

Some Generalized Sets and Mappings in Intuitionistic Topological Spaces

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ABSTRACT

In this paper we introduce new types of intuitionistic regular generalized closed set, intuitionistic generalized pre regular -closed set, intuitionistic weakly generalized closed set, intuitionistic strongly generalized semi closed set, intuitionistic weakly closed set, intuitionistic semi weakly generalized closed set, intuitionistic pre weakly generalized closed set, intuitionistic regular-weakly generalized closed set, intuitionistic regular w-closed set, intuitionistic regular generalized α -closed set and study the relations among them. Through the seconcepts we introduce a new class of mapping of intuitionistic generalized pre regular -closed map intuitionistic, weakly generalized closed map, intuitionistic strongly generalized semi closed map, intuitionistic weakly closed map, intuitionistic semi weakly generalized closed map intuitionistic pre weakly generalized closed map, intuitionistic regular weakly generalized closed map, intuitionistic regular w-closed map, strongly $Irg\alpha$ -continuous, $Irg\alpha$ -irresolute map, $Irg\alpha$ -continuous map. We study and investigate some characterizations and relationships among them.

Mathematics Subject Classification: 54XX .

Keywords: Intuitionistic regular generalized closed set, intuitionistic generalized pre regular -closed set, intuitionistic weakly generalized closed set.

INTRODUCTION

The concept of fuzzy set was introduced by Zadeh [15] in his classical paper 1965. After the discovery of the fuzzy sets much attention has been paid to generalize the basic concepts of classical topology in fuzzy setting and thus a subset naturally plays a very significant role in the study of fuzzy topology which was introduced by Chang 1968 [6], and later by Malghan and Benchalli in 1981 [10]. In 1983, Atanassov introduced the concept of "Intuitionistic fuzzy set" [1],[2],[3],[4] using a type of generalized fuzzy set. Later, the concept is used to define intuitionistic fuzzy special sets by Coker [7], and intuitionistic fuzzy topological spaces are introduced by Coker [8]. In this direction, the concept of separation axioms in intuitionistic fuzzy topological spaces which was introduced by Bayhan, S. and Coker, D [5]. Also concept of intuitionistic topological spaces which was introduced by Coker in 2000. [9]

In this paper and through this concepts of (intuitionistic regular generalized, intuitionistic generalized pre regular -closed set intuitionistic, weakly generalized closed set intuitionistic strongly generalized semi closed set intuitionistic weakly closed set, intuitionistic semi weakly generalized closed set, intuitionistic pre weakly generalized closed set, intuitionistic regular-weakly generalized closed set, intuitionistic regular w-closed set and intuitionistic regular generalized α -closed set) we introduce a new class of mapping of intuitionistic generalized pre regular -closed map intuitionistic, weakly generalized closed map, intuitionistic strongly generalized semi closed map, intuitionistic weakly closed map, intuitionistic semi weakly generalized closed map, intuitionistic pre-weakly generalized closed map, intuitionistic regular-weakly generalized closed map, intuitionistic regular w-closed map, strongly Irg α -continuous, Irg α -irresolute map, Irg α -continuous map. Also we study and investigate some characterizations and relationship among them.

Preliminaries

Definition 1.1 [7]

Let X be a non empty set . An intuitionistic set A is an object having the form $A = \langle x, A_1, A_2 \rangle$, where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \emptyset$. The set A_1 is called the set of members of A , while A_2 is called the set of nonmembers of A .

Remark

Any subset A of X can be regarded as intuitionistic set having the form $\tilde{A} = \langle x, A, A^c \rangle$.

Definition 1.2[7]

Let X be a nonempty set , and let $A = \langle x, A_1, A_2 \rangle$ and $B = \langle x, B_1, B_2 \rangle$ be intuitionistic sets respectively, furthermore, let $\{ A_i ; i \in J \}$ be an arbitrary family of intuitionistic sets in X , where $A_i = \langle x, A_i^{(1)}, A_i^{(2)} \rangle$, then

- 1) $A \subseteq B$ if and only if $A_1 \subseteq B_1$ and $B_2 \subseteq A_2$,
- 2) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
- 3) The complement of A is denoted by \bar{A} and defined by $\bar{A} = \langle x, A_2, A_1 \rangle$,
- 4) $\bar{A} = \langle x, A_1, A_1^c \rangle$, $\bar{S} = \langle x, A_2^c, A_2 \rangle$,
- 5) $\cup A_i = \langle x, \cup A_i^{(1)}, \cap A_i^{(2)} \rangle$, $\cap A_i = \langle x, \cap A_i^{(1)}, \cup A_i^{(2)} \rangle$,
- 6) $\tilde{\Phi} = \langle x, \emptyset, X \rangle$, $\tilde{X} = \langle x, X, \emptyset \rangle$.

Definition 1.3 [7]

Let X be a nonempty set , $p \in X$ a fixed element in X , and let $A = \langle x, A_1, A_2 \rangle$ be an intuitionistic set (IS, for short).The IS \dot{p} defined by $\dot{p} = \langle x, \{p\}, \{p\}^c \rangle$ is called an intuitionistic point (IP for short) in X .The IS $\ddot{p} = \langle x, \emptyset, \{p\}^c \rangle$ is called a vanishing intuitionistic point (VIP, for short) in X . The IS \dot{p} is said to be contained in A ($\dot{p} \in A$, for short) if and only if $p \in A_1$, and similarly IS \ddot{p} contained in A ($\ddot{p} \in A$, for short) if and only if $p \notin A_2$ For a given IS A in X , we may write $A = (\cup \{ \dot{p} : \dot{p} \in A \}) \cup (\cup \{ \ddot{p} : \ddot{p} \in A \})$, and whenever A is not a proper IS (i.e., if A is not of the form $A = \langle x, A_1, A_2 \rangle$ where $A_1 \cup A_2 \neq X$), then $A = \cup \{ \dot{p} : \dot{p} \in A \}$ hold . In general, any IS A in X can be written in the form $A = \dot{A} \cup \ddot{A}$ where $\dot{A} = \cup \{ \dot{p} : \dot{p} \in A \}$ and $\ddot{A} = \cup \{ \ddot{p} : \ddot{p} \in A \}$.

Definition 1.4 [7]

Let X and Y be two nonempty sets and $f : X \rightarrow Y$ be a function.

- a) If $B = \langle y, B_1, B_2 \rangle$ is an IS in Y , then the pre image (inverse image) of B under f is denoted by $f^{-1}(B)$ is an IS in X and defined by $f^{-1}(B) = \langle x, f^{-1}(B_1), f^{-1}(B_2) \rangle$.
- b) If $A = \langle x, A_1, A_2 \rangle$ is an IS in X , then the image of A under f denoted by $f(A)$ is an IS in Y defined by $f(A) = \langle y, f(A_1), \underline{f}(A_2) \rangle$, where $\underline{f}(A) = (f(A_2^c))^c$.

Definition 1.5[7],[8]

An intuitionistic topology on a nonempty set X is a family T of an intuitionistic sets in X satisfying the following conditions.

- (1) $\emptyset, X \in T$.
- (2) T is closed under finite intersections.
- (3) T is closed under arbitrary unions.

The pair (X, T) is called an intuitionistic topological space (ITS, for short) .Any element in T is usually called intuitionistic open set(IOS ,for short).

The complement of an IOS in a ITS (X, T) is called intuitionistic closed set (ICS ,for short).

