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The Complex of Lascoux in the Case of partition (7,7,3)

Nubras Y. Khudair [a] **,** ***Haytham R. Hassan*** [b]

[a, b] Department of Mathematics, College of Science, Mustansiriyah University, Iraq, Email : nubras_yasir@yahoo.com

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ABSTRACT

The purpose of this paper is to find and study the complex of Lascoux in the case of partition (7,7,3) as a diagram by utilize the concepts of mapping Cone, divided power of place polarization and Capllie identities.

MSC

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1. Introduction

Let $\mathbb{R}$ be a commutative ring with 1, $\mathbb{F}$ be a free module and $\mathcal{D}_b\mathbb{F}$ be the divided power of degree b. The technique used in this paper to study the complex of lascoux leaned heavily on the construction of an arithmetic koszul complex [2] and the certain sequence of skew-shapes together with mapping cone properties [7] of the complex. Recall the standard kind of maps which used in this work by place polarizations operators ∂_{21}, ∂_{32}, ∂_{31} and it's divided powers, where

$\partial_{21}^{(\mathfrak{k})}: \mathcal{D}_{\mathbb{p}+\mathfrak{k}}\,\mathbb{F} \otimes \mathcal{D}_{\mathbb{q}-\mathfrak{k}}\,\mathbb{F} \otimes \mathcal{D}_r\mathbb{F} \longrightarrow \mathcal{D}_{\mathbb{p}}\mathbb{F} \otimes \mathcal{D}_{\mathbb{q}}\mathbb{F} \otimes \mathcal{D}_r\mathbb{F}$ which is the place polarization from place 1 to place 2 and

$\partial_{32}^{(\mathfrak{k})}: \mathcal{D}_{\mathbb{p}}\mathbb{F} \otimes \mathcal{D}_{\mathbb{q}+\mathfrak{k}}\,\mathbb{F} \otimes \mathcal{D}_r\mathbb{F} \longrightarrow \mathcal{D}_{\mathbb{p}}\mathbb{F} \otimes \mathcal{D}_{\mathbb{q}}\mathbb{F} \otimes \mathcal{D}_r\mathbb{F}$ which is the place polarization from place 2 to place 3.

The complex of Lascoux within the below cases, (2,2,2), (3,3,2) and (4,4,3) are studied by the authors in [4], [5]and [6] as a graph by satisfying the notion of mapping Cone as in [7].

In this study we tend to discuss the complex of characteristic zero within the case of partition (7,7,3) as a diagram by utilize the speculation of mapping Cone [7].

Corresponding author Haytham R. Hassan

Email addresses : nubras_yasir@yahoo.com

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The map $\partial_{ij}^{(k)}$ which implies the divided power of the place polarization ∂_{ij} wherever j ought to be but i with its Capelli identities [2], during this paper we tend to solely used the below identities:

$$\partial_{32}^{(m)} \circ \partial_{21}^{(n)} = \sum_{\beta\geq 0} \partial_{21}^{(n-\beta)}\, \partial_{32}^{(m-\beta)} \partial_{31}^{(\beta)} \qquad (1.1)$$

$$\partial_{21}^{(n)} \circ \partial_{32}^{(m)} = \sum_{\beta\geq 0} (-1)^{\beta}\, \partial_{32}^{(m-\beta)} \circ \partial_{21}^{(n-\beta)} \circ \partial_{31}^{(\beta)} \qquad (1.2)$$

$$\partial_{21}^{(1)} \circ \partial_{31}^{(1)} = \partial_{31}^{(1)} \circ \partial_{21}^{(1)} \qquad (1.3)$$

$$\partial_{32}^{(1)} \circ \partial_{31}^{(1)} = \partial_{31}^{(1)} \circ \partial_{32}^{(1)} \qquad (1.4)$$

