) actual position of the stacker crane

Figure 8. Stacker crane moves

4. Conclusion

According to the important role of the transportation polytope in the real life, therefore a lot of open problem appear one of them is to prove that the 2_way transportation polytope is Hamiltonian, which is taken in this paper also the application that consolation its important is given.

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