Definition 1.6[13]

Let (X, T) be an ITS and let $A = \langle x, A_1, A_2 \rangle$ be an intuitionistic subset (IS's, forshort) in a set X . The interior (IntA, forshort) and closure (ClA, forshort) of a set A of X are defined : $\text{Int}A = \cup \{ G : G \subseteq A, G \in T \}$, $\text{Cl}A = \cap \{ F : A \subseteq F, \bar{F} \in T \}$. In other words: The IntA is the largest intuitionistic open set contained in A , and ClA is the smallest intuitionistic closed set contain A i.e., $\text{Int}A \subseteq A$ and $A \subseteq \text{Cl}A$.In the following definition we give a product of an intuitionistic set and a product of an intuitionistic topological space .

Definition 1.7.[13]

Let (X, T) be an ITS and let $A = \langle x, A_1, A_2 \rangle$ be an intuitionistic set , A is said to be Ig-closed set if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in X .

Definition 1.8.[13]

A map $f : (X, T) \rightarrow (Y, \gamma)$ is called Ig-continuous if the inverse image of every is Ig-closed set of (Y, γ) for every intuitionistic closed set of (Y, γ) .

Definition: 1.9.[5]

Let (X, T) be an ITS and $A = \langle x, A_1, A_2 \rangle$ be an intuitionistic set .Then A is said to be

- (i) intuitionistic regular open (intuitionistic regular closed) if $A = \text{Int}(\text{Icl}(A))$
where ($A = \text{Icl}(\text{Int}(A))$).
- (ii) intuitionistic generalized closed (Ig-closed) if $\text{Icl}(A) \subseteq U$ whenever $A \subseteq U$ and U is intuitionistic open in X .

intuitionistic regular α open (Ir α -open) if $\text{Icl}(A) \subseteq U$ whenever $A \subseteq U$ and U is (iii) (intuitionistic regular closed in X .

intuitionistic regular semi open (Irs-open) if $\text{IScl}(A) \subseteq U$ whenever $A \subseteq U$ and U is (iv) intuitionistic semi regular closed in X .

Section.2 Some Generalized Sets in Intuitionistic Topological Spaces

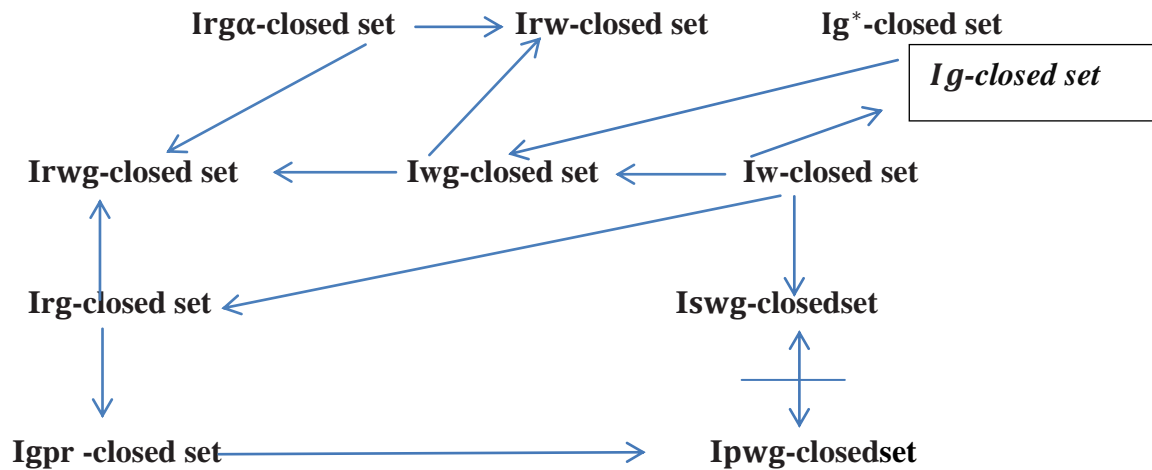
In this section we introduce new types of intuitionistic regular generalized (Irg-closed set, for short), intuitionistic generalized pre regular -closed set (Igpr-closed set, for short) intuitionistic weakly generalized closed set (Iwg-closed set, for short), intuitionistic strongly generalized semi closed set (Ig*-closed set, for short), intuitionistic weakly closed set (Iw-closed, for short), intuitionistic semi weakly generalized closed set (Iswg-closed, for short), intuitionistic pre weakly generalized closed set (Ipwg-closed, for short), intuitionistic regular-weakly generalized closed set (Irwg-closed, for short), intuitionistic regular w-closed set (Irw-closed, for short), intuitionistic regular generalized α -closed set (Irg α -closed, for short).

Definition 2.1 let (X, T) be an ITS, and let $A = \langle x, A_1, A_2 \rangle$ be an IS in X , then A is said to be:

- 1) intuitionistic regular generalized closed set (Irg-closed, for short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is intuitionistic regular open in X .
- 2) intuitionistic generalized pre regular closed set (Igpr-closed, for short) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is intuitionistic regular open in X .
- 3) intuitionistic weakly generalized closed set (Iwg-closed, for short) if $clint(A) \subseteq U$ whenever $A \subseteq U$ and U is intuitionistic open in X .
- 4) intuitionistic strongly generalized semi-closed set (Ig*-closed, for short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is intuitionistic g-open in X .
- 5) intuitionistic weakly closed set (Iw-closed, for short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is intuitionistic semi open in X .
- 6) intuitionistic semi weakly generalized closed set (Iswg-closed, for short) if $clint(A) \subseteq U$ whenever $A \subseteq U$ and U is intuitionistic semi-open in X .
- 7) intuitionistic pre weakly generalized closed set (Ipwg-closed, for short), if $clint(A) \subseteq U$ whenever $A \subseteq U$ and U is intuitionistic pre-open in X .
- 8) intuitionistic regular weakly generalized closed set (Irwg-closed, for short) if $clint(A) \subseteq U$ whenever $A \subseteq U$ and U is intuitionistic regular open in X .
- 9) intuitionistic regular w-closed set (Irw-closed, for short) if $cl(A) \subset U$ whenever $A \subset U$ and U is intuitionistic regular semi open in X .
- 10) intuitionistic regular generalized α -closed set (Irg α -closed, for short) if $\alpha cl(A) \subset U$ whenever $A \subset U$ and U is intuitionistic regular α -open in X .

The complements of the above mentioned closed sets are their respective open sets.

Proposition 2.2.let (X, T) be an ITS, then the following implications are valid:



Proof. $\text{Irg}\alpha\text{-closed set} \longrightarrow \text{Irw-closed set}$

Let $F = \langle x, F_1, F_2 \rangle$, such that $A = \langle x, A_1, A_2 \rangle \subseteq F$ then $\text{cl}(A) \subset F$. Since (X, T) is $\text{Irg}\alpha\text{-closed}$ then $\text{acl}(A) \subset F$, where F is intuitionistic regular α -open in X . Since every intuitionistic regular α -open in X is intuitionistic regular semi open in X . Therefore (X, T) is Irw-closed set. The other proofs are same way. The converse of the above Proposition 2.2. need not be true, as seen from the following examples

Example 2.3. Let $X = \{a, b, c\}$ with topology $T = \{\tilde{X}, \tilde{\emptyset}, A, B, C\}$, where $A = \langle x, \{a\}, \{b\} \rangle$, $B = \langle x, \{a, c\}, \emptyset \rangle$, $C = \langle x, \{a\}, \emptyset \rangle$ Then (X, T) is Irw-closed set but not $\text{Irg}\alpha\text{-closed}$ set.

Example 2.4. Let $X = \{a, b, c\}$ with topology $T = \{\tilde{X}, \tilde{\emptyset}, A, B\}$, where $A = \langle x, \{a\}, \emptyset \rangle$, $B = \langle x, \{a, c\}, \emptyset \rangle$. Then (X, T) is Irwg-closed set but not Irw-closed set, also Irwg-closed set but not Iwg-closed set.

