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First Chebyshev Wavelet in Numerical Solution and Signal Processing

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ABSTRACT

In this work, a kind of wavelet used in many mathematical problems and numerically solved, such as heterogeneity and integral equations, are presented. The solution is close to the exact solution using the matrix of integral operations. In this paper, the issues of limited value were solved and good solutions were found. Solved examples show that.
In addition, the proposed theory was used to process signals for one dimension and the noise and signal pressure were removed this was applied to some kind of signal.

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1 . Introduction

The wavelets have many uses in the scientific and engineering fields where they perform many functions because they contain the expansion and contraction parameters that make up the mother wavelet [1].
Today, there are many works on wavelets methods for approximating the solution of the problems, such as Hermite wavelets method [2], third kind Chebyshev wavelets [3], Haar wavelets method [4], and Sin and Cos wavelets method [5],
Several numerical methods have been proposed in the last years to solve (BVPs) which are based on orthogonal polynomials, also wavelets approach was used in several papers to solve (BVPs) [6].
Many of the works were processed using many signal wavelet such as Haar, Laguerre, db, etc [8-11].

2 . Chebyshev Wavelets (CW)

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The wavelets contain functions created from expansion and contraction parameters, d and f, and continuously expansion and contraction. The mother wavelet is created [6].

$$T_{d,f}(t)=|d|^{\frac{-1}{2}}T\left(\frac{t-f}{d}\right)\ d,f\in R,d\neq 0$$

Where

$$T(t)=[T_0(t),T_1(t),\dots,T_{m-1}(t)]^T$$

the basis wavelet $T_0(t)$, $T_1(t)$,..., $T_{M-1}(t)$ are orthogonal functions on [0, 1], Many problems were solved using the proposed theory such as differential equation and integral equation as well as fractional differential equation [7].

3 . First Chebyshev Wavelets (FCW)

FCW: $T^1_{n,m}(t)=T^1(t,n,m,k)$, where t is the normalized time, $n=1,2,\dots,2^k$, m is order for FCW polynomials and $k = 1,2,\dots$. They are defined on [0,1) by [7]

$$T^1_{n,m}(r)=\begin{cases}\frac{c_m 2^{\frac{k}{2}}}{\sqrt{\pi}}T_m(2^{k+1}t-2n+1) & \frac{n-1}{2^k}\leq t<\frac{n}{2^k}\\ 0 & \text{o.w}\end{cases} \tag{1}$$

$$\alpha_m=\begin{cases}\sqrt{2} & m=0\\ 2 & m=1,2,\dots\end{cases}$$

We known T_m is orthogonal with the weight function $w(r)\frac{1}{\sqrt{1-t^2}}$ the set of CW are orthogonal with the weight function $w_n(t)=w(2^{k+1}t-2n+1)$

An approximation function $f(t)\in L_2[0,1)$ may be expanded as:

$$f(t)=\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}A_{n,m}T^1_{n,m}(t) \tag{2}$$

$$A_{n,m}=\left(f(t),T^1_{n,m}(t)\right) \tag{3}$$

$(.,.)$ in (3) denoted to the inner product with weight function $w_n(t)$ on the Hilbert Space [0,1) If the infinite series in above equation is separated, then (2) can be written as:

$$f(t)=\sum_{n=1}^{2^k}\sum_{m=0}^{M-1}A_{n,m}T^1_{n,m}(t)=A^T T^1(t)$$

A and $T^1(t)$ are $2^kM\times 1$ matrices given by: (4)

$$A=\left[A_{10},A_{11},\dots,A_{1(M-1)},A_{20},\dots,A_{2(M-1)},\dots,A_{2^k},\dots,A_{2^kM-1}\right]^T \tag{5}$$

$$T^1(t)=\left[T^1_{10}(t),T^1_{11}(t),\dots,T^1_{1M-1}(t),T^1_{20}(t),\dots,T^1_{2^kM-1}(t),\dots T^1_{2^k0}(t),\dots T^1_{2^kM-1}(t)\right]^T \tag{6}$$