2. The Terms of Lascoux Complex in the Case of Partition (7,7,3)

The locations of the terms of the complex can be committed to the length of the permutation to that they correlate with [1],[4]. Currently within the case of the partition λ = (7,7,3), we have the subsequent matrix:

$$\begin{bmatrix} D_7\mathbb{F} & D_6\mathbb{F} & D_1\mathbb{F} \\ D_8\mathbb{F} & D_7\mathbb{F} & D_2\mathbb{F} \\ D_9\mathbb{F} & D_8\mathbb{F} & D_3\mathbb{F} \end{bmatrix}$$

The Lascux complex has the corresponding among its terms as below:

$D_7\mathbb{F} \otimes D_7\mathbb{F} \otimes D_3\mathbb{F} \leftrightarrow$ identity

$D_8\mathbb{F} \otimes D_6\mathbb{F} \otimes D_3\mathbb{F} \leftrightarrow (12)$

$D_7\mathbb{F} \otimes D_8\mathbb{F} \otimes D_2\mathbb{F} \leftrightarrow (23)$

$D_9\mathbb{F} \otimes D_6\mathbb{F} \otimes D_2\mathbb{F} \leftrightarrow (123)$

$D_8\mathbb{F} \otimes D_8\mathbb{F} \otimes D_1\mathbb{F} \leftrightarrow (132)$

$D_9\mathbb{F} \otimes D_7\mathbb{F} \otimes D_1\mathbb{F} \leftrightarrow (13)$

So, the complex of Lascoux in the case of the partition λ = (7,7,3) has the form:

$$D_9\mathbb{F} \otimes D_7\mathbb{F} \otimes D_1\mathbb{F} \to \begin{matrix} D_8\mathbb{F} \otimes D_8\mathbb{F} \otimes D_1\mathbb{F} \\ \oplus \\ D_9\mathbb{F} \otimes D_6\mathbb{F} \otimes D_2\mathbb{F} \end{matrix} \to \begin{matrix} D_7\mathbb{F} \otimes D_8\mathbb{F} \otimes D_2\mathbb{F} \\ \oplus \\ D_8\mathbb{F} \otimes D_6\mathbb{F} \otimes D_3\mathbb{F} \end{matrix} \to D_7\mathbb{F} \otimes D_7\mathbb{F} \otimes D_3\mathbb{F}$$

3. The Diagram of the Complex of Lascoux in the Case of Partition (7,7,3)

Look at the following diagram:

$$\begin{array}{ccccc} D_9\mathbb{F} \otimes D_7\mathbb{F} \otimes D_1\mathbb{F} & \xrightarrow{c_1} & D_8\mathbb{F} \otimes D_8\mathbb{F} \otimes D_1\mathbb{F} & \xrightarrow{c_2} & D_8\mathbb{F} \otimes D_6\mathbb{F} \otimes D_3\mathbb{F} \\ \downarrow t_1 & J & \downarrow t_2 & L & \downarrow t_3 \\ D_9\mathbb{F} \otimes D_6\mathbb{F} \otimes D_2\mathbb{F} & \xrightarrow{m_1} & D_7\mathbb{F} \otimes D_8\mathbb{F} \otimes D_2\mathbb{F} & \xrightarrow{m_2} & D_7\mathbb{F} \otimes D_7\mathbb{F} \otimes D_3\mathbb{F} \end{array}$$

Now, if we define

$$c_1 : D_9\mathbb{F} \otimes D_7\mathbb{F} \otimes D_1\mathbb{F} \longrightarrow D_8\mathbb{F} \otimes D_8\mathbb{F} \otimes D_1\mathbb{F}$$

as $\mathbb{c}_1(\mathbb{d}) = \partial_{21}(\mathbb{d})$;where $\mathbb{d} \in \mathcal{D}_9\mathbb{F} \otimes \mathcal{D}_7\mathbb{F} \otimes \mathcal{D}_1\mathbb{F}$

$\mathbb{t}_1 : \mathcal{D}_9\mathbb{F} \otimes \mathcal{D}_7\mathbb{F} \otimes \mathcal{D}_1\mathbb{F} \longrightarrow \mathcal{D}_9\mathbb{F} \otimes \mathcal{D}_6\mathbb{F} \otimes \mathcal{D}_2\mathbb{F}$

as $\mathbb{t}_1(\mathbb{d}) = \partial_{32}(\mathbb{d})$;where $\mathbb{d} \in \mathcal{D}_9\mathbb{F} \otimes \mathcal{D}_7\mathbb{F} \otimes \mathcal{D}_1\mathbb{F}$