Example 2.5. Let $X = \{a, b\}$ with topology $T = \{\tilde{X}, \tilde{\emptyset}, A, B\}$, where $A = \langle x, \emptyset, \{b\} \rangle$, $B = \langle x, \{a\}, \emptyset \rangle$, and $\text{SOX} = \{\tilde{X}, \tilde{\emptyset}, A, B, C, D\}$, where $C = \langle x, \{b\}, \emptyset \rangle$, $D = \langle x, \emptyset, \emptyset \rangle$. Then (X, T) is $\text{Ig}^*\text{-closed}$ set but not Iwg-closed set.

Example 2.6. Let $X = \{a, b\}$ with topology $T = \{\tilde{X}, \tilde{\emptyset}, A, B\}$, where $A = \langle x, \emptyset, \{b\} \rangle$, $B = \langle x, \emptyset, \{a\} \rangle$, $C = \langle x, \emptyset, \emptyset \rangle$. Then (X, T) is Iwg-closed set but not Iw-closed set, also Then (X, T) is Iwg-closed set but Ig-closed set

Example 2.7. Let $X = \{a, b, c\}$ with topology $T = \{\tilde{X}, \tilde{\emptyset}, A, B\}$, where $A = \langle x, \{a\}, \{b\} \rangle$, $B = \langle x, \{a\}, \emptyset \rangle$. Then (X, T) is Irg-closed set but not Iw-closed set.

Example 2.8. Let $X = \{a, b\}$ with topology $T = \{\tilde{X}, \tilde{\emptyset}, A, B, C\}$, where $A = \langle x, \{a\}, \{b\} \rangle$, $B = \langle x, \{a\}, \emptyset \rangle$, $C = \langle x, \emptyset, \emptyset \rangle$, and $\text{SOX} = \{\tilde{X}, \tilde{\emptyset}, A, B, C, D\}$, where $D = \langle x, \{b\}, \emptyset \rangle$. Then (X, T) is Iswg-closed set but not Iw-closed set.

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Example 2.9. Let $X = \{a, b\}$ with topology $T = \{\tilde{X}, \tilde{\emptyset}, A, B, C\}$, where $A = \langle x, \{a\}, \{b\} \rangle$, $B = \langle x, \{a\}, \emptyset \rangle$, $C = \langle x, \emptyset, \emptyset \rangle$, and $POX = \{\tilde{X}, \tilde{\emptyset}, A, B, C, D, E\}$, where $D = \langle x, \{b\}, \emptyset \rangle$, $E = \langle x, \emptyset, \{b\} \rangle$. Then (X, T) is I_{pwg} -closed set but not I_{gpr} -closed set.

Example 2.10. Let $X = \{a, b\}$ with topology $T = \{\tilde{X}, \tilde{\emptyset}, A, B, C\}$, where $A = \langle x, \{a\}, \{b\} \rangle$, $B = \langle x, \{b\}, \{a, c\} \rangle$, $C = \langle x, \{a, b\}, \emptyset \rangle$. Then (X, T) is I_g -closed set but not I_w -closed set.

Remark 2.11. by transitive: $I_{rg\alpha}$ -closed set \longrightarrow I_{rwg} -closed set,
 I_w -closed set \longrightarrow I_{rwg} -closed set, I_w -closed set \longrightarrow I_{rg} -closed set,
 I_{rg} -closed set \longrightarrow I_{pwg} -closed set, I_w -closed set \longrightarrow I_{gpr} -closed set and
 I_{gpr} -closed set \longrightarrow I_{g^*} -closed set.

Remark 2.12. The following examples show that the I_{swg} -closed set and I_{gpr} -closed set are independent.

Example 2.13. Recall **Example 2.8.** we see that (X, T) is I_{swg} -closed set but not I_{gpr} -closed set.

Example 2.14. Let $X = \{a, b, c\}$ with topology $T = \{\tilde{X}, \tilde{\emptyset}, A, B\}$, where $A = \langle x, \{a\}, \emptyset \rangle$, $B = \langle x, \emptyset, \emptyset \rangle$. Then (X, T) is I_{gpr} -closed set but not I_{swg} -closed set.

Remark 2.15. The following examples show that the I_w -closed set and I_{swg} -closed set are independent.

Example 2.16. Recall **Example 2.8.** we see that (X, T) is I_{swg} -closed set but not I_w -closed set.

Example 2.17. Recall **Example 2.6.** we see that (X, T) is I_w -closed set but not I_{swg} -closed set.

Remark 2.18. The following examples show that the I_{swg} -closed set and I_{pwg} -closed set are independent.

Example 2.19. Let $X = \{a, b, c\}$ with topology $T = \{\tilde{X}, \tilde{\emptyset}, A, B, C\}$, where $A = \langle x, \{a\}, \{b\} \rangle$, $B = \langle x, \{a\}, \emptyset \rangle$, $C = \langle x, \emptyset, \{b\} \rangle$ and $SOX = \{\tilde{X}, \tilde{\emptyset}, A, B, C, D\}$, where $D = \langle x, \emptyset, \emptyset \rangle$ and $POX = T$. Then (X, T) is I_{swg} -closed set but not I_{pwg} -closed set.

Example 2.20. Recall **Example 2.9.** we see that (X, T) is I_{pwg} -closed set but not I_{swg} -closed set.

Remark 2.21. The following examples show that the I_{g^*} -closed set and I_{rw} -closed set are independent.

Example 2.22. Recall **Example 2.5.** we see that (X, T) is I_{g^*} -closed set but not I_{rw} -closed set.

Example 2.23. Recall **Example 2.3.** we see that (X, T) is I_{rw} -closed set but not I_{g^*} -closed set.

Note : In general topology rw -closed set \longrightarrow rg -closed set. [15]

Section.3 Some Generalized Mappings in Intuitionistic Topological Spaces

We introduce the following definitions :

Definition 3.1. A map $f : (X, T) \rightarrow (Y, \gamma)$ is said to be intuitionistic regular generalized α -closed (briefly, Irg α -closed) maps if the image of every intuitionistic closed set in (X, T) is Irg α -closed in (Y, γ) .

Definition 3.2. A map $f : (X, T) \rightarrow (Y, \gamma)$ is called

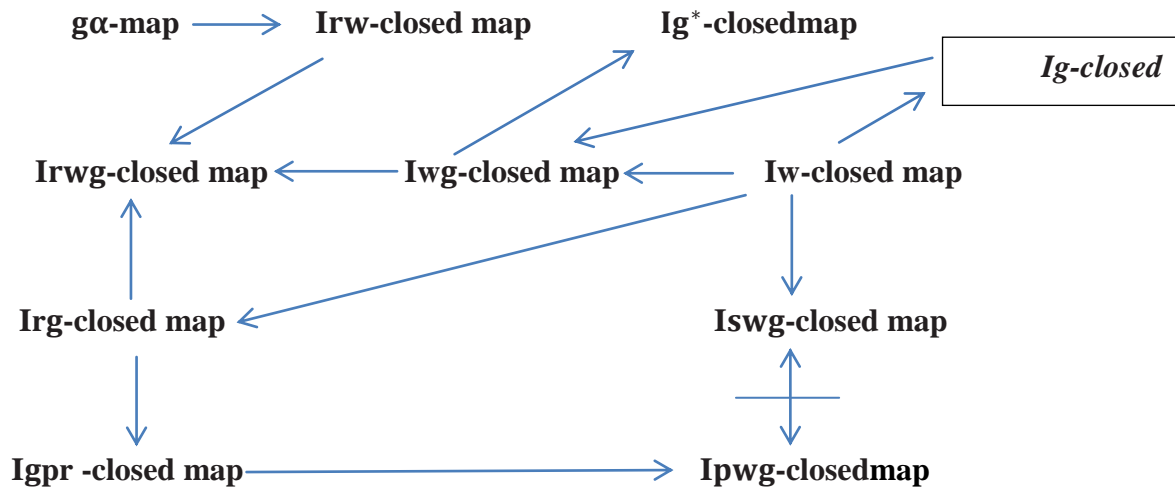
- (i) Irg α -continuous : if the inverse image of every intuitionistic closed set in set V of (Y, γ) , is Irg α -closed set in (X, T) ,
- (ii) Irg α -irresolute map: if the inverse image of every Irg α -closed set in (Y, γ) is Irg α -closed in (X, T) ,
- (iii) strongly Irg α -continuous : if the inverse image of every Irg α -open set in (Y, γ) is open in (X, T) .