4 . OMI for FCW

In this section, the OMI for FCW P_{T^1} was made. First, we find 6×6 matrix P_{Ψ^1}. The six basis functions are given by [7]. From equation (1) for k=1 and M=3 we obtain six functions then by integrating these six functions from 0 to r and using equation (3) we obtain the OMI P_{T^1} is:

$$P_{T^1_{6\times 6}} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4\sqrt{2}} & 0 & \vdots & \frac{1}{2} & 0 & 0 \\ -\frac{1}{8\sqrt{2}} & 0 & \frac{1}{16} & \vdots & 0 & 0 & 0 \\ -\frac{1}{6\sqrt{2}} & -\frac{1}{8} & 0 & \vdots & -\frac{\sqrt{2}}{6} & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \vdots & \frac{1}{4} & \frac{1}{4\sqrt{2}} & 0 \\ 0 & 0 & 0 & \vdots & -\frac{1}{8\sqrt{2}} & 0 & \frac{1}{16} \\ 0 & 0 & 0 & \vdots & -\frac{1}{6\sqrt{2}} & -\frac{1}{8} & 0 \end{bmatrix}$$

The matrix P_{Ψ^1} can be written as:

$$P_{T^1_{6\times 6}} = \begin{bmatrix} A_{3\times 3} & S_{3\times 3} \\ O_{3\times 3} & A_{3\times 3} \end{bmatrix}$$

In general have

$$\int_0^r T^1(t)dr = P_{T^1}T^1(t).$$

where $\Psi^1(r)$ has been given in equation (6) and $P_{\Psi^1}\left(2^k M\right) \times \left(2^k M\right)$ matrix given by:

$$P_{\Psi^1} = \begin{bmatrix} A & S & S & \dots & S \\ 0 & A & S & \dots & S \\ 0 & 0 & A & \dots & \vdots \\ \vdots & \vdots & \vdots & \ddots & S \\ 0 & 0 & 0 & \cdots & A \end{bmatrix};$$

Where S,A are $M \times M$ matrices as follows:

$$S = \frac{\sqrt{2}}{2^k}\begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ -\frac{1}{3} & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ -\frac{1}{15} & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\frac{1}{M(M-2)} & 0 & 0 & \dots & 0 \end{bmatrix};$$

$$A = \frac{1}{2^k}\begin{bmatrix} \frac{1}{2} & \frac{1}{2\sqrt{2}} & 0 & 0 & \dots & 0 & 0 & 0 \\ -\frac{1}{4\sqrt{2}} & 0 & \frac{1}{8} & 0 & \dots & 0 & 0 & 0 \\ -\frac{1}{3\sqrt{2}} & -\frac{1}{4} & 0 & \frac{1}{12} & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ -\frac{1}{2\sqrt{2}(M-1)(M-3)} & 0 & 0 & 0 & \dots & -\frac{1}{4(M-3)} & 0 & -\frac{1}{4(M-1)} \\ -\frac{1}{2\sqrt{2}M(M-2)} & 0 & 0 & 0 & \dots & 0 & -\frac{1}{4(M-2)} & 0 \end{bmatrix};$$

5. Applications of Matrices P_{Ψ^1} for Solving BVPs

In order to solve linear or nonlinear differential equation by using the OMI P_{Ψ^1} and, some numerical examples illustrate the procedure which will be given with:

$$y^{i}(t) = C^{T}\Psi_{n,m}(t)$$

$$y^{i-1}(t) = A^{T}\int_0^{r} T_{n,m}(t)dt + y^{i-1}(0) = C^{T}\,PT_{n,m}(t) + y^{i-1}(0).$$

$$\vdots$$

$$\vdots$$

$$y(t) = A^{T}P^{i}T_{n,m}(t) + y^{i-1}(0)t^{i-2} + y^{i-2}(0)t^{i-3} + \cdots + y(0).$$

5.1 Example

Consider the following BVP $z'' = -z$ with the boundary condition z(0)=0, z(1)=1, the exact solution for this problem is: $z(t) = \frac{\sin t}{\sin 1}$