And

$\mathbb{u}_1 : \mathcal{D}_9\mathbb{F} \otimes \mathcal{D}_6\mathbb{F} \otimes \mathcal{D}_2\mathbb{F} \longrightarrow \mathcal{D}_7\mathbb{F} \otimes \mathcal{D}_8\mathbb{F} \otimes \mathcal{D}_2\mathbb{F}$

as $\mathbb{u}_1(\mathbb{d}) = \partial_{21}^{(2)}(\mathbb{d})$;where $\mathbb{d} \in D_7\mathbb{F} \otimes D_8\mathbb{F} \otimes D_2\mathbb{F}$

Now, we must define the map:

$\mathbb{t}_2 : \mathcal{D}_8\mathbb{F} \otimes \mathcal{D}_8\mathbb{F} \otimes \mathcal{D}_1\mathbb{F} \longrightarrow \mathcal{D}_7\mathbb{F} \otimes \mathcal{D}_8\mathbb{F} \otimes \mathcal{D}_2\mathbb{F}$ by

$\mathbb{t}_2(\mathbb{d}) = (\frac{1}{2}\partial_{32}^{(1)} \circ \partial_{21}^{(1)} - \partial_{31}^{(1)})$

Lemma (3.1):

The diagram J is commute.

Proof:

To demonstrate that J is commute, we must prove $\mathbb{u}_1 \circ \mathbb{t}_1 = \mathbb{t}_2 \circ \mathbb{c}_1$

Now if we use Capelli identity we get:

$$\begin{aligned}
\mathbb{u}_1 \circ \mathbb{t}_1 &= \partial_{21}^{(2)} \circ \partial_{32}^{(1)} \\
&= \partial_{32}^{(1)} \circ \partial_{21}^{(2)} - \partial_{21}^{(1)} \circ \partial_{31}^{(1)} \\
&= (\frac{1}{2}\partial_{32}^{(1)} \circ \partial_{21}^{(1)} - \partial_{31}^{(1)}) \circ \partial_{21}^{(1)} \\
&= \mathbb{t}_2 \circ \mathbb{c}_1
\end{aligned}$$ □

On the other hand, if we define:

$\mathbb{u}_2 : \mathcal{D}_7\mathbb{F} \otimes \mathcal{D}_8\mathbb{F} \otimes \mathcal{D}_2\mathbb{F} \longrightarrow \mathcal{D}_7\mathbb{F} \otimes \mathcal{D}_7\mathbb{F} \otimes \mathcal{D}_3\mathbb{F}$

As $\mathbb{u}_2(\mathbb{d}) = \partial_{32}(\mathbb{d})$; where $\mathbb{d} \in \mathcal{D}_7\mathbb{F} \otimes \mathcal{D}_8\mathbb{F} \otimes \mathcal{D}_2\mathbb{F}$ and

$\mathbb{t}_3 : \mathcal{D}_8\mathbb{F} \otimes \mathcal{D}_6\mathbb{F} \otimes \mathcal{D}_3\mathbb{F} \longrightarrow \mathcal{D}_7\mathbb{F} \otimes \mathcal{D}_7\mathbb{F} \otimes \mathcal{D}_3\mathbb{F}$

as $\mathbb{t}_3(\mathbb{d}) = \partial_{21}(\mathbb{d})$;where $\mathbb{d} \in \mathcal{D}_8\mathbb{F} \otimes \mathcal{D}_6\mathbb{F} \otimes \mathcal{D}_3\mathbb{F}$ and

$\mathbb{c}_2 : \mathcal{D}_8\mathbb{F} \otimes \mathcal{D}_8\mathbb{F} \otimes \mathcal{D}_1\mathbb{F} \longrightarrow \mathcal{D}_8\mathbb{F} \otimes \mathcal{D}_6\mathbb{F} \otimes \mathcal{D}_3\mathbb{F}$

As $\mathbb{c}_2(\mathbb{d}) = \partial_{32}^{(2)}$

Lemma (3.2):

The diagram L is commute.