Definition 3.3. A map $f : (X, T) \rightarrow (Y, \gamma)$ is said to be

- (i) Irw-closed: if every image is Irw-closed in (Y, γ) for each intuitionistic regular semi open set of (X, T) ,
- (ii) Iw-closed: if every image is Iw-closed in (Y, γ) for each intuitionistic closed set of (X, T) ,
- (iii) Iwg-closed: if every image is Iwg-closed in (Y, γ) for each intuitionistic closed set of (X, T) ,
- (iv) Irwg-closed: if every image is Irwg-closed in (Y, γ) for each intuitionistic closed set of (X, T) ,
- (v) Irg-closed: if every image is Irg-closed in (Y, γ) for each intuitionistic closed set of (X, T) ,
- (vi) Igpr-closed: if every image is Igpr-closed in (Y, γ) for each intuitionistic closed set of (X, T) ,
- (vii) Ig*-closed if every image is Ig*-closed in (Y, γ) for each intuitionistic γ -closed set of (X, T) ,
- (viii) Irg α -closed if every image is Irg α -closed in (Y, γ) for each intuitionistic regular α -closed in (X, T) ,
- (viiii) Iswg-closed if every image is Iswg-closed in (Y, γ) for each intuitionistic semi-closed in (X, T) ,

(viii) I_{pwg} - closed if every image is I_{pwg} -closed in (Y, γ) for each intuitionistic pre-closed in (X, T) .

Proposition 3.4. Let $f : (X, T) \rightarrow (Y, \gamma)$ be a mapping, then the following implications are valid:



Proof: $Irg\alpha$ -closed map \longrightarrow Irw-closed map

The prove follows from the definitions and fact that every $Irg\alpha$ - closed is Irw-closed .

Irw-closed map \longrightarrow Igpr -closed map

The prove follows from the definitions and fact that every Irw-closed set is Igpr –closed.

Irw-closed map \longrightarrow Irwg-closed map

The proof follows from the definitions and fact that every Irw-closed set is Irwg-closed

Iwg-closed map \longrightarrow Irwg-closed map

The proof follows from the definitions and fact that every Iwg-closed set is Irwg-

closed. **Iwg-closed map \longrightarrow Ig*-closed map**

The proof follows from the definitions and fact that every Iwg-closed set is Ig*-closed. The other proofs are same way.

The converse of the above Proposition 3.4. need not be true, as seen from the following examples .

Example 3.5. Let $X = \{a, b, c\}$ with topology $T = \{\tilde{X}, \tilde{\emptyset}, A, B, C\}$, where $A = \langle x, \{a\}, \{b\} \rangle$, $B = \langle x, \{a, c\}, \emptyset \rangle$, $C = \langle x, \{a\}, \emptyset \rangle$, $Y = \{1, 2, 3\}$ with topology $\gamma = \{\tilde{Y}, \tilde{\emptyset}, D, E\}$, where $D = \langle x, \{1\}, \{2\} \rangle$, $E = \langle x, \{1, 2\}, \emptyset \rangle$. Define a function $f: X \rightarrow Y$ by $f(a) = 1$, $f(b) = 3$, $f(c) = 2$. Then f is Irw-closed map because every image is Irw-closed in (Y, γ) for every intuitionistic regular semi open set of (X, T) , but not $Irg\alpha$ -closed map because for every intuitionistic regular α -open in (X, T) there is no image satisfy $Irg\alpha$ -closed in (Y, γ) .

Example 3.6. Let $X = \{a, b, c\}$ with topology $T = \{\tilde{X}, \tilde{\emptyset}, A, B\}$, where $A = \langle x, \{a\}, \emptyset \rangle$, $B = \langle x, \{a, c\}, \emptyset \rangle$, $Y = \{1, 2, 3\}$ with topology $\gamma = \{\tilde{Y}, \tilde{\emptyset}, C, D\}$, where $C = \langle x, \{1\}, \{2\} \rangle$, $D = \langle x, \{1, 3\}, \emptyset \rangle$. Define a function by $f: X \rightarrow Y$ by $f(a) = 1, f(b) = 3, f(c) = 2$. Then f is Irwg-closed map because every image is Irwg-closed in (Y, γ) for every intuitionistic closed set of (X, T) , but not Iwg-closed map because for every intuitionistic closed set of (X, T) , there is no image satisfy Iwg-closed in (Y, γ) .

Example 3.7. Let $X = \{a, b, c\}$ with topology $T = \{\tilde{X}, \tilde{\emptyset}, A, B\}$, where $A = \langle x, \emptyset, \{b\} \rangle$, $B = \langle x, \{a\}, \emptyset \rangle$, $Y = \{1, 2, 3\}$ with topology $\gamma = \{\tilde{Y}, \tilde{\emptyset}, C, D\}$, where $C = \langle x, \emptyset, \{2\} \rangle$, $D = \langle x, \{1, 2\}, \emptyset \rangle$, $SOX = \{\tilde{X}, \tilde{\emptyset}, A, B, E, F\}$, where $E = \langle x, \{b\}, \emptyset \rangle$, $F = \langle x, \emptyset, \emptyset \rangle$, $SOY = \{\tilde{Y}, \tilde{\emptyset}, C, D, G, H\}$, where $G = \langle x, \{2\}, \emptyset \rangle$, $H = \langle x, \emptyset, \emptyset \rangle$. Define a function $f: X \rightarrow Y$ by $f(a) = 1, f(b) = f(c) = 2$. Then f is Ig^* -closed map because every image is Ig^* -closed in (Y, γ) for every intuitionistic g -closed set of (X, T) , but not Iwg-closed map because for every intuitionistic closed set of (X, T) , there is no image satisfy Iwg-closed in (Y, γ) .

Example 3.8. Let $X = \{a, b\}$ with topology $T = \{\tilde{X}, \tilde{\emptyset}, A, B\}$, where $A = \langle x, \emptyset, \{b\} \rangle$, $B = \langle x, \emptyset, \{a\} \rangle$, $C = \langle x, \emptyset, \emptyset \rangle$, $Y = \{1, 2, 3\}$ with topology $\gamma = \{\tilde{Y}, \tilde{\emptyset}, D, E, F\}$, where $D = \langle x, \emptyset, \{1\} \rangle$, $E = \langle x, \emptyset, \{3\} \rangle$, $F = \langle x, \emptyset, \emptyset \rangle$. Define a function $f: X \rightarrow Y$ by $f(a) = 1 = f(b)2$. Then f is Iwg-closed set because every image is Iwg-closed in (Y, γ) for every intuitionistic closed set of (X, T) , but not Iw-closed map because for every intuitionistic closed set of (X, T) , there is no image satisfy Iw-closed in (Y, γ) . Also f is not Ig -closed map because for every intuitionistic closed set of (X, T) , there is no image satisfy Ig -closed in (Y, γ) .