To solve this problem using an algorithm of FCW, assume that

$$z''(t) = A^{T}T_{n,m}^{1}$$

Find

$$z'(t) = A^{T}\int_0^{t} T_{n,m}^{1}(t)dt + z'(0).$$

$$z'(t) = A^{T}P_{T^1}T_{n,m}^{1}(t) + z'(0).$$

$$z(t) = A^{T}P_{T^1}\int_0^{t} T_{n,m}^{1}(t)dt + z'(0)\,t + z(0)\,.$$

$$z(t) = A^{T}P_{T^1}^{2}T_{n,m}^{1} + z'(0)r + z(0).$$

By using the above equations we get

$$A^{T}T_{n,m}^{1} + A^{T}P_{T^1}^{2}T_{n,m}^{1} + z'(0)t + z(0) = 0 \qquad (7)$$

$$A^{T}\left(1 + P_{T^1}^{2}\right)T_{n,m}^{1} + t = 0 \qquad (8)$$

r in equation (8) can be expressed in FCW as: $r = d^{T}\Psi_{n,m}^{1}$

then equation (8) can be written:

$$A^{T}\left(1 + P_{T^1}^{2}\right)T_{n,m}^{1} + d^{T}T_{n,m}^{1} = 0 \qquad (9)$$

for $M = 3$ and $k = 1$, we obtain

$d = [0.26329694 \quad 0.18617905 \quad 0 \quad 0.78989081 \quad 0.18617905 \quad 0]^{T}$

After substituting d in equations (7) and (8), good results will be obtained, table (1) shows it.

Table (1) Shows the results in example 5.1

t	Exact solution	FCW	Absolute Error (Exact-FCW)
0	0	-0.0007437	0.00074377
0.1	0.11864154	0.17897880	0.060337264
0.2	0.23609766	0.23649435	0.000396698
0.3	0.35119477	0.35073789	0.000456871
0.4	0.46278285	0.46206441	0.00071843
0.5	0.56974637	0.57047392	0.00072755
0.6	0.67101835	0.67151678	0.00049833
0.7	0.76558515	0.76582819	0.000243049
0.8	0.85250247	0.85210270	0.000399767
0.9	0.93090187	0.93034029	0.00056158
1	1	1.00054097	0.00054097

The comparison between the solutions obtained by using FCW and exact solution is made

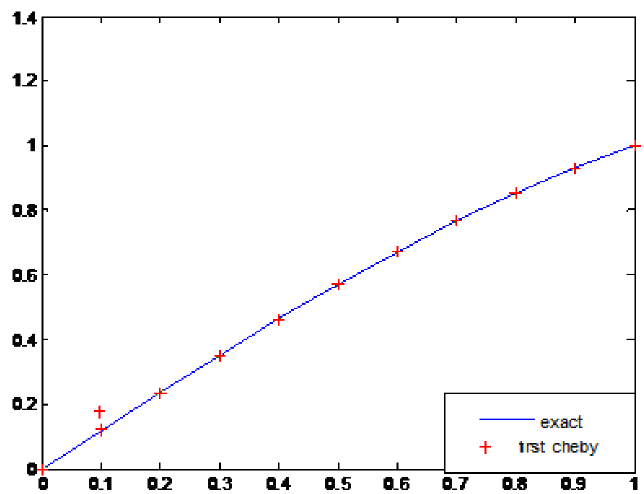

Figure 1 :comparison results between approximate and exact solution in example 5.1

5.2 Example

Consider the following boundary value problem $y'' = y$ with the boundary conditions $y(0) = 0, y(1) = 1$, the exact solution is: $y(t) = \frac{e^t - e^{-t}}{e^1 - e^{-1}}$

Similarity example (1) and by using our algorithms obtained following results

Table (2) show numerical results.in example 5.2

t	Exact solution	FCW	Absolute Error (Exact-FCW)
0	0	0.00057584	0.00057584
0.1	0.08523370	0.08469184	0.00541863
0.2	0.17132045	0.17097404	0.000346414
0.3	0.25912184	0.29422446	0.03510262
0.4	034951660	0.35003704	0.00052044
0.5	0.44340944	0.44281784	0.00059160
0.6	0.54174007	0.54103704	0.00070303
0.7	0.64549262	0.64500906	0.00048356
0.8	0.75570548	0.75603258	0.00032710
0.9	0.87348169	0.87410757	0.00062588
1	1	0.99923405	0.00076595