Proof:

To prove that L is commutative, we must prove $\mathbb{t}_3 \circ \mathbb{c}_2 = \mathbb{u}_2 \circ \mathbb{t}_2$ i.e.

$\mathbb{t}_3 \circ \mathbb{c}_2 = \partial_{21}^{(1)} \circ \partial_{32}^{(2)}$

by using Capelli identities we get

$= \partial_{32}^{(2)} \circ \partial_{21}^{(1)} - \partial_{32}^{(1)} \circ \partial_{31}^{(1)}$

$= \partial_{32}^{(1)} \circ (\frac{1}{2}\partial_{32}^{(1)} \circ \partial_{21}^{(1)} - \partial_{31}^{(1)})$

$= \mathbb{u}_2 \circ \mathbb{t}_2$

Lemma (3.3)

The totally diagram is commutative.

Proof:

To demonstrate that the totally diagram is commutative, we must prove that $\mathbb{t}_3 \circ \mathbb{c}_2 \circ \mathbb{c}_1 = \mathbb{u}_2 \circ \mathbb{u}_1 \circ \mathbb{t}_1$ i.e.

$\partial_{21}^{(1)} \circ \partial_{32}^{(2)} \circ \partial_{21}^{(1)} = \partial_{32}^{(1)} \circ \partial_{21}^{(2)} \circ \partial_{32}^{(1)}$

Now if we use Capelli identity we get:

$\partial_{21}^{(1)} \circ \partial_{32}^{(2)} \circ \partial_{21}^{(1)} = \partial_{21}^{(1)} \circ \partial_{21}^{(1)} \circ \partial_{32}^{(2)} + \partial_{21}^{(1)} \circ \partial_{32}^{(1)} \circ \partial_{31}^{(1)}$

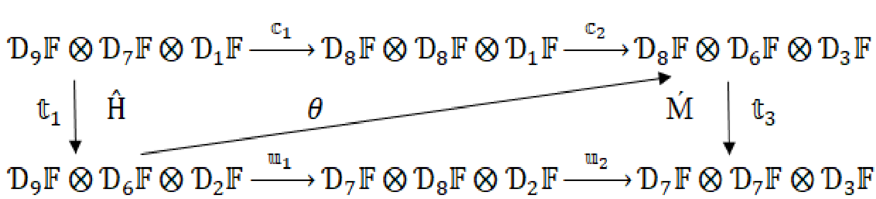
$= 2\ \partial_{21}^{(2)} \circ \partial_{32}^{(2)} + \partial_{21}^{(1)} \circ \partial_{32}^{(1)} \circ \partial_{31}^{(1)}$

And

$\partial_{32}^{(1)} \circ \partial_{21}^{(2)} \circ \partial_{32}^{(1)} = \partial_{21}^{(2)} \circ \partial_{32}^{(1)} \circ \partial_{32}^{(1)} + \partial_{21}^{(1)} \circ \partial_{32}^{(1)} \circ \partial_{31}^{(1)}$

$= 2\ \partial_{21}^{(2)} \circ \partial_{32}^{(2)} + \partial_{21}^{(1)} \circ \partial_{32}^{(1)} \circ \partial_{31}^{(1)}$

Now, consider the following diagram:



Define:

$\theta : \mathfrak{D}_9\mathbb{F} \otimes \mathfrak{D}_6\mathbb{F} \otimes \mathfrak{D}_2\mathbb{F} \longrightarrow \mathfrak{D}_8\mathbb{F} \otimes \mathfrak{D}_6\mathbb{F} \otimes \mathfrak{D}_3\mathbb{F}$, by:

$\theta = (\frac{1}{2}\partial_{32}^{(1)} \circ \partial_{21}^{(1)} + \partial_{31}^{(1)})$

Lemma (3.4)

The diagram Ĥ is commutative.