Example 3.9. Let $X = \{a, b, c\}$ with topology $T = \{\tilde{X}, \tilde{\emptyset}, A, B\}$, where $A = \langle x, \{a\}, \{b\} \rangle$, $B = \langle x, \{a\}, \emptyset \rangle$, $Y = \{1, 2, 3\}$ with topology $\gamma = \{\tilde{Y}, \tilde{\emptyset}, C, D\}$, where $C = \langle x, \{3\}, \{1\} \rangle$, $D = \langle x, \{3\}, \emptyset \rangle$. Define a function $f: X \rightarrow Y$ by $f(a) = 3, f(b) = 1, f(c) = 2$. Then f is Irg-closed map because every image is Irg-closed in (Y, γ) for every intuitionistic closed set of (X, T) , but not Iw-closed map because for every intuitionistic closed set of (X, T) , there is no image satisfy Iw-closed in (Y, γ) .

Example 3.10. Let $X = \{a, b\}$ with topology $T = \{\tilde{X}, \tilde{\emptyset}, A, B, C\}$, where $A = \langle x, \{a\}, \{b\} \rangle$, $B = \langle x, \{a\}, \emptyset \rangle$, $C = \langle x, \emptyset, \emptyset \rangle$, and $SOX = \{\tilde{Y}, \tilde{\emptyset}, A, B, C, D\}$, where $D = \langle x, \{b\}, \emptyset \rangle$, $Y = \{1, 2, 3\}$ with topology $\gamma = \{\tilde{Y}, \tilde{\emptyset}, E, F\}$, where $E = \langle x, \{1\}, \{2\} \rangle$, $F = \langle x, \{1\}, \emptyset \rangle$, $SOY = \{\tilde{Y}, \tilde{\emptyset}, C, E, F, G\}$, where $G = \langle x, \{2\}, \emptyset \rangle$. Define a function $f: X \rightarrow Y$ by $f(a) = 1, f(b) = 2, f(c) = 3$. Then f is Iswg-closed map because every image is Iswg-closed in (Y, γ) for every intuitionistic semi-closed in (X, T) , but not Iw-closed map because for every intuitionistic closed set of (X, T) , there is no image satisfy Iw-closed in (Y, γ) .

Example 3.11. Let $X = \{a, b\}$ with topology $T = \{\tilde{X}, \tilde{\emptyset}, A, B, C\}$, where $A = \langle x, \{a\}, \{b\} \rangle$, $B = \langle x, \{a\}, \emptyset \rangle$, $C = \langle x, \emptyset, \emptyset \rangle$, $POX = \{\tilde{X}, \tilde{\emptyset}, A, B, C, D\}$, where $D = \langle x, \{b\}, \emptyset \rangle$, $E = \langle x, \emptyset, \{b\} \rangle$, $Y = \{1, 2, 3\}$ with topology $\gamma = \{\tilde{Y}, \tilde{\emptyset}, C, E, F\}$, where $E = \langle x, \{1\}, \{2\} \rangle$, $F = \langle x, \{1\}, \emptyset \rangle$, $POY = \{\tilde{Y}, \tilde{\emptyset}, C, E, F, G, H\}$, where $G = \langle x, \{2\}, \emptyset \rangle$, $H = \langle x, \emptyset, \{2\} \rangle$. Define a function $f: X \rightarrow Y$ by $f(a) = 1, f(b) = f(c) = 3$. Then f is Ipwg-closed map because every image is Ipwg-closed in (Y, γ) for every intuitionistic pre-closed in (X, T) , but not $Igpr$ -closed map because for every intuitionistic closed set of (X, T) , there is no image satisfy $Igpr$ -closed in (Y, γ) .

Example 3.12. Let $X = \{a, b\}$ with topology $T = \{\tilde{X}, \tilde{\emptyset}, A, B\}$, where $A = \langle x, \{a\}, \{b, c\} \rangle$, $B = \langle x, \{b, c\}, \{a\} \rangle$, $Y = \{1, 2, 3\}$ with topology $\gamma = \{\tilde{Y}, \tilde{\emptyset}, C, D\}$, where $C = \langle x, \{1\}, \{2, 3\} \rangle$, $D = \langle x, \{1, 3\}, \{2\} \rangle$. Define a function $f: X \rightarrow Y$ by $f(a) = 1$, $f(b) = f(c) = 3$. Then f is Ig -closed map because every image is Ig -closed in (Y, γ) for every intuitionistic g -closed in (X, T) , but not Iw -closed map because for every intuitionistic closed set of (X, T) , there is no image satisfy Iw -closed in (Y, γ) .

Remark 3.13. by transitive: $Irg\alpha$ -closed map \longrightarrow $Irwg$ -closed map,

Iw -closed map \longrightarrow $Irwg$ -closed map, Iw -closed map \longrightarrow Irg -closed map,

Irg -closed map \longrightarrow $Ipwg$ -closed map, Iw -closed map \longrightarrow $Igpr$ -closed map

and $Igpr$ -closed set \longrightarrow Ig^* -closed set

Remark 3.14. The following examples show that the $Iswg$ -closed map and $Igpr$ -closed map are independent.

Example 3.15. Recall Example 3.10. we see that f is $Iswg$ -closed map because every image is $Iswg$ -closed in (Y, γ) for every intuitionistic semi-open in (X, T) , but not $Igpr$ -closed map because for every intuitionistic closed set of (X, T) there is no image satisfy $Igpr$ -closed in (Y, γ) .

Example 3.16. Let $X = \{a, b, c\}$ with topology $T = \{\tilde{X}, \tilde{\emptyset}, A, B\}$, where $A = \langle x, \{a\}, \emptyset \rangle$, $B = \langle x, \emptyset, \emptyset \rangle$, $Y = \{1, 2, 3\}$ with topology $\gamma = \{\tilde{Y}, \tilde{\emptyset}, C, D\}$, where $C = \langle x, \{3\}, \{1\} \rangle$, $D = \langle x, \{3\}, \emptyset \rangle$. Define a function $f: X \rightarrow Y$ by $f(a) = 3$, $f(b) = f(c) = 1$. Then f is $Igpr$ -closed map because every image is $Igpr$ -closed in (Y, γ) for every intuitionistic closed set of (X, T) , but not $Iswg$ -closed map because for every intuitionistic closed set of (X, T) , there is no image satisfy $Igpr$ -closed in (Y, γ) .

Remark 3.17. The following examples show that the Iwg -closed map and $Iswg$ -closed map are independent.

Example 3.18. Recall Example 3.10. we see that f is $Iswg$ -closed map because every image is $Iswg$ -closed in (Y, γ) for every intuitionistic semi-open in (X, T) , but not Iwg -closed map because for every intuitionistic closed set of (X, T) , there is no image satisfy Iwg -closed in (Y, γ) .

Example 3.19. Recall Example 3.8. we see that f is Iwg -closed map because every image is Iwg -closed in (Y, γ) for every intuitionistic closed set of (X, T) , but not $Iswg$ -closed map because for every intuitionistic semi-closed in (X, T) , there is no image satisfy $Iswg$ -closed in (Y, γ) .

Remark 3.20. The following examples show that the $Iswg$ -closed map and $Ipwg$ -closed map are independent.