The comparison between the solutions obtained by using FCW and exact solution is made

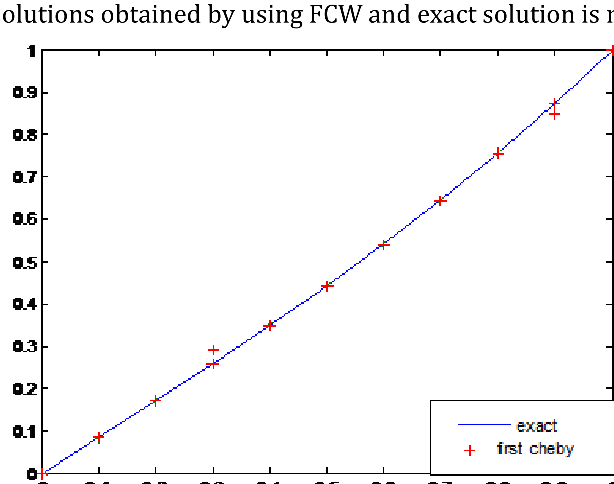

Figure 2: comparison results between approximate and exact solution in example 5.2

5.3 Example

Consider the following boundary value problem $g'' + g = -r$,
with the boundary conditions $g(0) = 1,\ g(1) = 2$, the exact solution is:

$$g(t) = \cos t \frac{3 - \cos 1}{\sin 1} \sin t - t$$

Similarity example (1) and by using our algorithms we obtained the following results

Table (3) Show numerical results in example 5.3

t	Exact solution	FCW	Absolute Error (Exact-FCW)
0	1	0.99832938	0.00167062
0.1	1.18682650	1.18836458	0.00153808
0.2	1.36079545	1.36161154	0.00081609
0.3	1.51916945	1.51807025	0.0010992
0.4	1.65936691	1.65774071	0.0016262
0.5	1.77898785	1.78062292	0.00163507
0.6	1.87583791	1.87315222	0.00268569
0.7	1.94795021	1.93676639	0.01118382
0.8	1.99360506	1.98134699	0.01225807
0.9	2.01134714	2.00689403	0.00445311
1	2	2.01340751	0.01340751

The comparison between the solutions obtained by using FCW and exact solution is made

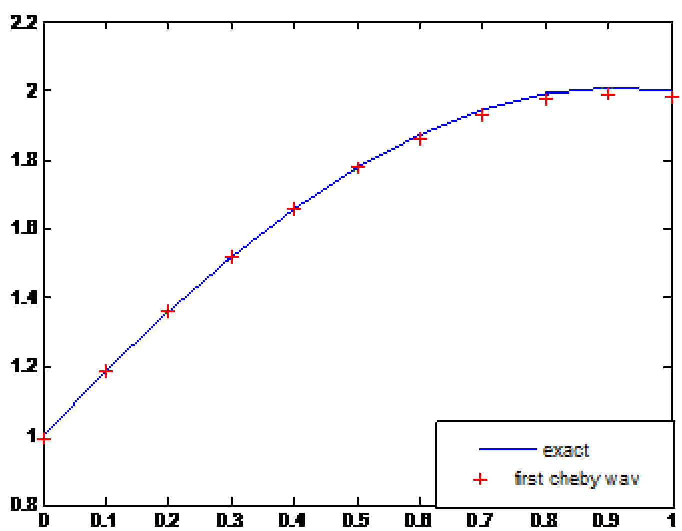

Figure 3: comparison results between approximate and exact solution in example 5.3

6. Signals are processed using wavelets

Electrical engineering has a subfield called signal processing such as sound, images and biological images which are processed by analysis, modification and synthesis. These processors help improve the transmitter as it depends on storage efficiency and quality. The following example used the proposed method for de noise signal by using MATLAB program, in Fig.4 S is a signal with noise by use CW in level 5 analyses to approximate coefficients and details coefficients can be illustrate the following statements.