Proof:

To demonstrate that Ĥ is commutative, we must prove $\mathbb{c}_2 \circ \mathbb{c}_1 = \theta \circ \mathbb{t}_1$

$\mathbb{c}_2 \circ \mathbb{c}_1 = \partial_{32}^{(2)} \circ \partial_{21}^{(1)}$

by using Capelli identities we get

$= \partial_{21}^{(1)} \circ \partial_{32}^{(2)} + \partial_{32}^{(1)} \circ \partial_{31}^{(1)}$

$= (\frac{1}{2}\partial_{32}^{(1)} \circ \partial_{21}^{(1)} + \partial_{31}^{(1)}) \circ \partial_{32}^{(1)}$

$= \theta \circ \mathbb{t}_1$

Lemma (3.5):

The diagram Ḿ is commutative.

Proof:

To demonstrate that Ĥ is commutative, we must prove $\mathbb{u}_2 \circ \mathbb{u}_1 = \mathbb{t}_3 \circ \theta$

$\mathbb{u}_2 \circ \mathbb{u}_1 = \partial_{32}^{(1)} \circ \partial_{21}^{(2)}$

by using Capelli identities we get

$= \partial_{21}^{(2)} \circ \partial_{32}^{(1)} + \partial_{21}^{(1)} \circ \partial_{31}^{(1)}$

$= \partial_{21}^{(1)} \circ (\frac{1}{2}\partial_{32}^{(1)} \circ \partial_{21}^{(1)} + \partial_{31}^{(1)})$

$= \mathbb{t}_3 \circ \theta$

lastly, we can define the maps σ_1, σ_2 and σ_3 where:

$$\sigma_3: \mathcal{D}_9\mathbb{F} \otimes \mathcal{D}_7\mathbb{F} \otimes \mathcal{D}_1\mathbb{F} \longrightarrow \begin{array}{c} \mathcal{D}_8\mathbb{F} \otimes \mathcal{D}_8\mathbb{F} \otimes \mathcal{D}_1\mathbb{F} \\ \oplus \\ \mathcal{D}_9\mathbb{F} \otimes \mathcal{D}_6\mathbb{F} \otimes \mathcal{D}_2\mathbb{F} \end{array}$$

by:

$\sigma_3(\varkappa) = (\mathbb{c}_1(\varkappa), \mathbb{t}_1(\varkappa))$

$= (\partial_{21}^{(1)}(\varkappa), \partial_{32}^{(1)}(\varkappa))$

;where $\varkappa \in \mathcal{D}_9\mathbb{F} \otimes \mathcal{D}_7\mathbb{F} \otimes \mathcal{D}_1\mathbb{F}$

$$\sigma_2: \begin{array}{c} \mathcal{D}_8\mathbb{F} \otimes \mathcal{D}_8\mathbb{F} \otimes \mathcal{D}_1\mathbb{F} \\ \oplus \\ \mathcal{D}_9\mathbb{F} \otimes \mathcal{D}_6\mathbb{F} \otimes \mathcal{D}_2\mathbb{F} \end{array} \longrightarrow \begin{array}{c} \mathcal{D}_8\mathbb{F} \otimes \mathcal{D}_6\mathbb{F} \otimes \mathcal{D}_3\mathbb{F} \\ \oplus \\ \mathcal{D}_7\mathbb{F} \otimes \mathcal{D}_8\mathbb{F} \otimes \mathcal{D}_2\mathbb{F} \end{array}$$

by:

$\sigma_2((\varkappa_1, \varkappa_2)) = (\mathbb{c}_2(\varkappa_1) - \theta(\varkappa_2)), \mathbb{u}_1(\varkappa_2) - \mathbb{t}_2(\varkappa_1);$

$$= (\partial_{32}^{(2)}(\varkappa_1) - (\tfrac{1}{2}\partial_{32}^{(1)} \circ \partial_{21}^{(1)} + \partial_{31}^{(1)})(\varkappa_2), (\partial_{21}^{(2)}(\varkappa_2) - (\tfrac{1}{2}\partial_{32}^{(1)} \circ \partial_{21}^{(1)} + \partial_{31}^{(1)})(\varkappa_1))$$

where $(\varkappa_1, \varkappa_2) \in \begin{matrix} \mathcal{D}_8\mathbb{F} \otimes \mathcal{D}_8\mathbb{F} \otimes \mathcal{D}_1\mathbb{F} \\ \oplus \\ \mathcal{D}_9\mathbb{F} \otimes \mathcal{D}_6\mathbb{F} \otimes \mathcal{D}_2\mathbb{F} \end{matrix}$

and

$$\sigma_1: \begin{matrix} \mathcal{D}_8\mathbb{F} \otimes \mathcal{D}_6\mathbb{F} \otimes \mathcal{D}_3\mathbb{F} \\ \oplus \\ \mathcal{D}_7\mathbb{F} \otimes \mathcal{D}_8\mathbb{F} \otimes \mathcal{D}_2\mathbb{F} \end{matrix} \longrightarrow \mathcal{D}_7\mathbb{F} \otimes \mathcal{D}_7\mathbb{F} \otimes \mathcal{D}_3\mathbb{F}$$

by

$\sigma_1((\varkappa_1, \varkappa_2)) = (\mathbb{t}_3(\varkappa_1) + \mathbb{u}_2(\varkappa_2))$

$= (\partial_{21}^{(1)}(\varkappa_1) + \partial_{32}^{(1)}(\varkappa_2))$

where $(\varkappa_1, \varkappa_2) \in \begin{matrix} \mathcal{D}_8\mathbb{F} \otimes \mathcal{D}_6\mathbb{F} \otimes \mathcal{D}_3\mathbb{F} \\ \oplus \\ \mathcal{D}_7\mathbb{F} \otimes \mathcal{D}_8\mathbb{F} \otimes \mathcal{D}_2\mathbb{F} \end{matrix}$

Lemma (3.6):

$$\mathcal{D}_9\mathbb{F} \otimes \mathcal{D}_7\mathbb{F} \otimes \mathcal{D}_1\mathbb{F} \rightarrow \begin{matrix} \mathcal{D}_8\mathbb{F} \otimes \mathcal{D}_8\mathbb{F} \otimes \mathcal{D}_1\mathbb{F} \\ \oplus \\ \mathcal{D}_9\mathbb{F} \otimes \mathcal{D}_6\mathbb{F} \otimes \mathcal{D}_2\mathbb{F} \end{matrix} \rightarrow \begin{matrix} \mathcal{D}_8\mathbb{F} \otimes \mathcal{D}_6\mathbb{F} \otimes \mathcal{D}_3\mathbb{F} \\ \oplus \\ \mathcal{D}_7\mathbb{F} \otimes \mathcal{D}_8\mathbb{F} \otimes \mathcal{D}_2\mathbb{F} \end{matrix} \rightarrow \mathcal{D}_7\mathbb{F} \otimes \mathcal{D}_7\mathbb{F} \otimes \mathcal{D}_3\mathbb{F}$$

is complex.

Proof:

As $\partial_{32}^{(1)}$ *and* $\partial_{21}^{(1)}$ are injective from its delineation [3], then we get σ_3 is injective

Now

$\sigma_2 \circ \sigma_3(\varkappa) = \sigma_2(\mathbb{c}_1(\varkappa), \mathbb{t}_1(\varkappa))$

$= \sigma_2(\partial_{21}^{(1)}(\varkappa), \partial_{32}^{(1)}(\varkappa))$

$= (\mathbb{c}_2(\partial_{21}^{(1)}(\varkappa)) - \theta(\partial_{32}^{(1)}(\varkappa)), \mathbb{u}_1(\partial_{32}^{(1)}(\varkappa)) - \mathbb{t}_2(\partial_{21}^{(1)}(\varkappa)))$.