Example 3.21. Let $X = \{a, b, c\}$ with topology $T = \{\tilde{X}, \tilde{\emptyset}, A, B, C\}$, where $A = \langle x, \{a\}, \{b\} \rangle$, $B = \langle x, \{a\}, \emptyset \rangle$, $C = \langle x, \emptyset, \{b\} \rangle$, $SOX = \{\tilde{X}, \tilde{\emptyset}, A, B, C, D\}$, where $D = \langle x, \emptyset, \emptyset \rangle$, $POX = TY = \{1, 2, 3\}$ with topology $\gamma = \{\tilde{Y}, \tilde{\emptyset}, E, F, G\}$, where $E = \langle x, \{1\}, \{2\} \rangle$, $F = \langle x, \{1\}, \emptyset \rangle$, $C = \langle x, \emptyset, \{b\} \rangle$, $SOY = \{\tilde{Y}, \tilde{\emptyset}, A, B, C, D\}$, $POY = T$. Then f is $Iswg$ -closed map because every image is $Iswg$ -closed in (Y, γ) for every intuitionistic semi-closed in (X, T) , but not $Ipwg$ -closed map because for every intuitionistic pre-open in (X, T) , there is no image satisfy $Ipwg$ -closed in (Y, γ) .

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Example 3.22. Recall Example 3.11. we see that f is I_{pwg} -closed map because every image is I_{pwg} -closed in (Y, γ) for every intuitionistic pre-open in (X, T) , but not I_{swg} -closed map because for every intuitionistic semi-closed in (X, T) , there is no image satisfy I_{swg} -closed in (Y, γ) .

Remark 3.23. The following examples show that the I_{g^*} -closed map and I_{rw} -closed map are independent.

Example 3.24. Recall Example 3.7. we see that f is I_{g^*} -closed map because every image is I_{g^*} -closed in (Y, γ) for every intuitionistic g -closed set of (X, T) , but not I_{rw} -closed map because for every intuitionistic regular semi closed set of (X, T) , there is no image satisfy I_{rw} -closed in (Y, γ) .

Example 3.25. Recall Example 2.3. we see that f is I_{rw} -closed map because every image is I_{rw} -closed in (Y, γ) for every intuitionistic regular semi closed set of (X, T) , but not I_{g^*} -closed because for every intuitionistic g -closed set of (X, T) , there is no image satisfy I_{g^*} -closed in (Y, γ) .

Proposition 3.26. If a mapping $f : (X, T) \rightarrow (Y, \gamma)$ is $I_{rg\alpha}$ -closed, then $I_{rg\alpha}\text{-cl}(f(A)) \subset f(\text{cl}(A))$ for every subset A of (X, T) .

Proof. Suppose that f is $I_{rg\alpha}$ -closed and $A \subset X$. Then $\text{cl}(A)$ is intuitionistic closed in X and so $f(\text{cl}(A))$ is $I_{rg\alpha}$ -closed in (Y, γ) . We have $f(A) \subset f(\text{cl}(A))$, so that $I_{rg\alpha}\text{-cl}(f(A)) \subset I_{rg\alpha}\text{-cl}(f(\text{cl}(A))) \rightarrow$ (i). Since $f(\text{cl}(A))$ is $I_{rg\alpha}$ -closed in (Y, γ) , so that $I_{rg\alpha}\text{-cl}(f(\text{cl}(A))) = f(\text{cl}(A)) \rightarrow$ (ii), From (i) and (ii), we have $I_{rg\alpha}\text{-cl}(f(A)) \subset f(\text{cl}(A))$ for every subset A of (X, T) .

Remark 3.7 The converse of the above Proposition 3.25. is not true in general as seen from the following example.

Example 3.28. Let $X = \{a, b, c\}$ with topology $T = \{\tilde{X}, \tilde{\emptyset}, A, B\}$, where $A = \langle x, \{a\}, \{b\} \rangle$, $B = \langle x, \{a\}, \emptyset \rangle$, $Y = \{1, 2, 3\}$ with topology $\gamma = \{\tilde{Y}, \tilde{\emptyset}, C, D\}$, where $C = \langle x, \{3\}, \{1\} \rangle$, $D = \langle x, \{3\}, \emptyset \rangle$. Define a function $f: X \rightarrow Y$ by $f(a) = 3$, $f(b) = 1$, $f(c) = 2$. Then $I_{rg\alpha}\text{-cl}(f(A)) \subset f(\text{cl}(A))$ for every subset A of (X, T) . But f is not $I_{rg\alpha}$ -closed, because there is not $I_{rg\alpha}$ -closed in (Y, γ) .

Theorem 3.29. Let (X, T) and (Y, γ) be two intuitionistic topological spaces where $I_{rg\alpha}\text{-cl}(A) = I_{w}\text{-cl}(A)$ for every subset A of Y and $f : (X, T) \rightarrow (Y, \gamma)$ be a map, then the following are equivalent.

- (i) f is $I_{rg\alpha}$ -closed map.
- (ii) $I_{rg\alpha}\text{-cl}(f(A)) \subset f(\text{cl}(A))$ for every subset A of (X, T) .

Proof. (i) \Rightarrow (ii) Follows from the Proposition 3.25.

(ii) \Rightarrow (i) Let A be any intuitionistic closed set of (X, T) . Then $A = \text{cl}(A)$ and so $f(A) = I_{rg\alpha}\text{-cl}(f(A)) \subset f(\text{cl}(A))$ by hypothesis. We have $f(A) \subset I_{rg\alpha}\text{-cl}(f(A))$. Therefore $f(A) = I_{rg\alpha}\text{-cl}(f(A))$. Also $f(A) = I_{rg\alpha}\text{-cl}(f(A)) = I_{w}\text{-cl}(f(A))$, by hypothesis. That is $f(A) = I_{w}\text{-cl}(f(A))$ and so $f(A)$ is I_{w} -closed in (Y, γ) . Thus $f(A)$ is $I_{rg\alpha}$ -closed set in (Y, γ) and hence f is $I_{rg\alpha}$ -closed map.

Theorem 3.30. A map $f : (X, T) \rightarrow (Y, \gamma)$ is $I_{rg\alpha}$ -closed if and only if for each subset S of (Y, γ) and each an intuitionistic open set U containing $f^{-1}(S) \subset U$, there is an $I_{rg\alpha}$ -open set V of (Y, γ) such that $S \subset V$ and $f^{-1}(V) \subset U$.

Proof. Suppose f is $Irg\alpha$ -closed. Let $S \subset Y$ and U be an intuitionistic open set of (X, τ_1, τ_2) such that $f^{-1}(S) \subset U$. Now $X - U = \langle x, U_2, U_1 \rangle$ is an intuitionistic closed set in (X, T) . Since f is $Irg\alpha$ -closed, $f(X - U)$ is $Irg\alpha$ -closed set in (Y, γ) . Then $V = Y - f(X - U)$ is an $Irg\alpha$ -open set in (Y, γ) . Note that $f^{-1}(S) \subset U$ implies $S \subset V$ and $f^{-1}(V) = X - f^{-1}(f(X - U)) \subset X - (X - U) = U$. That is $f^{-1}(V) \subset U$.

For the converse, let F be an intuitionistic closed set of (X, T) . Then $f^{-1}(f(F)^c) \subset F^c$ and F^c is an intuitionistic open in (X, T) . By hypothesis, there exists an $Irg\alpha$ -open set V in (Y, γ) such that $f(F)^c \subset V$ and $f^{-1}(V) \subset F^c$ and so $F \subset (f^{-1}(V))^c$. Hence $V^c \subset f(F) \subset f((f^{-1}(V))^c) \subset V^c$ which implies $f(V) \subset V^c$. Since V^c is $Irg\alpha$ -closed, $f(F)$ is $Irg\alpha$ -closed. That is $f(F)$ is $Irg\alpha$ -closed in (Y, γ) and therefore f is $Irg\alpha$ -closed.

Remark 3.31. The composition of two $Irg\alpha$ -closed maps need not be an intuitionistic closed map in general and this is shown by the following example.