The signal S is found its root started analysis signal from the approximation $A_0 = S$ in level n=0 second step will get the detail in level 1

$$D_1 = A_0 - A_1 = s - A_1$$
$$S = D_1 + A_1$$

And in level 2

$$D_2 = A_1 - A_2$$
$$= S - D_1 - A_2$$
$$S = D_2 + D_1 + A_2$$

In level 5 say

$$S = D_5 + D_4 + D_3 + D_2 + D_1 + A_4$$

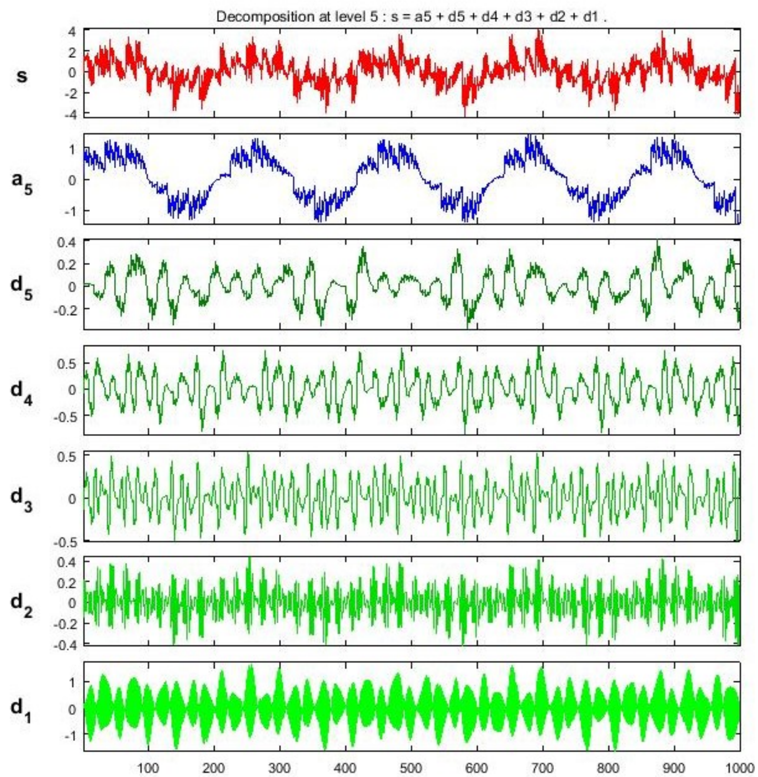

Figure 4: the analyses signal by using chebysheve wavelet in level 5

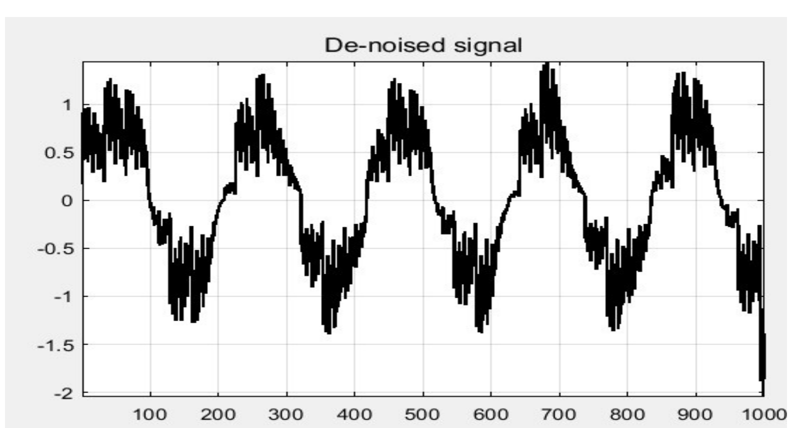

Figure 5 : the De noising signal by using chebysheve wavelet in level 5

7. Conclusion

In this paper presented for FCW and it' s OMI, the numerical results show an algorithm very efficient for the numerical solution of BVPs and we obtained a good approximate solution for these problems. Using FCW give high accuracy approximation of solution BVPs in examples above.

In addition, noise from the signal was raised using the proposed theory and acceptable results were obtained compared to other wavelets.

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