Now

$(\mathbb{c}_2(\partial_{21}(\varkappa))$ - $\theta(\partial_{32}(\varkappa)) = (\partial_{32}^{(2)} \circ \partial_{21}^{(1)} - \frac{1}{2}\partial_{32}^{(1)} \circ \partial_{21}^{(1)} \circ \partial_{32}^{(1)}$ - $\partial_{31}^{(1)} \circ \partial_{32})(\varkappa)$

$= (\partial_{32}^{(2)} \circ \partial_{21}^{(1)} - \partial_{32}^{(2)} \circ \partial_{21}^{(1)} + \partial_{31}^{(1)} \circ \partial_{32} - \partial_{31}^{(1)} \circ \partial_{32})(\varkappa)$

$= 0.$

and

$(\mathbb{u}_1(\partial_{32}(\varkappa))$ - $\mathbb{t}_2(\partial_{21}(\varkappa)) = (\partial_{21}^{(2)}) \circ \partial_{32}^{(1)}(\varkappa)$ - $(\frac{1}{2}\partial_{32}^{(1)} \circ \partial_{21}^{(1)}$- $\partial_{31}) \circ \partial_{21}^{(1)}(\varkappa)$

$= (\partial_{21}^{(2)} \circ \partial_{32}^{(1)}$- $\partial_{32}^{(1)} \circ \partial_{21}^{(2)} + \partial_{21}^{(1)} \circ \partial_{31})(\varkappa)$

$= (\partial_{21}^{(2)} \circ \partial_{32}^{(1)}$ - $\partial_{21}^{(2)} \circ \partial_{32}^{(1)}$ - $\partial_{21}^{(1)} \circ \partial_{31} + \partial_{21}^{(1)} \circ \partial_{31})(\varkappa)$

$= 0.$

So we get $\sigma_2 \circ \sigma_3(\varkappa) = 0$.

and

$\sigma_1 \circ \sigma_2(\varkappa_1, \varkappa_2) = \sigma_1(\mathbb{c}_2(\varkappa_1) - \theta(\varkappa_2), \mathbb{u}_1(\varkappa_2)$ - $\mathbb{t}_2(\varkappa_1))$

$= \sigma_1((\partial_{32}^{(2)}(\varkappa_1) - (\frac{1}{2}\,\partial_{32}^{(1)} \circ \partial_{21}^{(1)} + \partial_{31}^{(1)})(\varkappa_2), (\partial_{21}^{(2)} + \frac{1}{2}\,\partial_{32}^{(1)} \circ \partial_{21}^{(1)} - \partial_{31}^{(1)})(\varkappa_2))$

$= \partial_{21}^{(1)} \circ(\partial_{32}^{(2)}(\varkappa_1) - (\frac{1}{2}\,\partial_{32}^{(1)} \circ \partial_{21}^{(1)} + \partial_{31}^{(1)})(\varkappa_2) + \partial_{32}^{(1)} \circ (\partial_{21}^{(2)} + \frac{1}{2}\,\partial_{32}^{(1)} \circ \partial_{21}^{(1)} -$

$\partial_{31}^{(1)})(\varkappa_2))$

$= (\partial_{21}^{(1)} \circ \partial_{32}^{(2)}$- $\partial_{32}^{(2)} \circ \partial_{21}^{(1)} + \partial_{32}^{(1)} \circ \partial_{31})(\varkappa_1) + (-\partial_{21}^{(2)} \circ \partial_{32}^{(1)} - \partial_{21}^{(1)} \circ \partial_{31} + \partial_{32}^{(1)} \circ$

$\partial_{21}^{(2)})(\varkappa_2)$

$= (\partial_{32}^{(2)} \circ \partial_{21}^{(1)} - \partial_{32}^{(1)} \circ \partial_{31} - \partial_{32}^{(2)} \circ \partial_{21}^{(1)} + \partial_{32}^{(1)} \circ \partial_{31})(\varkappa_1) + (-\partial_{32}^{(1)} \circ \partial_{21}^{(2)}$- $\partial_{21}^{(1)} \circ$

$\partial_{31} - \partial_{21}^{(1)} \circ \partial_{31} + \partial_{32}^{(1)} \circ \partial_{21}^{(2)})(\varkappa_2)$

$= 0$ □

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