Example 3.32. Let $X = \{a, b, c\}$, with topology $T = \{\tilde{X}, \tilde{\emptyset}, A, B\}$, where $A = \langle x, \{a\}, \{b, c\} \rangle$, $B = \langle x, \{a\}, \emptyset \rangle$, $Y = \{1, 2, 3\}$ with topology $\gamma = \{\tilde{Y}, \tilde{\emptyset}, C, D\}$, where $C = \langle x, \{3, 2\}, \{1\} \rangle$, $D = \langle x, \{3\}, \emptyset \rangle$, $Z = \{e, f, g\}$ with topology $\beta = \{\tilde{Z}, \tilde{\emptyset}, E, F, G, H\}$, where $E = \langle x, \{e\}, \{g\} \rangle$, $F = \langle x, \{f\}, \emptyset \rangle$, $G = \langle x, \emptyset, \{g\} \rangle$, $H = \langle x, \{e, f\}, \emptyset \rangle$. Define $f : (X, T) \rightarrow (Y, \gamma)$ by $f(a) = 1, f(b) = 3$ and $f(c) = 2$ and $g : (Y, \gamma) \rightarrow (Z, \beta)$ by $f(1) = f(2) = f(3) = e$. Then f and g are $Irg\alpha$ -closed maps, but their composition $g \circ f : (X, T) \rightarrow (Z, \beta)$ is not $Irg\alpha$ -closed map, because $K = \langle x, \{b, c\}, \{a\} \rangle$ is an intuitionistic closed in (X, T) , but $g \circ f(K) = g \circ f(K) = g(f(K)) = g(K) = \langle x, \{f\}, \{g\} \rangle$ which is not $Irg\alpha$ -closed in (Z, β) .

Theorem 3.33. If $f : (X, T) \rightarrow (Y, \gamma)$ is an intuitionistic closed map and $g : (Y, \gamma) \rightarrow (Z, \beta)$ is $Irg\alpha$ -closed map, then the composition $g \circ f : (X, T) \rightarrow (Z, \beta)$ is $Irg\alpha$ -closed map.

Proof. Let F be any an intuitionistic closed set in (X, T) . Since f is an intuitionistic closed map, $f(F)$ is bi closed set in (Y, γ) . Since g is $Irg\alpha$ -closed map, $g(f(F))$ is $Irg\alpha$ -closed set in (Z, β) . That is $g \circ f(F) = g(f(F))$ is $Irg\alpha$ -closed and hence $g \circ f$ is $Irg\alpha$ -closed map.

Theorem 3.34. Let $(X, T), (Z, \beta)$ be two intuitionistic topological spaces, and (Y, γ) be topological spaces where every $Irg\alpha$ -closed subset is an intuitionistic closed. Then the composition $g \circ f : (X, T) \rightarrow (Z, \beta)$ of the $Irg\alpha$ -closed maps $f : (X, T) \rightarrow (Y, \gamma)$ and $g : (Y, \gamma) \rightarrow (Z, \beta)$ is $Irg\alpha$ -closed.

Proof. Let A be a an intuitionistic closed set of (X, T) . Since f is $Irg\alpha$ -closed, $f(A)$ is $Irg\alpha$ -closed in (Y, γ) . Then by hypothesis, $f(A)$ is an intuitionistic closed. Since g is $Irg\alpha$ -closed, $g(f(A))$ is $Irg\alpha$ -closed in (Z, β) and $g(f(A)) = g \circ f(A)$. Therefore $g \circ f$ is $Irg\alpha$ -closed.

Theorem 3.35. If a map $f : (X, T) \rightarrow (Y, \gamma)$ is $Irg\alpha$ -closed and A is an intuitionistic closed of X , then $f_A : (A, \tau_{1A}, \tau_{2A}) \rightarrow (Y, \sigma_1, \sigma_2)$ is $Irg\alpha$ -closed.

Proof. Let F be an intuitionistic closed set of A . Then $F = A \cap E$ for some an intuitionistic closed set E of (X, τ_1, τ_2) and so F is an intuitionistic closed set of (X, T) . Since f is $Irg\alpha$ -closed, $f(F)$ is $Irg\alpha$ -closed set in (Y, γ) . But $f(F) = f_A(F)$ and therefore $f_A : (A, \tau_{1A}, \tau_{2A}) \rightarrow (Y, \sigma_1, \sigma_2)$ is $Irg\alpha$ -closed. Analogous to $rg\alpha$ -closed maps, we define $rg\alpha$ -open map as follows.

Definition 3.36. A map $f : (X, T) \rightarrow (Y, \gamma)$ is called an $Irg\alpha$ -open map if the image $f(A)$ is $Irg\alpha$ -open in (Y, γ) for each an intuitionistic open set A in (X, T) . From the definitions we have the following results.

Corollary 3.37. (i) Every an intuitionistic open map is $Irg\alpha$ -open but not conversely.

(ii) Every Iw -open map is $Irg\alpha$ -open but not conversely.

(iii) Every $Irg\alpha$ -open map is Irg -open but not conversely.

(iv) Every $Ig\alpha$ -open map is $Irwg$ -open but not conversely.

(v) Every $Irg\alpha$ -open map is $Igpr$ -open but not conversely.

Theorem 3.38. For any bijection map $f : (X, T) \rightarrow (Y, \gamma)$, the following statements are equivalent:

(i) $f^{-1} : (X, T) \rightarrow (Y, \gamma)$ is $Irg\alpha$ -continuous.

(ii) f is $Irg\alpha$ -open map.

(iii) f is $Irg\alpha$ -closed map.

Proof. (i) \Rightarrow (ii) Let U be an intuitionistic open set of (X, T) . By assumption, $(f^{-1})(U) = f(U)$ is $Irg\alpha$ -open in (Y, γ) and so f is $Irg\alpha$ -open.

(ii) \Rightarrow (iii) Let $F = \langle x, F_2, F_1 \rangle$ be a an intuitionistic closed set of (X, T) . Then F^c is an intuitionistic open set in (X, T) . By assumption, $f(F^c)$ is $Irg\alpha$ -open in (Y, γ) . That is $f(F^c) = f(F)^c$ is $Irg\alpha$ -open in (Y, γ) and therefore $f(F)$ is $Irg\alpha$ -closed in (Y, γ) . Hence f is $Irg\alpha$ -closed.

(iii) \Rightarrow (i) Let F be an intuitionistic closed set of (X, T) . By assumption, $f(F)$ is $Irg\alpha$ -closed in (Y, γ) . But $f(F) = (f^{-1})^{-1}(F)$ and therefore f^{-1} is an intuitionistic continuous

Theorem 3.39. If a map $f : (X, T) \rightarrow (Y, \gamma)$ is $Irg\alpha$ -open, then $(int(A)) \subset Irg\alpha-int(f(A))$ for every subset A of (X, T) .

Proof. Let $f : (X, T) \rightarrow (Y, \gamma)$ be an intuitionistic open map and A be any subset of (X, T) . Then $int(A)$ is an intuitionistic open in (X, T) and so $f(int(A))$ is $Irg\alpha$ -open in (Y, γ) . We have $f(int(A)) \subset f(A)$. Therefore $f(int(A)) \subset Irg\alpha-int(f(A))$.

Remark 3.40. The converse of the above Theorem need not be true in general as seen from the following example.

Example 3.41. Let $X = \{a, b\}$, $T = \{\tilde{X}, \tilde{\emptyset}, A, B\}$, where $A = \langle x, \{a\}, \{b\} \rangle$, $B = \langle x, \{a\}, \emptyset \rangle$, $Y = \{1, 2\}$ with topology $\gamma = \{\tilde{Y}, \tilde{\emptyset}, C, D\}$, where $C = \langle x, \{2\}, \{1\} \rangle$. Define a function $f : X \rightarrow Y$ by $f(a) = 2$, $f(b) = 1$. In (Y, γ) , $Irg\alpha-int(f(A)) = f(A)$ for every subset A of (X, T) . So $f(int(A)) \subset f(A) = Irg\alpha-int(f(A))$ for every subset A of X . But f is not $Irg\alpha$ -open map, since for the an intuitionistic open set A of (X, T) , $f(A)$ is not $Irg\alpha$ -open in (Y, γ) .

Theorem 3.42. If a function $f : (X, T) \rightarrow (Y, \gamma)$ is $Irg\alpha$ -open, then $f^{-1}(Irg\alpha-cl(B)) \subset cl(f^{-1}(B))$ for each subset B of (Y, γ) .

Proof. Let $f : (X, T) \rightarrow (Y, \gamma)$ be an $Irg\alpha$ -open map and B be any subset of (Y, γ) . Then $f^{-1}(B) \subset cl(f^{-1}(B))$ and $cl(f^{-1}(B))$ is an intuitionistic closed set in (X, T) . So there exists a $Irg\alpha$ -closed set K of (Y, γ) such that $B \subset K$ and $f^{-1}(K) \subset cl(f^{-1}(B))$. Now $Irg\alpha-cl(B) \subset Irg\alpha-cl(K) = K$, as K is $Irg\alpha$ -closed set of (Y, γ) . Therefore $f^{-1}(Irg\alpha-cl(B)) \subset f^{-1}(K)$ and so $f^{-1}(Irg\alpha-cl(B)) \subset f^{-1}(K) \subset cl(f^{-1}(B))$. Thus $f^{-1}(Irg\alpha-cl(B)) \subset cl(f^{-1}(B))$ for each subset of B of (Y, γ) .

Remark 3.43. The converse of the above Theorem need not be true in general as seen from the following example.

Example 3.44. Let $X = \{a, b, c\}$, with topology $T = \{\tilde{X}, \tilde{\emptyset}, A, B\}$, where $A = \langle x, \{a, b\}, \{c\} \rangle$, $B = \langle x, \{b\}, \emptyset \rangle$, $Y = \{1, 2, 3\}$ with topology $\gamma = \{\tilde{Y}, \tilde{\emptyset}, C, D\}$, where $C = \langle x, \{3, 1\}, \emptyset \rangle$, $D = \langle x, \{3\}, \emptyset \rangle$. Let f be the identity map from (X, T) to (Y, γ) . In (Y, γ) , $\text{Irg}\alpha\text{-cl}(B) = B$ for every subset B of (Y, γ) . So $f^{-1}(\text{Irg}\alpha\text{-cl}(B)) = f^{-1}(B) \subset \text{cl}(f^{-1}(B))$ for every subset B of (Y, γ) . But f is not $\text{Irg}\alpha$ -open map, since for the intuitionistic open set $A = \langle x, \{a, b\}, \{c\} \rangle$ of (X, T) , $f(A) = f(\langle x, \{a, b\}, \{c\} \rangle)$ which is not $\text{Irg}\alpha$ -open in (Y, γ) . We define another new class of maps called $\text{bi-rg}\alpha^*$ -closed maps which are stronger than $\text{bi-rg}\alpha$ -closed maps.

Definition 3.45. A map $f : (X, T) \rightarrow (Y, \gamma)$ is said to be $\text{Irg}\alpha^*$ -closed map if the image $f(A)$ is $\text{Irg}\alpha$ -closed in (Y, γ) for every $\text{Irg}\alpha$ -closed set A in (X, T) .

Theorem 3.46. Every $\text{Irg}\alpha^*$ -closed map is $\text{Irg}\alpha$ -closed map but not conversely.

Proof. The proof follows from the definitions and fact that every intuitionistic closed set is $\text{Irg}\alpha$ -closed.

The converse of the above Theorem is not true in general as seen from the following example.

Example 3.47. Recall Example 3.41. we see that f $\text{Irg}\alpha$ -closed map but not $\text{Irg}\alpha^*$ -closed map. Since $\{a\}$ is $\text{bi-rg}\alpha$ -closed set in (X, T) , but its image under f is $\{a\}$, which is not $\text{bi-rg}\alpha$ -closed in (Y, γ) .

Theorem 3.48. If $f : (X, T) \rightarrow (Y, \gamma)$ and $g : (Y, \gamma) \rightarrow (Z, \beta)$ are $\text{Irg}\alpha^*$ -closed maps, then their composition $g \circ f : (X, T) \rightarrow (Z, \beta)$ is also $\text{Irg}\alpha^*$ -closed.

Proof. Let F be a $\text{rg}\alpha$ -closed set in (X, T) . Since f is $\text{Irg}\alpha^*$ -closed map, $f(F)$ is $\text{Irg}\alpha$ -closed set in (Y, γ) . Since g is $\text{Irg}\alpha^*$ -closed map, $g(f(F))$ is $\text{Irg}\alpha$ -closed set in (Z, β) . Therefore $g \circ f$ is $\text{Irg}\alpha^*$ -closed map.

We define another new class of maps called $\text{Irg}\alpha^*$ -open maps which are stronger than $\text{Irg}\alpha$ -open maps.

Definition 3.49. A map $f : (X, T) \rightarrow (Y, \gamma)$ is said to be $\text{Irg}\alpha^*$ -open map if the image $f(A)$ is $\text{Irg}\alpha$ -open set in (Y, γ) for every $\text{Irg}\alpha$ -open set A in (X, T) .

Remark 3.50. Since every intuitionistic open set is an $\text{Irg}\alpha$ -open set, we have every $\text{Irg}\alpha^*$ -open map is $\text{Irg}\alpha$ -open map. The converse is not true in general as seen from the following example.

Example 3.51. Let $X = \{a, b\}$, with topology $T = \{\tilde{X}, \tilde{\emptyset}, A, B, C\}$, where $A = \langle x, \{a\}, \emptyset \rangle$, $B = \langle x, \{b\}, \emptyset \rangle$, $C = \langle x, \emptyset, \emptyset \rangle$, $Y = \{1, 2, 3\}$ with topology $\gamma = \{\tilde{Y}, \tilde{\emptyset}, C, D\}$, where $D = \langle x, \{3\}, \emptyset \rangle$. Define $f : (X, T) \rightarrow (Y, \gamma)$ by $f(a) = 1$, $f(b) = 3$. Then f is $\text{Irg}\alpha$ -open map but not $\text{Irg}\alpha^*$ -open map, since for the $\text{Irg}\alpha$ -open set B in (X, T) $f(B) = f(\langle x, \{b\}, \emptyset \rangle) = \langle x, \{1\}, \emptyset \rangle$ which is not $\text{Irg}\alpha$ -open set in (Y, γ) .

Theorem 3.52. If $f : (X, T) \rightarrow (Y, \gamma)$ and $g : (Y, \gamma) \rightarrow (Z, \beta)$ are $\text{Irg}\alpha^*$ -open maps, then their composition $g \circ f : (X, T) \rightarrow (Z, \beta)$ is also $\text{Irg}\alpha^*$ -open.

Proof. Proof is similar to the Theorem 3.48.

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بعض تعميمات المجموعات والدوال في الفضاءات التوبولوجية الحدسية

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المستخلص :

في هذا البحث قدمنا انواع جديدة من تعميم المجموعة المغلقة السوية الحدسية ، تعميم المجموعة المغلقة شبه السوية الحدسية وتعميم المجموعة المغلقة الضعيفة الحدسية وتعميم المجموعة شبه المغلقة القوية الحدسية والمجموعة الضعيفة المغلقة الحدسية وانواع اخرى كثيرة . ومن خلال هذه المفاهيم عرفنا صفوف جديدة من الدوال الحدسية ودرسناها ، وتمر بنا بعض الخصائص والعلاقات فيما بينهما .